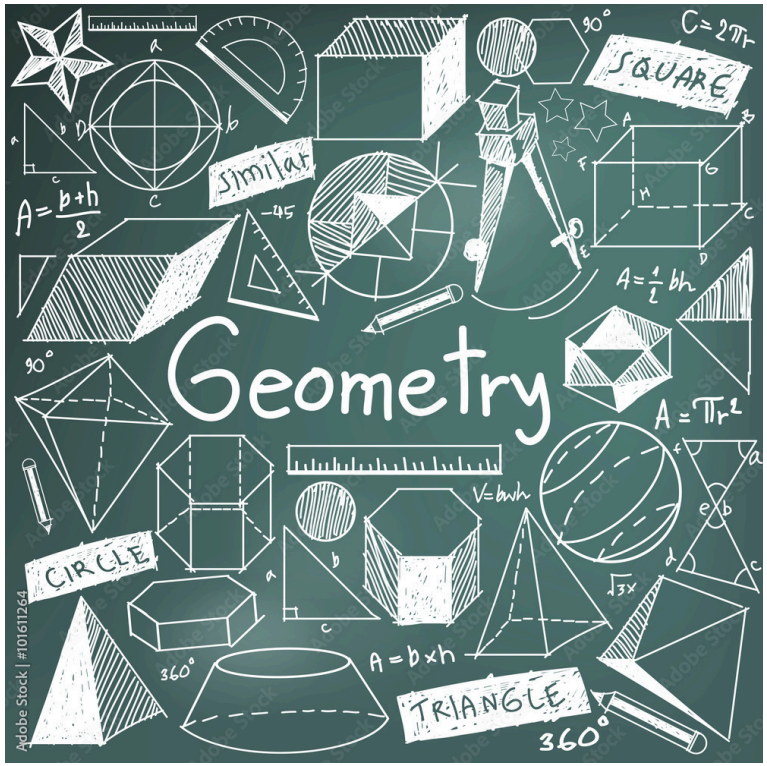
GEOMETRY CURRICULUM



Grade Level(s): 9-10

Curriculum Author(s): Jamie Lisevick and Kristen Pisano

Course Description: Geometry is a full-year course designed to increase the students' ability to reason intuitively, inductively, and deductively as they explore two and three-dimensional aspects of the world around them. Student learning in this course is investigative and cooperative with a heavy emphasis on data gathering, conjecture, and validation. Topics of study include: congruence, similarity, right triangle trigonometry, solid geometry, coordinate geometry, circles, and probability. The honors course is designed and implemented so that the depth of the topics discussed in the course surpass that of the college prep course to include the following: extensions of learning, accelerated pacing, greater complexity in topics, and greater responsibility, creativity, and reflection on the part of the students as their thinking is challenged.

Year At A Glance			
Unit Title	**Overarching Essential Question**	**Overarching Enduring Understanding**	**Vision of A Learner Attribute and Indicator**
Foundations of Geometry	What is Geometry?	Geometry is built on precise definitions, postulates, clearly defined terms and logical reasoning.	TCC1(9-12); CCE2(9-12); P3(9-12)
Tools for Reasoning and Constructions (Honors Only)	How can inductive and deductive reasoning help us form logical arguments?	Geometry is built on logical reasoning and proof involving definitions, postulates, and theorems.	TI3(9-12); AA1(9-12); AA2(9-12)
Properties of Triangles	In what ways can geometric figures be used to understand real-world situations?	Geometric definitions, properties and theorems allow one to describe, model, and analyze situations in the real-world.	TCC1(9-12); TCC4(9-12); CCE2(9-12); TI3(9-12)
Triangle Congruence	How can we use the fact that two triangles are congruent to solve problems?	Proving and applying congruence provides a basis for modeling situation geometrically	TCC1(9-12); TCC4(9-12); CCE2(9-12); TI3(9-12)
Similarity and Proportional Reasoning	What additional insights can be understood and used to solve problems once it is established that two figures are similar?	Dilations, similarity, and the properties of similar triangles allow for the application of trigonometric ratios to real-world situations (ie. scaling maps, architecture)	TCC1(9-12); TCC4(9-12); CCE2(9-12); TI3(9-12)
Right Triangles and Trigonometry	How do the properties of right triangles help us solve real-world situations?	Pythagorean theorem, Special Right Triangles and Trigonometric Ratios, and Geometric Means can be used to solve real world problems.	TCC1(9-12); TCC4(9-12); CCE2(9-12); TI3(9-12); AA2(9-12)

Circles and Conic Sections	How do the unique properties and relationships of circles help us model, measure and apply to the real-world?	Circles have special properties and relationships among angles, arcs, chords, tangents and segments that help us model, measure, and analyze round objects and patterns in the world around us.	CCE2(9-12); AA1(9-12)
Area, Surface Area, Volume & Probability	How can we measure and describe three-dimensional objects to understand, design, and solve problems in the real world?	Surface area and volume help us describe, measure, and analyze three-dimensional objects so we can understand their structure, compare their sizes, and solve real-world problems involving space and materials.	CCE2(9-12); AA1(9-12); TI3(9-12)

Unit 1 - Foundations of Geometry

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (*) indicates a modeling standard; (★ indicates greater emphasis in the Honors sections)

- **G.CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **G.CO.C.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
- **A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **A.REI.B.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **G.GPE.B.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. ★
- **A.REI.B.4.B** Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.
- **F.IF.C.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to...

- TCC4(9-12): I can integrate my learning to adapt to experiences in the classroom, career and life.
- P3(9-12): I can accept constructive feedback and use setbacks to adjust my learning journey in order to reach my goals.

Understandings: Students will understand that...	**Essential Questions:**

• Geometry is built on precise definitions, postulates, clearly defined terms and logical reasoning. • Vocabulary describes geometric terms presented both verbally and visually. • Geometric statements are represented by diagrams and specific labels. • Relationships between geometric figures are based on measurable properties. • For Coordinate Geometry the distance formula is derived from the Pythagorean Theorem. ★	• What is Geometry? • How do we develop and extend our foundational concepts of Geometry? • What role does vocabulary and notation play in the understanding of Geometry? • What is the difference between congruence and equality? • How do we use the concepts of Geometry to solve problems? • How do we utilize principles of Algebra to model Geometry? • How to divide a segment or angles into a given ratio. ★
Students will know... • State precise definitions of basic Geometry vocabulary. • How to identify, sketch basic vocabulary terms. • Read and write with geometric symbols (notation). • How to correctly name points, lines and planes, angles etc… • Recognize and apply basic properties of geometric figures • How to use and identify congruent segments and angles. • Understand and identify special pairs of angles, such as complementary, supplementary, vertical angles and linear pairs. • Measurements, definitions and labels must be precise in order to draw conclusions that are accurate. • Measure and compare segments and angles using appropriate tools (ruler, protractor).	**Students will be able to...** • Know and apply vocabulary. • Identify and label geometric concepts given a diagram. • Create a sketch given notation with correct markings for parallel and perpendicular lines. • Write geometric statements using correct notation. • Use the order of operations to calculate the distance between two points with the distance formula. ★ • Calculate the midpoint between two points. ★ • Calculate the endpoint of a segment given a midpoint and one endpoint. ★

Key Vocabulary:

Geometry	Trisect ★	Linear Pair
Point	Partition	Vertical Angles
Line	Measure	Complementary Angles
Plane	Degree	Supplementary Angles
Space	Equidistance	Postulate
Segment	Midpoint	Segment Addition Postulate
Ray	Collinear	Angle Addition Postulate

Parallel Lines/Planes	Coplanar	Theorem ★
Perpendicular Lines/Planes	Endpoint	Pythagorean Theorem ★
Skew Lines	Congruence	Distance Formula ★
Angle	Acute Angle	Midpoint Formula ★
Vertex	Obtuse Angle	
Adjacent Angles	Straight Angle	
Angle Bisector	Right Angle	
Bisect	Reflex Angle ★	

Assessment Evidence

Performance Tasks:

- **Dictionary Project:TCC4(9-12)**
  Students will create a personalized geometry dictionary to deepen their understanding of key geometric terms by defining, illustrating, and providing a real-world example for each.

- **Summative Unit Assessment:** Topics will include:
  - Identifying, writing definitions, sketching with new geometric vocabulary.
  - Creating equations based on geometric diagrams including the segment and angle addition postulates, and angle pair relationships and solving with algebraic methods.
  - Using protractors and rulers for accurate measurement.
  - Applying formulas to coordinate plane problems. ★

Other Evidence:

- **Algebra Check in Quiz 1-1**: This will assess Algebra I skills at the start of the year including solving equations, combining like terms. *For honors only solving quadratic equations by factoring and square root method.*★

- **Concept Checks: P3(9-12)**
  Students will complete a small concept check on several topics throughout the unit including vocabulary knowledge and sketching diagrams, utilizing the distance formula and midpoint formula in coordinate geometry with real world problems.

- **Quiz 1-2**: This will assess the comprehension of the vocabulary and notations in written form. It will also assess the proper markings of diagrams. (parallel/perpendicular/congruence)

- **Quiz 1-3**: Students will create equations from relationships of angle pairs, bisected segments and more. They will apply algebraic reasoning and correctly use the order or operations when solving. (Honors only will include partitioned segments/angles as well as factoring involved in solving quadratic equations)

	• **Vertical whiteboard warm-ups:** Student-led discussions regarding solutions or misconceptions. Given a multi-step problem which emphasizes depth or difficulty. These will be inquiry-based or application-heavy.

Learning Plan

Geometry is a branch of mathematics that studies shapes, sizes, positions, angles and dimensions of figures, objects and spaces. It focuses on the properties and relationships of points, lines, planes, and solids. It builds logical thinking and problem solving skills. The first unit covers the basic vocabulary, geometric notation, markings and basic postulates that are used in this branch of mathematics.

- Students will work on vertical white boards in small groups with the presentation: “Sketch It”. They will draw diagrams with markings based on the geometric notation given in written form.
- For the summative review students will work in small groups of 5 stations: Station 1: Vocabulary, Station 2: Drawing and Sketching, Station 3: Measurement, Station 4: Angels with Algebra, and Station 5: Coordinate Geometry. Answer keys are provided for each station and students can choose how much time and which different stations to work in. **CCE2(9-12)**
- Students will create a dictionary of terms from unit 1 (see “Dictionary Project” above). **TCC4(9-12)**
- Students will derive the distance formula from the pythagorean theorem by group led discussions. ★
- Students will be able to identify quadratic equations vs. linear equations. ★
- Students will review the structure of factoring. Factoring helps students to see structure and problem solve more efficiently. We will review a variety of techniques for factoring depending on the structure of the expression.★

Teacher Resources: *White boards, Desmos, IXL, Khan Academy, rulers, compass, protractor, TI Graphing Calculator, Interactive Notebooks, various Geometry textbooks.*

Unit 2 - Tools for Reasoning, Constructions & Transversals (Honors Only Unit)★

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (*) indicates a modeling standard; (★ indicates greater emphasis in the Honors sections)

- ***G.CO.D.12*** *Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
- **G.CO.B.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- ***G.CO.C.9*** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
- ***A.REI.A.1*** *Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.* ***Construct a viable argument to justify a solution method.***
- ***A.CED.A.3*** *Create equations that describe numbers or relationships. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
- ***G.GPE.B.5*** *Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).*

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to...

- AA1(9-12): I can evaluate different approaches and justify the best pathway to success.

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- AA2(9-12): I can assess my past successes and mistakes to change my approach.

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Understandings: Students will understand that...

- Geometry is built on logical reasoning and proof involving definitions, postulates, and theorems.
- Constructions are a physical form of proof, they demonstrate geometric truths without measurement, reinforcing the logic behind properties like congruent segments or perpendicular bisectors.
- Angle and line relationships reveal consistent patterns which can be proven using reasoning and used to solve real-world and abstract problems.
- Reasoning transfers across representations including visual diagrams, written proofs, and algebraic/graphical models (e.g., using slope to prove parallelism in the coordinate plane).
- There are some common misconceptions and missteps that students have/make with reasoning, constructions, and transversals:
  - Inductive reasoning leads to absolute truth (Theory vs Theorem).
  - Leaving out essential logical arguments from one step of the proof to the next. (i.e. assuming too much from a definition, postulate, theorem, and/or diagram)

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Essential Questions:

- How can inductive and deductive reasoning help us form logical arguments?
- How can the order and/or negation of a statement impact its truth and persuasiveness?
- What role does algebraic properties have in the creation and flow of a geometry proof?
- What strategies can be utilized when proving mathematical statements?
- How can you use formulas and equations to model geometric terms analytically?
- In what ways can two or more lines be described in space?
- What are the various angle relationships given two or more lines and a transversal?
- What conditions are necessary for angle relationships to be either congruent or supplementary?
- How do you prove lines parallel or perpendicular?

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Students will know…

- Inductive reasoning can lead us to believing a concept is true, but beware, we may have not thought of the counterexample yet.
- Deductive reasoning leads us to an absolute truth.
- Using the law of detachment simplifies proof writing by allowing the mathematician to utilize agreed upon notations and symbols.
- The law of syllogism is an essential strategy in proof writing to connect previous statements of logic to new ones.
- Mathematicians must be careful in the order and the phrasing of a statement as it may impact the truth of the statement.
- Proof writing is an art, taking on the personality of the writer,

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Students will be able to...

- Understand the difference between inductive and deductive reasoning.
- Use the law of detachment to “personalize” a geometric statement.
- Use the law of syllogism to develop connections between two mathematical or geometric concepts.
- Write a statement in conditional form and investigate the converse, inverse, contrapositive, and bi-conditional forms.
- Investigate the truth of various conditional statements.
- Write proofs using columns, flow, paragraph, or hybrid styles

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showcased through their style.

- Angle relationships can be corresponding, alternate interior, alternate exterior, consecutive interior, consecutive exterior, vertical, or linear pairs.
- Understand that an angle relationship exists regardless of the lines being parallel or perpendicular.
- Given two lines are parallel, angles pairs are either supplementary or congruent based on their relationships to the transversal.
- Slope can be calculated using the formula or by counting the rise and run.
- Given three points, find the slope between each to determine if the points are collinear (or find the equation through two of the points and determine if the third point lies on the line).
- Equations of lines can be written in standard form, point-slope form, and slope-intercept form.
- Parallel lines have the same slope. Perpendicular lines have opposite, reciprocal slopes.

involving previous unit concepts.

- Prove theorems such as the Linear Pair Congruent Theorem, the Congruent Complement Theorem, Supplement Theorem, etc.
- Know and apply the vocabulary of angle relationships, parallel, and perpendicular.
- Create and solve problems using the corresponding angles postulate, alternate interior angles theorem, alternate exterior angles theorem, consecutive interior angles theorem, consecutive exterior angles theorem
- Prove lines are parallel using postulates and theorems.
- Calculate the slope of a line given two points or a graph.
- Determine if 3 or more points are collinear (using slopes and/or equation of a line).
- Write the equation of a line given two points.
- Identify whether lines are parallel, perpendicular, or simply intersecting by comparing their slopes.
- Calculate the distance from a point to a line.
- Write the equation of a perpendicular bisector given a segment.

Key Vocabulary:

Inductive Reasoning
Deductive Reasoning
Proof
Theorem
Hypothesis
Conclusion
Negation
Conditional Statement
Biconditional Statement
Converse
Inverse
Contrapositive
Conjecture
Definition
Counterexample
Two-Column Proof
Constructions
Reflexive Property
Symmetric Property
Properties of Equality
Substitution Property
Transitive Property
Transversal
Corresponding Angles Postulate
Alternate Interior Angles
Consecutive Interior Angles
Alternate Exterior Angles
Consecutive Exterior Angles
Coordinate Geometry
Slope/y-intercept
Equation of a Line
Slope Intercept Form

Assessment Evidence	
Performance Tasks: ● **Unit Summative Assessment: AA2(9-12)** The topics will include inductive vs. deductive reasoning, they will show their mastery of writing and reading proofs. These proofs will include the angle pair relationships discovered in this unit with transversal and parallel lines. This summative will include error analysis questions, matching or fill in the blank, algebra and coordinate geometry with linear equations and systems of equations, as well as summary. ● **Proof competition: AA1(9-12)** Students will work in small groups and practice formal geometric proof writing (two-column). This will deepen understanding of postulates, definitions and theorems, build teamwork and communication around mathematical reasoning. The groups will present a proof to the class and others may find errors as part of the competition.	**Other Evidence:** ● **Construction Quiz:** Students will create congruent segments and angles, along with equilateral triangles and 30, 45, 60 and 90 degree angles with their straightedge and compass only. ● **Syllogism Story:** Students will start a hypothesis and pass it around the class for each student making a conclusion and using transitivity to create a syllogism. Once every student has contributed the story will be read aloud. ● **Concept Checks:** Students will complete a small concept check on several topics throughout the unit including logic & conditional statements, writing equations of parallel and perpendicular lines and writing proofs based on a diagram. ● **Vertical whiteboard warm-ups:** Student-led discussions regarding solutions or misconceptions. Given a multi-step problem which emphasizes depth or difficulty. These will be inquiry-based or application-heavy utilizing methods from Building Thinking Classrooms.

Learning Plan

Unit 2 covers basic reasoning and logic and how it can be used to communicate clearly with others and prove theorems. Students will begin to use this way of reasoning to discover angle relationships created by parallel lines and a transversal.

- *Students will learn the rules and how to use a compass and straightedge as well as the history of the importance of this skill.*
- *Students will discover the angles relationships of transversals by using deductive reasoning.*
- *Students will use two-column proofs to prove lines are parallel.* **AA1(9-12)**
- *Angle relationships challenge problem: "No Joking Around" Students will solve for many missing angle measures that travel through multiple sets of parallel lines and transversals. This would be best at white boards or in partner work at desks.*
- *To better understand how two-column proofs work students will complete "Uno Proofs" slides to get the understanding.*
- *Logic Puzzles are a fun way to use deductive reasoning.*
- *Syllogism Story (As described above)*

Teacher Resources: *White boards, Desmos, IXL, Khan Academy, rulers, compass, protractor, TI Graphing Calculator, Interactive Notebooks, various Geometry textbooks*

Unit 3 - Properties In Triangles

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (*) indicates a modeling standard; (★ indicates greater emphasis in the Honors sections)

- G.CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- G.CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
- G.MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- G.CO.C.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
- G.GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to...

- TCC4(9-12): I can integrate my learning to adapt to experiences in the classroom, career and life.
- CCE2(9-12): I can give and receive actionable and relevant feedback with openness to be able to determine meaningful revisions for

success.

Understandings: Students will understand that...

- Geometric definitions, properties and theorems allow one to describe, model, and analyze situations in the real-world.
- Proving and applying congruence provides a basis for modeling situations geometrically.
- There are some common misconceptions and missteps that students have/make with properties in triangles.
    - The sum of the two smaller sides of a triangle must be larger than the third side. Students often think that sides 4, 4, 8 make an isosceles triangle because they forget that the two smaller sides cannot be equal to the largest side.
    - Students forget to use the exterior angle theorem as an appropriate strategy.

Essential Questions:

- In what ways can geometric figures be used to understand real-world situations?
- In what ways can congruence be useful?
- How can the measures of a triangle be used to classify triangles?
- What do the Triangle Theorems tell you about the attributes of a triangle?
- How do you know which theorem applies to the given situation?
- How can the triangle inequalities be applied to the construction of the triangle and extension to the theorems?

Students will know...

- How to correctly interpret geometric diagrams (what can and cannot be assumed)
- How to use theorems, postulates, or definitions to prove theorems about triangle, including:
    - Points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
    - Measures of interior angles of a triangle sum to 180 degrees
    - Base angles of isosceles triangles are congruent.
    - The segment joining midpoints of two slides of a triangle is parallel to the third side and half the length.
    - The Medians of a triangle meet at a point. ★

Students will be able to...

- Classify Triangles by their sides and angles
- Determine largest and smallest sides and/or angles using the triangle inequality theorem
- Perform constructions of angles, segment bisectors, perpendicular bisectors ★
- Apply the properties of geometric figure to comparable real-world objects (e.g., The zipper of a tent is an altitude and perpendicular bisector which makes it equidistant from the base angles of an isosceles triangle.)
- Solve for missing segments and angles based on the interior and exterior angle theorems
- Solve for missing sides and angles using the properties of line segments inside a triangle (and their points of concurrency★)

Key Vocabulary:

Acute Triangle	Median	Base Angles
Obtuse Triangle	Angle Bisector	Legs
Right Triangle	Perpendicular Bisector	Remote Interior Angles
Equilateral Triangle	Points of Concurrency ★	Equidistant
Isosceles Triangle	Concurrent ★	Consecutive
Scalene Triangle	Centroid ★	Midpoint
Equiangular Triangle	Circumcenter ★	Midsegment
Triangle Sum Theorem	Incenter ★	Adjacent
Exterior Angle Theorem	Orthocenter ★	Congruent
Triangle Inequality Theorem	Isosceles Triangle Theorem	
Altitude	Converse Isosceles Triangle Theorem	

Assessment Evidence

Performance Tasks:

- **Fox Project Part 1 TCC4(9-12)**
  - Students have a picture of a fox that can be broken down into all triangles. Students must identify all special line segments inside a triangle based on the notation and special properties
  - Students must identify and classify 6 triangles based on the properties of their sides and angles. Students must explain their selections using deductive reasoning and proven theorems.
  - Students must solve for missing angles using the sum of interior angle theorem and exterior angle theorem
  - Students must identify the longest/shortest sides and largest/smallest angles of a triangle using the triangle inequality theorem.
- **Fox Project Part 2: CCE2(9-12)**
  - Students must create their own scaled drawing of a person, animal, or thing using only triangles. At least one each triangle used must contain the following attributes:
    - Altitude

Other Evidence:

- **Quiz 3-1**
  - Classify Triangles by Sides and angles: Scalene, Right, Obtuse, Acute, Equilateral, Isosceles
  - Give a range of possible 3rd sides given 2 sides of a triangle based on the triangle inequality theorem.
  - List sides in order from least to greatest given the angle measurements
  - List angles in order from least to greatest given the side measurements.
- **Quiz 3-2**
  - Solve for missing angles using the sum of interior angle theorem
  - Solve for missing angles using the exterior angle theorem
  - Solve for missing angles using properties of isosceles triangles.
- **Quiz 3-3**
  - Identifying special line segments in triangles
  - Solving for sides and angles using the properties of Altitude, Angle Bisector, Perpendicular Bisector, Median,

■ Midsegment ■ Angle Bisector ■ Acute Triangle ■ Obtuse Triangle ■ RIght Triangle ■ Equilateral Triangle ■ Isosceles Triangle ■ Median ■ Scalene Triangle ■ Perpendicular bisector ■ Accurate angle measurements ● **Airport Project ★ TCC4(9-12)** ○ Students will determine the cost of two different options for placing a new airport. They will use their knowledge of points of concurrency to locate the potential locations and through coordinate Geometry and constructions with their compass will determine the locations and costs for building new roads.	and Midsegment. ○ Apply theorems and definitions to real world applications such as camping equipment, flight plans, etc. ● **Daily warm-ups** ○ Here's where we've been and where are we going, what are the similarities and differences? ○ Student-led discussions regarding solutions ○ Discussions regarding student misconceptions ○ Examples of student work, find the mistake and describe ● **Daily homework discussion** ● **Concept Checks** ○ Students have the opportunity to receive feedback and rework questions to improve understanding

Learning Plan

- Students will review the properties of sides and angles in order to classify triangles as Scalene, Equilateral, Isosceles, Right, Obtuse, Acute.
- Students will discover the triangle inequality theorem using the pasta activity in which students explore what sides can or cannot make a triangle **TI3(9-12)**
- Students will Prove the sum of interior angles theorem using prior knowledge of parallel lines and transversals and supplementary angles★ and practice solving for angles algebraically **TI3(9-12), TCC1(9-12)**
- Students will participate in Vertical white board collaborative practice utilizing methods from Building Thinking Classrooms **TI3(9-12)**
- Students will discover the exterior angle theorem using knowledge of sum of interior angles and supplementary angles.
- Students will discover and learn special line segments in triangles and their properties using a Desmos Activity **TI3(9-12)**
- Students will construct special line segments in triangles using a ruler and a compass★ (Median, Altitude, Angle Bisector, Perpendicular Bisector, Midsegment)
- Students will solve for missing segments and angles in triangles using the above listed properties and theorems
- Students will solve for the distances between the vertices of a triangle and its points of concurrency.★

Teacher Resources: *White boards, Desmos, IXL, Khan Academy, rulers, compass, protractor, TI Graphing Calculator, Interactive Notebooks, various Geometry textbooks*

Unit 4 - Triangle Congruence

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (*) indicates a modeling standard; (★ indicates greater emphasis in the Honors sections)

- **G.CO.B.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- **G.CO.B.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
- **G.CO.C.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
- **G.SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- **G.CO.C.11** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to...

- TCC4(9-12): I can integrate my learning to adapt to experiences in the classroom, career and life.
- P3 (9-12): I can accept constructive feedback and use the setbacks to adjust my learning journey in order to reach my goals.

Understandings: Students will understand that...	**Essential Questions:**

• Proving and applying congruence provides a basis for modeling situation geometrically
• There are some common missteps that occur when proving congruence:
  ○ Students may reverse the roles the law of detachment and law of syllogism play in the development of proof writing.
  ○ Students may leave out essential logical arguments from one step of the proof to the next (i.e. assuming too much from a definition, postulate, theorem, and/or diagram.
  ○ Students may forget to mark all of the given information on the diagram.
	• In what ways is congruence useful?
	• What information is essential to prove triangles are congruent?
	• How can the movement of shapes preserve congruence and/or similarity?
	• How can we use the fact that two triangles are congruent to solve problems?

Students will know...	**Students will be able to...**
• Congruent figures are the same shape and same size regardless if they are translated, rotated or reflected. • How to interpret geometric diagrams • Think critically and deductively to write formal proofs to prove congruent triangles by SSS, SAS, ASA, AA, and HL. • Prove that parts of Triangles and parallelograms are congruent due to Corresponding parts of Congruent Triangles are congruent (CPCTC).	• Identify corresponding sides and corresponding angles of congruent figures • Explain that in a pair of congruent triangles, corresponding sides are congruent (distance is preserved) and corresponding angles are congruent (angle measure is preserved. • List sufficient conditions to prove triangles are congruent • Map a triangle with one of the sufficient conditions (SSS, SAS, ASA, AAS, HL) onto the original triangle and show that corresponding sides and corresponding angles are congruent • Write a formal proof providing statements with correlated reasons starting with a set of given information and ending with a proven statement. • Identify and use the properties of congruence and equality (reflexive property, symmetric, transitive, definition of midpoint, definition of angle bisector, parallel line relationships, vertical angles, etc) in proofs • Order statements based on inductive reasoning • Correctly interpret geometric diagrams by identifying what can and cannot be assumed

Key Vocabulary:

Congruence	SAS	Transversal
Distance	ASA	Alternate Interior Angles
Map	AAS	Corresponding Angles
Figure	SSS	Parallel
Corresponding Sides	HL	Angle Bisector
Corresponding Angles	Right Triangle	Segment Bisector
Triangle	CPCTC	Midpoint
Parallelogram	Proof	
Translation	Reflexive Property	

Assessment Evidence

Performance Tasks:

- **Unit Summative Assessment:** TCC4(9-12)
  - Using Angle relationships with parallel lines and transversals to solve for missing angles and prove congruent triangles
  - Identifying corresponding parts of Congruence triangles, interpreting and writing congruence statements
  - Explain whether a pair of triangles are congruent or not using one of the sufficient conditions (SSS, SAS, ASA, AAS)
  - Create formal proofs for congruent triangles using SSS, SAS, ASA,, AAS, HL, and CPCTC by placing the correct sticker statements with the correct sticker reason and accurately identifying congruent parts of a diagram and what information can be assumed.
  - Use properties of Congruent triangles to prove properties of parallelograms

Other Evidence:

- **Quiz 4-1 Intro to Congruent Triangles**
  - Identify corresponding parts of congruent figures
  - Solve algebraically for missing sides and angles of Congruent triangles using congruence statements, notation and corresponding parts.
  - Explain whether a pair of triangles are congruent or not using one of the sufficient conditions (SSS, SAS, ASA, AAS)
  - Write a congruence statement for congruent triangles
- **Quiz 4-2: Partner Sticker Quiz** TCC4(9-12)
  - Students collaborate to create formal proofs for congruent triangles using SSS, SAS, ASA, AAS, HL, and CPCTC by placing the correct sticker statements with the correct sticker reason and accurately identifying congruent parts of a diagram and what information can be assumed.
- **Quiz 4-3 Quadrilaterals and Parallelograms** TCC4(9-12)
  - Use properties of Congruent triangles to prove properties of parallelograms.
  - Use properties of parallelograms, kite, and trapezoid to solve for missing sides and angle measures algebraically

	○ Use midsegment of trapezoid to solve for missing parts ● **Daily warm-ups** ○ Here's where we've been and where are we going, what are the similarities and differences? ○ Student-led discussions regarding solutions ○ Discussions regarding student misconceptions ○ Examples of student work, find the mistake and describe ● **Daily homework discussion** ● **Concept Checks** P3(9-12) ○ Students have the opportunity to receive feedback and rework questions to improve understanding

Learning Plan

- Students will be utilizing previous knowledge on parallel lines and transversals from Unit 2 to prove congruent triangles.
- Students will work will discover Congruent triangles as same size and shape in Triangle Congruence Introduction Desmos TCC1(9-12)
- Students will discuss and discover Congruence shortcuts in a small group activity in which they must create a congruent triangle to their partner's by asking the fewest questions about the triangle. TI3(9-12)
- Students will practice using SSS, SAS, AAS, ASA, in Triangle congruence Practice Desmos
- Students will participate in Vertical white board collaborative practice utilizing methods from Building Thinking Classrooms TI3(9-12)
- Students will develop skills with a partner and gain immediate feedback completing Desmos: Fill in the Blanks Proofs
- Students will develop skills with a partner and gain immediate feedback completing Desmos: Proofs with CPCTC TI3(9-12)
- College Prep classes will complete interactive notebook pages for each skill set CCE2(9-12)
- Students will complete collaborative around the room Math Lib review activity CCE2(9-12)

Teacher Resources: *White boards, Desmos, IXL, Khan Academy, rulers, compass, protractor, TI Graphing Calculator, Interactive Notebooks, various Geometry textbooks*

Unit 5 - Similarity and Proportional Reasoning

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (*) indicates a modeling standard; (★ indicates greater emphasis in the Honors sections)

- **G.SRT.A.1.B** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- **G.SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- **G.SRT.A.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar
- **G.SRT.B.4** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
- **G.SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to...

- TCC4(9-12): I can integrate my learning to adapt to experiences in the classroom, career and life.
- CCE2(9-12): I can give and receive actionable and relevant feedback with openness to be able to determine meaningful revisions for success.

Understandings: Students will understand that...	**Essential Questions:**

• Dilations, similarity, and the properties of similar triangles allow for the application of trigonometric ratios to real-world situations (ie. scaling maps, architecture)
• There are some common misconceptions and missteps that students have/make with similarity and proportional reasoning:
  ○ Students often mistake area factors as being equivalent to the scale factor.★
  ○ Students Incorrectly identify corresponding parts of diagrams when setting up proportions.
  ○ Students often forget that the proportionality of the parallel lines in overlapping triangles are not equivalent to the proportionality of the parts (Side Splitter).
  ○ Students forget to factor before simplifying ratios.★
  ○ Students often try to cancel out terms such as canceling the x in (x+1)/x instead of canceling out factors. ★
	• How might the features of one figure be useful when solving problems about a similar figure?
	• What does it mean to be similar in the context of Geometry?
	• How can the movement of shapes preserve congruence and/or similarity?
	• How are ratios, scale factors, and proportions similar/different from each other and how can they be used to solve problems?
	• What additional insights can be understood and used to solve problems once it is established that two figures are similar?

Students will know...	**Students will be able to...**
• Use the cross-product property and algebraic properties (including factoring★) to solve proportions. • Understand ratios and proportionality to solve a wide variety of problems. • Use the fact that two figures are similar when corresponding angles are congruent and corresponding sides are proportional to solve problems. • Use the relationship that the perimeter ratio of two figures is equal to the scale factor and the area ratio of two similar figures is equal to the square of the scale factor. • SSS~, SAS~, and AA~ can be used to establish that two triangles are similar and students can use them to solve for missing values between the triangles. • The Angle Bisector of a Triangle and parallel lines within a triangle creates proportional relationships.	• Identify corresponding sides and corresponding angles of similar figures • Solve proportions • Solve real world problems (the golden ratios/recipes) using proportions. • Solve problems knowing two figures are similar involving angle measures, length of sides, perimeters, and areas. • Solve similarity problems involving algebra skills (including factoring a quadratic and solving a system★). • Prove two triangles are similar using AA, SAS, or SSS similarity. • Use the Proportionality of an Angle Bisector and the Proportionality of Parallel Lines (Side Splitter Theorem) to solve problems.

Key Vocabulary:

Dilation
Center
Scale Factor
Similarity
Corresponding Sides
Corresponding Angles
Triangle
Area Factor
SSS ~
SAS ~
AA ~
Center of Dilation
Cross Products
Ratio
Angle Bisector Theorem
Side Splitter Theorem
Parallel
Proportional
Reflection
Enlargement
Reduction

Assessment Evidence

Performance Tasks:

- **Similarity Choice Board Project CCE2(9-12)**
  - Students collaborate with a peer
  - Students choose 3 options of 7 mini projects
    - Scale a dinosaur
    - Similarity in pictures
    - Crazy horse: Real world architecture
    - Dinosaur Ratios
    - Scale a Barbie compared to a human
    - Scale a Business Logo
    - How much Dunkin: Use research, similarity and volume to determine how much coffee a Dunkin Truck can hold.
  - All projects assess student ability to apply the theorems of Similarity to think critically and solve real world problems.

Other Evidence:

- **Quiz 5-1 Proportional Reasoning TCC4(9-12)**
  - Use proportional reasoning for real world applications such as baking
  - Solve algebraically for missing sides and angles of Similar triangles
  - Explain what makes triangles similar using diagrams and notation
- **Quiz 5-2: Partner Sticker Quiz TCC4(9-12)**
  - Use similarity to determine heights of buildings and trees in real world applications
  - Prove triangles similar using SSS, SAS, AA
  - Use side splitter theorem and angle bisector theorem to solve for missing parts of similar triangles.
- **Daily warm-ups**
  - Here's where we've been and where are we going, what are the similarities and differences?
  - Student-led discussions regarding solutions
  - Discussions regarding student misconceptions
  - Examples of student work, find the mistake and describe
- **Daily homework discussion**

	● **Concept Checks** ○ Students have the opportunity to receive feedback and rework questions to improve understanding

Learning Plan

*Utilize this section for learning tasks that will be expected of **all** students enrolled in the course **no matter who is teaching the course**. These are the **common experiences** that all students will have. Essentially, these are the common formative assessments (CFAs) that all teachers will utilize. The expectation is that outside of these CFAs, teachers will use their autonomy to develop formative assessments that address their students' needs and interests. Modifications and accommodations should be made with the CFAs listed here for students with IEPs and 504s.*

- *Students must have the opportunity to practice and demonstrate the Vision of A Learner attributes throughout each unit, especially the skill-based attributes of "think critically and creatively," "collaborate and communicate effectively," and "take initiative."*
- *Utilize the VoL Teacher Website for ideas.*
- *Identify the attribute being practiced or demonstrated next to each learning task.*

- Students will discover the meaning of similarity through Scaling the GIant Desmos TCC1(9-12)
- Students will complete IXL activities using SImilarity Shortcuts with immediate feedback
- Students will practice using SSS, SAS, AA in Similarity Shortcuts Desmos
- Students will participate in Vertical white board collaborative practice utilizing methods from Building Thinking Classrooms TI3(9-12)
- Students will apply similarity to solve real world problems in indirect measurement Desmos TI3(9-12)
- Students will develop skills with a partner and gain immediate feedback completing Desmos: Proofs with CPCTC
- College Prep classes will complete interactive notebook pages for each skill set CCE2(9-12)
- Students will complete collaborative around the room Scavenger Hunt activity

Teacher Resources: *White boards, Desmos, IXL, Khan Academy, rulers, compass, protractor, TI Graphing Calculator, Interactive Notebooks, various Geometry textbooks*

Unit 6 - Right Triangles and Trigonometry

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (*) indicates a modeling standard; (★ indicates greater emphasis in the Honors sections)

- **G.SRT.B.4** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
- **G.SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- **G.SRT.C.7** Explain and use the relationship between the sine and cosine of complementary angles.
- **G.SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to...

- TCC4(9-12): I can integrate my learning to adapt to experiences in the classroom, career and life.
- CCE3(9-12): I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.
- AA2(9-12): I can assess my past successes and mistakes to change my approach.

Understandings: Students will understand that...	**Essential Questions:**
• Proving and applying congruence provides a basis for modeling situation geometrically • There are some common misconceptions and missteps that	• How do the ratios of special right triangles lead to the development of the trigonometric ratios? • How are side lengths or angle measures found in right

students have/make with right triangles and trigonometry: ○ Students forget to always put the value of the hypotenuse in for “c”, regardless of whether or not it is the unknown. ○ Students may mis identify the correct angle of depression when creating diagrams ○ Students often choose the incorrect trig function (sine, cosine, or tangent) when identifying the reference angle and corresponding sides (adjacent, opposite, hypotenuse).	triangles? ● How do the properties of right triangles help us solve real-world situations? ● How can you indirectly measure the height of an object that is too tall to measure directly?
Students will know... ● The Pythagorean Theorem can be used to derive the side ratio relationships for isosceles right triangles and 30-60-90 triangles. ● Apply the trigonometric ratios to acute angles. ● Trigonometric ratios can be used to find side lengths when angle measures are known. ● Sine (theta) = opposite/hypotenuse ● Cosine (theta) = adjacent/hypotenuse ● Tangent (theta) = opposite/adjacent ● Inverse trig ratios can be used to find the missing angle measure when sides are known. ● The angle of elevation and angle of depression are alternate interior (congruent) angles.	**Students will be able to...** ● Use pythagorean theorem to solve real world problems ● Use pythagorean theorem to prove the distance formula ● Simplify Radicals and rationalize the denominator to produce simplified exact answers when using Pythagorean Theorem and special right triangles. ● Recognize patterns utilizing the Pythagorean Theorem within two right triangles known as the special right triangles. ● Solve problems involving special right triangles in the most efficient way possible by utilizing the identified patterns. ● Find the sine, cosine, and tangent of an acute angle. ● Calculate side lengths using trigonometric ratios ● Find angle measures in right triangles using trigonometric ratios. ● Apply trig ratios to real world problems ● Identify the angle of elevation and angle of depression and use them to solve modeling problems.

Key Vocabulary:

Radical	Adjacent Side	Inverse Sine
Irrational	Special Right Triangles	Inverse Cosine
Exact	30-60-90 Triangle	Inverse Tangent
Approximate	45-45-90 Triangle	Angle of Elevation
Rationalizing the Denominator	Sine	Angle of Depression

Pythagorean Theorem Converse of the Pythagorean Theorem Distance Formula	Cosine Tangent Opposite Side	Reference Angle *Means* ★ *Extremes* ★

Assessment Evidence

Performance Tasks:

- Right Triangles Project Part 1: TCC4(9-12)
  - Real world applications in which student must decide what the appropriate method to solve is: Pythagorean theorem, special rights, or Trigonometry
  - Draw and label a diagram to represent each situation
- Right Triangles Project Part 2: Choice Project CCE3(9-12)
  - Students choose 2 projects from 4 mini project choices
    - Pythagorean Instagram: Prove Pythagorean theorem using real world examples
    - Trigonometry Brochure
    - Comic Education
    - Create an assessment with answer key

Other Evidence:

- **Quiz 6-1 Pythagorean Theorem and Its Converse**
  - Use Pythagorean theorem and distance formula to solve real world applications
  - Given three side measures of a triangle, identify if it is Right, Obtuse, or Acute using the converse of the Pythagorean theorem.
  - Simplify radicals, leave answers in exact simplified form
- **Quiz 6-2: Special Right Triangles**
  - Solve for missing sides of 30-60-90 and 45-45-90 right Triangles without using pythagorean theorem
  - Show exact simplified answers
  - Apply special right properties to real world situation regarding rectangles and squares
- **Quiz 6-3 Trig Functions**
  - Identify trig ratios using acute angles
  - Solve for sides with the variable in the numerator and denominator of the trig ratio using Sine, Cosine, Tangent
  - Solve for angles in right triangles using Inverse Sine, Inverse Cosine, Inverse Tangent.
- **Daily warm-ups**
  - Here's where we've been and where are we going, what are the similarities and differences?
  - Student-led discussions regarding solutions
  - Discussions regarding student misconceptions
  - Examples of student work, find the mistake and describe
- **Daily homework discussion**

	● **Concept Checks** AA2(9-12) ○ Students have the opportunity to receive feedback and rework questions to improve understanding

Learning Plan

- Students will simplify radicals and rationalize the denominator to get exact simplified answers using pythagorean Theorem Desmos
- Students will solve real world problems working collaboratively in partner self checking activity.
- Students will discover the special right triangles rules using pythagorean theorem and similar triangles. TI3(9-12)
- Students will complete IXL and Khan Academy Practice with immediate feedback on Special Right Triangles
- Students will participate in Vertical white board collaborative practice utilizing methods from Building Thinking Classrooms TI3(9-12)
- Students will explore right triangle Trig Functions in Intro to Trigonometry Desmos TCC1(9-12)
- Students will Solve for missing Sides of right triangles given an angle and one side using Sine, Cosine, and Tangent
- Students will discover how to solve for a missing angle in a right triangle using Math medic Exploratory activity introducing Arcsin, Arccos, Arctan TCC1(9-12)
- College Prep classes will complete interactive notebook pages for each skill set CCE2(9-12)
- Students will collaborate to complete a math lib around the room review AA2(9-12)

Teacher Resources: *White boards, Desmos, IXL, Khan Academy, rulers, compass, protractor, TI Graphing Calculator, Interactive Notebooks, various Geometry textbooks*

Unit 7 - Circles & Conic Sections

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (*) indicates a modeling standard; (★ indicates greater emphasis in the Honors sections)

- **G.CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **G.CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **G.C.A.1** Prove that all circles are similar.
- **G.C.A.2** Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
- **G.C.A.3** ★ Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- **G.C.A.4 (+)** ★ Construct a tangent line from a point outside a given circle to the circle.
- **G.C.B.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
- **F.TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **G.GPE.A.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- **G.GPE.B.4** For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to...

- TCC4(9-12): I can integrate my learning to adapt to experiences in the classroom, career and life.
- TI3(9-12): I can formulate and investigate probing questions to further my learning.
- AA1(9-12): I can evaluate different approaches and justify the best pathway to success.

Understandings: Students will understand that...

- A circle is defined as the set of all points in a plane that are a fixed distance (radius) from a given point (center).
- The relationship between lines and circles including the relationship between tangent lines and radii.
- The relationship between arcs and angles.
- How to calculate segment lengths and angle measures formed by chords, secants, tangents and intersecting lines.
- How to solve real world problems utilizing the properties of circles.
- There are some common misconceptions and missteps that students have/make with circles and conic sections:
    - Students may forget to square the radius when writing the equation of a circle.
    - Students may use central angle properties for inscribed angles.

Essential Questions:

- What are the characteristics of a circle?
- What notations do we use when working with circles?
- What types of angles exist in a circle and how do they relate to their intercepted arcs?
- Why is it important to know properties of chords and tangents of circles?
- How are the measures of central and inscribed angles related to their intercepted arc, and where is this used in geometry?
- How are the measures of interior and exterior floating angles related to their intercepted arcs, and where is this used in geometry?
- What is the distinction between arc measure and arc length and how is arc length related to circumference?
- What types of lines and segments exist in a circle and what relationships exist in terms of their measures?
- How do you represent a circle using Analytical Geometry?

Students will know...

- A circle is made up of a set of points equidistant from a center.
- Notation communicates the knowledge you are sharing.
- The location of angles created by segments about a circle determine how you calculate arc measure.

Students will be able to...

- Identify characteristics of a circle.
- Use appropriate notation for circles, arcs, chords, tangents, and secants.
- Use central, inscribed, inside, and outside angles to determine

• Power Theorems allow you to find lengths of chords and pieces of secants. • The definition of a circle can be modeled with algebra.	arc measure • Use Power Theorems to find lengths of chords and pieces of tangents and secants. ★ • Students will represent a circle using Analytical Geometry. • Determine the location of a point given a circle equation.

Key Vocabulary:

Circle	Chord	Circumscribed Circle
Center	Sector	Segments of a Chord
Radius	Tangent	Standard Equation of a Circle
Diameter	Point of Tangency	External Segment
Arc Length	Central Angle	Focus
Arc Measure	Inscribed Angle	Directrix ★
Degrees	Inscribed Polygon	Parabola ★
Radians	Cyclic Quadrilateral	Hyperbola ★
Major Arc	Intercepted Arc	Ellipse ★
Minor Arc	Semicircle	

Assessment Evidence

Performance Tasks:	**Other Evidence:**
• **Unit Summative Assessment**: **AA1(9-12)** This will assess circle vocabulary and definitions, angles in circles both interior, on, and outside the circle, arcs and segments in circles (measure vs. length). Using radians as a form of measuring angles ★. It will also cover coordinate geometry with equations of circles, where students will create and read equations to locate center/radius and determine if a point is located on,interior or exterior the circle.As well as real-world problems. This assessment will be set up with vocabulary matching or fill in the blank, error analysis question, multiple choice questions, Use the 4-step Process (Diagram, Geometric Statement and Reason, Algebra, and Summary) to	• **Quiz 7-1:** Students will identify new circle vocabulary. They will solve for missing angles and segments utilizing the properties of circles. • **Quiz 7-2:** Students will be assessed on finding arc length including problems with more than one revolution. Honors will include radian measure. ★ • **Vertical whiteboard warm-ups:** Student-led discussions regarding solutions or misconceptions. Given a multi-step problem which emphasizes depth or difficulty. These will be inquiry-based or application-heavy utilizing methods from

solve problems. ● **Conic Sections Project: "Conics in the Real World" ★** (If time allows) **TCC4(9-12)** ○ Students will explore four conic sections - circle, ellipse, parabola, and hyperbola. ○ They will demonstrate their understanding by connecting the equations, features, and real-world applications of each through a visual written presentation.	Building Thinking Classrooms. **TI3(9-12)** ● **Daily homework discussion** ● **Exit ticket** (Reflective Questions on what the student learned and what they need additional support on), Google Form, Concept Check, etc.

Learning Plan

Unit 7 is the study of circles and their properties and how they can be used in the real-world. Some applications include the fields of engineering, construction, art and physics. (E.g. Ferris wheels, gears, mandalas, domes, arches, roundabouts and turntables, pizza/pies/cakes, cans, cups, circular motion)

- *Students will use precise mathematical vocabulary to describe relationships between circle elements.*
- *Students will calculate arc measures and the measures of central angles.*
- *Students will solve for unknown angles formed by intersecting chords, tangents and secants in and around circles.* **TI3(9-12)**
- *Students will derive the formula for the equation of a circle through the pythagorean theorem.*
- *Students will communicate their understanding through discussion, diagrams, and written explanations.* **TCC4(9-12)**
- *Demonstration of radians, have the students use the wooden cut out unit circle to demonstrate the relationship between the radius and the circumference of a circle. ★*

Teacher Resources: *White boards, Desmos, IXL, Khan Academy, rulers, compass, protractor, TI Graphing Calculator, Interactive Notebooks, various Geometry textbooks*

Unit 8 - Surface Area & Volume

Desired Results - Goals, Transfer, Meaning, Acquisition
Established Goals: **Common Core State Standards** (*) indicates a modeling standard; (★ indicates greater emphasis in the Honors sections) ● **G.GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. ● **G.GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* ● **G.GMD.B.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. ● **G.MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* ● **G.MG.A.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* ● **G.SRT.D.9 ★(+)** Derive the formula A = (1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. ● **G.CO.D.13 ★** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. ● **S.CP.B.6** Find the conditional probability of A given B as a fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ● **7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Large numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely or likely, and a probability near 1 indicates a likely event. ● **N.Q.A.2** Define appropriate quantities for the purpose of descriptive modeling. **Standards for Mathematical Practice** ● MP1: Make sense of problems and persevere in solving them. ● MP2: Reason abstractly and quantitatively. ● MP3: Construct viable arguments and critique the reasoning of others. ● MP4: Model with mathematics. ● MP5: Use appropriate tools strategically. ● MP6: Attend to precision. ● MP7: Look for and make use of structure. ● MP8: Look for and express regularity in repeated reasoning.
Vision of A Learner Attributes: Students will be able to independently use their learning to...

- CCE2(9-12): I can give and receive actionable and relevant feedback with openness to be able to determine meaningful revisions for success.
- AA1(9-12): I can evaluate different approaches and justify the best pathway to success.
- TI3(9-12): I can formulate and investigate probing questions to further my learning.

Understandings: Students will understand that…

- Area formulas come from geometric reasoning. (for example how regular polygons can be derived by breaking it into congruent isosceles triangles.
- When doubling the side length, the effect on the perimeter is equivalent, but the area is four times as great.
- Three-dimensional shapes can be understood by their two-dimensional components such as nets and cross-sections.
- Formulas for surface area and volume are derived from geometric reasoning, rather than memorizing formulas, students should understand why they work and be able to adjust them as needed for irregular solids.
- Units matter and change with dimension, scaling dimensions affects surface area and volume differently.
- Connections to real-world modeling, packaging, construction, engineering, and efficiency problems.
- There are some common misconceptions and missteps that students have/make with surface area and volume:
    - Students may use the formula incorrectly using a side vs base of the solid
    - Students may mistakenly use the height of the face vs height of the solid.
    - Students may forget to include all parts of the formula in real-world problems (I.e. interior surface of a pool and they put the top in their solution).

Essential Questions:

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- What do surface area and volume tell us about a 3D solid?
- In what ways can the sides of a solid be described?
- How do you calculate the surface area and/or volume of a solid when considering a real world situation?
- What are the similarities/differences between a solid and a polyhedron?★
- What unique polyhedrons exist and what makes them unique?★
- How can you use Geometric Probabilities to solve problems?

Students will know...

- The area of a sector is related to the area of a circle because of the relationship between the circle and the

Students will be able to…

- Determine if a polygon is convex or concave
- Determine if a polygon is regular

sector. • Convex polygons have angle measures that are less than 180°. Concave polygons contain at least one angle larger than 180°. • The exterior angles of a polygon always add to 360° • At each vertex, the exterior and interior angles form a linear pair. • Irregular polygons can be broken down into components of regular polygons. Estimation can be applied. • Surface area is the measurement of all the two-dimensional space that the surface of a three-dimensional shape takes up. • Volume is the amount of space an object occupies. • Composite figures help to model real-world phenomena. • Surface Area and Volume formulas need to be modified and/or adjusted when solving composite figures or real world problems. • *Geometric probability can be applied to length and area problems with connections made to the real-world. (i.e. time passed is similar to the concept of length).*	• Find the sum of the measures of the interior angles and exterior angles of a polygon • Find the measures of one interior angle of a regular polygon • Find the number of sides a regular polygon has given one interior or exterior angle. • Calculate the area of irregular polygons. • Estimate area using a grid strategy. • Determine the change on perimeter and/or area given a dimension change in a polygon. • Calculate the area of a regular polygon given a radius or an apothem. • Derive the equilateral triangle volume formula. ★ • Discuss cross-sections and Cavalieri's principle. • Calculate the surface area of three dimensional figures. • Calculate the volume of three dimensional figures. • Calculate the surface area and volume of composite solids. • Solve real world problems. • To derive Euler's formula through inductive reasoning.★ • Compare and contrast solids, polyhedrons, platonic solids, and archimedean solids. ★ • Explain how probability can be represented as a ratio of areas in 2D figures or volumes in 3D figures • Understand that geometric probability involves comparing a target region to a total region. • Calculate the probability of a point landing in a specific region of a 2D shape by using area. • Solve real-world problems involving geometric probability in two dimensions (e.g., dartboard problems, shaded regions).

Key Vocabulary:

Polygon	Volume	Right Prism
Central angle of a polygon	Surface Area	Oblique
Apothem of a polygon	Net	Cross Section

Radius of a polygon	Face	Hemisphere
Pentagon	Edge	Sphere
Hexagon	Vertex	Great Circle
Heptagon	Euler's Formula	Similar Solids
Octagon	Base	Platonic solids
Nonagon	Lateral Area	Polyhedron
Decagon	Prism	
Dodecagon	Cylinder	
N-gon	Pyramid	
Convex	Cone	
Concave	Right Cone	

Assessment Evidence

Performance Tasks:

- **Unit Summative Assessment**: **CCE2(9-12)**
  This test will include calculating and adjusting formulas for area/surface area and volumes of solids. They will also solve real-world problems involving surface area and volume. They will calculate or analyze similar solids adjusting the scale factor for two or three-dimensions. They demonstrate conversions of units in 1/2/3 dimensions. This assessment will be set up with vocabulary matching or fill in the blank, error analysis question, multiple choice questions, Use the 4-step Process (Diagram, Geometric Statement and Reason, Algebra, and Summary) to solve problems.

- ***Polyhedron Project:*** **AA1(9-12)**
  *S*tudents will research a polyhedron that is not a platonic solid. They will choose, research and create a physical solid for their polyhedron. The volume will be calculated by breaking it up into pyramids with a vertex at the center and will research a volume formula to compare. They will calculate the surface area in an organized fashion with correct formulas, units and labels. ★ (If

Other Evidence:

- **Quiz 8-1:** Students will calculate the area of polygons and solve problems involving interior and exterior angles of different polygons.

- **Quiz 8-2:** Students will be assessed on calculating surface area of prisms, cylinders, pyramids, cones and spheres. This check in will include irregular solids and applications of real-world problems.

- **Volume Discovery: TI3(9-12)** Students will use inductive reasoning to determine the formulas for volume of pyramids, cones and spheres using water or rice

- **Quiz 8-3:** Students will be assessed on calculating the volume of prisms, cylinders, pyramids, cones and spheres. This check in will include irregular solids and applications of real-world problems.
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- **Vertical whiteboard warm-ups:** Student-led discussions

time allows)	regarding solutions or misconceptions. Given a multi-step problem which emphasizes depth or difficulty. These will be inquiry-based or application-heavy utilizing methods from Building Thinking Classrooms. ● **Daily homework discussion** ● **Exit tickets** (Reflective Questions on what the student learned and what they need additional support on), Google Form, Concept Check, etc.

Learning Plan

Unit 8 is the study of Solid Geometry, the branch of Geometry that deals with three-dimensional solids.

- Students will identify and describe three-dimensional figures, including prisms, pyramids, cylinders, cones and spheres.
- Students will calculate the surface area and volumes of three-dimensional figures, including prisms, pyramids, cylinders, cones and spheres.
- Students will compare and contrast surface area and volume, and explain how they measure different aspects of a 3D shape.
- Students will use inductive reasoning to determine the formulas for volume of pyramids, cones and spheres using water or rice. **TI3(9-12)**
- Students will solve real-world problems involving surface area and volume. **CCE2(9-12)**
- Students will apply formulas in multi-step and word problems that involve composite solids.
- Students will sketch and interpret nets of three-dimensional figures to visualize surface area.
- Students will determine the effect of changing dimensions (scaling) on surface area and volume. **AA1(9-12)**
- Students will apply dimensional analysis and use correct units (square units for area, cubic units for volume).
- Students will justify their work and communicate reasoning through labeled diagrams and written explanations.
- Students will use spatial reasoning to identify cross-sections and build understanding of 3D geometry.
- Students will model solids using physical or digital tools and connect volume/surface area to real-world applications.
- Students will calculate the probability of a randomly chosen point landing in a target region of a 2D and 3D figure.
- Students will collaborate with a peer to solve real-world problems involving geometric probability in two dimensions

Teacher Resources: *White boards, Desmos, IXL, Khan Academy, rulers, compass, protractor, TI Graphing Calculator, Interactive Notebooks, various Geometry textbooks*