

Modern Mathematics A: Math in Society

Grade(s):	11 & 12
Discipline/Course:	Mathematics
Course Title:	Modern Mathematics – A
Prerequisite(s):	Algebra 2
Course Description: <i>Program of Studies</i>	Modern Mathematics is a fourth-year launch course that differs from the courses that precede it in that the mathematics is focused on discrete topics instead of continuous functions. This post-Algebra II course is heavily based in modeling with mathematics and includes topics like elections and weighted voting, graph theory, game theory, and apportionment. Students engage in problem-based learning where problems are ill defined and may have varying outcomes. In this course, reasoning and modeling are primary drivers of instruction.
Course Essential Questions:	Where is mathematics used? What is the mathematics connection to history, art, computer science, etc?
Course Enduring Understandings:	● A quantity can be represented numerically in various ways. ● Problem solving depends upon choosing wise ways. ● There are multiple algorithms for finding a solution.
Duration:	Semester
Course Materials/Resources:	Excursions in modern mathematics (9th), Peter Tannenbaum, Pearson

***Note: Topics listed in the units may evolve over time based on adaptations to implementation. However, the overall content of the entire course will not change**

Grade(s):	☐ K ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5	☐ 6 ☐ 7 ☐ 8	☐ 9 ☐ 10 ☒ 11 ☒ 12 ☐ Other _____
Discipline/Course:	Discipline: Mathematics Course: Modern Mathematics		
Course Title:	Modern Mathematics - A: Math in Society		
Prerequisite(s):	Algebra 2 with a C or higher or Algebra II Honors		
Course Description: <i>Program of Studies</i>	Modern Mathematics is a discrete mathematics survey course that explores contemporary mathematics topics, mostly non-algebraic. The topics were chosen because they provide glimpses into other ways of thinking mathematically, and because they have interesting applications to everyday life. Together, they highlight algorithmic, graphical, algebraic, statistical, and analytic approaches to solving problems. Modern Mathematics is a course for students who want to learn about mathematical topics that have been discovered in the past 100 years and can be applied in modern times. In this course, students develop quantitative reasoning, mathematical modeling skills, and communication of ideas. In Modern Mathematics A: Math in Society, students learn the mathematical applications to politics, political science, and government. Topics include the mathematics of elections, the paradox of democracy, and the mathematics of power in weighted voting. Students explore the fairness of elections, voting methods, and the power of voting groups. Using the U.S. House of Representatives, students learn about the mathematics of apportionment and fair representation. Students explore mathematics in authentic situations to decide what is fair, what is right, what is possible, and what is impossible. They explore the mathematics of sharing and fair division games and how to make decisions that will be fair for all groups. Students engage in problem-based situations which may have varying outcomes. Their challenge is how to		

	make the best decision when there is no single right answer.	
Course Essential Questions:	• What does it mean for a process, system, or decision to be "fair"? • How can mathematical modeling help us analyze, structure, and resolve real-world conflicts in society, politics, and business? • Is fairness always mathematically possible, or must every system involve trade-offs? • How do different rules, algorithms, and voting methods influence who holds power and who wins? • How can quantitative reasoning help us make decisions when there is no single "correct" answer?	
Course Enduring Understandings:	• Fairness is mathematically complex and relative. • Mathematical impossibilities exist in social choice. • Systems and rules shape outcomes. • Discrete mathematics models authentic, real-world decisions.	
Duration: Credit:	☒ Semester ☐ Full-Year	☒ 0.5 Credit (s) ☐ 1.0 Credit(s) ☐ 1.5 Credit(s) ☐ N/A
Course Materials/Resources:	Excursions in Modern Mathematics (10th), Peter Tannenbaum, Pearson	
FPS Course Academic Expectation(s):	☒ Exploring and Understanding (EU) ☐ Synthesizing and Evaluating (SE) ☐ Creating and Constructing (CC) ☒ Conveying Ideas (CI) ☐ Collaborating Strategically (CS) ☐ Using Communication Tools (UCT)	
Year at a Glance	Unit 1: The Mathematics of Elections (~ 8 classes)	

(Units):	Unit 2: The Mathematics of Weighted Voting (~10 classes) Unit 3: The Mathematics of Apportionment (~ 10 classes) Unit 3: The Mathematics of Sharing: Fair Division Games (~ 10 classes)
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Academic Expectations The Fairfield Public Schools describe a variety of cross-curricular expectations that all students should exemplify during their time within the schooling experience. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:		
Standards	Explanations	Example
1. Exploring and Understanding	When students engage in problem-solving situations, they should be able to understand the problem, determine relevant information, and ask relevant additional questions.	Students should be able to answer the following questions when approaching a problem: 1. Do you understand all the words used in stating the problem? 2. What are you asked to find or show? 3. Can you restate the problem in your own words? 4. Can you think of a picture or diagram that might help you understand the problem?
2. Synthesizing and Evaluating	Engaging in a problem solving situation, students should be able to analyze the most efficient approach, and reflect on the process used to solve the problem.	Students should be able to answer the following questions when analyzing how to approach a problem, and also reflect on the result: 1. Is there enough information to enable you to find a solution? If not, what additional information is needed? 2. Are there multiple ways to complete the task? Which approach do you think is most efficient, and why? 3. Do you know a related problem? Look at the unknown and try to think of a familiar problem having the same or similar unknown. Can you use it? 4. Was your strategy effective? What worked? What didn't?

		~~5. Was there another approach that could have been more efficient?~~ ~~6. Is your answer reasonable? How do you know?~~ ~~7. Was your presentation approach effective? If not, what would you change?~~ ~~8. How did the communication tools allow you to get the message across to the intended audience?~~
3. Creating and Constructing	Engaged in a problem solving situation, students should implement a plan.	Students should be able to answer the following question to implementing their plan to solve a problem: 1. What strategy will you use to complete the task?
4. Conveying Ideas	Students should be able to use correct mathematical language, logically display their work for the desired problem.	Students should be able to answer the following questions to convey their mathematical thinking to solve a problem: 1. How will you present your information to your intended audience? 2. Does your response illustrate the correct terms and work to the problem?
5. Using Communication Tools	Students should be able to choose the correct tools to illustrate their mathematical work to solve a specific problem.	Students should be able to answer the following question to use specific communication tools to solve a problem: 1. If applicable, what communication tools will you use to convey your ideas and solution?
6. Collaborating Strategically	Students should be able to work collaboratively to solve problems.	Students should be able to answer the following question to collaboratively solve problems: 1. In what ways did you work together to help solve the desired problem?

Unit Number and Title:	Unit 1: The Mathematics of Elections ~~r-(The Paradoxes of Democracy & Weighted Voting)~~
Duration:	~8 classes
Resource(s):	~~Textbook:~~ Excursions in Modern Mathematics (10th) Chapter 1, Chapter 2
Unit overview:	In Unit 1, students explore how elections are decided. Students learn about elections and ballots. They explore four voting methods and how they are used in a variety of real-life contexts, beyond politics. Students will learn the basic principles of fairness for voting methods and use that to analyze the four voting methods.
Learning Goals	
Standard(s):	CC Standards: ~~CC 2.4 HS.B.1: Summarize, represent, and interpret data on a single count or measurement variable~~ ~~— Use data displays in problem-solving settings and/or make predictions~~ ~~CC.2.HS.C.1: Use the concept and notation of functions to interpret and apply them in terms of their context.~~ ~~— Analyze and/or use patterns or relations.~~ ~~— Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.~~ ~~Compare and/or order any real numbers.~~ ~~CC 2.2.HS.D.2:~~ A-SSE.3 Write expressions in equivalent forms to solve problems S- CPB.9 (+) Use permutations and combinations to compute compound events and solve problems. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others.

	MP.4 Model with mathematics. MR.8 Look for and express regularity in repeated reasoning. <i>Note: The concepts in Unit 1 extend beyond the CCSS for high school mathematics</i>
Essential Question(s):	• What is meant by a fair election? • Is any method of voting fair? • How can a method of voting be used to favor an outcome in an election? • How does the method of voting in the United States compare to methods used elsewhere? • What are the shortcomings of each method? • How can the voter be satisfied in the face of the Impossibility Theorem? • How are elections decided?
Enduring Understanding(s):	• Choosing the best voting method for your a situation. • Elections are ubiquitous • There are many different voting methods. • Different methods produce different outcomes. • A method of determining election results that is always fair is a mathematical impossibility.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Concepts (Students will know...) • elements of a basic election. ○ single choice ballot: a ballot in which a voter only has to choose one candidate. ○ preference ballot: a ballot in which the voter has to rank all candidates in order of preference. ○ truncated preference ballot: a ballot in which a voter has to rank the top k choices rather than all the choices. ○ ranking (full ranking): in an election, an outcome that lists all the candidates in order of preference. ○ partial ranking: in an election, an outcome where just the top k candidates are ranked. ○ preference schedules: a table that summarizes the preference ballots of all the voters.

- the plurality method.
 - plurality method: a voting method that ranks candidates based on the number of first-place votes they receive.
 - insincere voting: voting for candidates in a manner other than the voter's real preference with the purpose of manipulating the outcome of the election.
 - Condorcet candidate: a candidate that beats all others in a pairwise comparison.
- Borda count method: a voting method that assigns points to positions on the ballot and ranks candidates according to the number of positions.
- plurality with elimination method: a voting method that chooses the candidate with a majority of the votes. Where there is one, it eliminates the candidate(s) with the fewest votes and transfers those votes to the next highest candidate until there is a majority candidate.
 - ranked choice voting (instant runoff voting): a variation of the plurality with elimination based on a truncated preference ballot.
- pairwise comparisons: a voting method based on head-to-head comparisons between candidates.
- fairness criteria: basic rules that define formal requirements for fairness- a fair voting method is expected to always satisfy these basic rules.
 - majority criterion: if a candidate receives a majority of the first-place votes, then that candidate should be the winner of the election.
 - Condorcet criterion: a fairness criterion that says that if there is a Condorcet candidate, then that candidate should be the winner.
 - monotonicity criterion: a candidate who would otherwise win an election should not lose merely because some voters changed their ballots in a manner that favors that candidate.
 - independence of irrelevant alternatives criterion: a candidate who would otherwise win an election should not lose the election merely because one of the losing candidates withdraws from the race.
 - Arrow's Impossibility Theorem: it is mathematically impossible to satisfy all of the fairness criteria.

Skills: (Students will be able to...)

- use preference ballots.

- create a preference schedule from the total number of ballots for any given election.
- determine the winner of an election using any method: Plurality, Majority, Borda Count, Pairwise Comparison, Plurality with Elimination (Runoff and Sequential Runoff), and Approval.
- compare and contrast the different voting procedures.
- recognize and discuss the implications of Arrow's impossibility theorem.

- ~~1. use preference ballots~~
- ~~2. create a preference schedule from the total number of ballots for any given election.~~
- ~~3. determine the winner of an election using any method: Plurality, Majority, Borda Count, Pairwise Comparison, Plurality with Elimination (Runoff and Sequential Runoff), and Approval.~~
- ~~4. compare and contrast the different voting procedures~~
- ~~5. recognize and discuss the implications of Arrow's impossibility theorem.~~

Unit Number and Title:	~~Unit 1: The Mathematics of Elections r (The Paradoxes of Democracy & Weighted Voting)~~ Unit 2: The Mathematics of Weighted Voting
Duration:	~ 10 classes
Resource(s):	~~Textbook:~~ Excursions in Modern Mathematics (10th) Chapter 1, Chapter 2
Unit overview:	In Unit 2, students extend their learning about voting to weighted voting. They examine how the power of individuals, groups, or voting blocs is measured.
Learning Goals	
Standard(s):	~~CC Standards:~~ ~~CC 2.4 HS.B.1: Summarize, represent, and interpret data on a single count or measurement variable~~ ~~— Use data displays in problem-solving settings and/or make predictions~~ ~~CC.2.HS.C.1: Use the concept and notation of functions to interpret and apply them in terms of their context:~~ ~~— Analyze and/or use patterns or relations.~~ ~~— Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.~~ ~~Compare and/or order any real numbers.~~ ~~CC 2.2.HS.D.2:~~ A-SSE.3 Write expressions in equivalent forms to solve problems MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MR.8 Look for and express regularity in repeated reasoning. <i>Note: The concepts in Unit 2 extend beyond the CCSS for high school mathematics</i>
Essential	~~What is meant by a fair election?~~

Question(s):	● ~~Is any method of voting fair?~~ ● ~~How can a method of voting be used to favor an outcome in an election?~~ ● ~~How does the method of voting in the United States compare to methods used elsewhere?~~ ● ~~What are the shortcomings of each method?~~ ● ~~How can the voter be satisfied in the face of the Impossibility Theorem?~~ ● How are elections decided? ● How can the power of individuals, groups, or voting blocks be measured? ● How do we measure a voter's power in a weighted voting situation?
Enduring Understanding(s):	● In weighted voting, a voter's power can be deceiving. ● Mathematics does not give us the answer, just the tools to help us make an informed decision. ● A method of determining election results that are always fair is a mathematical impossibility.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content (Students will know...) ● weighted voting: a formal voting arrangement where each player controls a given number of votes. ● Banzhaf Power: in weighted voting, looking at the power of a player and its impact on the votes. ○ coalition: any set of players who decide to join forces and vote in the same way. ○ winning coalition: when a coalition has enough votes to win. ○ critical player: in a winning coalition, the player whose votes are needed for the coalition to win. ○ critical count: for each player, the count of the number of times that player is a critical player. ○ Banzhaf Power Index (BPI): the size of a player's share of the power, often represented as a percentage. ● Shapley-Shubik power distribution. ○ sequential coalition: an ordered list of the players. ○ factorial: for any positive integer N, the product of all the integers between 1 and N. A weighted voting system with N players has $N!$ sequential coalitions. ○ pivotal player: in a sequential coalition, the votes are tallied from left to right; the first

player whose votes make the total equal to or higher than the quota.

- pivotal count: for each player, the number of sequential coalitions in which the player is a pivotal player.
- Shapley-Shubik Power Index (SSPI): for each player, the ratio of the player's pivotal count to the number of players.

Skills: (Students will be able to...)

- determine the results based on different partnerships and coalitions.
- measure who is a critical player in a coalition.
- measure who is the pivotal player in a sequential coalition.
- measure a voter's power in a weighted voting situation.
- create a list of players and their power.

- ~~1. use preference ballots~~
- ~~2. create a preference schedule from the total number of ballots for any given election.~~
- ~~3. determine the winner of an election using any method: Plurality, Majority, Borda Count, Pairwise Comparison, Plurality with Elimination (Runoff and Sequential Runoff), and Approval.~~
- ~~4. compare and contrast the different voting procedures~~
- ~~5. recognize and discuss the implications of Arrow's impossibility theorem.~~

Unit Number and Title:	Unit 3: The Mathematics of Apportionment: ~~Making The Rounds~~
Duration:	~ 10 classes
Resource(s):	~~Textbook:~~ Excursions in Modern Mathematics (10th), Chapter 4
Unit Overview:	In Unit 3, students learn about apportionment. They will explore the dividing and allocating of things, and doing this on a proportional basis and in a planned, rule-based fashion. Students will journey into the apportionment of the U.S. House of Representatives and the mathematics behind it. They will see parallels between apportionment and voting. They will learn the various apportionment methods named after prominent figures in American History: Hamilton, Jefferson, Adams, and Webster. Students will examine the current method used in the U.S. House, the Huntington-Hill apportionment method. Students apply the various apportionment methods to other contexts beyond the U.S. House of Representatives, such as hospitals, schools, and imaginary countries and planets.
Learning Goals	
Standard(s):	~~N/A~~ MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MR.8 Look for and express regularity in repeated reasoning. <i>Note: The concepts in Unit 3 extend beyond the CCSS for high school mathematics</i>
Essential Question(s):	• Why were different apportionment methods applied in the United States government? • What are the benefits and shortcomings of the various methods of fair division? ~~• What method is used currently?~~ • How can we share on a large scale (apportion)?

Enduring Understanding(s):	- Sharing is not always fair. - Sharing on a large scale (apportioning) is always flawed.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content (students will know...) - apportionment problems: any problem in which there are M identical, indivisible objects (“seats”) that must be divided among N parties (“states”), using some proportionality criterion (“populations”). - standard divisor: the ratio of the total population of seats to the number of seats being apportioned. - standard quota (quota): the fair apportionment to the state if the seats were divisible into fractional parts. - lower quota: the largest integer smaller than or equal to the quota. - upper quotas: the smallest integer greater than or equal to the quota. - rounding rules. - remainders. - apportionment methods. - Hamilton’s method: the method of largest remainders. - Jefferson’s method: the method of greatest divisors. - Adam’s method: the method of smallest divisors. - Webster’s method: using conventional rounding. - Huntington - Hill Method: the equal proportions method, a sophisticated variation of Webster’s method. A divisor method that produces an apportionment by using modified quotas that are rounded according to the Huntington-Hill rounding rule. - geometric mean: the geometric mean of two positive numbers a and b is the square root of their product. - Huntington-Hill rounding rule: given a quota, q, the cutoff for rounding is given by the geometric mean of the lower quota and the upper quota. - Quota rule and apportionment paradoxes. - The quota rule: every state should be apportioned by either its lower quota or its upper quota.

- Alabama paradox: a paradox where a state may lose seats to another state merely because of an increase in the number of seats being apportioned.
- Population paradox: a paradox where a state may lose seats to another state merely because its population increased at a higher rate.
- new-states paradox: a paradox where a state may lose seats to another state merely because of the introduction of a new state (with its fair seats) into the apportionment calculations.
- Balinski and Young's Impossibility Theorem: a perfect apportionment method (no violations of the quota rule and no apportionment paradoxes) is a mathematical impossibility.

Skills (students will be able to ...)

- relate apportionment to fair division and define the players and the goods.
 - calculate the standard divisor using the total population and the total number of seats available to apportion.
 - determine each player's (state's) standard quota based on the standard divisor.
 - find the lower and upper quotas based on the standard quota.
 - understand and apply the apportionment methods: Hamilton, Jefferson, Adams, Webster, and Huntington-Hill.
 - compare and contrast each of the different methods and the benefits of using each one.
 - apply each of the methods of apportionment to specific problems to determine the allocation of goods
- ~~1. Relate apportionment to fair division and define the players and the goods.~~
 - ~~2. Calculate the standard divisor using the total population and the total number of seats available to apportion.~~
 - ~~3. Determine each player's (state's) standard quota based on the standard divisor.~~
 - ~~4. Find the lower and upper quotas based on the standard quota~~
 - ~~5. Understand and apply the apportionment methods: Hamilton, Jefferson, Adams, Webster and Huntington-Hill~~
 - ~~6. Compare and contrast each of the different methods and the benefits of using each one.~~
 - ~~7. Apply each of the methods of apportionments to specific problems to determine the allocation~~

	of goods
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Unit Number and Title:	Unit 4: The Mathematics of Sharing: Fair Division Games
Duration:	~ 10 classes
Resource(s):	~~Textbook:~~ Excursions in Modern Mathematics, 10th edition, Chapter 3
Unit overview:	In Unit 4, students explore how commonly owned assets can be divided in a fair and equitable manner. The problem of dividing an object or set of objects is a practical problem that comes up regularly in our everyday lives. When we are dividing a pizza, a cake, or a bunch of candy, we don't always pay attention to fairness. However, when we are dividing up real estate, jewelry, or other valuable assets, dividing things fairly is a critical issue. Students will explore a variety of methods to divide assets fairly.
Learning Goals	
Standard(s):	— N/A MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MR.8 Look for and express regularity in repeated reasoning. <i>Note: The concepts in Unit 2 extend beyond the CCSS for high school mathematics</i>
Essential Question(s):	• What is fair division? • What is the difference between discrete and continuous schemes in fair division? • What methods can be used in each scheme? • How can a person be satisfied that the outcome of a method is fair? • How are assessed common-owned items divided in a fair and equitable manner?
Enduring Understanding(s):	~~• Voting results may vary depending on voting methodology.~~ • Fair division involves rules the players must obey.

	- There are multiple methods to divide assets. - Mathematics and game theory can help humans share their common assets in a rational and fair way
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) - fair-division games. - assets: items or things being divided. - players: the parties that jointly own the assets and wish to divide them among themselves. - value system: an option that each player has regarding the value of the assets or any parts thereof. - fair share (proportional fair share): a share that, in the player's opinion, is worth at least $1/N$ of the total value of the assets (N is the number of players). - fair division: a division of the assets that gives each player a fair share. - fair-division method: a procedure that guarantees as its outcome a fair division of assets. - continuous fair division: a division involving assets that can be divided in infinite ways and by making arbitrarily small changes. - discrete fair division: a division involving assets consisting of indivisible objects that can only be divided up to a point. - divider-chooser methods: two players, one cuts the assets into two shares, the other chooses one of the shares. - lone-divider method: the lone divider cuts the assets into N shares, and the choosers declare which shares they consider to be fair. - lone-chooser method: all but one player divide the assets fairly among themselves, then each subdivides his or her shares into sub-shares. The remaining player (chooser) picks one sub-share from each divider. - method of sealing bids: each player bids a dollar value for each item, with the item going to the highest bidder. The other players get cash from the winning bid for their equity in the item. - sealed bid auction: an auction of an item of positive value. Only one secret bid per bidder; the highest bidder gets the item.

- reverse auction: an auction where the item being auctioned is a job or a chore. The lowest bidder gets the amount of his or her bid as payment for doing the job or chore.
- method of markers: a method of division that requires no cash, but only works with a fine-grained, discrete, fair-division game.

Skills: (Students will be able to...)

- define and identify a player, game, goods, and scheme type (continuous, discrete, or mixed).
 - define and identify fair division.
 - investigate and describe situations using continuous division, or infinitely divisible sets, using the following Fair Division schemes: divider/chooser, lone divider, lone chooser, and last diminisher.
 - investigate and describe situations involving discrete division using methods of sealed bids and method of markers.
 - compare and contrast the differences between the two types of fair division schemes.
 - solve fair division problems that consist of n individuals or players who must partition some set of goods, s , into n disjoint sets.
- ~~1. Define player, game, goods and scheme type (continuous, discrete or mixed).~~
 - ~~2. Define what is meant by fair division~~
 - ~~3. Investigate and describe situations using continuous division, or infinitely divisible sets using the following Fair Division schemes: divider/chooser, lone divider, lone chooser, and last diminisher~~
 - ~~4. Investigate and describe situations involving discrete division using methods of sealed bids and method of markers~~
 - ~~5. Compare and contrast the differences between the two types of fair division schemes~~
 - ~~6. Solve fair division problems that consist of n individuals or players who must partition some set of goods, s , into n disjoint sets.~~