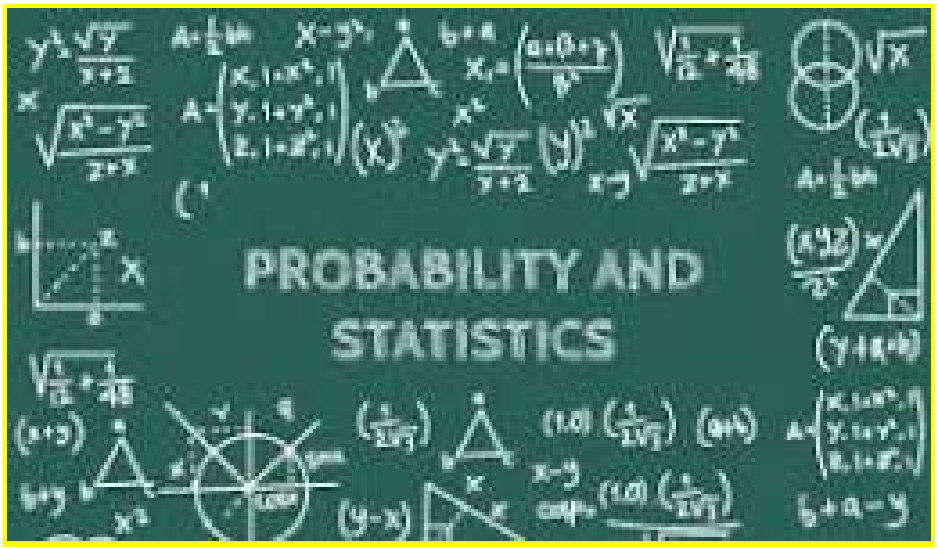
PROBABILITY AND STATISTICS CURRICULUM



Grade Level(s): 11-12

Curriculum Author(s): Nicole Cowles

Probability & Statistics is a full-year course designed to introduce students to the fields of mathematics which deals with the study of purely chance phenomena (probability) and the science of collecting, organizing, analyzing, and interpreting data(statistics). Through many hands-on activities, students will study topics such as surveying and sampling, the normal distribution, probability theory, confidence intervals and hypothesis testing, and linear regression. There will be a heavy emphasis on problem solving and real world applications.

Year At A Glance			
Unit Title	**Overarching Essential Question**	**Overarching Enduring Understanding**	**Vision of A Learner "I Can" Statements**
Probability (7-8 weeks)	What is probability and how is it used?	A solid understanding of probability enhances one's ability to critically evaluate information, assess risks, and make data-driven decisions.	CCE3(9-12); CCE4(9-12); AA4(9-12)
Univariate Data (8-9 weeks)	Why are some types of graphs and summary statistics more appropriate to display a set of data?	Different types of graphs are appropriate for different types of data and different statistics (measures of central tendency and spread) are appropriate for different sets of data.	CCE4(9-12)
Bivariate Data (4 weeks)	How can linear regression be used to make predictions, and what factors should be considered when making these predictions?	Linear regression is a powerful statistical tool used to model the relationship between two quantitative variables and make predictions based on that relationship.	TCC3(9-12); CCE4(9-12)
Collecting Data (4 weeks)	Why is it important to follow appropriate methods of data collection and how do we design an experiment to ensure that the results are valuable?	Data is a fundamental component in making informed decisions in various fields, from scientific research to business and public policy.	TCC1(9-12); CCE1(9-12); CCE4(9-12)
Central Limit Theorem (2-3 weeks)	What is the Central Limit Theorem and why is it fundamental in statistics?	The Central Limit Theorem (CLT) shows that, regardless of the population distribution, the distribution of the sample means becomes approximately normal as the sample size increases.	TCC1(9-12); TI2(9-12)

Inference Involving Means (5-6 weeks)	How do we use the sample mean to make conclusions about the population mean?	Confidence intervals provide a range of plausible values for a population mean based on sample data, offering a more informative estimate than a single point estimate; significance tests are tools for making informed decisions about population means based on sample data	TCC1(9-12); TCC2(9-12); CCE1(9-12); CCE4(9-12); TI2(9-12); TI3(9-12)
Inference Involving Proportions (3 weeks)	How does inference for data involving proportions differ from inference for data involving means?	Confidence intervals provide a range of plausible values for a population proportion based on sample data, offering a more informative estimate than a single point estimate; significance tests are tools for making informed decisions about population proportions based on sample data.	TCC1(9-12); TCC3(9-12); CCE3(9-12); TI3(9-12)

Unit 1 - Probability

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards:
S.ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
S.CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
S.CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
S.CP.A.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
S.CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
S.CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
S.CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
S.CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
S.CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
S.MD.B6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
S.MD.B7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.

- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.
CCE3(9-12): I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.
CCE4(9-12): I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
AA4(9-12): I can create opportunities to extend my learning by remaining open-minded in any situation.

Understandings: Students will understand that…

- Probability is a mathematical framework for quantifying uncertainty and making informed predictions about future events.
- Theoretical probability provides a model for predicting the likelihood of events, while experimental probability is derived from actual data and observations.
- The fundamental rules of probability, including the addition and multiplication rules, govern the likelihood of single and compound events.
- Understanding the concepts of independence and dependence is critical for correctly applying probability rules to different scenarios.
- A solid understanding of probability enhances one's ability to critically evaluate information, assess risks, and make data-driven decisions.

Essential Questions:

- What is probability and how is it used?
- Why is it important to understand the difference between theoretical and experimental probability?
- How does probability theory relate to decision-making in uncertain situations?
- What is the difference between a permutation and a combination?
- How do odds differ from probability?
- How is determining a conditional probability different from determining a simple probability?
- What are the key differences between independent and dependent events in probability?

Students will know...	Students will be able to...
• A simple probability of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes. Theoretical probability and experimental results help us make predictions and decisions. • Theoretical probability is what should occur in an experiment, while experimental probability is what actually occurs when repeating a probability experiment. Experimental probability can vary while theoretical is constant. • The Law of Large Numbers tells us that the more times a controlled experiment is performed, the closer the result will approach the theoretical probability. • Permutations are arrangements that include all possible orders while combinations have no regard for order. • To calculate odds, we compare favorable outcomes to unfavorable outcomes, while calculating probability compares favorable outcomes to the entire sample space. • A conditional probability is the probability that an event will occur under a particular circumstance. • Sometimes it is easier to calculate the probability of the complement of an event and to use that result to calculate the probability of the event. • A union involves all events in either sample space while an intersection involves the overlap between two or more sample spaces. • An independent event does not rely on what happened previously, while a dependent event does.	• Calculate and compare theoretical and experimental probabilities. • Use permutations and combinations to compute probabilities of events and solve problems. • Distinguish between probability and odds and calculate each. • Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to calculate conditional probabilities. • Identify the complement of an event and use complements to calculate probabilities. • Understand the meaning of the words “and” and “or” and calculate probabilities involving intersections and unions. • Distinguish between and calculate probabilities involving independent and dependent events

Key Vocabulary: Probability, Theoretical Probability, Experimental Probability, Outcomes, Sample Space, Permutation, Combination, Odds, Two-Way Table, Conditional Probability, Complementary Events, And/Or, Replacement, Independent and Dependent Events

Assessment Evidence	
Performance Tasks: Skittles problem -Students check the validity of a Skittles commercial claim regarding the number of flavor combinations. **AA4(9-12)** Final project - Students work in small groups to create a children's book about probability. **CCE4(9-12)**	**Other Evidence:** Quiz 1 - introduction to probability Quiz 2 - write and solve permutation problems **CCE3(9-12)** Quiz 3 - permutations and combinations Quiz 4 - conditional probability and odds Collect data problem - students collect data and write and solve many types of probability problems **TCC1(9-12)** Quiz 5 - complements, and/or probability, independent and dependent events

Learning Plan
This unit begins with an informal introduction to the idea of probability. Next students learn how to solve permutation and combination problems and to calculate probabilities involving these quantities. As part of this study, students write and solve their own permutation problems in pairs. **CCE3(9-12)** Students also evaluate the validity of a Skittles commercial related to the number of flavor combinations. **AA4(9-12)** Next students learn the difference between probability and odds, terms which are often used interchangeably but which have different meanings. Students learn how to read data from and create a two-way table, as well as how to calculate conditional probabilities. Next we study complements and the ability to use a complement to calculate a probability that would be difficult to calculate directly. Students learn the difference between intersections and unions and calculate the probability of each. In the "collect data problem", students collect data and use it to create a two-way table. They create and solve problems involving odds, conditional probability, and unions and intersections. **TCC1(9-12)** The final topic of study is the difference between independent and dependent events, and calculating the probability of each. The culminating project involves students working in small groups to create a children's book about probability that includes all of the topics that have been studied. **CCE4(9-12)**
Teacher Resources: Teacher created worksheets and assessments; Desmos; Khan academy; IXL; Socrative; Blooket; Quizizz; Kahoot; videos; applets

Unit 2 - Univariate Data

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards:

S.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).
S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
S.ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

CCE4(9-12): I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.

Understandings: Students will understand that...	**Essential Questions:**
• Different types of graphs are appropriate for different types of data. • Different statistics (measures of central tendency and spread) are appropriate for different sets of data. • The Normal distribution can be used to model many real world data sets.	• Why are some types of graphs more appropriate to display a set of data? • How and why are graphs made to be misleading? • Why are some measures of central tendency and spread more appropriate to describe a set of data? • Why are Normal Distributions important in statistics?
Students will know…	**Students will be able to…**
• An appropriate representation of data effectively presents and communicates information. The method of presentation may be selected based on the intended audience. • Vertical scaling creates different visual effects which can cause a graph to be intentionally or unintentionally misleading. • Measures of central tendency and spread are ways to represent the general data in a succinct way. Measures of central tendency and spread are affected differently by extreme values. • The normal curve can describe many real world phenomena. • A Z-score is a standard score, independent of units of measure, which tells the position the raw data has relative to the mean, measured in standard deviations. • If data is normally distributed, given percentages of the data will fall within 1, 2, and 3 standard deviations of the mean.	• Interpret graphs and represent data with appropriate graphs (circle graph, bar graph, dot plot, stem and leaf plot, histogram, box and whisker plot) based on the type of data (qualitative vs quantitative and discrete vs continuous) • Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets • Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers) • Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use empirical rule and standard normal table to estimate areas under the normal curve.
Key Vocabulary: Quantitative Data, Qualitative Data,Circle Graph, Bar Graph, Dot Plot, Stem & Leaf Plot, Discrete, Continuous, Histogram, Left Skewed, Right Skewed, Symmetric, Uniform, Mean, Median, Mode, Range, Standard Deviation, Quartile, Five Number Summary, Interquartile Range (IQR), Box & Whisker Plot, Normal Distribution, Z-Score, Empirical (68-95-99.7) Rule, Standard Normal Table, Percentile	

Assessment Evidence	
Performance Tasks: Family Ages Project - students collect data related to ages of members of their family; data is analyzed and compared through graphs and statistics **CCE4(9-12)** Unit 2 test - summative assessment covering all topics from this unit	**Other Evidence:** Quiz 1 - qualitative vs quantitative; discrete vs continuous; graphs Dot plot and stem plot assessment - students collect data appropriate for each type of graph; they create and discuss the graphs Histogram assessment - students collect data to create a histogram and discuss the graph Quiz 2 - measures of center and range Quiz 3 - boxplots and standard deviation Quiz 4 - normal curve, empirical rule, z-scores Quiz 5 - normal curve - calculating percentiles using std normal table
Learning Plan	
This unit begins with an understanding of the difference between qualitative and quantitative data. Next we study the types of graphs used to depict each type of data. Students distinguish between discrete and continuous data, and learn about histograms. Students discuss the shape of graphs and learn about the qualities of misleading graphs. Next we shift to numerical ways to describe data, specifically one variable statistics. Students review mean, median, mode, and range. They learn about choosing the best measure of center to describe a data set. Next students learn about the five number summary and box and whisker plots. Students complete the Family Ages Project as a summative assessment for the first part of the unit. **CCE4(9-12)** The final one variable statistic that we study is the standard deviation. This leads into a complete study of the normal distribution. Students learn how to calculate probabilities and percentiles using the Empirical (68-95-99.7) rule as well as using z-scores and the standard normal table.	
Teacher Resources: Teacher created worksheets and assessments; Desmos; Khan academy; IXL; Socrative; Blooket; Quizizz; Kahoot; videos	

Unit 3 - Bivariate Data

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards:

S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
S.ID.B.6.B Informally assess the fit of a function by plotting and analyzing residuals.
S.ID.B.6.C Fit a linear function for a scatter plot that suggests a linear association.
S.ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S.ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
S.ID.C.9 Distinguish between correlation and causation.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

TCC3(9-12): I can integrate relevant information to produce multiple valid solutions.
CCE4(9-12): I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.

Understandings: Students will understand that…	**Essential Questions:**
• Linear regression is a powerful statistical tool used to model the	• What is linear regression, and how does it model the

relationship between two quantitative variables and make predictions based on that relationship. ● The strength and direction of the relationship between two variables can be quantified using correlation and the slope of the regression line. ● A linear model may not always be appropriate for every dataset, and the accuracy of predictions depends on the quality of the data and the appropriateness of the model. ● Linear regression models have limitations and should be used with caution when making predictions, particularly when extrapolating beyond the range of the data.	relationship between two variables? ● How can we determine if a linear model is appropriate for a given set of data? ● What do the slope and y-intercept of a linear regression line represent in a real-world context? ● What does it mean when a data point is considered an outlier in the context of linear regression, and how does it affect the model? ● How can the correlation coefficient help us understand the strength and direction of a linear relationship? ● How can linear regression be used to make predictions, and what factors should be considered when making these predictions? ● How do changes in the data, such as adding or removing points, affect the regression line and correlation?
Students will know… ● A scatterplot is used to graph bivariate data. ● Although scatter plots and regression lines may reveal a pattern, the relationship of the variables may indicate a correlation, but not causation. ● The slope and the y-intercept have real world meanings. ● If data contains one or more outliers, assumptions based on the trend line might not be valid.	**Students will be able to…** ● Identify the explanatory and response variable for a set of data. ● Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ● Compute (using technology) and interpret the correlation coefficient of a linear fit. ● Distinguish between correlation and causation. ● Obtain the equation of a trend line by hand and of the line of best fit using a graphing calculator. ● Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ● Evaluate the effect of an outlier or an influential observation on the slope and correlation coefficient.
Key Vocabulary: Scatterplot, Explanatory Variable, Response Variable, Positive Association, Negative Association, Correlation, Causation, Correlation Coefficient R, Line of Best Fit, Residual, Outlier, Influential Observation	
Assessment Evidence	
Performance Tasks: Unit 3 project - students measure height and forearm of students in	**Other Evidence:** Quiz 1 - sketching scatterplots, explanatory vs response variable,

class and use this data to determine the line of best fit **TCC3(9-12)**	correlation Quiz 2 - write equation of line of best fit and analyze **CCE4(9-12)**

Learning Plan

This unit builds off of the linear functions unit in Algebra I. Students start by reviewing scatterplots, learning the new terms explanatory and response variable. Students describe scatterplots and sketch scatterplots to illustrate pairs of variables. Students learn about correlation and the caution that correlation does not mean causation. Then students evaluate the best placement for a trend line and review writing equations of lines. **CCE4(9-12)**

Finally students use the TI graphing calculator to find the equation of the best fit line and use that line to answer questions about the variables. The culminating project requires students to measure their classmates height and forearm and create a scatterplot. They use a graphing calculator to find the equation of the line of best fit and use that equation to answer questions about the relationship between the variables. **TCC3(9-12)**

Teacher Resources: Teacher created worksheets and assessments; Desmos; Khan academy; IXL; Socrative; Blooket; Quizizz; Kahoot; videos; applets; TI calculator and emulator

Unit 4 - Collecting Data

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards:

S.IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
S.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
S.IC.B.6 Evaluate reports based on data.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.
CCE1(9-12): I can initiate discussions with my peers and teachers about a variety of topics, respecting differing viewpoints, actively listening to others, and responding thoughtfully with peer-reviewed evidence that is free of bias.
CCE4(9-12): I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.

Understandings: Students will understand that...	**Essential Questions:**
● Data is a fundamental component in making informed decisions in various fields, from scientific research to business and public	● Why is it important to follow appropriate methods of data collection?

policy. ● A well-designed experiment or study reduces bias and increases the reliability of results. ● Bias can systematically affect the results of a study, leading to incorrect conclusions, while variability is inherent in data and must be accounted for.	● How do we design an experiment to ensure that the results are valuable?
Students will know… ● Data must be collected in an appropriate way so that the results that are drawn are meaningful. ● Experiments must use large sample size, randomization, replication, and control groups to ensure valuable results. ● Well designed experiments can prove causation.	**Students will be able to…** ● Identify the population and the sample in a given situation. ● Determine the sampling technique used in a given situation. ● Explain how bias can affect the results of data collection. ● Analyze an experiment that incorporates blocking, randomization, blinding, control, and replication. ● Design an experiment that incorporates blocking, randomization, blinding, control, and replication.
Key Vocabulary: Population, Sample, Simple Random Sample, Stratified Random Sample, Cluster Sample, Convenience Sample, Systematic Sample, Census, Voluntary Response Sample, Bias, Observational Study, Experiment, Treatment, Control Group, Treatment Group, Blocking, Placebo (Effect), Experimental Units, Random Assignment, Replication, Blinding, Double Blinding	
Assessment Evidence	
Performance Tasks: Design own experiment - students describe the design of an experiment of their choosing, using appropriate vocabulary **CCE4(9-12)** Unit 4 test - summative assessment covering all topics from this unit	**Other Evidence:** Quiz 1 - population vs sample, types of sampling methods, bias Design experiment introduction - students design an experiment to determine if eating M&Ms reduces stress prior to learning about experimental design **TCC1(9-12) CCE1(9-12)**
Learning Plan	
This unit focuses on collecting data that is useful for statistical analysis. The two main types of data collection are samples and experiments. The unit begins with an exploration of the difference between a population and a sample. Students learn the different types of sampling methods and which ones are more likely to lead to biased results. Students begin their study of experiments by working in pairs to design an experiment to determine if eating M&Ms reduces stress. This is a formative assessment to see what students already know about experimental design. **TCC1(9-12) CCE1(9-12)**	

After students share their experimental designs, we discuss the vocabulary associated with experiments. Students analyze experiments, making sure that they understand the elements of a good design. Students design their own experiment as a summative assessment for the experiment portion of this unit. **CCE4(9-12)** The unit ends with a summative test which covers all topics.
Teacher Resources: Teacher created worksheets and assessments; Desmos; Khan academy; IXL; Socrative; Blooket; Quizizz; Kahoot; videos

Unit 5 - Central Limit Theorem

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards:

S.ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.
TI2(9-12): I can evaluate my objectives and a variety of credible resources to find the best solutions for any challenge.

Understandings: Students will understand that…	**Essential Questions:**
• The Central Limit Theorem (CLT) shows that, regardless of the population distribution, the distribution of the sample means becomes approximately normal as the sample size increases. • As sample size increases, the variability of the sample mean	• What is the Central Limit Theorem and why is it fundamental in statistics? • Why does the shape of the population distribution become less important as sample size increases?

decreases, making it a more accurate reflection of the population mean. • The CLT relies on random sampling, which helps ensure that the sample mean is an unbiased estimator of the population mean. • The normal distribution plays a central role in statistics because of the CLT, which explains why many statistical procedures rely on the assumption of normality.	
Students will know… • The sample distribution of sample means will guarantee that data is approximated by a normal distribution, even if the original population is not normally distributed, if the sample size is sufficiently large. • The Central Limit Theorem forms the foundation for the inferential branch of statistics; this theorem describes the relationship between the sampling distribution of sample means and the population that the samples are taken from.	**Students will be able to…** • Determine if the sample size is large enough to ensure that the distribution of the sample mean is normal. • Use the Central Limit Theorem to solve problems involving the distribution of the sample mean.
Key Vocabulary: Sampling Distribution, Central Limit Theorem	
Assessment Evidence	
Performance Tasks: Unit 5 project - in pairs, students go online to find distributions (mean and standard deviation) and then write and solve problems involving the distribution of the sample mean (CLT) **TCC1(9-12) TI2(9-12)**	**Other Evidence:** Quiz - conceptual understanding of CLT, along with solving problems
Learning Plan	
This unit begins with a review of the normal curve. Students then investigate the idea of the central limit theorem informally, through desmos (release the hounds) and in class data collection (phone numbers). Students practice problems involving the distribution of the sample mean. Students are formally assessed with a quiz (individual) and a project which they complete in pairs. **TCC1(9-12) TI2(9-12)**	
Teacher Resources: Teacher created worksheets and assessments; Desmos; Khan academy; IXL; Socrative; Blooket; Quizizz; Kahoot; videos	

Unit 6 - Inference Involving Means

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards:

S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
S.IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0. 5. Would a result of 5 tails in a row cause you to question the model?
S.IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
S.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
S.IC.B.6 Evaluate reports based on data.
S.MD.B7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
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- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.
TCC2(9-12): I can evaluate evidence from multiple perspectives, and recognize their limitations and implications, in order to justify new conclusions.

CCE1(9-12): I can initiate discussions with my peers and teachers about a variety of topics, respecting differing viewpoints, actively listening to others, and responding thoughtfully with peer-reviewed evidence that is free of bias.
CCE4(9-12): I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
TI2(9-12): I can evaluate my objectives and a variety of credible resources to find the best solutions for any challenge.
TI3(9-12): I can formulate and investigate probing questions to further my learning.

Understandings: Students will understand that…	**Essential Questions:**
• Confidence intervals provide a range of plausible values for a population mean based on sample data, offering a more informative estimate than a single point estimate. • A confidence interval provides a range within which the true population mean is likely to fall, but it does not guarantee that the population mean is within that range for any specific sample. • The width of a confidence interval is determined by the confidence level and the sample size. • Significance tests are tools for making informed decisions about population means based on sample data. • The p-value measures the strength of evidence against the null hypothesis, but it does not prove whether the null hypothesis is true or false. • Statistical significance does not always imply practical importance, and real-world context must be considered when interpreting the results of a significance test.	• What does a confidence interval for means represent, and how does it help us estimate the population mean? • What factors influence the accuracy and reliability of a confidence interval? • How do we interpret and communicate the meaning of a confidence interval for means in context? • What does a significance test for means tell us, and how does it help us make decisions about a population mean? • How do we interpret the results of a significance test for means in context, and what are the potential limitations?
Students will know…	**Students will be able to…**
• A confidence interval for means is a range of values used to estimate the true value of a population mean. • A hypothesis (significance) test for means is a process by which a decision is made between two opposing hypotheses. • Sample statistics are used to test a claim about the value of the population parameter.	• Distinguish between a parameter and a statistic. • Create a confidence interval based on a sample mean to estimate a population mean. • Interpret a confidence interval in context. • Determine the sample size needed for a desired margin of error. • Write hypotheses for a significance test for means. • Calculate the test statistic and the p-value, and use this to determine if the result is significant or not. • Make a conclusion based on the significant or not significant

	result.

Key Vocabulary: Parameter, Statistic, Confidence Interval, Margin of Error, z-star value, Hypothesis Test, Null Hypothesis, Alternative Hypothesis, One-Sided Hypothesis, Two-Sided Hypothesis, Test Statistic, P-value, Alpha-Level (level of significance)

Assessment Evidence

Performance Tasks:	**Other Evidence:**
Unit 6 project - working in pairs, students look online for data involving means; they analyze how the data was collected and create a confidence interval based on the data; they create hypotheses for which they would and would not expect a significant result and test these hypotheses **TCC1(9-12) TCC2(9-12) CCE1(9-12) TI2(9-12) TI3(9-12)**	Chips Ahoy! Problem - students work in pairs to complete a project evaluating the accuracy of a Chips Ahoy! commercial which claims 1000 chips in a bag; students use a confidence interval to test this claim, thus introducing students to the idea of a confidence interval **TCC1(9-12) TCC2(9-12) CCE1(9-12) CCE4(9-12)** Quiz 1 - confidence intervals for means Group quiz - students collect quantitative data from their classmates and create a confidence interval and perform a significance test based on this data **TCC1(9-12) TI3(9-12)** Quiz 2 - hypothesis testing for means

Learning Plan

This unit starts with the distinction between parameters and statistics. Then students complete a project evaluating the accuracy of a Chips Ahoy! commercial which claims 1000 chips in a bag. Students use a confidence interval to test this claim, thus introducing students to the idea of a confidence interval. **TCC1(9-12) TCC2(9-12) CCE1(9-12) CCE4(9-12)**

Students construct and interpret confidence intervals for means, and they determine the sample size needed for a desired margin of error. Next students learn about hypothesis testing (significance testing), the other main idea in inferential statistics. The culminating project requires students to find quantitative data online, analyze how the data was collected, create a confidence interval for the data, and create hypotheses for which they would and would not expect a significant result and test those hypotheses **TCC1(9-12) TCC2(9-12) CCE1(9-12) TI3(9-12)**

Teacher Resources: Teacher created worksheets and assessments; Desmos; Khan academy; IXL; Socrative; Blooket; Quizizz; Kahoot; videos

Unit 7 - Inference Involving Proportions

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards:

S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
S.IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0. 5. Would a result of 5 tails in a row cause you to question the model?
S.IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
S.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
S.IC.B.6 Evaluate reports based on data.
S.MD.B7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.
TCC3(9-12): I can integrate relevant information to produce multiple valid solutions.
CCE3(9-12): I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.

TI3(9-12): I can formulate and investigate probing questions to further my learning.

Understandings: Students will understand that…

- Confidence intervals provide a range of plausible values for a population proportion based on sample data, offering a more informative estimate than a single point estimate.
- A confidence interval provides a range within which the true population proportion is likely to fall, but it does not guarantee that the population proportion is within that range for any specific sample.
- The width of a confidence interval is determined by the confidence level and the sample size.
- Significance tests are tools for making informed decisions about population proportions based on sample data.
- The p-value measures the strength of evidence against the null hypothesis, but it does not prove whether the null hypothesis is true or false.
- Statistical significance does not always imply practical importance, and real-world context must be considered when interpreting the results of a significance test.

Essential Questions:

- How does inference for data involving proportions differ from inference for data involving means?
- What does a confidence interval for proportions represent, and how does it help us estimate the population proportion?
- What factors influence the accuracy and reliability of a confidence interval?
- How do we interpret and communicate the meaning of a confidence interval for proportions in context?
- What does a significance test for proportions tell us, and how does it help us make decisions about a population proportion?
- How do we interpret the results of a significance test for proportions in context, and what are the potential limitations?

Students will know…

- A confidence interval for proportions is a range of values used to estimate the true value of a population proportion.
- A hypothesis (significance) test for proportions is a process by which a decision is made between two opposing hypotheses.
- Sample statistics are used to test a claim about the value of the population parameter.
- We work with means for quantitative data; we work with proportions for qualitative data.

Students will be able to…

- Create a confidence interval based on a sample proportion to estimate a population proportion.
- Interpret a confidence interval in context.
- Write hypotheses for a significance test for proportions.
- Calculate the test statistic and the p-value, and use this to determine if the result is significant or not.
- Make a conclusion based on the significant or not significant result.
- Determine if a problem involves means or proportions.

Key Vocabulary: Parameter, Statistic, Confidence Interval, Margin of Error, z-star value, Hypothesis Test, Null Hypothesis, Alternative Hypothesis, One-Sided Hypothesis, Two-Sided Hypothesis, Test Statistic, P-value, Alpha-Level (level of significance)

Assessment Evidence	
Performance Tasks: Unit 6, 7 group problems - students work in pairs to solve mixed problems from units 6 and 7; they must first identify if the problem involves means or proportions and if it involves a confidence interval or a significance test; then they solve the problem and interpret the result in context **TCC3(9-12) CCE3(9-12)**	**Other Evidence:** Quiz - students collect qualitative data from their classmates; they create a confidence interval and perform a significance test based on this data **TCC1(9-12) TI3(9-12)**
Learning Plan	
This unit continues the work from the previous unit, but switches the focus to qualitative data and proportions. Students begin by creating confidence intervals for proportions, then perform significance tests for proportions. For an interim assessment, students collect qualitative data from their classmates and create a confidence interval and perform a significance test based on the data **TCC1(9-12) TI3(9-12)** For a summative assessment, students work in pairs to solve mixed problems from units 6 and 7. They must first identify if the problem involves means or proportions and if it involves a confidence interval or a significance test; then they solve the problem and interpret the result in context **TCC3(9-12) CCE3(9-12)**	
Teacher Resources: Teacher created worksheets and assessments; Desmos; Khan academy; IXL; Socrative; Blooket; Quizizz; Kahoot; videos	