

Mathematics

Grade 5

Grade(s):	Grade 5
Discipline/Course:	Mathematics
Course Title:	Grade 5 Mathematics
Prerequisite(s):	Grade 4 Mathematics
Course Description: <i>Program of Studies</i>	This year, students focus intensively on the three critical areas specified by the Common Core State Standards for Mathematics in Grade 5: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.
Course Essential Questions:	• How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? • How are quantitative relationships represented by numbers? • How do geometric relationships and measurements help us to solve problems and make sense of our world? • How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decisions?
Course Enduring Understandings:	Insights learned from exploring generalizations through the essential questions. Students will understand that: • Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technologies. • Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. • Shapes and structures can be analyzed, visualized, measured and transformed using a variety of

	strategies, tools, and technologies. • Data can be analyzed to make informed decisions using a variety of strategies, tools, and technologies.
Duration:	1 year
Course Materials/Resources:	Bridges 2nd ed.
FPS Course Academic Expectation(s):	☒ Exploring and Understanding (EU) ☐ Synthesizing and Evaluating (SE) ☐ Creating and Constructing (CC) ☒ Conveying Ideas (CI) ☐ Collaborating Strategically (CS) ☐ Using Communication Tools (UCT)
At a Glance	Unit 1: Expressions, Equations, and Volume (Bridges Unit 1 and August/September Number Corner) Unit 2: Adding and Subtracting Fractions (Bridges Unit 2 and October Number Corner) Unit 3: Place Value and Decimals (Bridges Unit 3 and November/December Number Corner) Unit 4: Multiplying & Dividing Whole Numbers and Decimals (Bridges Unit 4 and January Number Corner) Unit 5: Multiplying and Dividing Fractions (Bridges Unit 5 and February Number Corner) Unit 6: Graphing, Geometry and Volume (Bridges Unit 6 and March Number Corner) Unit 7: Division and Decimals (Bridges Unit 7 and April, May/June Number Corner)

Unit 1: Expressions, Equations, and Volume	In Unit 1, an exploration of volume serves as a bridge between fourth and fifth grade. Working with volume provides the context in which students review and extend skills and concepts from fourth grade, while introducing skills and concepts that are central to this year's studies. Students use the study of volume to review and extend a host of skills and concepts related to multiplication. Using rectangular prisms, they look for associative and commutative properties of multiplication as students use expressions with parentheses to represent different rectangular prisms. Students find the surface area of boxes to further develop an understanding of volume (and the ways in which it differs from area), as well as the use of the associative property in expressions with parentheses. Students develop major multi-digit multiplication strategies to solve real-world and mathematical problems in elegant and efficient ways. The link between multiplication and division is revisited through the lens of the area model and extended into dividing 3-digit by 2-digit numbers. Over the course of the unit, students build multiplicative thinking—a key component for success with division and fractions throughout the rest of the year.
Standard(s):	Operations and Algebraic Thinking Write and interpret numerical expressions. 5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product. Number and Operations in Base Ten Perform operations with multi-digit whole numbers and with decimals to hundredths. 5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations,

	rectangular arrays, and/or area models. Measurement and Data Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. 5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. 5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and improvised units. 5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
Essential Question(s):	• How do benchmark numbers help to estimate and justify reasonableness of answers to computation problems? • How do understanding factors and factor relationships help with understanding fractional relations? • Which strategy is the most efficient in a specific situation for computing and why? • How do written expressions represent mathematical problems? • How can using all the prime number factors and the associative property help you find all the other factors of a number? • Why is it important to consider the numbers before selecting a strategy to solve a problem?

	What is volume and how can volume be measured and compared?
Enduring Understanding(s):	- Benchmark numbers help to flexibly and efficiently add and subtract and multiply and divide numbers to mentally compute and estimate reasonableness of answers. - Commutative property for addition and for multiplication. - Associative property for addition and for multiplication. - Distributive property to simplify multiplication and partial factors to simplify division problems. - Division is partitive and quotative (sharing or grouping: 12 cookies shared by 3 people, or 12 cookies in bags of 3). - Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) - products to 100. - factor pairs between 1 and 100. - whether a whole number is a multiple of each of its factors. - whether a given whole number in the range 1–100 is a multiple of a given one-digit number. - whether a whole number in the range of 1-100 is prime or composite. - place value and property of operations strategies to multiply 2-, 3-, or 4-digit whole number by a 1-digit whole number or multiply two 2-digit numbers. - place value, the properties of operations, or the relationship between multiplication and division strategies to divide a 3-digit whole number by a 2-digit whole number. - volume can be written as an equation of the area of the base times the height $V = (l \times w) \times h$. - strategies - over strategy - partial products - doubling and halving - five is half of ten - models: - area or array model

- ratio table

Skills: (Students will be able to...)

- multiply a 2-, 3-, or 4-digit whole number by a 1-digit whole number or multiply two 2-digit numbers using strategies based on place value and the properties of operations.
- write numerical expressions with parentheses.
- evaluate numerical expressions that contain parentheses.
- write a simple expression to record calculations with numbers.
- interpret numerical expressions without evaluating them.
- divide a 3-digit whole number by a 2-digit whole number using strategies based on place value, the properties of operations, or the relationship between multiplication and division.
- compare the size of a product to the size of one of its factors on the basis of the size of the other factor, without performing the indicated multiplication.
- demonstrate an understanding that a solid figure that can be packed without gaps or overlaps by n unit cubes has a volume of n cubic units.
- measure the volume of a solid figure by counting the number of cubic units that fill it, with no gaps or overlaps.
- show that the volume of a right rectangular prism with whole number edge lengths can be found by multiplying the edge lengths or by multiplying the area of the base by the height.
- represent the product of three whole numbers as the volume of a right rectangular prism whose edge lengths are equal to those three whole numbers.
- make sense of problems to use order of operations to evaluate equations.
- reason abstractly and quantitatively by building models to represent and solve for volume of an object.
- ask questions and investigate prime and composite numbers to deepen an understanding of number relationships..
- use tools and clear and precise language to defend or counter one's reasoning or the reasoning of others.

Unit 2: Adding and Subtracting Fractions	The purpose of this unit is to deepen the understanding of fractions. Understanding equivalence is critical to adding and subtracting fractions. Students understand the larger the denominator, the smaller the equal unit parts and the numerator is the count of these unit parts. Students understand the connection between fractions and division and can explain why the fraction symbol indicates division. Benchmark fractions are used to help estimate solutions to problems. Equivalence is different representations of the same quantity, and equivalent forms can be substituted for each other. The commutative, associative, and distributive properties used with whole numbers also apply to fractions.
Standard(s):	Number and Operations—Fractions Use equivalent fractions as a strategy to add and subtract fractions. 5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.) 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators ,e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$. Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.3 Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share

	a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)
Essential Question(s):	• What are different ways fractions can be represented and how do they relate to each other? • How can fractions (mixed numbers, improper fractions) be represented visually, e.g. on the number line or ruler? • Why can equivalent fractional forms be substituted for each other? • How do benchmark fractions help you to estimate, compare, compute, and judge reasonableness of answers? • How can mathematical problems involving fractions be represented using basic operations? • How do you interpret division of a unit fraction by a non-zero whole number or a whole number by a unit fraction and compute?
Enduring Understanding(s):	• Benchmark fractions are helpful for estimating, comparing and computing. • When comparing fractions, the size of the whole matters. • The algebraic properties with whole numbers continue to be applied to fractions. • Fractions indicate division and the denominator is the divisor. • The more the whole is divided into equal parts, the smaller the parts (12ths are smaller than 4ths). • Equivalent fractions are different representations of the same quantity. • Equivalent fractions can be thought of as a ratio relationship (1 out of 2, 2 out of 4). • Knowing the factors of composite numbers helps us to identify common factors. This allows us to compare and find equivalent fractions. • Equivalent fractions can be substituted for each other. • Addition and subtraction of fractions must have like denominators.

	- Strategies are developed to divide fractions in general, by reasoning about the relationship between multiplication and division. - Interpret division of a whole number by a unit fraction or a unit fraction by a non-zero whole number and compute, e.g., $\div 4$ (Four students were sharing of a pan of brownies. How much of the pan will each student get if they share the brownies equally?)
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) - how to add and subtract fractions efficiently with and without finding common denominators. - fractions, mixed numbers, and improper fractions. - common denominators in order to compare, add, and subtract fractions. - double number lines and tables can be used to rewrite fractions with common denominators. - double number lines can be used to add and subtract fractions with unlike denominators. - solving a variety of story problems, and making generalizations about finding common denominators. - greatest common factors and least common multiples to find common denominators and learn to simplify fractions. - models: - money (quarter, dollar, nickel, dime) - time (minutes, fraction of an hour) - double number line - tables of equivalent fractions - strategies - finding equivalent fractions - least common multiple and least common denominator Skills: (Students will be able to...) - model with mathematics to investigate equivalent fractions and fractions as division. - reason abstractly and quantitatively to add and subtract fractions with like and unlike denominators.

- construct viable arguments to compare fractions and mixed numbers to conceptualize fractions, estimate fractions, and use benchmark fractions.
- attend to the precision of language, refine reasoning, explain thinking to others, and respond to others' thinking related to understanding fractions.
- add fractions with unlike denominators, including mixed numbers.
- subtract fractions with unlike denominators, including mixed numbers.
- rewrite fractions with unlike denominators as equivalent fractions with a common denominator in order to find their sum or difference.
- solve story problems involving addition of fractions referring to the same whole, with like and unlike denominators.
- solve story problems involving subtraction of fractions referring to the same whole, with like and unlike denominators.
- mentally estimate the answers to story problems involving addition and subtraction of fractions with like and unlike denominators.
- assess the reasonableness of answers to story problems involving addition or subtraction of fractions with like and unlike denominators.
- solve story problems involving division of whole numbers with fraction or mixed number quotients.
- multiply a whole number by a fraction.
- solve story problems involving multiplying a whole number by a fraction.

Unit 3: Place Value and Decimals ~~Place Value~~ ~~Concepts,~~ ~~Estimation and~~ ~~Computation~~	In Unit 3, the focus of instruction shifts from fractions to decimals, which invites a closer look at the base ten place value system. The purpose of this unit is to deepen the understanding of place value by recognizing that the base-ten system extends infinitely in two directions. The understanding of multiples of 10 with whole numbers is extended to numbers less than one. Students use patterns in our number system for numbers between 1,000,000 and 0.001. Students use algebraic properties and their knowledge of the structure of our place value system to flexibly estimate and compute with large and small numbers. Open number lines, area models, time measurement, and the use of benchmark numbers as referent are helpful to visualize fractions and operations with fractions.
Standard(s):	Number and Operations in Base Ten Understand the place value system 5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. 5.NBT.3 Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$. b. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. 5.NBT.4 Use place value understanding to round decimals to any place. Perform operations with multi-digit whole numbers and with decimals to hundredths. 5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations,

	rectangular arrays, and/or area models. 5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Number and Operations—Fractions Use equivalent fractions as a strategy to add and subtract fractions. 5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.) Measurement and Data Convert like measurement units within a given measurement system. 5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
Essential Question(s):	• What patterns emerge in the base-ten system when using powers of 10, and on the right side of the decimal? • How are these patterns helpful when composing and decomposing numbers? • How do benchmark numbers help to estimate, mentally compute, and solve problems involving decimal numbers? • How are two expressions equivalent or not equivalent? • How do partial products and partial quotients (factors) make it easier to estimate and perform mental computations? • How does using the expanded form make it easier to estimate, compute and judge the reasonableness of answers? • How can visual models be helpful in understanding the concepts and processes when adding, subtracting, multiplying and dividing decimals?
Enduring	• Place value patterns extend infinitely in both directions.

Understanding(s):	• Equivalent quantities can be represented differently (expanded form), e.g., $1,472 =$ - o $1,000 + 400 + 70 + 2 = (1 \text{ thousand}) + (4 \text{ hundreds}) + (7 \text{ tens}) + (2 \text{ units})$ - o $(1 \times 1,000) + (4 \times 100) + (7 \times 10) + (2 \times 1)$, or - o $(1 \times 10 \times 10 \times 10) + (4 \times 10 \times 10) + (7 \times 10) + (2 \times 1)$, or - o $(1 \times 10^3) + (4 \times 10^2) + (7 \times 10) + (2 \times 1)$ • These equivalent forms may be used to generalize rules for efficient computing. • Expanded form allows us to represent and substitute numbers, e.g., $4232 = (4 \times 1,000) + (2 \times 100) + (3 \times 10) + 2$, or $42.32 = (4 \times 10) + (2 \times 1) + (3 \times 0.1) + (2 \times 0.01)$ • The algebraic properties (associative, commutative, and distributive) with whole numbers applies to decimal fractions as it does with whole numbers. • Concrete and pictorial models can help make visible the processes of adding, subtracting, multiplying and dividing decimals to hundredths.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) • the connection between decimals, fractions, and money amounts. • decimals can be placed, ordered and rounded using a metric number line. • conversions of bytes, kilobytes, megabytes and gigabytes and metric conversions. • in a multi-digit number, each digit represents one-tenth what it represents in the place to its left. • the number of zeroes in the product when multiplying by powers of 10. • the placement of the decimal point when multiplying or dividing by powers of 10. • decimals can be written with base ten numerals, words, and in expanded form. • long division can be used to divide numbers. • strategies to divide a 3-digit whole number by a 2-digit whole number based on place value, the properties of operations, or the relationship between multiplication and division. • strategies based on place value and properties of operations, and the relationship between addition and subtraction to add and subtract decimals. • models: - o base ten area pieces - o specialized grid paper - o open number line

- ratio table
- strategies
 - add or subtract by place value
 - give and Take
 - finding the difference and removing
 - constant difference
 - place value shift

Skills: (Students will be able to...)

- model with mathematics to read, write, order, and compare decimals.
- reason abstractly and quantitatively with place value patterns when multiplying and dividing by powers of ten.
- look for and make use of structure to understand fraction/decimal equivalence.
- compare and describe relative magnitude of fractions, decimals, and mixed numbers using visual models.
- demonstrate an understanding that in a multi-digit number, each digit represents one-tenth what it represents in the place to its left.
- explain patterns in the number of zeroes in the product when multiplying by powers of 10 and in the placement of the decimal point when multiplying or dividing by powers of 10.
- write decimals to thousandths with base ten numerals, words, and in expanded form.
- compare pairs of decimals to thousandths, based on an understanding of what the digit in each place represents.
- use $>$, $=$, and $<$ symbols to record comparisons of two decimals to thousandths.
- round decimals to the nearest one, tenth, and hundredth.
- divide a 3-digit whole number by a 2-digit whole number using strategies based on place value, the properties of operations, or the relationship between multiplication and division.
- add and subtract decimals to hundredths, using concrete models or drawings and strategies based on place value and properties of operations, and the relationship between addition and subtraction.
- convert among different sized standard measurement units within a given measurement system.

Unit 4: Multiplication and Division of Whole Numbers and Decimals	During Unit 4, students make extensive use of such familiar tools as ratio tables, rectangular arrays, the area model, and money to review and develop a variety of strategies for multiplying and dividing whole numbers and decimals. The purpose of this unit is to develop proficiency with the properties of operations (commutative, associative, and distributive properties) with whole numbers and decimals. Students develop the understanding that algebraic properties apply to decimal numbers in the same way they apply to whole numbers. Students use partial quotients and the inverse relationship with multiplication to solve problems. Students understand that there are division patterns when dividing with remainders.
Standard(s):	Operations and Algebraic Thinking Write and interpret numerical expressions. 5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. <i>For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.</i> Perform operations with multi-digit whole numbers and with decimals to hundredths. 5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm. 5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings

	and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.) Measurement and Data Convert like measurement units within a given measurement system. 5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
Essential Question(s):	• What pattern do you notice in our base-ten system when using powers of 10? • How are these patterns helpful when composing and decomposing numbers? • How do benchmark numbers help to estimate, mentally compute, and solve problems? • How do you know if two expressions are equivalent? • How do partial products and partial quotients (factors) make it easier to estimate and perform mental computations? • How does using the expanded form make it easier to estimate, compute and judge the reasonableness of answers? • How can visual models help understand the concepts and processes when adding, subtracting, multiplying and dividing decimals?
Enduring Understanding(s):	• Place value patterns extend infinitely in both directions. • These equivalent forms may be used to generalize rules for efficient computing. • Expanded form allows us to represent and substitute numbers, e.g., $4232 = (4 \times 1,000) + (2 \times 100) + (3 \times 10) + 2$, or $42.32 = (4 \times 10) + (2 \times 1) + (3 \times 0.1) + (2 \times 0.01)$

	• The algebraic properties (associative, commutative, and distributive) with whole numbers applies to decimal fractions as it does with whole numbers. • Concrete and pictorial models can help make visible the processes of adding, subtracting, multiplying and dividing decimals to hundredths, e.g., 3 tenths subtracted from 4 wholes. The whole number must be divided into tenths.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) • strategies to multiply a 2, 3, or 4-digit whole number by a 1-digit whole number based on place value and the properties of operations. • strategies to multiply two 2-digit numbers using strategies based on place value and the properties of operations. • strategies for multiplying with multi-digit numbers. • the standard algorithm to multiply multi-digit whole numbers. • Strategies to divide a 2- or 3-digit whole number by a 2-digit whole number based on place value, the properties of operations, or the relationship between multiplication and division. • strategies for computing with decimals to hundredths to written methods. • the relationships between fractions, decimals, and whole numbers. • models: ○ ratio tables ○ the area model • strategies ○ using quarters ○ over and under ○ doubling and halving ○ half-ten facts Skills: (Students will be able to...) • reason abstractly and quantitatively to explore properties of operations (Commutative, Associative and Distributive properties) to make computation easier. • use appropriate tools strategically (including estimation) to add, subtract, multiply and divide

whole numbers and decimals to hundredths.

- look for and make use of structure to find whole number quotients based on place value, the properties of operations, and/or the relationship between multiplication and division.
- multiply two 2-digit numbers using strategies based on place value and the properties of operations.
- interpret numerical expressions without evaluating them.
- use the standard algorithm with fluency to multiply multi-digit whole numbers.
- divide a 2- or 3-digit whole number by a 2-digit whole number using strategies based on place value, the properties of operations, or the relationship between multiplication and division.
- multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value and properties of operations.
- use written numbers and symbols to represent strategies for computing with decimals to hundredths.
- relate strategies for computing with decimals to hundredths to written methods.
- multiply a whole number by a fraction.

Unit 5: Multiplying and Dividing Fractions	In Unit 5, students extend their understanding of multiplication and division to working with fractions. The purpose of this unit is to connect the understanding that algebraic properties (associative, commutative, and distributive) used with whole numbers continue to be applied to fractional numbers. Estimation is important for judging the reasonableness of comparisons and computations with fractions and decimals. Students extend the use of addition and subtraction with fractions and use the distributive property to multiply fractions and mixed numbers. They use equivalence to work with mixed numbers and improper fractions. Students write expressions, equations, and use formulas with fractions and mixed numbers to estimate and solve problems. Students are provided opportunities to use proportional reasoning to justify solutions to problems.
Standard(s):	Use equivalent fractions as a strategy to add and subtract fractions. 5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.) 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$. Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(\frac{a}{b}) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(\frac{2}{3}) \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. (In general, $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$.)

	5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. Operations and Algebraic Thinking Write and interpret numerical expressions. 5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product. Measurement and Data Represent and interpret data. 5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.
Essential Question(s):	• What different strategies can be used to compute fractions? • Which strategy is the most efficient for estimating and computing fractions for a given situation and why? • What needs to be considered to estimate with fractions? • How do benchmark fractions help you to mentally compute? • How does the distributive property help to estimate and multiply fractions and mixed numbers? • How can equivalence be maintained if the form of the fraction or decimal is changed?
Enduring Understanding(s):	• Equivalent fractions and/or decimals can be substituted when operating on them. • Benchmark fractions help to estimate, compare, compute numbers mentally, and judge the reasonableness of answers. • The distributive property and arrays provide visual models for multiplying fractions and mixed

	numbers. • When multiplying two rational numbers the product may be equal to, greater than, or less than the factors, e.g., $\frac{4}{5} \times \frac{2}{2} = \frac{8}{10}$, $1\frac{1}{2} \times 3 = 4\frac{1}{2}$, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ • The algebraic properties with whole numbers apply to fractions. • Equivalent fractions are different representations of the same quantity. • Equivalent fractions can be thought of as a ratio relationship (1 out of 2, 2 out of 4) • Knowing the factors of composite numbers helps us to identify common factors in the denominator. This allows us to compare and find equivalent fractions. • Equivalent fractions can be substituted for each other.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) • whole-number-by-fraction multiplication. • multiplying whole numbers by unit and common fractions. • fraction-by-fraction multiplication problems. • division of whole numbers by unit fractions. • partitive and measurement division (sharing and grouping). • dividing whole numbers by unit fractions. • division of unit fractions by whole numbers. • improper fractions and mixed numbers. • ways to visualize multiplication and division with fractions. • models: ○ area model ○ number line ○ discrete objects • strategies ○ scaling up (e.g. $\frac{3}{4} \times 9$ by finding $\frac{1}{4}$ of 9 and then scaling up by a factor of 3) ○ break into pieces (e.g. $\frac{1}{4} \times 9$ as $(\frac{1}{4} \times 8) + (\frac{1}{4} \times 1)$. ○ commutative and associative properties (students interpret $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations, $a \times (q / b)$.)

$$\circ \quad a/b \times c/d = ac/bd$$

Skills: (Students will be able to...)

- use equivalent fractions as a strategy to add and subtract fractions.
- develop and use models to represent proportional reasoning and equivalence (arrays, clock model, money model).
- plan and conduct investigations to multiply fractions by fractions using models.
- use the distributive property to multiply fractions and mixed numbers.
- use tools and clear and precise language to explain that equivalent fractions, including decimal fractions, are a representation of the same quantity.
- add and subtract fractions with unlike denominators, including mixed numbers.
- multiply a whole number by a fraction.
- multiply a fraction by a fraction.
- solve story problems involving multiplying a whole number or a fraction by a fraction.
- demonstrate that the area of a rectangle with fractional side lengths can be found through tiling or by multiplying the side lengths.
- represent the product of two fractions as an array whose dimensions are the two fractions being multiplied.
- compare the size of a product to the size of one of its factors on the basis of the size of the other factor, without performing the indicated multiplication.
- explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number.
- divide a unit fraction by a whole number; use a visual model to represent division of a unit fraction by a whole number.
- divide a whole number by a unit fraction; use a visual model to represent division of a whole number by a unit fraction.
- solve story problems involving division of a unit fraction by a whole number, and division of a whole number by a fraction.

Unit 6: Graphing, Geometry, and Volume	The purpose of this unit is to use points on a coordinate plane to represent and then interpret real world and mathematical problems. Two-dimensional figures can be categorized in a hierarchy based on their properties. Students use attributes to compare polygons and solids and identify commonalities and subsets. Students understand that the relationship among attributes helps to define polygons and solids.
Standard(s):	Graph points on the coordinate plane to solve real-world and mathematical problems. 5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). 5.G.2 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. Classify two-dimensional figures into categories based on their properties. 5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. 5.G.4 Classify two-dimensional figures in a hierarchy based on properties Operations and Algebraic Thinking Analyze patterns and relationships.

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Number and Operations - Fractions

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Measurement and Data

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and improvised units.

	5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems
Essential Question(s):	• How can estimations and actual measurement of parts help to classify polygons and solids? • What attribute can you use to classify polygons and solids? • What structures, patterns, and/or relationships emerge when graphing data? • What does a graph represent information and how can data inform decisions?
Enduring Understanding(s):	• The first number indicates how far to travel from the origin in the direction of one axis. • The second number indicates how far to travel from the origin in the direction of the second axis. • The names of the two axes and the coordinates correspond. • Data can represent and illuminate information in problems. • Attributes of polygons and solids enable us to categorize and classify them. • Polygons and solids can be described through measures of their parts. • Polygons can be moved in a plane or space. • Polygons and solids may be viewed from different perspectives.

Learning Goal(s):

Students will be able to use their learning to:

Content: (Students will know...)

- volume formula for a rectangular prism ($V = l \times w \times h$ and $V = b \times h$).
- coordinate graphing in the first quadrant.
- the hierarchies to classify two-dimensional shapes.
- names of common two- and three-dimensional shapes, including the various types of triangles and quadrilaterals.
- basic elements: points, rays, line segments, lines, and degrees of rotation.
- angle types (right, obtuse, and acute), as well as angles of specified measure.
- parallel and perpendicular lines.
- reflective or line symmetry.
- names of the parts of the coordinate grid (origin, horizontal distance is the x-coordinate, vertical distance is the y-coordinate), grid lines.
- models:
 - coordinate grids
 - shape hierarchies
 - paper nets
 - area model for multiplication

Skills: (Students will be able to...)

- analyze and interpret the structures, patterns, and/or relationships that emerge when graphing data.
- ask questions and investigate to represent real world mathematical problems by graphing points in the first quadrant of the coordinate plane.
- use clear and precise language to defend or counter one's reasoning or the reasoning of others to describe that attributes belonging to a category of two-dimensional figures also belong to all subcategories.
- generate two numerical patterns given two different rules.
- identify relationships between corresponding terms in two numerical patterns generated according to two different rules.

- graph on a coordinate plane ordered pairs consisting of the corresponding terms in two numerical patterns generated according to two different rules.
- divide a 3-digit whole number by a 2-digit whole number using strategies based on place value, the properties of operations, or the relationship between multiplication and division.
- multiply a fraction by a fraction.
- demonstrate that the area of a rectangle with fractional side lengths can be found through tiling or by multiplying the side lengths.
- solve story problems involving multiplication of fractions and mixed numbers.
- use the formula $V = l \times w \times h$ to find the volume of a right rectangular prism with whole-number edge lengths.
- use the formula $V = b \times h$ to find the volume of a right rectangular prism with whole-number edge lengths.
- solve story problems involving finding the volume of a solid figure composed of two nonoverlapping right rectangular prisms.
- locate a point on a coordinate plane based on its ordered pair of coordinates.
- write the x- and y-coordinates of a given point in a coordinate plane as an ordered pair.
- graph points in the first quadrant of the coordinate plane to represent a problem.
- describe the meaning of the values of coordinate points based on the context of a problem or situation.
- demonstrate an understanding that attributes of a category of dimensional figures also belong to all subcategories of that category.
- classify two-dimensional figures based on their attributes.
- classify two-dimensional figures within a hierarchy based on properties.

Unit 7: Division and Decimals	In this unit, students continue their study of division, including its relationship to multiplication. This is a unit-long investigation of division, including multi-digit division with whole numbers and decimals, and fraction division. While students have already worked with these operations in several different units, this unit introduce some new strategies, including the use of partial quotients and equivalent ratios. There is a consistent focus throughout the module on partitive and quotative—or sharing and grouping—division students review and extend their understandings of number and operations in base ten. Students are introduced to the use of whole number exponents to denote the powers of 10, looking at patterns when multiplying or dividing by 10 and apply these findings to decimals. Students multiply and divide decimals using familiar models.
Standard(s):	Number and Operations in Base Ten Understand the place value system 5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. Perform operations with multi-digit whole numbers and with decimals to hundredths. 5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

	Number and Operations - Fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.) Operations and Algebraic Thinking Write and interpret numerical expressions. 5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols
Essential Question(s):	• How can visual models help understand the concepts and processes when adding, subtracting, multiplying and dividing decimals? • How can the standard algorithm be used to divide multi-digit numbers accurately and efficiently? • What is the relationship between the placement of the decimal point in the factors and the placement in the product or quotient? • How does estimation help verify the reasonableness of a calculated decimal answer? • How can partial quotients be used as a bridge to understanding the standard division algorithm?

	● In what ways can division with remainders be interpreted in the context of a real-world problem?
Enduring Understanding(s):	● The standard algorithm for division is a highly efficient way to divide multi-digit numbers, built upon the foundation of partial quotients and place value. ● Multiplication and division are inverse operations; this relationship allows for the checking of work and the use of multiplication to solve complex division problems. ● Decimals can be added, subtracted, multiplied, and divided using strategies similar to those used with whole numbers, provided the placement of the decimal point is managed through place value logic and estimation. ● Contextual situations involving measurement, money, and shared quantities can be modeled and solved using division with whole numbers and decimals.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) ● partial quotients can be used to divide 3- and 4-digit dividends by 2-digit divisors. ● equivalent ratios can be found to solve division problems. ● rates can be represented as fractions. ● dividing unit fractions by whole numbers. ● dividing whole numbers by unit fractions. ● remainders are kept, rounded up or down depending on context. ● estimation of results for decimal operations. ● in a multi-digit number, each digit represents 10 times what it represents in the place to its right and one-tenth what it represents in the place to its left. ● the number of zeros in the product when multiplying by powers of 10. ● the placement of the decimal point when multiplying or dividing by powers of 10. ● strategies for dividing multi-digit whole numbers. ● strategies to divide a 2-, 3- or 4- digit whole number by a 2-digit whole number based on place value, the properties of operations, or the relationship between multiplication and division. ● strategies for dividing multi-digit whole numbers. ● strategies - division ○ multiplying to divide

- partial quotients
- overs and unders
- equivalent ratios
- models:
 - ratio tables
 - area models
 - clock faces
 - rectangular closed arrays

Skills: (Students will be able to...)

- demonstrate an understanding that in a multi-digit number, each digit represents 10 times what it represents in the place to its right and one-tenth what it represents in the place to its left.
- explain patterns in the number of zeros in the product when multiplying by powers of 10.
- explain patterns in the placement of the decimal point when multiplying or dividing by powers of 10.
- denote powers of 10 with whole-number exponents.
- divide a 2, 3, or 4-digit whole number by a 2-digit whole number using strategies based on place value, the properties of operations, or the relationship between multiplication and division.
- use equations and rectangular arrays to explain strategies for dividing multi-digit whole numbers.
- multiply and divide decimals to hundredths, using strategies based on place value, and explain the reasoning behind these strategies.
- divide a unit fraction by a whole number.
- write story problems involving division of a unit fraction by a whole number.
- divide a whole number by a unit fraction.
- write story problems involving division of a whole number by a unit fraction.
- solve story problems involving division of a whole number by a unit fraction.
- solve story problems involving division of a unit fraction by a whole number.