CALCULUS HONORS CURRICULUM



Grade Level(s): 12

Curriculum Author(s): Raymond Robillard

Course Description: Calculus is a full-year course designed to provide students with a basic understanding of Differential and Integral Calculus as it applies to business, finance, the social sciences, environmental sciences, physical sciences, etc. Students will engage in the study of Calculus through real-world application and prepare for the post-secondary study of Calculus.

Year At A Glance			
Unit Title	**Overarching Essential Question**	**Overarching Enduring Understanding**	**Vision of A Learner "I Can" Statements**
Limits	How can limits help understand the behavior of functions?	A limit describes the function value approached as the independent variable infinitely approaches some value or infinity.	TCC1(9-12); AA1(9-12)
Derivatives	What is a derivative?	The derivative is more than the slope of the tangent line. It represents an instantaneous rate of change for modeled quantities relative to some other variable, often time.	TCC3(9-12); AA2(9-12):
More Derivatives	Are there types of functions for which we cannot find a derivative?	There are differentiation shortcuts for all families of functions learned in high school math.	TI3(9-12); AA1(9-12)
Applications of Derivatives	What real world problems can be solved using differentiation?	Functions allow us to optimize processes across a vast array of fields and careers.	TCC1(9-12); TI1(9-12)
Integration	What does the integral represent in a real world situation?	An integral allows us to calculate the total change in any measurable variable for any real world application.	TI3(9-12); AA2(9-12)
Slope Fields, Euler's Method, U-Substitution, and L'Hopital's Rule	How can we analyze the behavior of a function (sometimes without knowing what it is)?	Advanced Calculus techniques can be used to analyze differential equations, integrate composite functions, and determine limits of indeterminate forms.	TI1(9-12); TI3(9-12)
Geometry of Calculus	How can we use Calculus to measure the size of things in the real world?	Calculus allows us to measure the length, area, and volume of infinitely complex shapes.	TCC1(9-12), TI1(9-12)
***Units in this document do not appear to match numbers indicated on assignments. This is because the first chapter of our textbook is a precalculus review unit that is not taught in this course. For example, the first assignment in Unit 1 is labeled number 2.1 because it corresponds to the first section of chapter 2 in our textbook.**			

Unit 1 - Limits

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards
HSN.Q.A.3-Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
HSA.SSE.B.3-Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
HSF.IF.C.7-Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Standards for Mathematical Practice
MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

- TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.
- AA1(9-12): I can evaluate different approaches and justify the best pathway to success.

Understandings: Students will understand that...

- A limit describes the function value approached as the independent variable infinitely approaches some value or infinity.
- A two-sided limit exists if and only if the one-sided limits exist and are equal.
- A function is continuous at a point if and only if the two sided limit exists at the point and is equivalent to the function value at that point.
- A function is continuous if and only if it is continuous at every

Essential Questions:

- How can limits help understand the behavior of functions?
- How can limits be used to define continuity of a function?
- How can we calculate slope at a single point on a function?

point. • A limit can be used to find an instantaneous rate of change(slope of a tangent line)	
Students will know… • The definition of a limit • Proper notation of a limit • Concept of limit can be extended to include one-sided limits • A limit may not exist for some functions at a particular x value • End behavior of a graph can be described using limits • Definition of continuity • Different types of discontinuity • The slope between two points is an average rate of change • The slope at one point is an instantaneous rate of change	**Students will be able to...** • Find limits numerically • Find limits graphically • Find limits algebraically • Use a limit at infinity to describe end behavior • Determine continuity at a point • Classify types of discontinuities • Find the slope of a tangent line at a point
Key Vocabulary: Limit; One-sided limit; Two-sided limit; End Behavior; Continuous/Discontinuous; Removable discontinuity; Infinite discontinuity; Jump discontinuity; Average rate of change; Instantaneous rate of change	
Assessment Evidence	
Performance Tasks: **Summative Unit Assessment** TCC1(9-12) and AA1(9-12) Topics will include: • Evaluating limits at a constant • Evaluating limits at infinity • Evaluating limits with a graph/table • Identifying and describing types of discontinuities • Calculate and average rate of change • Finding equations of lines tangent to a function at a point	**Other Evidence:** Book Practice (Classwork and Homework Assignments): • 2.1- P. 65-67 #3,7,9,11,13,25,27,29 • 2.1- P. 65-67 #43,44,45,47,49 • 2.2- P. 76-77 #1,5,13,15,19,21,22,23,27,35-38 • 2.3- P. 84-85 #1,2,4,5,7,23,25,48 • 2.4- P. 93-95 #1,9,23,26 Interim: • Quiz on Evaluating Limits
Learning Plan	
In this unit, students will be introduced to the concept of the limit. Students will evaluate limits with the informal understanding of “getting as close to a point as possible without actually getting there”. The idea of calculating a slope between two points has become second nature to students since Algebra 1, so the concept of finding a slope of a tangent line from a single point will seem impossible before seeing how a limit allows us to solve such a problem. • 2.1- Students will evaluate limits at a point using substitution in cases where a function is continuous at that point. They will also use	

algebraic techniques to remove discontinuities. Students will evaluate limits graphically, extending their knowledge to one and two-sided limits. **AA1(9-12)**

- 2.2- Students will evaluate limits at infinity and limits that produce infinite discontinuities. End behavior and asymptotes will be reinforced.**TCC1(9-12)**
- 2.3- Students will define continuity, both at a point, and across an entire function.**TCC1(9-12)**
- 2.4- Students will calculate the average rate of change using the slope formula and generalize the formula to find the slope of a tangent line at a single point. **TCC1(9-12)**

Teacher Resources: Textbook: Calculus: Graphical, Numerical, Algebraic; TI Smartview Software and individual calculators; Teacher created in-class notes, activities, and problem sets

Unit 2 - Derivatives

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards
HSN.RN.A.1-Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
HSN.RN.A.2-Rewrite expressions involving radicals and rational exponents using the properties of exponents.
HSA.SSE.A.2-Use the structure of an expression to identify ways to rewrite it.
HSA.SSE.B.3-Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
HSF.IF.B.6-Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Standards for Mathematical Practice
MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

- TCC3(9-12): I can integrate relevant information to produce multiple valid solutions.
- AA2(9-12): I can assess my past successes and mistakes to change my approach.

Understandings: Students will understand that...

- The derivative is more than the slope of the tangent line. It represents an instantaneous rate of change for modeled quantities relative to some other variable, often time.

Essential Questions:

- What is a derivative?
- What happens to the slope between two points as they come infinitely close to one another?

• There are many techniques to determine derivative functions, and in some cases, multiple ways to reach these solutions. • The derivative of a function is also a function (and thus has a corresponding graph).	
Students will know… • Proper derivative notation. • The first derivative is velocity of a distance function • A function is non-differentiable at a cusp, corner, vertical tangent line, or a discontinuity. • The general forms of the power rule, product rule, and quotient rule. • The derivatives of the 6 basic trigonometric functions.	**Students will be able to...** • Evaluate the derivative by taking the limit of the difference quotient • Calculate the derivative using differentiation rules (including trigonometric functions) • Write the equation of the tangent line to a specific point on a curve • Solve rate of change application problems using the derivative • Determine velocity of a moving object
Key Vocabulary: Limit definition of the derivative; Instantaneous rate of change; Position/Velocity; Cusp; Corner; Vertical Tangent Line; Discontinuity; Power Rule; Product Rule; Quotient Rule	
Assessment Evidence	
Performance Tasks: **Summative Unit Assessment** TCC3(9-12) and AA2(9-12) Topics will include: • Defining a derivative using the limit definition • Determining the value of a derivative at a point, or across a whole function, using the limit definition of the derivative • Explain the four types of ways a function can fail to be differentiable • Sketch graphs of derivatives of functions • Use the shortcuts of differentiation to determine f'(x) given f(x)	**Other Evidence:** Book Practice (Classwork and Homework Assignments): • 3.1- P. 107-108 #1-3, 13-16 • 3.2- P. 116-117 #3,8,10,12,15 • 3.3- P. 126 #1,2,5,6,9,15,16,17,19,20,29,31,33 • 3.5- P. 147-148 #1,2,4,8,11,13,21,23 Interim: • Quiz on Differentiability • Product and Quotient Rule Matching Game Activity
Learning Plan	
In this unit, students will formalize their evaluation of a slope of a tangent line from a point to the concept of the derivative. They will start to discuss what, exactly, a derivative is and see applications for its use. They will learn shortcuts for differentiation and discover ways that differentiation doesn't hold for a function f'(x). This knowledge will extend to trigonometric functions.	

- 3.1- Students will use the limit definition of the derivative to evaluate a derivative value at a point.**AA2(9-12)**
- 3.2- Explore and classify ways the a function could fail to be differentiable at a point.**AA2(9-12)**
- 3.3- Find derivatives, f'(x), of functions, f(x), using shortcuts of differentiation including the power rule, product rule, multiplication rule, and by rewriting radicals and rational functions as powers with negative or fractional exponents.They will also begin to sketch graphs of derivatives. **TCC3(9-12)**
- 3.5- Students will determine derivatives of basic trigonometric functions.**TCC3(9-12)**

Teacher Resources: Textbook: Calculus: Graphical, Numerical, Algebraic; TI Smartview Software and individual calculators; Teacher created in-class notes, activities, and problem sets

Unit 3 - More Derivatives

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards
HSN.Q.A.2-Define appropriate quantities for the purpose of descriptive modeling.
HSA.SSE.A.2-Use the structure of an expression to identify ways to rewrite it.
HSA.SSE.B.3-Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
HSA.CED.A.2-Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
HSF.IF.B.4-For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
HSF.BF.B.5-Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
HSF.IF.B.6-Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
HSF.IF.C.7-Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
HSF.BF.A.1-Write a function that describes a relationship between two quantities.*
HSF.BF.B.5-Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
HSF.LE.A.3-Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Standards for Mathematical Practice
MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- TI3(9-12): I can formulate and investigate probing questions to further my learning.
- AA1(9-12): I can evaluate different approaches and justify the best pathway to success.

Understandings: Students will understand that...

- When differentiating a composite function, one cannot simply differentiate only the inner and outer function.
- Implicit differentiation is necessary for differentiating a relation in terms of multiple variables
- There are differentiation shortcuts for all families of functions learned in high school math.

Essential Questions:

- Are there types of functions for which we cannot find a derivative?
- Can we find the derivatives of relations that are not functions
- How are the derivatives of functions and the derivatives of their inverses related?

Students will know...

- Given a function f(x) = g(h(x)), then f'(x) = g'(h(x))*h'(x).
- The derivative of a relation of multiple variables will require the use of the chain rule.
- The derivative of f(x) = a^x is f'(x) = (log a) a^x. e^x is a special case.
- The derivative of $f(x) = \log_a x$ is f'(x) = 1/ (x(ln a)). ln x is a special case.
- Given f(x), the derivative of $f^{-1}(x)$ is $(f^{-1})'(x) = 1/ f'(f^{-1}(x))$. ln x is a special case.

Students will be able to...

- Find a derivative of a composite function
- Find a derivative of a relation in multiple variables
- Find a derivative of an exponential function
- Find a derivative of a logarithmic function
- Find a derivative of an inverse function for f(x)

Key Vocabulary: Chain Rule; Newton's Notation/Leibniz's Notation; Implicit Differentiation; Euler's Number; Exponential Function; Logarithmic Function; Inverse Function

Assessment Evidence

Performance Tasks:
Summative Unit Assessment TI3(9-12) and AA1(9-12)
Topics will include:

- Differentiating functions using the chain rule
- Differentiating relations using implicit differentiation
- Differentiating exponential and logarithmic functions
- Finding a general derivative of an inverse function

Other Evidence:
Book Practice (Classwork and Homework Assignments):

- 4.1- P. 160-161 #1,5,6,13,15,16,29,33
- 4.2- P. 170 #1-4,9-13,15-22,27-40
- 4.4- P. 186 #1-35(Odds)

Interim:

- Quiz on Chain Rule
- Implicit Differentiation "Around the Room" Activity
- Quiz on Implicit Differentiation and Logarithmic/Exponential Derivatives

Learning Plan
In this unit, students will expand on their knowledge of derivatives and differentiation techniques. They will be able to differentiate composite functions, relations in multiple variables, and exponential and logarithmic functions. These topics, seemingly unrelated, build closely on each other, leading students to an understanding of how differentiation techniques, in general, came to be developed. ● 4.1- Students will discover and utilize the Chain Rule, a technique for differentiating composite functions. Students will be given the real world example of unwrapping a present to emphasize the "removal of the outer layer, while keeping the inner contents unchanged, before moving onto the inner layer".**AA1(9-12)** ● 4.2- The chain rule, represented with Leibniz's notation, will serve as the basis for differentiating relations in multiple variables. Students will see the chain rule as a way to differentiate a relation in terms of a variable other than the one the derivative is being taken in terms of.**AA1(9-12)** ● 4.3- Students will be shown the technique for differentiating an exponential function. Then, the chain rule, and an understanding of inverse functions, allows students to build upon this knowledge to find the derivatives of logarithmic functions. The process can be generalized to find the derivative of any inverse function, such as inverse trigonometric functions.**TI3(9-12)**
Teacher Resources: Textbook: Calculus: Graphical, Numerical, Algebraic; TI Smartview Software and individual calculators; Teacher created in-class notes, activities, and problem sets

Unit 4 - Applications of Derivatives

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards
HSN.Q.A.1-Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
HSN.Q.A.2-Define appropriate quantities for the purpose of descriptive modeling.
HSA.SSE.A.1-Interpret expressions that represent a quantity in terms of its context.
HSA.CED.A.2-Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
HSA.CED.A.3-Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
HSA.CED.A.4-Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
HSF.BF.A.1-Write a function that describes a relationship between two quantities.
HSG.MG.A.3-Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Standards for Mathematical Practice
MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

- TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.
- TI1(9-12): I can implement a realistic plan and adapt when necessary to achieve my goals.

Understandings: Students will understand that…	**Essential Questions:**
● Functions allow us to optimize processes across a vast array of	● What real world problems can be solved using differentiation?

fields and careers. ● A function can only have, at most, one absolute maximum and one absolute minimum, although these can occur in multiple places. ● An inflection point is where the concavity of a function changes. ● A related rates problem involves 2 or more derivatives. ● Proper notation is critical for communicating clear mathematics.	● How does the instantaneous rate of change over a function relate to the average value of a function? ● What do the derivative and the second derivative tell us about a function?
Students will know... ● Endpoints and critical numbers must be evaluated when seeking absolute extrema ● A function must be continuous on some interval [a,b] and differentiable on (a,b) for the Mean Value Theorem to be applied. ● The values of f''(x) tell us about the concavity of f(x) ● An optimization problem with a real world context frequently involves extraneous answers. ● There is a significant difference between d/dx and dy/dx.	**Students will be able to...** ● Find the absolute and relative extrema of a function, either over an interval, or including end behavior. ● Justify the validity of the Mean Value Theorem. ● Sketch a graph of f''(x) given a function f(x). ● Solve an optimization problem. ● Solve a related rates problem.
Key Vocabulary: Extrema; Local(relative) Maximum/Minimum; Global(absolute) Maximum/Minimum; Hypothesis; Conclusion; Mean Value Theorem; Acceleration; Concavity; Inflection Points; Optimization; Related Rates	
Assessment Evidence	
Performance Tasks: **Summative Unit Assessment TCC1(9-12) and TI1(9-12)** Topics will include: ● Determining extrema from a function. ● Determining extrema from a graph. ● Explaining what we know of a situation based on the Mean Value Theorem. ● Matching the graphs of f, f', and f'' and providing rationale for why graphs connected with each other. ● Solving optimization problems in context ● Solving related rates problems in context	**Other Evidence:** Book Practice (Classwork and Homework Assignments): ● 5.1- P. 200 #1,3,5,7,9,11,15,19,23,29,35,37 ● 5.2- P. 208 #1,4,6,11,12 ● 5.3- P. 220 #7,9,12,13,16,20 ● 5.3- P. 220 #21,22,23,24,25,29 ● 5.3- P. 220 #31,33,35,37 ● 5.4- P. 231 #1,7,9,17,23 ● 5.6- P. 257 #1,3,11,13,17,31 Interim: ● Matching game for graphs of f, f', and f'' ● Quiz on extrema, Mean Value Theorem, and graphs of f, f', and

	f” ● Quiz on optimization and related rates

Learning Plan

In this unit, students will utilize all they have learned so far about differentiation to solve a variety of real world problems. The Mean Value Theorem, connecting position, velocity, and acceleration, optimization, and related rates are concepts that professionals in all fields use to make informed decisions. An understanding of extrema sets up optimization and so it starts out this unit.

- 5.1- Students will use the first derivative of a function to determine the local(relative) and global(absolute) extrema of a function. They will do this both algebraically and graphically and compare and contrast the results.**TCC1(9-12)**
- 5.2- Students will learn and utilize the Mean Value Theorem to make claims and validate statements about the instantaneous rate of change of a function based on the average rate of change of the function. The real world example of speeding on the Massachusetts Pike will serve as the backbone for students grasping this concept.**TCC1(9-12)**
- 5.3- Up to this point, students have studied the connections between the graph of a function f(x) and its derivative f’(x). Ideally, by this point, a student has inquired about the graph of the derivative of f’(x), or f’’(x) and how this might provide us information about the original function f(x). Position, velocity, and acceleration graphs are used here to demonstrate these connections. Students learn about concavity and inflection points. **TCC1(9-12)**
- 5.4- In the third of four application driven sections in this unit, students will build upon the techniques learned in section 5.1 and solve real world optimization problems. Topics will include optimizing profit, minimizing materials needed to make containers, maximizing area for a fixed perimeter, and more.**TI1(9-12)**
- 5.5- In this section,students will build upon their knowledge of implicit differentiation from section 4.2 to solve problems involving related rates. Such topics include, the rate at which a snowball melts, how the viewing angle changes as a plane flies overhead, and how full a bathtub is as it drains. A point of emphasis that helps students here is to clarify the difference between the notation of d/dx and dy/dx.**TI1(9-12)**

Teacher Resources: Textbook: Calculus: Graphical, Numerical, Algebraic; TI Smartview Software and individual calculators; Teacher created in-class notes, activities, and problem sets

Unit 5 - Integration

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards
HSA.SSE.A.2-Use the structure of an expression to identify ways to rewrite it.
HSA.SSE.B.3-Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Standards for Mathematical Practice
MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

- TI3(9-12): I can formulate and investigate probing questions to further my learning.
- AA2(9-12): I can assess my past successes and mistakes to change my approach.

Understandings: Students will understand that...

- An integral allows us to calculate the total change in any measurable variable for any real world application.
- An area under a curve can be approximated by an infinitely large number of infinitely thin rectangles.
- The antiderivative of a function f is a function g whose derivative is f.
- A definite integral represents an area under a curve. (FToC part 1)
- The accumulation of area under a curve up to some variable point produces an accumulation function related to the

Essential Questions:

- Why is it necessary to have an inverse process to differentiation?
- What does the integral represent in a real world situation?

antiderivative of the integrand. (FToC part 2)	
Students will know... ● How to determine the width of a rectangle when finding a Riemann Sum. ● An antiderivative produces a family of functions and thus requires a constant of integration ● Proper notation of integration. ● Both parts of the FToC.	**Students will be able to...** ● Approximate an area under a curve with a Riemann Sum. ● Find an exact area under a curve using Geometry. ● Find a general and specific antiderivative of a function. ● Evaluate a definite integral. ● Apply the FToC part 2 to produce a new function (the accumulation function)
Key Vocabulary: Riemann Sum; RRAM (Right Rectangle Approximation Method); LRAM (Left Rectangle Approximation Method); Antiderivative; Constant of Integration; Integral; Integrand; Limits of Integration; Definite Integral; Indefinite Integral; Fundamental Theorem of Calculus; Accumulation function	
Assessment Evidence	
Performance Tasks: **Summative Unit Assessment TI3(9-12) and AA2(9-12)** Topics will include: ● Approximating the area under the curve with a Riemann Sum. ● Find the exact area under the curve of a function using formulas from Geometry. ● Finding antiderivatives, F(x), from a function, f(x). ● Evaluating a definite integral using the FToC part 2. ● Producing a function for an interval with variable limit of integration using the FToC part 1.	**Other Evidence:** Book Practice (Classwork and Homework Assignments): ● 6.1- P. 277 #7,8,19 ● 6.2- P. 291 #7-25(Odds) ● 6.3- P. 304 #3-33(Odds) ● 6.4- P. 310 #1-19(Odds) ● 6.4- P. 311 #27-39(Odds) Interim: ● Basic Integration Quiz ● Quiz on area under the curve, definite integrals, and the Fundamental Theorem of Calculus
Learning Plan	
In this unit, students will be introduced to the integral, a formalization of an area under the curve using an antiderivative. This unit is a pivotal bridge between Differential and Integral Calculus and is typically when students begin to make strong connections between all the key components of Calculus. Care should be taken in this section when working with students on antiderivatives, especially those with rational or negative exponents, as they tend to be quite error prone(the students, not the exponents!). ● 6.1- In this section, students are introduced to the concept of the Riemann Sum. The concept of finding the area of a rectangle is elementary to them, but the concept of doing so with an infinitely increasing number of increasingly thin rectangles reinforces that Calculus	

is a study of extremes. This section begins with an activity relating to estimating the area of the main island of Hawaii.
- 6.2- Students diverge from using rectangles to approximate an area to studying the area under a curve using a variety of geometric shapes. This section often is taught with a review sheet of area formulas from Geometry.
- 6.3- The antiderivative is introduced in this section. Students have experience with the concept of an antiderivative, naturally working in reverse of the differentiation process they have mastered over the course of semester 1. What is different is the introduction of the constant of integration, C, that shows the solution to an antiderivative problem is not just a single function, but, in fact, an entire family of functions.
- 6.4- Combining the aspects of sections 6.1, 6.2, and 6.3, students arrive at the most important theorem in Calculus: the Fundamental Theorem of Calculus. Starting with the new notation for an integral, and given in two parts, the first part demonstrates the connection between an integral and area under the curve whereas the second part provides an algebraic technique for using the antiderivative to calculate the value of a definite integral.

TI3(9-12) and AA2(9-12) - Summative Unit Assessment

Teacher Resources: Textbook: Calculus: Graphical, Numerical, Algebraic; TI Smartview Software and individual calculators; Teacher created in-class notes, activities, and problem sets

Unit 6 - Slope Fields, Euler's Method, U-Substitution, and L'Hopital's Rule

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards
HSA.SSE.A.2-Use the structure of an expression to identify ways to rewrite it.
HSA.SSE.B.3-Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
HSF.IF.C.7-Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Standards for Mathematical Practice
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MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- TI1(9-12): I can implement a realistic plan and adapt when necessary to achieve my goals.
- TI3(9-12): I can formulate and investigate probing questions to further my learning.

Understandings: Students will understand that...

- Advanced Calculus techniques can be used to analyze differential equations, integrate composite functions, and determine limits of indeterminate forms.
- Slope Fields are a graphical representation of a family of antiderivatives. Euler's Method "traces out" these particular antiderivatives using small line segments of a chosen size.
- A good choice for the expression of u, when using u-substitution, is one whose derivative is also part of the

Essential Questions:

- How can we analyze the behavior of a function (sometimes without knowing what it is)?
- Does the chain rule have a corresponding integration technique?
- Are there different sizes of infinity?

integrand, give or take a constant multiple. ● Indeterminate Forms can yield different limit values depending on the structure of the form.	
Students will know... ● Slope Fields are more accurate as we graph more “slope pieces” ● Euler’s Method is the basis for modern day GPS navigation. ● U-substitution is ideal when an integrand includes an expression AND a form of its derivative. ● L’Hopital’s Rule focuses on the rate at which expressions approach 0 or infinity.	**Students will be able to...** ● Match slope fields to differential equations. ● Estimate a function value with Euler’s Method. ● Use u-substitution to find an antiderivative. ● Evaluate a limit that produces an indeterminate form
Key Vocabulary: Slope Field; Differential Equation; Euler’s Method; U-Substitution; Indeterminate Form; L’Hopital’s Rule; Cardinalities of Infinity	
Assessment Evidence	
Performance Tasks: **Summative Unit Assessment TI1(9-12) and TI3(9-12)** Topics will include: ● Matching slope fields with corresponding differential equations ● Estimating a function value using Euler’s Method ● Evaluating indefinite integrals using u-substitution ● Evaluating limits using L’Hopital’s Rule	**Other Evidence:** Book Practice (Classwork and Homework Assignments): ● 7.1- P. 335 #29-46,51,57 ● 7.2- P. 343 #17,20,21,22,32,39,56,60 ● 9.2- P. 458 #17,23,33,39,49,51,65 Interim: ● Quiz on slope fields and Euler’s Method ● Quiz on U-Substitution and L’Hopital’s Rule
Learning Plan	
This unit is a bit of a hodgepodge of topics that bridge this course and Calculus 2 or Calculus 3 courses that students may take in college. Slope FIelds and Euler’s Method are an introduction to the concept of differential equations, a core topic of Calculus 3. U-Substitution and L’Hopital’s Rule extend the concepts of integration and limits to more complex tasks. These topics are included due to their accessibility as extensions of concepts learned in this class. ● 7.1- Slope fields and Euler’s Method allow students to investigate concepts relating to differential equations. The family of functions produced by a slope field demonstrates, graphically, the need for a constant of integration. Students look for key features of a slope field to match it to its corresponding differential equation. Euler’s Method provides a way of estimating function values in the absence of an expression for the function, assuming we know its derivative.**TI3(9-12)** ● 7.2- Students have previously learned how to differentiate a composite function using the chain rule. Here they learn how to	

antidifferentiate to a composite function. A real world metaphor of a shopping cart carrying watermelons helps students see the components and purpose of the process as it focuses on the "container" (the expression set equal to u) that allows the process to be simplified before antidifferentiation and "unpacked" afterwards.**TI1(9-12)**

- 9.2- L'Hopital's Rule provides a method for evaluating limits that result in indeterminate forms such as 0/0 or infinity/infinity. Students discover that, equally important as the values an expression approaches, is the rate at which it approaches these values.**TI3(9-12)**

TI1(9-12) and TI3(9-12) - Summative Unit Assessment

Teacher Resources: Textbook: Calculus: Graphical, Numerical, Algebraic; TI Smartview Software and individual calculators; Teacher created in-class notes, activities, and problem sets

Unit 7 - Geometry of Calculus

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards
HSA.SSE.A.1-Interpret expressions that represent a quantity in terms of its context.
HSN.Q.A.2-Define appropriate quantities for the purpose of descriptive modeling.
HSG.GMD.A.3-Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
HSG.GMD.B.4-Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
HSG.MG.A.1-Use geometric shapes, their measures, and their properties to describe objects

Standards for Mathematical Practice
MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.
- TI1(9-12): I can implement a realistic plan and adapt when necessary to achieve my goals.

Understandings: Students will understand that…	**Essential Questions:**
• Calculus allows us to measure the length, area, and volume of infinitely complex shapes. • Area is the summation of an infinite number of lengths and volume is the summation of an infinite number of areas. • The processes of finding area, volume, and length can be done relative to either the x or y axis.	• How can we use Calculus to measure the size of things in the real world? • How are the concepts of length, area, and volume related?
Students will know... • The area between two curves can always be written as the absolute value of the area under one curve minus the area of the other. • All 3 dimensional solids can be reduced to an infinite number of 2 dimensional cross sections. • Arc length techniques allow us to measure a curved distance.	**Students will be able to...** • Find the area between two curves relative to either the x or y axis. • Find the volume formed by a function rotated around some axis of revolution using the disc method. • Find the length of a curve relative to either the x or y axis.

Key Vocabulary: Area between curves; Disc method; Axis of rotation; Arc length

Assessment Evidence

Performance Tasks:	**Other Evidence:**
Summative Unit Assessment TCC1(9-12) and TI1(9-12) Topics will include: • Setting up and solving an integral to find the area between two curves • Setting up and solving an integral to find the volume of a solid of revolution using the disc method • Setting up and solving an integral to find the length of a curve relative to either the x or y axis	Book Practice (Classwork and Homework Assignments): • 8.2- P. 402 #1,5,6,9,13,15,17 • 8.3- P. 415 #7,10,11,14,15,17,18 • 8.4- P. 424 #1,4,9,11,15

Learning Plan

This unit is a synthesis of concepts from Geometry, through the lens of Calculus. Each of these processes takes a geometric technique and utilizes it on an infinitely small scale with a summation that produces a valid solution. What appeals most to students is when they can prove the

seemingly random formulas and techniques of days past through the power of integration.

- 8.2- Students will first solve problems involving integrals to find the area between two curves in a way similar to Geometry problems like the infamous “circle cut of a square” problem. **TI1(9-12)**
- 8.3- Volume formulas in Geometry were memorized, but here students create integrals that can find the volume of even the most irregular solids by considering them as an infinite number of infinitely thin discs. **TCC1(9-12)**
- 8.4- Finally, students utilize the distance formula, learned a few years ago, as part of an integral that sums an infinite number of infinitely small segments to determine arc length. **TCC1(9-12)**

TCC1(9-12) and TI1(9-12) - Summative Unit Assessment

Teacher Resources: Textbook: Calculus: Graphical, Numerical, Algebraic; TI Smartview Software and individual calculators; Teacher created in-class notes, activities, and problem sets