Wall Township Public Schools	Board Approval Date: 8/18/2026

Course Title: Calculus Honors

Course Description: Calculus is not only the language for expressing physical laws in precise mathematical terms, but it is also a tool for studying these laws. This course involves a comprehensive study of differential and integral calculus. The concepts of limits and continuity are analyzed as the basis for the study of calculus. A balance is maintained between theory, applications and manipulative techniques. This course prepares students for future study in college level mathematics.

New Jersey Student Learning Standards - Mathematics

Vision for Mathematics Education in New Jersey

A New Jersey education in Mathematics builds quantitatively and analytically literate citizens prepared to meet the demands of college and career, and to engage productively in an information-driven society. All students will have access to a high-quality mathematics education that fosters a population that:

- leverages data in decision-making and as a lens for discussing, analyzing, and responding to practical questions.
- persists to make sense of and model problems arising in everyday life, society, and the workplace.
- thinks critically and strategically to assess quantitative relationships and to solutions to complex problems.
- employs precise reasoning and constructs viable arguments to deduce conclusions, recognize false statements and assess peers' reasoning.

- interprets, evaluates and critiques the mathematics embedded in social, scientific and commercial systems, as well as the claims made in the private and public sectors.
- communicates precisely when conveying, representing, and justifying both qualitative and quantitative perspectives.

Standards for Mathematical Practice*

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

**See end of document for additional information related to the mathematical practices.*

Inclusion of Climate Change Opportunities

() notes opportunities to integrate specific examples of climate change education provided by additional age-appropriate resources

The goal of inclusion of climate change education implementation is to foster generations of New Jersey students that can analyze, question, interpret, to think independently, and bring critical deduction to fulfill, and to lead in jobs created by burgeoning industries of the future green economy.

Course Sequence & Pacing

Unit Title	Estimated Number of Instructional Periods**
Unit 1: Limits • Define limits and interpret limit notation. • Estimate limits from graphs. • Estimate limits from tables. • Evaluate limits using limit laws and algebraic properties. • Evaluate limits using algebraic manipulation. • Select and apply appropriate limit-solving strategies. • Apply the Squeeze Theorem to evaluate limits. • Connect graphical, numerical, algebraic, and verbal representations of limits. • Identify and classify discontinuities. • Determine continuity at a point. • Determine continuity over an interval. • Remove removable discontinuities. • Evaluate infinite limits and identify vertical asymptotes. • Evaluate limits at infinity and identify horizontal asymptotes. • Apply the Intermediate Value Theorem to justify the existence of solutions.	17

Unit 2: Definition of Derivatives • Define average and instantaneous rates of change. • Define the derivative and use derivative notation. • Write the equation of a tangent line. • Estimate derivatives from graphs and tables. • Relate differentiability and continuity. • Apply the Power Rule to differentiate polynomial functions. • Apply the constant, sum, difference, and constant multiple rules. • Determine horizontal tangent lines. • Write the equation of a normal line. • Determine the differentiability of piecewise functions.	15
Unit 3: Differentiation • Apply differentiation rules to find derivatives of functions, including the Constant Rule, Power Rule, Sum/Difference Rules, Product Rule, Quotient Rule, and Chain Rule. • Find derivatives of trigonometric, exponential, and logarithmic functions. • Recognize the structure and type of a function to determine the appropriate differentiation rule. • Identify and decompose composite functions into inside and outside functions before applying the Chain Rule. • Find higher-order derivatives and interpret their meaning in context. • Write equations of tangent and normal lines using derivatives. • Apply differentiation shortcuts to polynomial, trigonometric, exponential, and logarithmic functions. • Recognize that e^x is the unique exponential function whose derivative is itself. • Use derivatives to analyze and interpret rates of change in real-world applications. • Use graphing calculators and technology to graph functions, explore patterns, verify solutions, and support mathematical conclusions.	17

• Communicate mathematical reasoning using appropriate notation, vocabulary, and representations.	
Unit 4: Applications of Differentiation • Apply the Mean Value Theorem to analyze relationships between average and instantaneous rates of change. • Understand the conditions required for the Mean Value Theorem and interpret its meaning graphically. • Apply the Extreme Value Theorem to determine when absolute extrema exists. • Identify critical points of functions. • Distinguish between local (relative) extrema and absolute (global) extrema. • Determine intervals where functions are increasing or decreasing using the first derivative. • Use the First Derivative Test to identify relative maximum and minimum values. • Use the Candidates Test to determine absolute maximum and minimum values on closed intervals. • Determine intervals of concavity and identify points of inflection. • Apply the Second Derivative Test to determine relative extrema. • Analyze and sketch graphs of functions using information from the first and second derivatives. • Describe the relationship between a function, its first derivative, and its second derivative. • Use graphing calculators and technology to analyze functions, verify solutions, and support conclusions. • Communicate mathematical reasoning using appropriate notation, vocabulary, and representations.	17
Unit 5: Implicit Differentiation • Distinguish between functions written in implicit form and explicit form.	15

• Find derivative of a function expressed implicitly.
• When differentiating implicitly, utilize:
  ○ Power Rule
  ○ Chain Rule
  ○ Product Rule
  ○ Quotient Rule
• Solve related rates
• Approximating Values of a Function Using Local Linearity and Linearization	

Unit 6: Definition of Integration • Explain accumulation of change. • Approximate areas with Riemann Sums. • Use left, right, and midpoint Riemann Sums. • Use summation (sigma) notation. • Evaluate definite integrals. • Interpret definite integrals. • Connect Riemann Sums and definite integrals. • Apply definite integrals to real-world problems. • Apply properties of definite integrals. • Apply the Fundamental Theorem of Calculus. • Find antiderivatives. • Evaluate indefinite integrals. • Apply basic integration rules and notation.	15

Unit 7: Integration
 	
• Evaluate **indefinite integrals** using:
  ○ Trigonometric rules
  ○ Logarithmic rules
  ○ Exponential rules
	17

○ U-Substitution • Evaluate **definite integrals** using appropriate integration techniques. • Apply integration to solve particle motion problems. • Use integrals to determine position, velocity, and acceleration. • Find and interpret the average value of a function over an interval. • Use accumulation functions and definite integrals to represent quantities that change over time. • Determine the area between curves with respect to xxx. • Interpret integrals as accumulated change in real-world applications.	
Cumulative Exams and Knowledge Checks	10
Total:	123 periods

**Instructional periods are scheduled for 56 minutes. Classes meet 3 times over 4 days.

Unit 1

Desired Results

Unit 1 Title: Limits

Unit Summary: In this unit, students will evaluate limits numerically, graphically, and analytically as a means of transitions from Pre-Calculus procedures to the Calculus process of differentiation.

Learning Standards

NJSLS Math:
Arithmetic with Polynomials and Rational Expressions

A -APR A. Perform arithmetic operations on polynomials
B. Understand the relationship between zeros and factors of polynomials
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Seeing Structure in Expressions
A-SSE B. Write expressions in equivalent forms to solve problems
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
★ a. Factor a quadratic expression to reveal the zeros of the function it defines.
Interpreting Functions
F-IF A. Understand the concept of a function and use function notation
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
B. Interpret functions that arise in applications in terms of the context
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★

Math Practices:
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
5 Use appropriate tools strategically.
6 Attend to precision.

Interdisciplinary Connections
Career Readiness, Life Literacies, & Key Skills *(CLKS)*: 9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice. 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task. 9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.
Computer Science & Design Thinking (*CS & DT***):** 8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors. 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change. 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena. 8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets	
Unit Essential Questions: What does the slope of a line represent? What is calculus? What is a limit?	**Unit Enduring Understandings:** The concept of limits is one of the foundations of calculus.

When does a limit not exist? What role do limits play in calculus? How are limits and continuity related?	The concept of limits can be applied throughout mathematics.

Vocabulary:
average rate of change ,secant line, instantaneous rate of change, tangent line limits, continuity, Intermediate Value Theorem

Students will be able to:

- Define limits and interpret limit notation.
- Estimate limits from graphs.
- Estimate limits from tables.
- Evaluate limits using limit laws and algebraic properties.
- Evaluate limits using algebraic manipulation.
- Select and apply appropriate limit-solving strategies.
- Apply the Squeeze Theorem to evaluate limits.
- Connect graphical, numerical, algebraic, and verbal representations of limits.
- Identify and classify discontinuities.
- Determine continuity at a point.
- Determine continuity over an interval.
- Remove removable discontinuities.
- Evaluate infinite limits and identify vertical asymptotes.
- Evaluate limits at infinity and identify horizontal asymptotes.
- Apply the Intermediate Value Theorem to justify the existence of solutions

Learning Activities: *Suggested or Potential Learning Activities*
Building Thinking Classroom Tasks, Collaborative Problem Solving, Exit Passes, WarmUps, Flipped Classroom Activity, Graphing Investigations, FRQ Practice, Multiple Choice Practice

Climate Change: **Performance Task** Students will analyze authentic climate datasets (such as NASA global temperature anomalies or NOAA atmospheric CO_2 data) by:

- Estimating limits from graphs and tables.
- Determining whether the data appears continuous.
- Identifying any discontinuities and explaining possible causes.
- Using limit notation to describe long-term trends.
- Writing a brief explanation of how calculus helps scientists model and understand climate change.

Assessment Evidence
Formative Assessments: Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Delta Math Check for understanding
Summative Assessments: Cumulative Assessments
Common Benchmark Assessments: Common Unit Assessments, LinkIt! Benchmarks
Alternative Assessments: online practice, Modified Unit Assessments, Modified Projects, Other Modified Assessments

Core Instructional & Supplemental Materials
DeltaMath, Khan Academy, Desmos, Graphing Calculators
LinkIt! Test Bank

Accommodations and Modifications: Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students

Specific Strategies and Practices that Support Students with Disabilities:
Students receiving Special Education programming have specific goals, objectives, accommodations, and modifications outlined in their Individualized Education Plans (IEPs) due to an identified disability and/or diagnosis. In addition to access to the general education curriculum, instruction is differentiated based on individual student needs. The IEP serves as a supplemental curriculum guide, inclusive of instructional strategies tailored to support each learner. *Examples of supports and accommodations include* small group or one-to-one instruction, additional time, review and clarification of directions, students restating instructions to demonstrate understanding, space for movement or scheduled breaks, extra visual and verbal cues and prompts, preferential seating, consistent routines and schedules, rest breaks, visual and verbal reminders for directions and task focus, checklists, immediate feedback, and alternate or reduced assignments within digital platforms such as Desmos, Delta Math, and LinkIt.

Specific Strategies and Practices that Support English Language Learners
An English Language Learner (ELL) is a national-origin minority student who is limited-English proficient. The term "ELL" is preferred over "limited-English proficient" as it emphasizes potential and progress rather than deficits. *Instructional strategies to support ELLs include* pre-teaching vocabulary and concepts, use of personal glossaries, extended time, simplified and verbal instructions, frequent breaks, visual aids including graphic organizers, use of cognates to support comprehension, teacher modeling, peer pairing (pairing beginning ELLs with more advanced ELLs), scaffolding techniques, word walls, think-pair-share activities, cooperative learning groups, and alternate or reduced assignments within digital platforms such as Desmos, Delta Math, and LinkIt.

Students at Risk of Failure
Students at risk of failure benefit from targeted supports that address both academic and social-emotional needs, providing multiple access points to learning and increasing the likelihood of success. *Support strategies include* strengthening the home/school connection, alternative assessments, learning contracts, homogeneous grouping, supplemental instruction, think-pair-share by readiness, homework options, differentiated online practice, leveled rubrics, tiered levels of support and activities, small group instruction, choice

to work alone or in groups, alternate or reduced assignments within digital platforms such as Delta Math, and LinkIt, opportunities to make up work or reduce the number of assignments, and access to extra help when available.

Students with 504 Plans
Teachers are responsible for implementing the accommodations and strategies outlined in a student's 504 Plan. A 504 Plan is a formal document designed to provide support for students with physical or mental impairments that significantly limit one or more major life activities. Accommodations are individualized and legally binding, and support must be consistently provided as outlined in the plan.

Specific Strategies and Practices that Support Gifted & Talented Students
It is essential that teachers can identify high-ability students who may require more depth and complexity in instruction. These learners benefit from opportunities to accelerate and enrich their learning. *Strategies to support gifted and talented students include* homogeneous grouping, small group in-depth instruction, differentiated online practice, alternative assessments, pre-assessment to guide instruction and accelerate learning, and enrichment and extension activities within digital platforms such as Desmos, Delta Math, and LinkIt.

Unit 2

Desired Results

Unit 2 Title: Definition of Derivatives

Unit Summary: In this unit, students will differentiate all types of functions in order to find instantaneous rates of change.

Learning Standards

NJSLS Math:
Seeing Structure in Expressions
A-SSE B. Write expressions in equivalent forms to solve problems
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity

represented by the expression. ★
c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as (1.151/12)^(12t) ≈1.01212^t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
Interpreting Functions F-IF B. Interpret functions that arise in applications in terms of the context
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
The Real Number System N -RN A. Extend the properties of exponents to rational exponents.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Math Practices:
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
5 Use appropriate tools strategically.
6 Attend to precision.

Interdisciplinary Connections

Career Readiness, Life Literacies, & Key Skills *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

Computer Science & Design Thinking *(CS & DT)*:

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8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets

Unit Essential Questions:	**Unit Enduring Understandings:**
What is a derivative? What role do derivatives play in calculus? How are limits and derivatives related? How are differentiability and continuity related? What is the relationship between derivatives and tangent lines? What is the difference between average and instantaneous velocity**?**	Calculus can be used to solve real world problems. Algebra is the foundation of all levels of mathematics. The derivative is used to represent the instantaneous rate of change.

Vocabulary:
Continuity, Derivative, Constant Rule, Power Rule, Tangent Line, Normal Line, Limits, Point-Slope

Students will be able to:

- Define average and instantaneous rates of change.
- Define the derivative and use derivative notation.

- Write the equation of a tangent line.
- Estimate derivatives from graphs and tables.
- Relate differentiability and continuity.
- Apply the Power Rule to differentiate polynomial functions.
- Apply the constant, sum, difference, and constant multiple rules.
- Determine horizontal tangent lines.
- Write the equation of a normal line.
- Determine the differentiability of piecewise functions.

Learning Activities: *Suggested or Potential Learning Activities*
Building Thinking Classroom Tasks, Collaborative Problem Solving, Exit Passes, WarmUps, Flipped Classroom Activity, Graphing Investigations, FRQ Practice, Multiple Choice Practice

Climate Change Example: **Performance Task** Students will analyze authentic climate datasets (NASA, NOAA, or EPA) to:

- Calculate average and instantaneous rates of change.
- Estimate derivatives from graphs and tables.
- Interpret the meaning of the derivative in the context of climate science.
- Use tangent lines to approximate future values.
- Explain how derivatives help scientists monitor changes in global temperature, sea level rise, atmospheric CO_2, or Arctic sea ice.

Assessment Evidence

Formative Assessments: Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Delta Math Check for understanding
Summative Assessments: Cumulative Assessments
Common Benchmark Assessments: Common Unit Assessments, LinkIt! Benchmarks
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routines and schedules, rest breaks, visual and verbal reminders for directions and task focus, checklists, immediate feedback, and alternate or reduced assignments within digital platforms such as Desmos, Delta Math, and LinkIt.

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Students at Risk of Failure
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Unit 3

Desired Results

Unit 3 Title: Differentiation Fundamentals and Composite Functions.

Unit Summary: Using derivatives to describe the rate of change of one variable with respect to another variable allows students to understand change in a variety of contexts. With their understanding of functions, students will recognize that the slopes of the tangents at the given points represent a relationship between the two quantities. Students will call this function the derivative of a function. The derivative is the key to modeling instantaneous change. Students should be able to estimate derivatives from tables and graphs, and apply various derivative rules and properties

Learning Standards

NJSLS Math:
Functions
Interpreting Functions F.IF
B. Interpret functions that arise in applications in terms of the context.
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★
C. Analyze functions using different representations.
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Algebra

Seeing Structure in Expressions A.SSE

A. Interpret the structure of expressions.

2. Use the structure of an expression to identify ways to rewrite it

Creating Equations ★ A.CED

A. Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Reasoning with Equations and Inequalities A.REI

A. Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality

Functions

Building Functions F.BF

A. Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.

B. Build new functions from existing functions.

3. Identify the effect on the graph of replacing f(x)f(x)f(x) by f(x)+kf(x)+kf(x)+k, kf(x)kf(x)kf(x), f(kx)f(kx)f(kx), and f(x+k)f(x+k)f(x+k) for specific values of k.

Math Practices:

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

6 Attend to precision.

Interdisciplinary Connections

Career Readiness, Life Literacies, & Key Skills *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

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9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

Computer Science & Design Thinking (*CS & DT***):**

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8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets

Unit Essential Questions:	**Unit Enduring Understandings:**
What does the derivative tell us about a function?	Derivatives measure instantaneous rates of change.
How do different differentiation rules simplify finding derivatives?	Different function types require different differentiation techniques.
How can recognizing function structure help determine the appropriate differentiation method?	Complex functions can often be broken into simpler parts before differentiation.
Why is the Chain Rule necessary for composite functions?	The Chain Rule extends differentiation to composite functions.
How can derivatives be used to describe and analyze graphs?	

How can technology be used to verify and interpret derivatives?	Derivatives provide information about slopes, tangent lines, normal lines, and graph behavior. Technology is a valuable tool for verifying solutions and exploring mathematical relationships.

Vocabulary: Derivative, Instantaneous Rate of Change, Tangent Line, Normal Line, Composite Function, Higher-Order Derivative, Product Rule, Quotient Rule, Chain Rule, Exponential Function, Natural Logarithm, Euler's Number (e), Trigonometric Functions, Differentiability, Slope Function

Students will be able to:

- Apply differentiation rules to find derivatives of functions, including the Constant Rule, Power Rule, Sum/Difference Rules, Product Rule, Quotient Rule, and Chain Rule.
- Find derivatives of trigonometric, exponential, and logarithmic functions.
- Recognize the structure and type of a function to determine the appropriate differentiation rule.
- Identify and decompose composite functions into inside and outside functions before applying the Chain Rule.
- Find higher-order derivatives and interpret their meaning in context.
- Write equations of tangent and normal lines using derivatives.
- Apply differentiation shortcuts to polynomial, trigonometric, exponential, and logarithmic functions.
- Recognize that e^x is the unique exponential function whose derivative is itself.
- Use derivatives to analyze and interpret rates of change in real-world applications.
- Use graphing calculators and technology to graph functions, explore patterns, verify solutions, and support mathematical conclusions.
- Communicate mathematical reasoning using appropriate notation, vocabulary, and representations.
- Straight-Line Motion: Connecting Position, Velocity, and Acceleration
- Solve optimization

Learning Activities: *Suggested or Potential Learning Activities*
Building Thinking Classroom Tasks, Collaborative Problem Solving, Exit Passes, WarmUps, Flipped Classroom Activity, Graphing Investigations, FRQ Practice, Multiple Choice Practice

Climate Change Example: Scientists monitor atmospheric carbon dioxide (CO_2) levels because changes in CO_2 concentration provide important information about climate trends. The amount of atmospheric CO_2 (measured in parts per million, ppm) can be modeled by the function: $C(t) = 0.012t^2 + 1.9t + 369$, where t represents the number of years after 2000. Use calculus to analyze how the rate of atmospheric carbon dioxide change is evolving.

Assessment Evidence
Formative Assessments: Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Delta Math Check for understanding
Summative Assessments: Cumulative Assessments
Common Benchmark Assessments: Common Unit Assessments, LinkIt! Benchmarks
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Core Instructional & Supplemental Materials
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LinkIt! Test Bank

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Students at Risk of Failure
Students at risk of failure benefit from targeted supports that address both academic and social-emotional needs, providing multiple access points to learning and increasing the likelihood of success. *Support strategies include* strengthening the home/school connection, alternative assessments, learning contracts, homogeneous grouping, supplemental instruction, think-pair-share by readiness, homework options, differentiated online practice, leveled rubrics, tiered levels of support and activities, small group instruction, choice

to work alone or in groups, alternate or reduced assignments within digital platforms such as Delta Math, and LinkIt, opportunities to make up work or reduce the number of assignments, and access to extra help when available.

Students with 504 Plans
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Specific Strategies and Practices that Support Gifted & Talented Students
It is essential that teachers can identify high-ability students who may require more depth and complexity in instruction. These learners benefit from opportunities to accelerate and enrich their learning. *Strategies to support gifted and talented students include* homogeneous grouping, small group in-depth instruction, differentiated online practice, alternative assessments, pre-assessment to guide instruction and accelerate learning, and enrichment and extension activities within digital platforms such as Desmos, Delta Math, and LinkIt.

Unit 4

Desired Results

Unit 4 Title: Application of Differentiation

Unit Summary: In this unit, students extend their understanding of derivatives by analyzing the relationship between a function and its derivatives. Students apply differentiation concepts to determine key characteristics of functions, including extrema, intervals of increase and decrease, concavity, and points of inflection. Students use the Mean Value Theorem, Extreme Value Theorem, First Derivative Test, and Second Derivative Test to analyze functions and justify conclusions. Students will also connect graphical, numerical, and analytical representations of a function, its first derivative, and its second derivative.

Learning Standards

NJSLS Math:

Functions
Interpreting Functions F.IF
B. Interpret functions that arise in applications in terms of the context.
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★
C. Analyze functions using different representations.
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Algebra
Reasoning with Equations and Inequalities A.REI
D. Represent and solve equations and inequalities graphically.
11. Explain why the x-coordinates of the points where the graphs of the equations intersect are the solutions of the equation; find the solutions approximately using technology.

Functions
Building Functions F.BF.
B. Build new functions from existing functions.
3. Identify the effect on the graph of replacing f(x) by related functions.

Math Practices:
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
5 Use appropriate tools strategically.
6 Attend to precision.

Interdisciplinary Connections

Career Readiness, Life Literacies, & Key Skills *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

Computer Science & Design Thinking (*CS & DT***):**

8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets

Unit Essential Questions:	**Unit Enduring Understandings:**

How can derivatives be used to analyze the behavior of functions? How do the first and second derivatives provide information about the shape of a graph? How can calculus be used to identify maximum and minimum values in real-world situations? How can mathematical reasoning and technology be used to analyze and interpret functions?	The derivative provides information about the rate of change and behavior of a function. Critical points and derivatives can be used to identify relative and absolute extrema. The first derivative describes where a function is increasing or decreasing. The second derivative describes the concavity and behavior of the first derivative. Graphs of functions and their derivatives are connected and provide different perspectives of the same relationship.
Vocabulary: Average Rate of Change, Candidates Test, Concave Down, Concave Up, Concavity, Critical Points, Critical Numbers, Derivative, Extreme Value Theorem (EVT), First Derivative, First Derivative Test, Global Maximum, Global Minimum, Increasing Function, Inflection Point, Local Extrema, Local Maximum, Local Minimum, Mean Value Theorem (MVT), Relative Extrema, Second Derivative, Second Derivative Test, Secant Line, Tangent Line, Turning Point, Decreasing Function, Rate of Change, Function Behavior, Graph Analysis, Intervals of Increase, Intervals of Decrease, Critical Values, Differentiability, Continuity, Domain, Endpoints.	
Students will be able to: ● Apply the Mean Value Theorem to analyze relationships between average and instantaneous rates of change. ● Understand the conditions required for the Mean Value Theorem and interpret its meaning graphically. ● Apply the Extreme Value Theorem to determine when absolute extrema exists. ● Identify critical points of functions. ● Distinguish between local (relative) extrema and absolute (global) extrema. ● Determine intervals where functions are increasing or decreasing using the first derivative. ● Use the First Derivative Test to identify relative maximum and minimum values.	

- Use the Candidates Test to determine absolute maximum and minimum values on closed intervals.
- Determine intervals of concavity and identify points of inflection.
- Apply the Second Derivative Test to determine relative extrema.
- Analyze and sketch graphs of functions using information from the first and second derivatives.
- Describe the relationship between a function, its first derivative, and its second derivative.
- Use graphing calculators and technology to analyze functions, verify solutions, and support conclusions.
- Communicate mathematical reasoning using appropriate notation, vocabulary, and representations.

Learning Activities: *Suggested or Potential Learning Activities*
Building Thinking Classroom Tasks, Collaborative Problem Solving, Exit Passes, WarmUps, Flipped Classroom Activity, Graphing Investigations, FRQ Practice, Multiple Choice Practice

Climate Change Example: Scientists monitor changes in global sea levels to better understand the effects of climate change. The average sea level increase (in millimeters) over a period of time can be modeled by the function: $S(t) = t^3 - 6t^2 + 9t + 20$, where t represents the number of years after 2000.nUsing calculus, analyze the sea level model S(t)S(t)S(t) by examining the relationship between the function and its first and second derivatives. Determine how the rate of sea level change is behaving over time, including any intervals of increase or decrease, relative extrema, and changes in concavity. Explain what your findings indicate about the pattern of sea level change.

Assessment Evidence
Formative Assessments: Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Delta Math Check for understanding
Summative Assessments: Cumulative Assessments
Common Benchmark Assessments: Common Unit Assessments, LinkIt! Benchmarks

Alternative Assessments: online practice, Modified Unit Assessments, Modified Projects, Other Modified Assessments

Core Instructional & Supplemental Materials
DeltaMath, Khan Academy, Desmos, Graphing Calculators
LinkIt! Test Bank

Accommodations and Modifications: Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students

Specific Strategies and Practices that Support Students with Disabilities:
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Specific Strategies and Practices that Support English Language Learners
An English Language Learner (ELL) is a national-origin minority student who is limited-English proficient. The term "ELL" is preferred over "limited-English proficient" as it emphasizes potential and progress rather than deficits. *Instructional strategies to support ELLs include* pre-teaching vocabulary and concepts, use of personal glossaries, extended time, simplified and verbal instructions, frequent

breaks, visual aids including graphic organizers, use of cognates to support comprehension, teacher modeling, peer pairing (pairing beginning ELLs with more advanced ELLs), scaffolding techniques, word walls, think-pair-share activities, cooperative learning groups, and alternate or reduced assignments within digital platforms such as Desmos, Delta Math, and LinkIt.

Students at Risk of Failure
Students at risk of failure benefit from targeted supports that address both academic and social-emotional needs, providing multiple access points to learning and increasing the likelihood of success. *Support strategies include* strengthening the home/school connection, alternative assessments, learning contracts, homogeneous grouping, supplemental instruction, think-pair-share by readiness, homework options, differentiated online practice, leveled rubrics, tiered levels of support and activities, small group instruction, choice to work alone or in groups, alternate or reduced assignments within digital platforms such as Delta Math, and LinkIt, opportunities to make up work or reduce the number of assignments, and access to extra help when available.

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Unit 5

Desired Results

Unit 5 Title: Implicit Differentiation

Unit Summary: In this unit, students will enhance their understanding of differentiation by finding the derivative of implicit functions. Additionally, students will apply differentiation to real world problems.

Learning Standards

NJSLS Math:

Functions
Interpreting Functions (F.IF)
B. Interpret functions that arise in applications in terms of the context.
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship.
6. Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph. ★
C. Analyze functions using different representations.
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
Building Functions (F.BF)
A. Build a function that models a relationship between two quantities.
1. Write a function that describes a relationship between two quantities. ★

Algebra
Reasoning with Equations and Inequalities (A.REI)
D. Represent and solve equations and inequalities graphically.
11. Explain why the x-coordinates of the points where the graphs of the equations intersect are the solutions of the equation; find the solutions approximately using technology.

Math Practices:

1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
5 Use appropriate tools strategically.
6 Attend to precision.

Interdisciplinary Connections

Career Readiness, Life Literacies, & Key Skills *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

Computer Science & Design Thinking (*CS & DT***):**

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8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets	
Unit Essential Questions:	**Unit Enduring Understandings:**

How can derivatives be found when functions cannot be solved explicitly? Why is implicit differentiation useful in mathematics and science? How are changing quantities mathematically related? Why do tangent lines provide accurate local approximations? When are linear approximations appropriate, and what are their limitations? How can derivatives be used to evaluate difficult limits?	Not all functions can be written explicitly, yet derivatives can still be determined. Implicit differentiation relies on repeated application of derivative rules and the Chain Rule. Rates of change between related variables are interconnected. The tangent line provides the best linear approximation to a function near a given point. Linearization simplifies complicated functions while maintaining accuracy near the point of approximation. L'Hôpital's Rule provides an effective method for evaluating indeterminate limits by relating limits to derivatives.
Vocabulary: Implicit Function, Explicit Function, Implicit Differentiation, Dependent Variable, Independent Variable, Related Rates, Rate of Change, Chain Rule, Power Rule, Product Rule, Quotient Rule, Local Linearity, Linear Approximation, Linearization, Tangent Line Approximation, Indeterminate Form, L'Hôpital's Rule, and Instantaneous Rate of Change.	
Students will be able to: Distinguish between functions written in implicit form and explicit form. Find derivative of a function expressed implicitly. When differentiating implicitly, utilize: ● Power Rule ● Chain Rule ● Product Rule ● Quotient Rule Solve related rates Approximating Values of a Function Using Local Linearity and Linearization	

Using L'Hopital's Rule for Determining Limits of Indeterminate Forms
Learning Activities: *Suggested or Potential Learning Activities* Building Thinking Classroom Tasks, Collaborative Problem Solving, Exit Passes, WarmUps, Flipped Classroom Activity, Graphing Investigations, FRQ Practice, Multiple Choice Practice As global temperatures rise, glaciers melt and lose ice over time. Suppose a glacier is modeled as a cylinder with radius rrr meters and thickness hhh meters. The volume of the glacier is given by $V = \pi r^2 h$. At a certain moment, the radius is decreasing at 0.4 m/year while the radius is 200 m and the thickness is 50 m. Use implicit differentiation and related rates to determine how fast the glacier's volume is changing at that instant. Explain what your answer means in the context of climate change.

Assessment Evidence
Formative Assessments: Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Delta Math Check for understanding
Summative Assessments: Cumulative Assessments
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Students at Risk of Failure

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Unit 6

Desired Results

Unit 6 Title: Definition of Integration

Unit Summary:
In this unit, students will take a graphical and analytic approach to the area under a curve. Students will approximate the area as well as find the exact area via the Fundamental Theorem of Calculus

Learning Standards

NJSLS Math:
Interpreting Functions
F-IF A. Understand the concept of a function and use function notation
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
B. Interpret functions that arise in applications in terms of the context
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★

Math Practices:
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.
6 Attend to precision.

Interdisciplinary Connections

Career Readiness, Life Literacies, & Key Skills *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

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9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

Computer Science & Design Thinking ***(****CS & DT****)*:

8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets

Unit Essential Questions:	**Unit Enduring Understandings:**
How can area under a curve be used to represent accumulated change? How do Riemann Sums approximate definite integrals?	Definite integrals represent accumulated change and can be used to model real-world situations.

What is the relationship between derivatives, antiderivatives, and integrals? How does the Fundamental Theorem of Calculus simplify the process of evaluating definite integrals? How can integrals be used to model and solve real-world problems involving accumulation?	Riemann Sums approximate definite integrals, and increasing the number of subintervals improves the approximation. The Fundamental Theorem of Calculus connects differentiation and integration, allowing definite integrals to be evaluated using antiderivatives.
Vocabulary: Accumulation, Riemann Sum, Left Riemann Sum, Right Riemann Sum, Midpoint Riemann Sum, Summation Notation (Σ), Definite Integral, Integrand, Net Area, Fundamental Theorem of Calculus, Antiderivative, Indefinite Integral, Constant of Integration, Integration, Accumulation Function	
Students will be able to: • Explain accumulation of change. • Approximate areas with Riemann Sums. • Use left, right, and midpoint Riemann Sums. • Use summation (sigma) notation. • Evaluate definite integrals. • Interpret definite integrals. • Connect Riemann Sums and definite integrals. • Apply definite integrals to real-world problems. • Apply properties of definite integrals. • Apply the Fundamental Theorem of Calculus. • Find antiderivatives. • Evaluate indefinite integrals. • Apply basic integration rules and notation.	
Learning Activities: *Suggested or Potential Learning Activities* Building Thinking Classroom Tasks, Collaborative Problem Solving, Exit Passes, WarmUps, Flipped Classroom Activity, Graphing Investigations, FRQ Practice, Multiple Choice Practice	

Climate change example: Performance Task Students analyze authentic climate data from organizations such as **NASA, NOAA, or the EPA** to estimate the total accumulation of carbon emissions, sea level rise, or glacier melt over a specified period. Students will:

- Estimate accumulation using Riemann Sums.
- Compare estimates to exact values using definite integrals.
- Apply the Fundamental Theorem of Calculus to determine total change.
- Interpret the meaning of the accumulated value in the context of climate change.
- Explain how integration helps scientists quantify long-term environmental impacts and supports informed decision-making.

Assessment Evidence
Formative Assessments: Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Delta Math Check for understanding
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Unit 7

Desired Results

Unit 7 Title: Integration

Unit Summary: In this unit, students will be able to find antiderivatives of functions through the process of indefinite and definite integration.

Learning Standards

NJSLS Math:
Functions
Interpreting Functions (F.IF)
B. Interpret functions that arise in applications in terms of the context.
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship.
6. Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph. ★

Math Practices:
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
5 Use appropriate tools strategically.
6 Attend to precision.

Interdisciplinary Connections

Career Readiness, Life Literacies, & Key Skills *(CLKS)*:

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Computer Science & Design Thinking (*CS & DT*):

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8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets	
Unit Essential Questions: How can integration be used to measure accumulated change? How are derivatives and integrals connected? How can integrals help us understand motion and real-world phenomena? Why is the average value of a function useful when analyzing changing quantities?	**Unit Enduring Understandings:** Integration is the inverse process of differentiation and allows us to find accumulated change. Definite integrals represent the net accumulation of a quantity over an interval. Position, velocity, and acceleration are connected through integration and differentiation.

How can the area between curves represent meaningful information?	Average value provides a way to summarize the behavior of a changing function. Accumulation functions allow mathematicians to model real-world situations where quantities build over time. The area between curves can represent meaningful measurements such as distance, volume, and differences between models.
Vocabulary: Integration, Indefinite Integral, Definite Integral, Trigonometric Integrals, Exponential Functions, Logarithmic Functions, u-Substitution, Antiderivative, Constant of Integration, Particle Motion, Position, Velocity, Acceleration, Average Value of a Function, Accumulation Function, Definite Integral, Area Between Curves, Limits of Integration, Integrand, Initial Value, Net Change	
Students will be able to: ● Evaluate **indefinite integrals** using: ○ Trigonometric rules ○ Logarithmic rules ○ Exponential rules ○ U-Substitution ● Evaluate **definite integrals** using appropriate integration techniques. ● Apply integration to solve particle motion problems. ● Use integrals to determine position, velocity, and acceleration. ● Find and interpret the average value of a function over an interval. ● Use accumulation functions and definite integrals to represent quantities that change over time. ● Determine the area between curves with respect to xxx.	

- Interpret integrals as accumulated change in real-world applications.

Learning Activities: *Suggested or Potential Learning Activities*
Building Thinking Classroom Tasks, Collaborative Problem Solving, Exit Passes, WarmUps, Flipped Classroom Activity, Graphing Investigations, FRQ Practice, Multiple Choice Practice

Climate change: **Performance Task** Students will analyze authentic climate data from organizations such as **NASA, NOAA, or the EPA** to compare two climate scenarios (for example, current emissions versus reduced-emissions policies). Students will:

- Evaluate definite integrals to determine total accumulated emissions.
- Calculate the average value of a climate variable over time.
- Use accumulation functions to model environmental change.
- Interpret the area between two curves as the cumulative environmental impact of different climate policies.
- Explain how integration helps scientists quantify and predict the long-term effects of climate change and evaluate potential mitigation strategies.

Melting Ice Sheets and Accumulated Change

Scientists monitor the melting rate of an ice sheet to estimate how much ice is lost over time. The rate of ice loss is modeled by the function: $R(t) = 50e^{0.04t}$, where R(t) represents the amount of ice lost per year (in billions of tons) and t is the number of years after measurements begin.

1. Use a definite integral to determine the total amount of ice lost during the first 10 years.

2. Explain what the value of the integral represents in the context of climate change.

3. Scientists estimate that the rate of ice loss will continue increasing. Explain how an accumulation function could be used to predict future changes in sea level.

Assessment Evidence
Formative Assessments: Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Delta Math Check for understanding
Summative Assessments: Cumulative Assessments
Common Benchmark Assessments: Common Unit Assessments, LinkIt! Benchmarks
Alternative Assessments: online practice, Modified Unit Assessments, Modified Projects, Other Modified Assessments

Core Instructional & Supplemental Materials
DeltaMath, Khan Academy, Desmos, Graphing Calculators
LinkIt! Test Bank

Accommodations and Modifications: Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students
Specific Strategies and Practices that Support Students with Disabilities: Students receiving Special Education programming have specific goals, objectives, accommodations, and modifications outlined in their Individualized Education Plans (IEPs) due to an identified disability and/or diagnosis. In addition to access to the general education curriculum, instruction is differentiated based on individual student needs. The IEP serves as a supplemental curriculum guide, inclusive of instructional strategies tailored to support each learner. *Examples of supports and accommodations include* small group or one-to-one instruction, additional time, review and clarification of directions, students restating instructions to demonstrate understanding, space for movement or scheduled breaks, extra visual and verbal cues and prompts, preferential seating, consistent

routines and schedules, rest breaks, visual and verbal reminders for directions and task focus, checklists, immediate feedback, and alternate or reduced assignments within digital platforms such as Desmos, Delta Math, and LinkIt.

Specific Strategies and Practices that Support English Language Learners
An English Language Learner (ELL) is a national-origin minority student who is limited-English proficient. The term "ELL" is preferred over "limited-English proficient" as it emphasizes potential and progress rather than deficits. *Instructional strategies to support ELLs include* pre-teaching vocabulary and concepts, use of personal glossaries, extended time, simplified and verbal instructions, frequent breaks, visual aids including graphic organizers, use of cognates to support comprehension, teacher modeling, peer pairing (pairing beginning ELLs with more advanced ELLs), scaffolding techniques, word walls, think-pair-share activities, cooperative learning groups, and alternate or reduced assignments within digital platforms such as Desmos, Delta Math, and LinkIt.

Students at Risk of Failure
Students at risk of failure benefit from targeted supports that address both academic and social-emotional needs, providing multiple access points to learning and increasing the likelihood of success. *Support strategies include* strengthening the home/school connection, alternative assessments, learning contracts, homogeneous grouping, supplemental instruction, think-pair-share by readiness, homework options, differentiated online practice, leveled rubrics, tiered levels of support and activities, small group instruction, choice to work alone or in groups, alternate or reduced assignments within digital platforms such as Delta Math, and LinkIt, opportunities to make up work or reduce the number of assignments, and access to extra help when available.

Students with 504 Plans
Teachers are responsible for implementing the accommodations and strategies outlined in a student's 504 Plan. A 504 Plan is a formal document designed to provide support for students with physical or mental impairments that significantly limit one or more major life activities. Accommodations are individualized and legally binding, and support must be consistently provided as outlined in the plan.

Specific Strategies and Practices that Support Gifted & Talented Students
It is essential that teachers can identify high-ability students who may require more depth and complexity in instruction. These learners benefit from opportunities to accelerate and enrich their learning. *Strategies to support gifted and talented students include* homogeneous grouping, small group in-depth instruction, differentiated online practice, alternative assessments, pre-assessment to guide instruction and accelerate learning, and enrichment and extension activities within digital platforms such as Desmos, Delta Math, and LinkIt.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. Reason abstractly and quantitatively Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own,

without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered$7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$ older students can see the 14 as $2 \times 7$and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For

example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)\left(x^2 + x + 1\right)$ and $(x - 1)\left(x^3 + x^2 + x + 1\right)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.