Modern Mathematics B: Management Science

Grade(s):	**11 & 12**
Discipline/Course:	**Mathematics**
Course Title:	**Modern Mathematics - B**
Prerequisite(s):	Algebra 2
Course Description: *Program of Studies*	Modern Mathematics is a fourth-year launch course that differs from the courses that precede it in that the mathematics is focused on discrete topics instead of continuous functions. This post-Algebra II course is heavily based in modeling with mathematics and includes topics like elections and weighted voting, graph theory, game theory, and apportionment. Students engage in problem-based learning where problems are ill defined and may have varying outcomes. In this course, reasoning and modeling are primary drivers of instruction.
Course Essential Questions:	**Where is mathematics used?** **What is the mathematics connection to history, art, computer science, etc?**
Course Enduring Understandings:	● A quantity can be represented numerically in various ways. ● Problem solving depends upon choosing wise ways. ● There are multiple algorithms for finding a solution.
Duration:	Semester
Course Materials/Resources:	Excursions in modern mathematics (9th), Peter Tannenbaum, Pearson

***Note: Topics listed in the units may evolve over time based on adaptations to implementation. However, the overall content of the entire course will not change**

Grade(s):	☐ K ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5	☐ 6 ☐ 7 ☐ 8	☑ 9 ☑ 10 ☑ 11 ☑ 12 ☐ Other _____
Discipline/Course:	**Discipline:** Mathematics **Course:** Modern Mathematics		
Course Title:	Modern Mathematics B: Management Science		
Prerequisite(s):	Algebra 2 and Teacher Recommendation Advised		
Course Description: *Program of Studies*	Modern Mathematics is a discrete mathematics survey course that explores mostly non-algebraic contemporary mathematics topics. The topics provide glimpses into other ways of thinking mathematically, and have interesting applications to everyday life. Together, they highlight algorithmic, graphical, algebraic, statistical, and analytic approaches to solving problems. Modern Mathematics is a course for students who want to learn about mathematical topics that have been discovered in the past 100 years and can be applied in modern times. In this course, students develop quantitative reasoning, mathematical modeling skills, and communication of ideas. In Modern Mathematics B: Management Science, students learn the mathematical approaches to efficiency and how to manage valuable resources, like time, money, or energy. Students learn the mathematics of getting around, travel and touring, networks, and scheduling. Students learn to design efficient pathways, manage valuable resources, and create networks that connect people and things. They learn effective ways to manage a project through scheduling. Students engage in problem-based situations and may have varying outcomes. The challenge is to get the optimal solution when dealing with in actual situations.		

Course Essential Questions:	• What happens when we put mathematical theory into practice? • How can mathematical approaches help us manage effectively? • How can models help us describe and solve complex real-life situations?	
Course Enduring Understandings:	• Using mathematical algorithms, theorems, and graphical models, we can create more efficient solutions. • Using models, we turn a real-life problem into a mathematical one. • Reaching an optimal solution depends on evaluating options and choosing strategically. • Modeling allows us to take a large-scale issue and make it a simpler problem to solve. The methods of solving the model can be applied to solve the large-scale problem using technology.	
Duration: **Credit:**	☑ Semester ☐ Full-Year	☑ 0.5 Credit (s) ☐ 1.0 Credit(s) ☐ 1.5 Credit(s) ☐ N/A
Course Materials/Resources:	Excursions in Modern Mathematics (10th), Peter Tannenbaum, Pearson	
FPS Course Academic Expectation(s):	☑ Exploring and Understanding (EU) ☐ Synthesizing and Evaluating (SE) ☐ Creating and Constructing (CC) ☑ Conveying Ideas (CI) ☐ Collaborating Strategically (CS) ☐ Using Communication Tools (UCT)	
Year at a Glance (Units):	**Unit 1:** The Mathematics of Getting Around (~8 classes) **Unit 2:** The Mathematics of Touring (~8 classes) **Unit 3:** The Mathematics of Networking (~8 classes) **Unit 4:** The Mathematics of Scheduling (~12 classes)	

Academic Expectations The Fairfield Public Schools describe a variety of cross curricular expectations that all students should exemplify during their time within the schooling experience. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:		
Standards	Explanations	Example
1. Exploring and Understanding	When students engage in problem solving situations, they should be able to understand the problem, determine relevant information, and ask relevant additional questions.	Students should be able to answer the following questions when approaching a problem: 1. Do you understand all the words used in stating the problem? 2. What are you asked to find or show? 3. Can you restate the problem in your own words? 4. Can you think of a picture or diagram that might help you understand the problem?
2. Synthesizing and Evaluating	Engaging in a problem solving situation, students should be able to analyze the most efficient approach, and reflect on the process used to solve the problem.	Students should be able to answer the following questions when analyzing how to approach a problem, and also reflect on the result: 1. Is there enough information to enable you to find a solution? If not, what additional information is needed? 2. Are there multiple ways to complete the task? Which approach do you think is most efficient, and why? 3. Do you know a related problem? Look at the unknown and try to think of a familiar problem having the same or similar unknown. Can you use it? 4. Was your strategy effective? What worked? What didn't? 5. Was there another approach that could have been more efficient? 6. Is your answer reasonable? How do you know? 7. Was your presentation approach effective? If not, what would you change?

		8. How did the communication tools allow you to get the message across to the intended audience?
3. Creating and Constructing	Engaged in a problem solving situation, students should implement a plan.	Students should be able to answer the following question to implementing their plan to solve a problem: 1. What strategy will you use to complete the task?
4. Conveying Ideas	Students should be able to use correct mathematical language, logically display their work for the desired problem.	Students should be able to answer the following questions to convey their mathematical thinking to solve a problem: 1. How will you present your information to your intended audience? 2. Does your response illustrate the correct terms and work to the problem?
5. Using Communication Tools	Students should be able to choose the correct tools to illustrate their mathematical work to solve a specific problem.	Students should be able to answer the following question to use specific communication tools to solve a problem: 1. If applicable, what communication tools will you use to convey your ideas and solution?
6. Collaborating Strategically	Students should be able to work collaboratively to solve problems.	Students should be able to answer the following question to collaboratively solve problems: 1. In what ways did you work together to help solve the desired problem?

Unit Number and Title:	~~**Unit 1: The Mathematics of Paths and Touring (Euler Paths and Circuits, Traveling Salesman Problems, & The Cost of Being Connected)**~~ **Unit 1:** The Mathematics of Getting Around
Duration:	~ 8 classes
Resource(s):	~~Textbook:~~ Excursions in Modern Mathematics (10th). Chapter 5
Unit overview:	In Unit 1, students are introduced to Graph Theory. Using street routing problems, students explore ways to create efficient travel routes. They look at different settings and applications, in a scaled-down size, knowing that the learning can be applied on a larger scale. They look at graphs, a key mathematical tool that allows them to tackle street routing problems. They apply mathematical theorems and algorithms to these problems to develop strategies to solve street-routing problems in general.
Learning Goals	
Standard(s):	~~CC 2.3.HS.A.14: Apply geometric concepts to model and solve real world problems~~ ~~CC.2.3.HS.A.13: Analyze relationships between two-dimensional and three dimensional objects~~ ~~CC 2.3.HS.A.2: Apply rigid transformations to determine and explain congruence~~ MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MR.8 Look for and express regularity in repeated reasoning. *Note: The concepts in Unit 1 extend beyond the CCSS for high school mathematics*
Essential Question(s):	~~• What is the difference between a path and a circuit?~~ ~~• What distinguishes an Euler path or circuit from a Hamilton path or circuit?~~

	~~● What types of real-world problems can be solved by Euler algorithms?~~ ~~● What types of real-world problems can be solved by Hamilton algorithms?~~ ● What happens when we put mathematical theory into practice? ● How can mathematical approaches help us manage effectively? ● How can models help us describe and solve complex real-life situations?
Enduring Understanding(s):	~~● Maneuvering around the real world requires math.~~ ~~● The difference between trips and tours in real-world logistics.~~ ~~● Real-world logistics is really moving through a network.~~ ● Using mathematical algorithms, theorems, and graphical models, we can create more efficient solutions. ● Using models, we turn a real-life problem into a mathematical one. ● Modeling allows us to take a large-scale issue and make it a simpler problem to solve. The methods of solving the model can be applied to solve the large-scale problem using technology.
Learning Goal(s): *Students will be able to use their learning to:*	Content (Students will know…) ● street routing problems: finding ways to route the delivery of goods and/or services to an assortment of destinations. ○ graph concepts and types. ■ simple connected graph. ■ simple disconnected graph. ■ graphs with names and not letters. ■ airline route maps. ■ friendship graphs. ■ bridges. ○ Euler's paths and circuits. ○ Euler's circuit theorem. ○ Euler's path theorem.

- Fleury's algorithm.

Skills: (Students will be able to…)

- find the valence of each vertex in a graph.
- represent the vertices and edges of a graph as an adjacency matrix, and use the matrix to solve problems.
- investigate and describe valence and connectedness.
- determine whether a graph is planar or non-planar.
- determine that every planar graph has a chromatic number that is less than or equal to four, based on the four-color map theorem.
- apply the four-color map theorem to multiple maps.
- apply the elements of graph theory to solve street routing problems.
- solve the Bridges of Königsberg problem.
- explore the differences between a path and a circuit.
- determine if a graph has an Euler path or circuit and find one if either exists.
- apply the Euler circuit algorithm to solve optimization problems.
- determine the number of edges in a complete graph.

~~1. find the valence of each vertex in a graph~~
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~~6. apply the four-color map theorem to multiple maps~~
~~7. apply the elements of graph theory to solve street routing problems.~~
~~8. solve the Bridges of Königsberg problem.~~
~~9. define the following vocabulary words: tree, spanning tree, shortest network, network,~~

	~~minimum spanning tree~~ ~~10. use Kruskal's, Prim's, and Dijkstra's algorithms to find the shortest spanning tree~~ ~~11. determine the degree of each of the vertices in a directed graph (digraph) as well as the in-degree and out-degree~~ ~~12. model projects consisting of several subtasks~~ ~~13. identify the critical path to determine the earliest completion of time (minimum project time)~~ ~~14. solve scheduling problems with critical path algorithms~~ ~~15. explore the differences between a path and a circuit~~ ~~16. determine if a graph has an Euler path or circuit and find one if either exists~~ ~~17. apply the Euler circuit algorithm to solve optimization problems~~ ~~18. determine if a graph has a Hamilton path or circuit and find one if either exists~~ ~~19. count the number of Hamiltonian circuits for a complete graph with *n* vertices.~~ ~~20. determine the number of edges in a complete graph~~ ~~21. compare and contrast the differences between Euler and Hamilton~~ ~~22. define the different types of algorithms that exist: optimal, inefficient, efficient, and approximate~~ ~~23. calculate the optimal solutions to graphs and charts that have Hamilton circuits using one of the following algorithms: brute-force, nearest neighbor, repetitive nearest neighbor, and cheapest link~~

Unit Number and Title:	~~**Unit 1: The Mathematics of Paths and Touring (Euler Paths and Circuits, Traveling Salesman Problems, & The Cost of Being Connected)**~~ **Unit 2:** The Mathematics of Touring
Duration:	~8 classes
Resource(s):	~~Textbook:~~ Excursions in Modern Mathematics (10th), Chapter 6
Unit overview:	In Unit 2, students build on the previous unit through the lens of optimizing the route of the traveler. The traveler has to visit specific locations and may have a set of costs. The street-routing problems that were used in unit 1 now change to a traveling salesman problem. The solution to the traveling salesman problem is a "trip" that starts and ends at one site and visits all the other sites. The optimal solution is a trip of minimal cost. Using knowledge about paths and circuits, students apply various algorithms to find the best routes possible.
Learning Goals	
Standard(s):	~~CC 2.3.HS.A.14: Apply geometric concepts to model and solve real world problems~~ ~~CC.2.3.HS.A.13: Analyze relationships between two-dimensional and three dimensional objects~~ ~~CC 2.3.HS.A.2: Apply rigid transformations to determine and explain congruence~~ MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MR.8 Look for and express regularity in repeated reasoning. *Note: The concepts in Unit 2 extend beyond the CCSS for high school mathematics*
Essential Question(s):	~~● What is the difference between a path and a circuit?~~

	● ~~What distinguishes an Euler path or circuit from a Hamilton path or circuit?~~ ● What happens when we put mathematical theory into practice? ● How can mathematical approaches help us manage effectively? ● How can models help us describe and solve complex real-life situations?
Enduring Understanding(s):	● ~~Maneuvering around the real world requires math.~~ ● ~~The difference between trips and tours in real-world logistics.~~ ● ~~Real-world logistics is really moving through a network.~~ ● Using mathematical algorithms, theorems, and graphical models, we can create more efficient solutions. ● Using models, we turn a real-life problem into a mathematical one. ● Modeling allows us to take a large-scale issue and make it a simpler problem to solve. The methods of solving the model can be applied to solve the large-scale problem using technology.
Learning Goal(s): *Students will be able to use their learning to:*	Concepts (Students will know…) ● traveling salesman problems: a metaphor for problems that have a common mathematical structure dealing with paths, sites, and costs. ○ Hamilton circuit and paths. ■ Brute-Force algorithm. ○ Nearest Neighbor algorithm. ○ Repetitive Nearest Neighbor algorithm. ○ Cheapest Link algorithm. **Skills:** (Students will be able to…) ● determine if a graph has a Hamilton path or circuit and find one if either exists. ● count the number of Hamiltonian circuits for a complete graph with *n* vertices. ● compare and contrast the differences between Euler and Hamilton.

	• define the different types of algorithms that exist: optimal, inefficient, efficient, and approximate. • calculate the optimal solutions to graphs and charts that have Hamilton circuits using one of the following algorithms: brute-force, nearest neighbor, repetitive nearest neighbor, and cheapest link. ~~1. find the valence of each vertex in a graph~~ ~~2. represent the vertices and edges of a graph as an adjacency matrix, and use the matrix to solve problems.~~ ~~3. investigate and describe valence and connectedness~~ ~~4. determine whether a graph is planar or non-planar~~ ~~5. determine that every planar graph has a chromatic number that is less than or equal to four based on the four-color map theorem~~ ~~6. apply the four-color map theorem to multiple maps~~ ~~7. define the following vocabulary words: tree, spanning tree, shortest network, network, minimum spanning tree~~ ~~8. use Kruskal's, Prim's, and Dijkstra's algorithms to find the shortest spanning tree~~ ~~9. determine the degree of each of the vertices in a directed graph (digraph) as well as the in-degree and out-degree~~ ~~10. model projects consisting of several subtasks~~ ~~11. identify the critical path to determine the earliest completion of time (minimum project time)~~ ~~12. solve scheduling problems with critical path algorithms~~ ~~13. explore the differences between a path and a circuit~~ ~~14. determine if a graph has an Euler path or circuit and find one if either exists~~ ~~15. apply the Euler circuit algorithm to solve optimization problems~~ ~~16. determine if a graph has a Hamilton path or circuit and find one if either exists~~ ~~17. count the number of Hamiltonian circuits for a complete graph with n vertices.~~ ~~18. determine the number of edges in a complete graph~~ ~~19. compare and contrast the differences between Euler and Hamilton~~ ~~20. define the different types of algorithms that exist: optimal, inefficient, efficient, and~~

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Unit Number and Title:	~~**Unit 1: The Mathematics of Paths and Touring (Euler Paths and Circuits, Traveling Salesman Problems, & The Cost of Being Connected)**~~ **Unit 3:** The Mathematics of Networks
Duration:	~8 classes
Resource(s):	~~Textbook:~~ Excursions in Modern Mathematics (10th), Chapter 7
Unit overview:	In Unit 3, students are introduced to networks and related structures such as trees and spanning trees. The unit starts with real-life networks: social networks, six degrees of separation, power grids, and small networks that are easy to manage. They are introduced to the concept of trees and spanning trees of a network. They then look at optimization of networks and an algorithm for finding an optimal solution.
Learning Goals	
Standard(s):	~~CC.2.3.HS.A.14: Apply geometric concepts to model and solve real-world problems~~ ~~CC.2.3.HS.A.13: Analyze relationships between two-dimensional and three-dimensional objects~~ ~~CC.2.3.HS.A.2: Apply rigid transformations to determine and explain congruence~~ MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MR.8 Look for and express regularity in repeated reasoning. *Note: The concepts in Unit 3 extend beyond the CCSS for high school mathematics*
Essential Question(s):	~~● What is the difference between a path and a circuit?~~ ~~● What distinguishes an Euler path or circuit from a Hamilton path or circuit?~~

	~~● What types of real-world problems can be solved by Euler algorithms?~~ ~~● What types of real-world problems can be solved by Hamilton algorithms?~~ ● What happens when we put mathematical theory into practice? ● How can mathematical approaches help us manage effectively? ● How can models help us describe and solve complex real-life situations?
Enduring Understanding(s):	~~● Maneuvering around the real world requires math.~~ ~~● The difference between trips and tours in real-world logistics.~~ ~~● Real-world logistics is really moving through a network.~~ ● Using mathematical algorithms, theorems, and graphical models, we can create more efficient solutions. ● Using models, we turn a real-life problem into a mathematical one. ● Modeling allows us to take a large-scale issue and make it a simpler problem to solve. The methods of solving the model can be applied to solve the large-scale problem using technology.
Learning Goal(s): *Students will be able to use their learning to:*	Concepts (Students will know…) ● networks: a connected graph, simple networks. ○ social networks. ○ six degrees of separation. ○ power grids. ● trees: a network with no circuits. ○ spanning trees: a subtree of the network that spans all the vertices. ○ minimum spanning tree (MST): in a weighted network, a spanning tree with the least total weight. ○ maximum spanning tree: in a weighted network, the spanning tree with the highest total weight. ○ efficient and optimal algorithms for MSTs: Kruskal's algorithm.

	Skills: (Students will be able to…) ● define the following vocabulary words: tree, spanning tree, shortest network, network, minimum spanning tree. ● use Kruskal's, Prim's, and Dijkstra's algorithms to find the shortest spanning tree. ~~1. find the valence of each vertex in a graph~~ ~~2. represent the vertices and edges of a graph as an adjacency matrix, and use the matrix to solve problems.~~ ~~3. investigate and describe valence and connectedness~~ ~~4. determine whether a graph is planar or non-planar~~ ~~5. determine that every planar graph has a chromatic number that is less than or equal to four based on the four-color map theorem~~ ~~6. apply the four-color map theorem to multiple maps~~ ~~7. define the following vocabulary words: tree, spanning tree, shortest network, network, minimum spanning tree~~ ~~8. use Kruskal's, Prim's, and Dijkstra's algorithms to find the shortest spanning tree~~ ~~9. determine the degree of each of the vertices in a directed graph (digraph) as well as the in-degree and out-degree~~ ~~10. model projects consisting of several subtasks~~ ~~11. identify the critical path to determine the earliest completion of time (minimum project time)~~ ~~12. solve scheduling problems with critical path algorithms~~ ~~13. explore the differences between a path and a circuit~~ ~~14. determine if a graph has an Euler path or circuit and find one if either exists~~ ~~15. apply the Euler circuit algorithm to solve optimization problems~~ ~~16. determine if a graph has a Hamilton path or circuit and find one if either exists~~ ~~17. count the number of Hamiltonian circuits for a complete graph with *n* vertices.~~ ~~18. determine the number of edges in a complete graph~~ ~~19. compare and contrast the differences between Euler and Hamilton~~ ~~20. define the different types of algorithms that exist: optimal, inefficient, efficient, and approximate~~

	~~21. calculate the optimal solutions to graphs and charts that have Hamilton circuits using one of the following algorithms: brute-force, nearest neighbor, repetitive nearest neighbor, and cheapest link~~

Unit Number and Title:	~~**Unit 2:**~~ **Unit 4:** The Mathematics of Scheduling: Chasing the Critical Path
Duration:	~12 classes
Resource(s):	~~Textbook~~: Excursions in Modern Mathematics, 10th edition, Chapter 8
Unit overview:	In Unit 4, students explore the coordination and organization of a project, which is about scheduling. In any scheduling problem, there are processors, tasks, processing times, and formal restrictions. Scheduling then becomes a directed graph. Students learn the common algorithms for solving scheduling problems, keeping in mind the general rules of creating schedules- that there is a priority list.
Learning Goals	
Standard(s):	A.CED.1: Create equations and inequalities in one variable and use them to solve problems. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MR.8 Look for and express regularity in repeated reasoning. *Note: The concepts in Unit 4 extend beyond the CCSS for high school mathematics*
Essential Question(s):	~~• Will increasing the number of processors decrease completion time?~~ ~~• Will the decreasing time list algorithm guarantee an optimal solution?~~ ~~• What situations can be best modeled using order-requirement digraphs or a machine scheduling problem?~~

	~~● What is the maximum number of colors needed to color any map? How can the vertex coloring algorithm be applied to resolve conflict?~~ ● What happens when we put mathematical theory into practice? ● How can mathematical approaches help us manage effectively? ● How can models help us describe and solve complex real-life situations?
Enduring Understanding(s):	~~● Every scheduler's dream: the "best" path through a network~~ ● Using mathematical algorithms, theorems, and graphical models, we can create more efficient solutions. ● Reaching an optimal solution depends on evaluating options and choosing strategically. ● Modeling allows us to take a large-scale issue and make it a simpler problem to solve. The methods of solving the model can be applied to solve the large-scale problem using technology.
Learning Goal(s): *Students will be able to use their learning to:*	**Content:** (Students will know…) ● scheduling "characters." ○ processors or "workers." ○ the tasks. ○ processing times. ○ precedence relations (formal restrictions). ○ optimal schedules. ○ optimal finishing times. ● directed graphs or digraphs: a graph in which the edges have directions associated with them. ● priority-list scheduling: tasks prioritized in the order we prefer to execute them. ● decreasing time algorithm: prioritize bigger jobs first, then leave small jobs to last. ● critical paths and the critical path algorithm: the greater the amount of work that lies ahead of that task, the sooner that task should be started. **Skills:** (Students will be able to…)

- schedule tasks on two or three “machines.”
- create critical paths.
- schedule independent tasks.
- find efficient ways to pack “bins.”
- solve conflicts using colors.
- solve scheduling problems by:
    - determining precedence relations.
    - creating a priority list.
    - creating a project digraph.
    - using one or more processors or workers.
- optimize solutions via the:
    - decreasing time algorithm.
    - critical path algorithm.
- discuss the efficiency of these algorithms.

~~1. Schedule task on two or three “machines”~~
~~2. Create critical paths~~
~~3. schedule independent tasks~~
~~4. Find efficient ways to pack “bins”~~
~~5. Solve conflict using colors~~
~~6. Solve scheduling problems by:~~
    ~~a. Determining precedence relations.~~
    ~~b. Creating a priority list.~~
    ~~c. Creating a project digraph.~~
    ~~d. Using one or more processors or workers.~~
~~7. Optimize solutions via the:~~
    ~~a. Decreasing Time algorithm.~~
    ~~b. Critical Path algorithm~~
~~8. Discuss the efficiency of these algorithms.~~

Unit Number and Title:	**Unit 3: Fractals**
Resource(s):	Textbook: Chapter 12
	Learning Goals
Standard(s):	CCSS.MATH.CONTENT.HSG.MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). CCSS.MATH.CONTENT.HSG.MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). CCSS.MATH.CONTENT.HSG.SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides
Essential Question(s):	• What is a fractal? • Where can you find one in nature? • How can you create fractals?
Enduring Understanding(s):	• The simple, yet complex, similarities of nature: Fractals.
Learning Goal(s): *Students will be able to use their learning*	1. Understand the geometry, representation, and vast application of fractals embedded in broccoli, clouds, coastlines, arteries, antennas, movie backgrounds, and other applications. 2. Express the contributions to the field of famous mathematicians Mandelbrot, Cantor, Koch, and

~~*to:*~~	~~Sierpinski to name a few.~~ ~~3. Draw a fractal given an initiator & generator.~~ ~~4. Use technology to generate fractal iterations.~~ ~~5. Write explicit and recursive formulae for measurements of fractals.~~ ~~6. Determine the fractal dimension of objects as a measure of roughness~~

Unit Number and Title:	**Unit 4: Fibonacci Numbers and the Golden Ratio**
Resource(s):	Textbook: Chapter 13
Learning Goals	
Standard(s):	CCSS.MATH.CONTENT.HSF.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
Essential Question(s):	• What are Fibonacci Numbers? • How does is a Fibonacci sequence used in the "real world?"
Enduring Understanding(s):	• Recursive sequences, the golden ratio, and their ubiquitous nature
Learning Goal(s): *Students will be able to use their learning to:*	1. Use and solve problems with the Fibonacci sequence and other recursive sequences. 2. Use the Fibonacci sequence to develop the Golden Ratio. 3. Discover Fibonacci patterns in nature. 4. Explain Fibonacci Numbers and their origin. 5. Generate the next numbers in a Fibonacci sequence. 6. Recursive sequences, the golden ratio, and their ubiquitous nature