

~~Probability and Statistics ECE~~
Statistics with Probability

Grade(s):	11-12
Discipline/Course:	Mathematics
Course Title:	Probability & Statistics
Prerequisite(s):	Algebra-2
Course Description: <i>Program of Studies</i>	Probability and statistics is recommended for those who want an elective that will be beneficial to many academic, medical, social science, and business careers. Statistics topics studied include: describing data with graphs, distributions, histograms and other graphical techniques, and measures of center and spread. Probability topics include: probability rules, probability distributions—discrete and normal. Additional topics include: sampling design, experimental design, sampling distributions, linear regression, and an introduction to inference testing and confidence intervals. Probability & Statistics is an excellent option for students anticipating statistics requirements in college. 1. Unit one focuses on acquiring data, understanding measurement, and critiquing data. Students will learn good sampling techniques to conduct experiments and studies to give results about a given population that is as accurate as possible. In choosing a sampling method it is important to avoid bias and other sources of error by properly using random sampling and through careful design of a sample survey. A properly designed and executed experiment can give good evidence for causation. Control and randomization are the most important aspects of the statistical design of experiments, without them, data collected can be misleading and lead to invalid results. 2. In unit two, the student will be able to describe patterns of univariate and bivariate data as well as observations that do not fit those patterns. Students will learn to organize sets of data into graphs, calculate numerical summaries, and then critically analyze data by describing, interpreting, and comparing important features. Choosing the most appropriate data display and quantitative description of data is an important skill in correctly analyzing and comparing data. 3. In this third unit, the students will apply probability principles to calculate likelihoods, to build probability models, and to calculate expected values. Students will apply the multiplication

	principle, counting techniques, tree diagrams, simulations, and other probability rules. Differentiation will be made between discrete and continuous random variables and the rules and properties that apply to both. 4. In the fourth unit, students will be introduced to hypothesis testing and confidence intervals and they will use this concept to estimate population parameters and draw conclusions about situations.
Course Essential Questions:	How can collecting, validating, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?
Course Enduring Understandings:	- Observe and describe patterns and departures from patterns. - Plan and conduct a study using samples, experiments, and simulations. - Explore random phenomena using the normal distribution, probability, and simulation. - Use statistical inference to make conclusions with confidence. - Estimate population parameters and test hypotheses.
Duration:	One year
Course Materials/Resources:	<i>Statistics and Probability with Applications</i> (Starnes & Tabor)

***Note: Topics listed in the units may evolve over time based on adaptations to implementation. However, the overall content of the entire course will not change**

Grade(s):	☐ K ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5	☐ 6 ☐ 7 ☐ 8	☒ 9 ☒ 10 ☒ 11 ☒ 12 ☐ Other _____
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Discipline/Course:	Discipline: Mathematics Course: Statistics and Probability
Course Title:	Statistics with Probability
Prerequisite(s):	Algebra 2 and Teacher Recommendation Advised
Course Description: <i>Program of Studies</i>	Statistics with Probability introduces the fundamental nature of mathematics and its applications in modern life through an introduction to the concepts of statistics. This course covers fundamental concepts in descriptive statistics, probability, sampling distributions, confidence intervals, hypothesis testing, correlation, and regression. Statistics with Probability prepares students for a wide variety of future areas of study, including liberal arts, business, science, social sciences, and health sciences.
Course Essential Questions:	• How can collecting, validating, organizing, and displaying data help us analyze information and make reasonable predictions and informed decisions? • How is mathematics used to measure, model, and calculate data? • How can a solid understanding of statistics help people make informed decisions based on data in their daily lives? • How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving? • How can we communicate mathematical ideas clearly and effectively?
Course Enduring Understandings:	• Statistics provides tools for describing variability in data and for making informed decisions that take it into account. • Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. • Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. • Random processes can be described mathematically by using a probability model. • Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in

	a short amount of time. - When encountering statistics in daily life, it is important to consider the study design, how the data were gathered, and the analyses employed, as well as the data summaries and the conclusions drawn. - A statistic of an unbiased sample is an estimator of a population parameter.	
Duration: Credit:	☐ Semester ☒ Full-Year	☐ 0.5 Credit (s) ☒ 1.0 Credit(s) ☐ 1.5 Credit(s) ☐ N/A
Course Materials/Resources :	<i>Statistics and Probability with Applications</i> . Fifth Edition. (Starnes, Tabor, and Wilcox), BFW Publishers.	
FPS Course Academic Expectation(s):	☒ Exploring and Understanding (EU) ☐ Synthesizing and Evaluating (SE) ☐ Creating and Constructing (CC) ☐ Conveying Ideas (CI) ☐ Collaborating Strategically (CS) ☐ Using Communication Tools (UCT)	
Year at a Glance (Units):	Unit 1: Analyzing One-Variable Data (~10 classes) Unit 2: Modeling One-Variable Quantitative Data (~9 classes) Unit 3: Analyzing Two-Variable Data (~11 classes) Unit 4: Collecting Data (~10 classes) Unit 5: Probability (~10 classes) Unit 6: Random Variables (~8 classes) Unit 7: Sampling Distributions (~8 classes) Unit 8: Estimating a Population Parameter (~9 classes) Unit 9: Testing a Claim about a Population Parameter (~9 classes)	

Academic Expectations

The Fairfield Public Schools describe a variety of cross-curricular expectations that all students should exemplify during their time within the schooling experience. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:

Standards	Explanations	Example
1. Exploring and Understanding [MPI]	When students engage in problem-solving situations, they should be able to understand the problem, determine relevant information, and ask relevant additional questions.	Students should be able to answer the following questions when approaching a problem: 1. Do you understand all the words used in stating the problem? 2. What are you asked to find or show? 3. Can you restate the problem in your own words? 4. Can you think of a picture or diagram that might help you understand the problem?
2. Synthesizing and Evaluating	Engaging in a problem solving situation, students should be able to analyze the most efficient approach, and reflect on the process used to solve the problem.	Students should be able to answer the following questions when analyzing how to approach a problem, and also reflect on the result: 1. Is there enough information to enable you to find a solution? If not, what additional information is needed? 2. Are there multiple ways to complete the task? Which approach do you think is most efficient, and why? 3. Do you know a related problem? Look at the unknown and try to think of a familiar problem having the same or similar unknown. Can you use it? 4. Was your strategy effective? What worked? What didn't? 5. Was there another approach that could have been more efficient? 6. Is your answer reasonable? How do you know? 7. Was your presentation approach effective? If not, what would you change? 8. How did the communication tools allow you to get the message across to the intended audience?

3. Creating and Constructing	Engaged in a problem solving situation, students should implement a plan:	Students should be able to answer the following question to implementing their plan to solve a problem: 1. What strategy will you use to complete the task?
4. Conveying Ideas	Students should be able to use correct mathematical language, logically display their work for the desired problem:	Students should be able to answer the following questions to convey their mathematical thinking to solve a problem: 1. How will you present your information to your intended audience? 2. Does your response illustrate the correct terms and work to the problem?
5. Using Communication Tools	Students should be able to choose the correct tools to illustrate their mathematical work to solve a specific problem:	Students should be able to answer the following question to use specific communication tools to solve a problem: 1. If applicable, what communication tools will you use to convey your ideas and solution?
6. Collaborating Strategically	Students should be able to work collaboratively to solve problems:	Students should be able to answer the following question to collaboratively solve problems: 1. In what ways did you work together to help solve the desired problem?

Unit Number and Title:	Unit 1: Producing Data,
Duration:	9 weeks
Resource(s):	<i>Statistics and Probability with Applications</i> (Starnes & Tabor)
Learning Goals	
Standard(s):	N/A
Essential Question(s):	- How do you design and carry out an experiment to allow valid conclusion of causation? - How do you design and carry out a survey to minimize bias and random error? - How do you characterize the results of a study to reflect both the basic findings and appropriate qualifications?
Enduring Understanding(s):	- Observational studies and controlled experiments allow conclusions regarding the population based on data from a carefully selected group of subjects. - Well-designed observational studies can provide information about the characteristics of the population whereas randomized controlled experiments can provide information about treatment effects on the population.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	- Understand different methods of data collection; for example a census, sample survey, experiment and observational study. - Critique the reliability, validity, accuracy of measurements. - Design, conduct and interpret surveys and experiments. - Differentiate between the many sources of bias in sampling and surveys and determine how to avoid them. - Summarize survey results with a confidence statement. - Determine the sample size corresponding to a given margin of error for a 95% confidence

	level using the $1/\sqrt{n}$ approximation. ~~7. Distinguish between different sampling methods including simple random sampling, stratified random sampling, and other methods.~~ ~~8. Apply the terms of treatments, control groups, and subjects.~~ ~~9. Understand the principles of good experimental design: control of variables, random assignments, and replication.~~ ~~10. Apply techniques for addressing bias and confounding, including placebos and blinding in experiments.~~ ~~11. Differentiate between a basic completely randomized design and other designs.~~ ~~12. Understand the limitations on generalizing results and the types of conclusions that can be drawn from observational studies, experiments, and surveys.~~ ~~13. Critique ethical issues involved in the design of a study.~~
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Unit Number and Title:	~~Unit 2: Organizing Data~~ Unit 1: Analyzing One-Variable Data
Duration:	~~11 Weeks~~ ~10 classes
Resource(s):	<i>Statistics and Probability with Applications</i> Fifth Edition (Starnes, & Tabor, and Wilcox)
Unit Overview:	In this unit, students learn about one-variable data. They examine the ways of displaying categorical and quantitative data, measuring the center of a data set, measuring the variability in a data set, and summarizing quantitative data. Students will use methods of calculation by hand and through technology. They will be able to interpret the differences in shape, center, and spread in the context of data sets and look at the effects of outliers.
Learning Goals	
Standard(s):	~~N/A~~ Interpreting Categorical and Quantitative Data ● S-ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). ● S-ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets ● S-ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
Essential Question(s):	● ~~Which calculations provide the most appropriate characterization of distribution?~~ ● ~~How do you fully describe a data set in context, using appropriate terminology, calculations, and graphs?~~ ● ~~How can technology display and create models?~~ ● ~~How do you create a model for bivariate data and how do you describe, interpret, and~~

	~~analyze the model?~~ ~~How can you use the properties of a normal distribution to analyze a data set?~~ - How can collecting, validating, organizing, and displaying data help us analyze information and make reasonable predictions and informed decisions? - How is mathematics used to measure, model, and calculate data? - How can a solid understanding of statistics help people make informed decisions based on data in their daily lives? - How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving? - How can we communicate mathematical ideas clearly and effectively?
Enduring Understanding(s):	~~Complete analysis of data makes use of correct terminology, varied graphs, and appropriate numerical techniques to study patterns and departures from patterns.~~ - Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. - Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. - Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. - Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) - the definitions of the measures of center: mean, median, and mode of distribution of data and how to compute each of them in particular situations using formulas and/or technology. - the measures of variability: range, standard deviation, interquartile range, first quartile, third quartile, and how to compute them using formulas and/or technology. - the different methods of organizing and comparing data: frequency tables, relative frequency tables, bar graphs, stem and leaf displays, histograms, box plots, dot plots, pie

	charts, side-by-side bar graphs. - the shapes of data: symmetric, skewed left, skewed right. - the ways to look at the distribution of data: shape, center, spread, outlier, variability. Skills: (Students will be able to...) - identify the individuals and variables in a data set, then classify the variables as categorical or quantitative. - summarize the distribution of a variable with a frequency table or a relative frequency table. - make and interpret bar charts or pie charts of categorical data. - compare distributions of categorical data. - identify what makes some graphs of categorical data misleading. - make and interpret dotplots of quantitative data. - describe the shape of a distribution of quantitative data. - describe or compare distributions of quantitative data with dotplots. - make stemplots and histograms of quantitative data. - describe stemplots and histograms of quantitative data. - compare distributions of quantitative data with stemplots and histograms. - find and interpret the median of a distribution of quantitative data. - calculate the mean of a distribution of quantitative data. - compare the mean and median of a distribution, and choose the more appropriate measure of center in a given setting. - find the range of a distribution of quantitative data. - calculate and interpret the standard deviation of a distribution of quantitative data. - find and interpret the interquartile range of a distribution of quantitative data. - use the 1.5xIQR rule to identify outliers. - make and interpret boxplots of quantitative data. - compare distributions of quantitative data with boxplots. ~~1. Know the definitions of the mean, median, and mode of distribution of data and how to compute each of them in particular situations using formulas and/or technology.~~
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	<del>2. Organize and compare univariate data: ~~a. Organize data using a number of different methods, including frequency tables, histograms, standard line graphs and bar graphs, stem-and-leaf displays, and box plots.~~ ~~b. Describe the center, shape, spread and unusual features (outliers) of univariate data.~~ ~~c. Graphically compare distributions of univariate data.~~ </del> ~~3. Explore categorical data using bar charts, pie charts, and frequency tables.~~ ~~4. Determine the mean and the standard deviation of a normally distributed random variable.~~ ~~5. Understand the meaning of the standard deviation of a distribution of data and compute it using technology.~~ ~~6. Describe the form, strength, and direction of bivariate data as displayed in a scatterplot.~~ ~~7. Find the line of best fit to a given distribution of data by using least squares regression.~~ ~~8. Analyze the relationship characterized by a least squares regression, discussing possible relationships of variables and interpreting intercept, slope, r, and r^2.~~ ~~9. Understand properties of the normal distribution and how to use, 68-95-99.7 rule, tables, and technology to solve problems.~~
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Unit Number and Title:	Unit 2: ~~Organizing Data~~ Modeling One Variable Quantitative Data
Duration:	~~11 Weeks~~ ~9 classes
Resource(s):	<i>Statistics and Probability with Applications</i> , Fifth Edition (Starnes, ~~&~~ Tabor, and Wilcox), Chapter 2
Unit Overview:	In this unit, students model one-variable quantitative data sets. They learn to describe the location in a distribution, use density curves to model a distribution of data, and draw normal curves to model a distribution of quantitative data. They identify the mean and median in a distribution of data. Students use the empirical rule to estimate proportion values in an interval in a normal distribution and determine if a data set is approximately normal. They use the normal distribution to calculate means and standard deviations, and to calculate area under the curve and boundary values. Students will standardize values using the mean and standard deviation, creating the normal distribution when the mean is 0 and the standard deviation is 1. In all cases, students model with authentic situations and data.
Learning Goals	
Standard(s):	~~N/A~~ Summarize, represent, and interpret data on a single count or measurement variable. ● S.ID-2: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ● S-ID.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ● S-ID.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a

	procedure is not appropriate.
Essential Question(s):	~~Which calculations provide the most appropriate characterization of distribution?~~ ~~How do you fully describe a data set in context, using appropriate terminology, calculations, and graphs?~~ ~~How can technology display and create models?~~ ~~How do you create a model for bivariate data and how do you describe, interpret and analyze the model?~~ ~~How can you use the properties of a normal distribution to analyze a data set?~~ - How can collecting, validating, organizing, and displaying data help us analyze information and make reasonable predictions and informed decisions? - How is mathematics used to measure, model, and calculate data? - How can a solid understanding of statistics help people make informed decisions based on data in their daily lives? - How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving? - How can we communicate mathematical ideas clearly and effectively?
Enduring Understanding(s):	~~Complete analysis of data makes use of correct terminology, varied graphs, and appropriate numerical techniques to study patterns and departures from patterns.~~ - Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. - Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. - Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.
Learning Goal(s): <i>Students will be able to use their learning to: σ</i>	~~Know the definitions of the mean, median, and mode of distribution of data and how to compute each of them in particular situations using formulas and/or technology.~~ ~~Organize and compare univariate data.~~

- a. ~~Organize data using a number of different methods, including frequency tables, histograms, standard line graphs and bar graphs, stem-and-leaf displays, and box plots.~~
- b. ~~Describe the center, shape, spread and unusual features (outliers) of univariate data.~~
- e. ~~Graphically compare distributions of univariate data.~~
- 3. ~~Explore categorical data using bar charts, pie charts, and frequency tables.~~
- 4. ~~Determine the mean and the standard deviation of a normally distributed random variable.~~
- 5. ~~Understand the meaning of the standard deviation of a distribution of data and compute it using technology.~~
- 6. ~~Describe the form, strength, and direction of bivariate data as displayed in a scatterplot.~~
- 7. ~~Find the line of best fit to a given distribution of data by using least squares regression.~~
- 8. ~~Analyze the relationship characterized by a least squares regression, discussing possible relationships of variables and interpreting intercept, slope, r , and r^2 .~~
- 9. ~~Understand properties of the normal distribution and how to use, 68-95-99.7 rule, tables, and technology to solve problems.~~

Content: (Students will know...)

- the two ways of describing an individual data value's location in a distribution of quantitative data: percentiles and standardized scores.
- the density curve describes the overall pattern of some distributions of quantitative data.
- the mean of a density curve is the point at which the density curve would balance if it were made of solid material.
- the median density curve is the equal-area point. The point that divides the area under the curve in half.
- mean of the density curve is denoted by μ and the standard deviation by σ .
- a normal curve is a symmetric, single-peaked, bell-shaped density curve.
- any normal distribution is specified by its mean μ and its standard deviation σ .
- the empirical rule describes what percentage of values in any normal distribution fall within 1, 2, and 3 standard deviations of the mean: about 68%, 95%, and 99.7%.
- a distribution is normal when the shape is observed to be a normal curve, and the data is fit to the empirical rule.

- all normal distributions are the same when the values are standardized.
- standard normal distribution is when the mean is 0 and the standard deviation is 1.
- the mean and the standard deviation of a normal distribution are found using the boundary value and the corresponding area.
- the area of a normal distribution is found using a table of standardized values or using technology without standardizing.

Skills: (Students will be able to...)

- find and interpret a percentile in a distribution of quantitative data.
- find and interpret a standardized score (z-score) in a distribution of quantitative data.
- use percentiles or standardized scores (z-scores) to compare the location of values in distributions of quantitative data.
- use a density curve to model a distribution of quantitative data.
- identify the relative locations of the mean and median of a distribution from a density curve.
- draw a normal curve to model a distribution of quantitative data.
- use the empirical rule to estimate the proportion of values in a specified interval in a normal distribution.
- determine whether a distribution of quantitative data is approximately normal using graphical and numerical evidence.
- find the proportion of values to the left of a boundary in a normal distribution.
- find the proportion of values to the right of a boundary in a normal distribution.
- find the proportion of values between two boundaries in a normal distribution.
- find the boundary value corresponding to a given area in a normal distribution.
- calculate the mean or standard deviation of a normal distribution given a boundary value and the corresponding area.

Unit Number and Title:	~~Unit 2: Organizing Data~~ Unit 3: Analyzing Two-Variable Data
Duration:	~~11 Weeks~~ ~11 classes
Resource(s):	<i>Statistics and Probability with Applications</i> Fifth Edition (Starnes, & Tabor, & Wilcox), Chapter 3
Unit Overview:	In this unit, students analyze two-variable data sets. They learn to describe the relationship between two categorical variables and the relationship between two quantitative variables. They will make use of data displays such as bar charts and two-way tables for categorical variables, and scatterplots for quantitative variables. Students will distinguish correlation from causation. They will describe the effect of outliers on correlation between two quantitative variables. Students will make predictions using regression lines. They will make interpretations of residuals, slopes, and y-intercepts. They will calculate the least squares regression line using technology, describe how outliers affect this regression line, and explain the concept of regression to the mean. Students will assess a regression model, interpreting the standard deviation of the residuals, and the coefficient of determination. In all cases, students model with authentic situations and data.
Learning Goals	
Standard(s):	~~N/A~~ Summarize, represent, and interpret data on two categorical and quantitative variables. ● S-ID.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ● S-ID.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

	a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. Interpret linear models • S-ID.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. • S-ID.8: Compute (using technology) and interpret the correlation coefficient of a linear fit. • S-ID.9: Distinguish between correlation and causation. <i>Note: Unit 3 extends beyond high school math standards into inference for regression.</i>
Essential Question(s):	• ~~Which calculations provide the most appropriate characterization of distribution?~~ • ~~How do you fully describe a data set in context, using appropriate terminology, calculations, and graphs?~~ • ~~How can technology display and create models?~~ • ~~How do you create a model for bivariate data and how do you describe, interpret and analyze the model?~~ • ~~How can you use the properties of a normal distribution to analyze a data set?~~ • How can collecting, validating, organizing, and displaying data help us analyze information and make reasonable predictions and informed decisions? • How is mathematics used to measure, model, and calculate data? • How can a solid understanding of statistics help people make informed decisions based on data in their daily lives? • How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving? • How can we communicate mathematical ideas clearly and effectively?

Enduring Understanding(s):	- Complete analysis of data makes use of correct terminology, varied graphs, and appropriate numerical techniques to study patterns and departures from patterns. - Association does not imply causation. - Statistics provides tools for describing variability in data and for making informed decisions that take it into account. - Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. - Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. - Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	- Know the definitions of the mean, median, and mode of distribution of data and how to compute each of them in particular situations using formulas and/or technology. - Organize and compare univariate data: - Organize data using a number of different methods, including frequency tables, histograms, standard line graphs and bar graphs, stem-and-leaf displays, and box plots. - Describe the center, shape, spread and unusual features (outliers) of univariate data. - Graphically compare distributions of univariate data. - Explore categorical data using bar charts, pie charts, and frequency tables. - Determine the mean and the standard deviation of a normally distributed random variable. - Understand the meaning of the standard deviation of a distribution of data and compute it using technology. - Describe the form, strength, and direction of bivariate data as displayed in a scatterplot. - Find the line of best fit to a given distribution of data by using least squares regression. - Analyze the relationship characterized by a least squares regression, discussing possible relationships of variables and interpreting intercept, slope, r, and r^2. - Understand properties of the normal distribution and how to use, 68-95-99.7 rule, tables, and technology to solve problems.

Content: (Students will know...)

- a response variable measures the outcome of a study.
- an explanatory variable may help predict or explain changes in a response variable.
- a two-way table is a table of counts or relative frequencies that summarizes data on the relationship between two categorical variables for some group of individuals.
- a segmented bar graph displays the possible values of a categorical variable as segments of a bar, with the area of each segment being proportional to the number of individuals in the corresponding category.
- association does not imply causation.
- two variables have an association if knowing the value of one variable helps us to predict the value of the other.
- a scatterplot shows the relationship between quantitative variables measured on the same individuals.
- in a scatterplot, the values of the explanatory variable appear on the horizontal axis, and the values of the response variable on the vertical axis.
- scatterplot patterns can be described by the direction (e.g. positive, negative), form (e.g. linear, nonlinear), and strength (e.g. strong, weak), and through unusual features (outliers, clusters).
- correlation r gives the direction and measures the strength of a linear relationship between two quantitative variables.
- a regression line models how a response variable y changes as an explanatory variable x changes. It is expressed as $\hat{y} = a + bx$.
- extrapolation is the use of a regression line for prediction outside the interval of x -values used.
- a residual is the difference between an actual value of y and the predicted value of y .
- the slope of a regression line is the amount by which the predicted value of y changes when x increases by one unit.
- the y -intercept of a regression line is the predicted value of y when $x=0$.
- the least-squares regression line is the line that makes the sum of the squared residuals as small as possible.
- the coefficient of determination, r^2 measures the percentage of variability in the response

variable that is accounted for by the linear model.

Skills: (Students will be able to...)

- make a scatterplot to display the relationship between two quantitative variables.
- describe the direction, form, and strength of a relationship displayed in a scatterplot, and identify unusual features.
- estimate the correlation between two quantitative variables from a scatterplot.
- interpret the correlation between two quantitative variables.
- distinguish correlation from causation.
- calculate the correlation between two quantitative variables.
- apply the properties of the correlation.
- describe how outliers influence the correlation between two quantitative variables.
- make predictions using regression lines, keeping in mind the dangers of extrapolation.
- interpret the slope and y-intercept of a regression line.
- calculate the equation of the least-squares regression line using technology.
- describe how outliers affect the least-squares regression line.
- use a residual plot to determine whether a regression model is appropriate.
- interpret the coefficient of determination, " r^2 ".

Unit Number and Title:	~~Unit 1: Producing Data~~ Unit 4: Collecting Data
Duration:	~~9 weeks~~ ~10 classes
Resource(s):	<i>Statistics and Probability with Applications</i> Fifth Edition (Starnes, & Tabor, and Wilcox), Chapter 4
Unit Overview:	In this unit, students learn that depending on how data are collected, they may or may not be able to generalize findings or establish evidence of causal relationships. Students will learn the methods of collecting data and methods to reduce bias in data collection. For data collected using well-designed experiments, statistically significant differences between or among experimental treatment groups are evidence that the treatments caused the effect.
Learning Goals	
Standard(s):	~~N/A~~ Understand and evaluate random processes underlying statistical experiments. ● S-IC.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ● S-IC.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. ● S-ID.6b: Evaluate reports based on data.
Essential Question(s):	● ~~How do you design and carry out an experiment to allow valid conclusion of causation?~~

	• How do you design and carry out a survey to minimize bias and random error? • ~~How do you characterize the results of a study to reflect both the basic findings and appropriate qualifications?~~ • How can collecting, validating, organizing, and displaying data help us analyze information and make reasonable predictions and informed decisions? • How is mathematics used to measure, model, and calculate data? • How can a solid understanding of statistics help people make informed decisions based on data in their daily lives? • How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving? • How can we communicate mathematical ideas clearly and effectively?
Enduring Understanding(s):	• ~~Observational studies and controlled experiments allow conclusions regarding the population based on data from a carefully selected group of subjects.~~ • Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. ○ Well-designed observational studies can provide information about the characteristics of the population whereas randomized controlled experiments can provide information about treatment effects on the population • Statistics provides tools for describing variability in data and for making informed decisions that take it into account. • Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. • Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. • Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.
Learning Goal(s): <i>Students will be able to</i>	1. ~~Understand different methods of data collection; for example a census, sample survey, experiment and observational study.~~

use their learning to:

- ~~2. Critique the reliability, validity, accuracy of measurements.~~
- ~~3. Design, conduct and interpret surveys and experiments.~~
- ~~4. Differentiate between the many sources of bias in sampling and surveys and determine how to avoid them.~~
- ~~5. Summarize survey results with a confidence statement.~~
- ~~6. Determine the sample size corresponding to a given margin of error for a 95% confidence level using the $1/\sqrt{n}$ approximation.~~
- ~~7. Distinguish between different sampling methods including simple random sampling, stratified random sampling, and other methods.~~
- ~~8. Apply the terms of treatments, control groups, and subjects.~~
- ~~9. Understand the principles of good experimental design: control of variables, random assignments, and replication.~~
- ~~10. Apply techniques for addressing bias and confounding, including placebos and blinding in experiments.~~
- ~~11. Differentiate between a basic completely randomized design and other designs.~~
- ~~12. Understand the limitations on generalizing results and the types of conclusions that can be drawn from observational studies, experiments, and surveys.~~
- ~~13. Critique ethical issues involved in the design of a study.~~

Content: (Students will know...)

- the statistical problem-solving process involves four steps: ask questions, collect/consider data, analyze data, and interpret results.
- a valid statistical question is based on data that vary.
- there are many data collection methods, including sample surveys, observational studies, and experiments.
- the scope of inference determines how we interpret results.
 - random selection of individuals allows inference about the population from which the sample was collected.
 - random assignment of individuals to groups allows for inference about cause and effect.

- the population in a statistical study is the entire group of individuals about whom we want information.
- a sample is a subset of individuals from the population from which we collect data.
- a random sample consists of individuals in the population who are selected using a chance process.
- a statistical study shows bias if it is very likely to underestimate or very likely to overestimate the sample you want to know.
 - types of sample bias: convenience sample, voluntary response sample, undercoverage, nonresponse, response bias.
- the fact that different random samples of the same size from the same population produce different estimates is called sampling variability. It can be reduced by increasing the sample size.
- observational studies observe individuals and measure variables of interest but do not attempt to influence the responses.
- an experiment deliberately imposes treatments (conditions) on individuals to measure their responses. Results from a well-designed experiment can be used to make inferences about cause and effect.
- when an observed difference in responses between the groups in an experiment is so large that it is unlikely to be explained by chance variation in the random assignment, we say the result is statistically significant.

Skills: (Students will be able to...)

- understand different methods of data collection, for example, a census, sample survey, experiment, and observational study.
- distinguish statistical questions from other types of questions.
- identify the population and the sample in a statistical study.
- distinguish between an observational study and an experiment.
- distinguish between good and bad samples.
- critique the reliability, validity, and accuracy of measurements.
- describe how convenience sampling can lead to bias.

- describe how voluntary response sampling can lead to bias.
- explain how random sampling can help to avoid bias.
- determine other sources of bias in surveys.
- differentiate between the many sources of bias in sampling and surveys and determine how to avoid them.
- explain how undercoverage can lead to bias.
- explain how nonresponse can lead to bias.
- explain how response bias can affect the results of surveys.
- determine the sample size corresponding to a given margin of error for a 95% confidence level using the $\frac{1}{\sqrt{n}}$ approximation.
- distinguish between different sampling methods, including simple random sampling, stratified random sampling, and other methods.
- describe how to select a simple random sample.
- explain the concept of sampling variability and the effect of changing sample size.
- use simulation to test a claim about a population proportion.
- identify the treatments in an experiment.
- explain the purpose of a control group in an experiment.
- describe how to randomly assign treatments in an experiment.
- explain the purpose of random assignment in an experiment.
- identify other sources of variability in an experiment and explain the benefits of keeping these variables the same for all experimental units.
- outline an experiment that uses a completely randomized design.
- explain the concept of statistical significance in the context of an experiment.
- identify the population about which we can make inferences from a sample.
- determine when it is appropriate to make an inference about cause and effect.
- evaluate if a statistical study has been carried out in an ethical manner.

Unit Number and Title:	~~Unit 3: Probability~~ Unit 5: Probability
Duration:	~~9 weeks~~ ~10 classes
Resource(s):	<i>Statistics and Probability with Applications</i> Fifth Edition (Starnes, & Tabor, & Wilcox), Chapter 5
Unit Overview:	In this unit, students will explore the mathematics of chance behavior, known as probability. They will interpret probability as a long-run relative frequency, dispel myths about randomness, and use simulations to model chance behavior. They will calculate a probability based on observed proportions, simulation, or theoretical analysis, and solve probability problems using the rules for addition (“or”) and multiplication (“and”). Students will learn the role of the “Law of Large Numbers” in improving estimates of probabilities. Students will use computations of combinations and permutations to calculate probabilities.
Learning Goals	
Standard(s):	~~N/A~~ Summarize, represent, and interpret data on two categorical and quantitative variables. ● S-ID.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. Understand independence and conditional probability and use them to interpret data. ● S-CP.1: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). ● S-CP.2: Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

	● S-CP.3: Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. ● S-CP.4: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. ● S-CP.5: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. Use the rules of probability to compute probabilities of compound events. ● S-CP.6: Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ● S-CP.7: Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. ● S-CP.8 (+): Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$, and interpret the answer in terms of the model. ● S-CP.9 (+): Use permutations and combinations to compute probabilities of compound events and solve problems. Use probability to evaluate outcomes of decisions. ● S-MD.6 (+): Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ● S-MD.7 (+): Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
Essential Question(s):	● ~~How do you use probability rules to evaluate chance behavior in real-world contexts?~~

	● ~~How can you use expected value for decision making?~~ ● ~~How can simulation be used to calculate expected values and probabilities?~~ ● ~~How can technology be applied to create and interpret models?~~ ● How can collecting, validating, organizing, and displaying data help us analyze information and make reasonable predictions and informed decisions? ● How is mathematics used to measure, model, and calculate data? ● How can a solid understanding of statistics help people make informed decisions based on data in their daily lives? ● How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving? ● How can we communicate mathematical ideas clearly and effectively?
Enduring Understanding(s):	● ~~expected value is a key tool for rational decision making.~~ ● ~~Simulation allows us to overcome difficulties with other means of calculating probabilities and expected values.~~ ● ~~The normal distribution can be applied to many real-life situations to solve probability problems.~~ ● Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. ● Random processes can be described mathematically by using a probability model. ○ Probability is the tool used for anticipating future outcomes based on past events, theoretical speculation, or simulation. ○ In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. ○ Interpreting probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. ● Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Learning Goal(s):

Students will be able to use their learning to:

- ~~1. Identify a random process.~~
- ~~2. Calculate a probability based on observed proportions, simulation, or theoretical analysis.~~
- ~~3. Solve probability problems using the rules for addition (“or”) and multiplication (“and”) rules.~~
- ~~4. Understand properties of the normal distribution and how to use tables and technology to solve probability problems.~~
- ~~5. Know the role of the “Law of Large Numbers” in improving estimates of probabilities.~~
- ~~6. Know the definition of the mean (expected value) of a discrete random variable and determine the mean for a particular set of random events.~~
- ~~7. Understand and differentiate probability distributions.~~
- ~~8. Design and execute simulations to calculate probabilities and expected values.~~

Content: (Students will know...)

- probability is the long-run relative frequency of a chance outcome.
- probability can be calculated using observed proportions, simulation, or theoretical analysis
- the law of large numbers says that in many repetitions of the same chance process, the proportion of times that a particular outcome occurs will approach its probability.
- the role of the “Law of Large Numbers” in improving estimates of probabilities.
- a simulation can be used to imitate chance behavior and to estimate probabilities.
- tabular and graphical representations for two categorical variables.
 - a two-way table is a table of counts or relative frequencies that summarizes data on the relationship between two categorical variables for some group of individuals.
 - a segmented bar graph displays the possible values of a categorical variable as segments of a bar, with the area of each segment being proportional to the number of individuals in the corresponding category.
- summary statistics for two categorical variables.
- probability models and probability rules.
 - general addition rule $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
 - addition rule for mutually exclusive events $P(A \text{ or } B) = P(A) + P(B)$.

- conditional probability, multiplication rules, and independence.
 - conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$
 - General Multiplication Rule: $P(A \cap B) = P(A) \cdot P(B|A)$
 - independent events
- counting and probability.
 - the number of permutations of an entire group of individuals is n factorial.
 - the number of permutations of r individuals from a group of n individuals can be calculated using the multiplication counting principle ${}_nP_r = \frac{n!}{(n-r)!}$
 - if there are n individuals, you can calculate the number of different combinations of r individuals chosen from a group of n individuals ${}_nC_r = \frac{{}_nP_r}{(n-r)!}$

Skills: (Students will be able to...)

- interpret probability as a long-run relative frequency.
- dispel common myths about randomness.
- use simulation to model chance behavior.
- give a probability model for a chance process with equally likely outcomes and use it to find the probability of an event.
- confirm that a probability model is valid and use the complement rule to find probabilities.
- use the addition rule for mutually exclusive events to find probabilities.
- use a two-way table to find probabilities.
- calculate probabilities with the general addition rule.
- use a Venn diagram to find probabilities.
- find and interpret conditional probabilities using two-way tables.
- use the conditional probability formula to calculate probabilities.
- determine whether two events are independent.

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| | <ul style="list-style-type: none"> • use the general multiplication rule to calculate probabilities. • use a tree diagram to model a chance process involving multiple stages and to find probabilities. • calculate conditional probabilities by working backward on a tree diagram. • use the multiplication rule for independent events to calculate probabilities. • calculate $P(\text{at least one})$ using the complement rule and the multiplication rule for independent events. • determine whether it is appropriate to use the multiplication rule for independent events in a given setting. • use the multiplication counting principle to determine the number of ways to complete a process involving several steps. • use factorials to count the number of permutations of a group of individuals. • compute the number of permutations of n individuals taken r at a time. • compute the number of combinations of n individuals taken r at a time. • use combinations to calculate probabilities. • use the multiplication counting principle and combinations to calculate probabilities. |
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Unit Number and Title:	~~Unit 3: Probability~~ Unit 6: Random Variables
Duration:	~~9 weeks~~ 8 classes
Resource(s):	<i>Statistics and Probability with Applications</i> Fifth Edition (Starnes, & Tabor, & Wilcox), Chapter 6
Unit Overview:	Building on unit 5, students will use probability to represent, interpret, and calculate parameters for theoretical probability distributions for discrete and continuous random variables. They will identify a random process and use tables and technology to solve probability problems. They will learn the definition of the mean (expected value) of a discrete random variable and determine the mean for a particular set of random events. Students will learn about probability distributions for discrete random variables.
Learning Goals	
Standard(s):	Calculate expected values and use them to solve problems ● S-MD.1 (+): Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. ● S-MD.2 (+): Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ● S-MD.3 (+): Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. ● S-MD.4 (+): Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
Essential Question(s):	● ~~How do you use probability rules to evaluate chance behavior in real-world contexts?~~ ● ~~How can you use expected value for decision making?~~ ● ~~How can simulation be used to calculate expected values and probabilities?~~ ● ~~How can technology be applied to create and interpret models?~~ ● How can collecting, validating, organizing, and displaying data help us analyze information and

	make reasonable predictions and informed decisions? • How is mathematics used to measure, model, and calculate data? • How can a solid understanding of statistics help people make informed decisions based on data in their daily lives? • How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving? • How can we communicate mathematical ideas clearly and effectively?
Enduring Understanding(s):	• ~~Expected value is a key tool for rational decision-making.~~ • Random processes can be described mathematically by using a probability model. ○ Probability is the tool used for anticipating future outcomes based on past events, theoretical speculation, or simulation. • Statistics provides tools for describing variability in data and for making informed decisions that take it into account. • Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. • Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. • Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. ○ Simulation allows us to overcome difficulties with other means of calculating probabilities and expected values. • ~~The normal distribution can be applied to many real-life situations to solve probability problems.~~
Learning Goal(s): <i>Students will know and will be able to use their</i>	~~1. Identify a random process.~~ ~~2. Calculate a probability based on observed proportions, simulation, or theoretical analysis.~~ ~~3. Solve probability problems using the rules for addition (“or”) and multiplication (“and”) rules.~~

<i>learning to:</i> (Content/ Skills)	~~4. Understand properties of the normal distribution and how to use tables and technology to solve probability problems.~~ ~~5. Know the role of the “Law of Large Numbers” in improving estimates of probabilities.~~ ~~6. Know the definition of the mean (expected value) of a discrete random variable and determine the mean for a particular set of random events.~~ ~~7. Understand and differentiate probability distributions.~~ ~~8. Design and execute simulations to calculate probabilities and expected values.~~ Content: (Students will know...) • discrete random variable takes a fixed set of numerical values with gaps between them on a number line. • the probabilities of a discrete random variable can be displayed as a histogram. • the probability of any event of a discrete random variable is the sum of the probabilities of all the values that make up the event. • a continuous random variable can take any value in a specified interval on the number line. • the display of the probability distribution of a continuous random variable is a density curve, with a total area underneath of 1. • the mean is the center of a probability distribution. The mean of a random variable is the average value of the variable after many repetitions of the chance process, known as the expected value. • the standard deviation is used to summarize the variability of a probability distribution. • the binomial distribution is the probability distribution of a binomial random variable. • a binomial random variable is a special type of discrete random variable that has binary outcomes, independent trials, fixed number of trials, and the same probability of success on each trial. Skills: (Students will be able to...)
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| | <ul style="list-style-type: none">• classify a random variable as discrete or continuous.• calculate probabilities involving a discrete random variable.• calculate probabilities involving a continuous random variable.• make a histogram to display the probability distribution of a discrete random variable and describe its shape.• calculate and interpret the mean, or expected value, of a discrete random variable.• calculate and interpret the standard deviation of a discrete random variable.• determine whether a random variable is a binomial random variable.• calculate probabilities involving a single value of a binomial random variable.• calculate and interpret the mean of a binomial random variable. |
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Unit Number and Title:	~~Unit 3: Probability~~ Unit 7: Sampling Distributions
Duration:	~~9 weeks~~ ~8 classes
Resource(s):	<i>Statistics and Probability with Applications</i> Fifth Edition (Starnes, & Tabor, & Wilcox), Chapter 7
Unit Overview:	In unit 7, students will explore a sampling distribution. They will distinguish between a parameter and a statistic. Students will create a sampling distribution using all possible samples from a small population. Students will calculate the sample mean and sample standard deviation of a sampling distribution of a sample proportion and interpret the values.
Learning Goals	
Standard(s):	Understand and evaluate random processes underlying statistical experiments. ● S-IC.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. <i>Note: Unit 7 extends beyond high school math standards into sampling distributions.</i>
Essential Question(s):	● How do you use probability rules to evaluate chance behavior in real-world contexts? ● ~~How can you use expected value for decision making?~~ ● ~~How can simulation be used to calculate expected values and probabilities?~~ ● ~~How can technology be applied to create and interpret models?~~ ● How can collecting, validating, organizing, and displaying data help us analyze information and make reasonable predictions and informed decisions? ● How is mathematics used to measure, model, and calculate data? ● How can a solid understanding of statistics help people make informed decisions based on data in their daily lives? ● How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving? ● How can we communicate mathematical ideas clearly and effectively?

Enduring Understanding(s):	● ~~Probability is the tool used for anticipating future outcomes based on past events, theoretical speculation, or simulation.~~ ● ~~Expected value is a key tool for rational decision making.~~ ● ~~Simulation allows us to overcome difficulties with other means of calculating probabilities and expected values.~~ ● Statistics provide tools for describing variability in data and for making informed decisions that take it into account. ● Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. ● Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. ● Random processes can be described mathematically by using a probability model. ● Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. ● When encountering statistics in daily life, it is important to consider the study design, how the data were gathered, and the analyses employed, as well as the data summaries and the conclusions drawn ● A statistic of an unbiased sample is an estimator of a population parameter. ● ~~The normal distribution can be applied to many real-life situations to solve probability problems.~~
Learning Goal(s): <i>Students will know and will be able to use their learning to:</i> (Content/ Skills)	1. ~~Identify a random process.~~ 2. ~~Calculate a probability based on observed proportions, simulation, or theoretical analysis.~~ 3. ~~Solve probability problems using the rules for addition (“or”) and multiplication (“and”) rules.~~ 4. ~~Understand properties of the normal distribution and how to use tables and technology to solve probability problems.~~ 5. ~~Know the role of the “Law of Large Numbers” in improving estimates of probabilities.~~ 6. ~~Know the definition of the mean (expected value) of a discrete random variable and determine the mean for a particular set of random events.~~ 7. ~~Understand and differentiate probability distributions.~~

~~8. Design and execute simulations to calculate probabilities and expected values.~~

Content: (Students will know...)

- a parameter is a number that describes some characteristic of a population.
- a statistic is a number that describes some characteristic of a sample.
- we use statistics to estimate parameters.
- we can use sampling distributions to determine which values of a statistic are likely to happen by chance alone and how much a statistic typically varies from the parameter it is trying to estimate.
- a statistic used to estimate a parameter is an unbiased estimator if the mean of its sampling distribution is equal to the value of the parameter being estimated.
- the sampling distribution of a sample proportion describes the distribution of values taken by the sample proportion in all possible samples of the same size from the same population.
- the sampling distribution of the sample mean describes the distribution of values taken by the sample mean in all possible samples of the same size from the same population.
- the shape of the sampling distribution will be approximately normal when the population distribution is approximately normal or when the sample size is sufficiently large.

Skills: (Students will be able to...)

- distinguish between a parameter and a statistic.
- use the sampling distribution of a statistic to evaluate a claim about a parameter.
- determine whether a statistic is an unbiased estimator of a population parameter.
- describe the relationship between sample size and the variability of a statistic.
- calculate the mean and standard deviation of the sampling distribution of a sample proportion “p-hat” and interpret the standard deviation.
- determine whether the sampling distribution of “p-hat” is approximately normal.
- if appropriate, use a normal distribution to calculate probabilities involving “p-hat”.

Unit Number and Title:	~~Unit 4: Inference~~ Unit 8: Estimating a Population Parameter
Duration:	~~7 weeks~~ ~9 classes
Resource(s):	<i>Statistics and Probability with Applications</i> Fifth Edition (Starnes, & Tabor, & Wilcox), Chapter 8
	In unit 8, students are learning how to estimate a population parameter. They learn the idea of a confidence interval and use a confidence interval to make decisions. Students look at the effects of a margin of error when interpreting the confidence interval. They learn conditions for estimating a population proportion and calculating a population proportion confidence interval. By the end of unit 8, students put it all together using a four-step process for constructing and interpreting a confidence interval and determining the sample size required for a specific confidence interval.
Learning Goals	
Standard(s):	~~N/A~~ Understand and evaluate random processes underlying statistical experiments. ● S-IC.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. ● S-IC.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ● S-IC.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. <i>Note: Unit 8 extends beyond high school math standards</i>
Essential Question(s):	● ~~How do you use inferential models to draw statistically significant conclusions from data and make inferences about populations?~~

	- How can the language of statistics be used to communicate mathematical ideas coherently and precisely? - How can technology be applied to create and interpret models? - How can improperly applied inference procedures lead to bad conclusions? - How can collecting, validating, organizing, and displaying data help us analyze information and make reasonable predictions and informed decisions? - How is mathematics used to measure, model, and calculate data? - How can a solid understanding of statistics help people make informed decisions based on data in their daily lives? - How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving? - How can we communicate mathematical ideas clearly and effectively?
Enduring Understanding(s):	- The foundation of an inference procedure is the hypotheses, null and alternative, which are determined by carefully considering the question of interest. - The p-value determines acceptance of the null hypothesis or rejection of it in favor of the alternative hypothesis based on the likelihood of the observed result. - There are many inference procedures to select from based on the nature of the problem at hand. - Statistics provides tools for describing variability in data and for making informed decisions that take it into account. - Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. - The foundation of an inference procedure is the hypotheses, null and alternative, which are determined by carefully considering the question of interest. - Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. - Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. - When encountering statistics in daily life, it is important to consider the study design, how the

	data were gathered, and the analyses employed, as well as the data summaries and the conclusions drawn • A statistic of an unbiased sample is an estimator of a population parameter. ◦ A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	~~1. Formulate appropriate null and alternative hypotheses and properly choose a test to conduct in order to reach a conclusion regarding the null and alternative.~~ ~~a. State hypotheses using proper notation.~~ ~~b. Determine whether the test is one-sided or two-sided based on the problem setting.~~ ~~2. Select an appropriate testing procedure (i.e. confidence interval or z-test)~~ ~~3. Determine and interpret confidence intervals on proportions and means.~~ ~~a. Determine confidence intervals for a simple random sample from a normal distribution of data.~~ ~~b. Determine the sample size required for a desired margin of error.~~ ~~c. Interpret confidence intervals in context of a given situation.~~ ~~4. Use a z-procedure to determine the p-value for a proportion or mean for a simple random sample from a normal distribution.~~ ~~a. Use the p-value to draw a conclusion about the null and alternative hypothesis.~~ ~~b. Interpret the conclusion in context of a given situation.~~ ~~5. Critique an application of inference procedures.~~ Content: (Students will know...) • a confidence interval gives a set of plausible values for a parameter based on sample data • the confidence level c gives the approximate percentage of confidence intervals that will capture the population parameter in repeated random sampling with the sample size • to construct a confidence interval for a population proportion, there are two conditions: ◦ the data comes from a random sample from the population of interest, ◦ the observed number of successes and the observed number of failures are both at least 10.

Skills: (Students will be able to...)

- interpret a confidence interval.
- determine the point estimate and margin of error from a confidence interval.
- use a confidence interval to make a decision about the value of a parameter.
- interpret a confidence level.
- describe how the confidence level and sample size affect the margin of error.
- explain how practical issues like nonresponse, undercoverage, and response bias can affect the interpretation of a confidence interval.
- check the random and large counts conditions for constructing a confidence interval for a population proportion.
- determine the critical value for calculating a C% confidence interval for a population proportion.
- calculate a C% confidence interval for a population proportion.
- use the four-step process to construct and interpret a confidence interval for a population proportion.
- determine the sample size required to obtain a C% confidence interval for a population proportion with a specified margin of error.

Unit Number and Title:	~~Unit 4: Inference~~ Unit 9: Testing a Claim about a Population Parameter
Duration:	~~7 weeks~~ ~ 9 classes
Resource(s):	<i>Statistics and Probability with Applications</i> Fifth Edition (Starnes, & Tabor, & Wilcox), Chapter 9
Unit Description	In unit 7, students measured the values for a sample of a population. In unit 8, students estimated population parameters using samples. In Unit 9, students will perform tests using hypotheses, null and alternative, and will use procedures to determine which hypothesis is likely based on the observed results.
Learning Goals	
Standard(s):	N/A <i>Note: Unit 9 extends beyond high school math standards</i>
Essential Question(s):	~~● How do you use inferential models to draw statistically significant conclusions from data and make inferences about populations?~~ ~~● How can the language of statistics be used to communicate mathematical ideas coherently and precisely?~~ ~~● How can technology be applied to create and interpret models?~~ ~~● How can improperly applied inference procedures lead to bad conclusions?~~ ● How can collecting, validating, organizing, and displaying data help us analyze information and make reasonable predictions and informed decisions? ● How is mathematics used to measure, model, and calculate data? ● How can a solid understanding of statistics help people make informed decisions based on data in their daily lives? ● How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving? ○ How can improperly applied inference procedures lead to bad conclusions? ● How can we communicate mathematical ideas clearly and effectively?
Enduring	~~● The foundation of an inference procedure is the hypotheses, null and alternative, which are~~

Understanding(s):	determined by carefully considering the question of interest. ● ~~The p-value determines acceptance of the null hypothesis or rejection of it in favor of the alternative hypothesis based on the likelihood of the observed result.~~ ● ~~There are many inference procedures to select from based on the nature of the problem at hand.~~ ● Statistics provides tools for describing variability in data and for making informed decisions that take it into account. ● Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. ○ The foundation of an inference procedure is the hypotheses, null and alternative, which are determined by carefully considering the question of interest. ● Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. ● Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. ● When encountering statistics in daily life, it is important to consider the study design, how the data were gathered, and the analyses employed, as well as the data summaries and the conclusions drawn ● A statistic of an unbiased sample is an estimator of a population parameter. ○ A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	~~1. Formulate appropriate null and alternative hypotheses and properly choose a test to conduct in order to reach a conclusion regarding the null and alternative.~~ ~~a. State hypotheses using proper notation.~~ ~~b. Determine whether the test is one-sided or two-sided based on the problem setting.~~ ~~2. Select an appropriate testing procedure (i.e. confidence interval or z-test)~~ ~~3. Determine and interpret confidence intervals on proportions and means.~~ ~~a. Determine confidence intervals for a simple random sample from a normal distribution of data.~~ ~~b. Determine the sample size required for a desired margin of error.~~

- e. ~~Interpret confidence intervals in context of a given situation.~~
4. ~~Use a z-procedure to determine the p-value for a proportion or mean for a simple random sample from a normal distribution.~~
- a. ~~Use the p-value to draw a conclusion about the null and alternative hypothesis.~~
- b. ~~Interpret the conclusion in context of a given situation.~~
5. ~~Critique an application of inference procedures.~~

Content: (Students will know...)

- the significance test is a formal procedure for using observed data to decide between two competing claims called hypotheses.
- the p-value of a test is the probability of getting evidence for the alternative hypothesis as strong or stronger than the observed evidence by chance alone when the null hypothesis is true.
- we can make a conclusion in a significance test based on the p-value.
- when we make a conclusion in a significance test, there are two kinds of mistakes we can make: a Type I error and a Type II error.
- a Type I error occurs when there is convincing statistical evidence that the alternative hypothesis is true (due to the small p-value), but it is not.
- a Type II error occurs when there is not convincing statistical evidence that the alternative hypothesis is true (due to the large p-value), but it is.
- in some studies, making a Type I error may have more serious consequences than making a Type II error. In other studies, making a Type II error may have more serious consequences than making a Type I error. The consequences of each error should be considered prior to conducting the study.
- confidence intervals provide additional information that significance tests do not, namely a set of plausible values for the population proportion based on sample data.
- to determine if a p-value should be considered small, we compare it to a boundary value called a significance level.

Skills: (Students will be able to...)

- state appropriate hypotheses for a significance test about a population parameter.

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| | <ul style="list-style-type: none">• interpret a p-value.• make an appropriate conclusion for a significance test based on a p-value.• make an appropriate conclusion in a significance test using a significance level.• check the random and large counts conditions for performing a significance test about a population proportion.• calculate the standardized test statistic for a significance test about a population proportion.• find the p-value for a one-sided significance test about a population proportion.• use the four-step process to perform a one-sided significance test about a population proportion.• calculate the p-value for a two-sided significance test about a population proportion.• use the four-step process to perform a two-sided significance test about a population proportion. |
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