

Mathematics

Grade 4

Grade(s):	Grade 4
Discipline/Course:	Mathematics
Course Title:	Grade 4 Mathematics
Prerequisite(s):	Grade 3 Mathematics
Course Description: <i>Program of Studies</i>	This year, students focus intensively on the three critical areas specified by the Common Core State Standards for Mathematics in Grade 4: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.
Course Essential Questions:	• How do expressions represent mathematical problems and vs.? • How can four operations apply to multi-digit computation and what are efficient ways to mentally calculate solutions to problems? • How can decomposing multi-digit numbers based on place value help to make computation efficient? • What are prime and composite numbers and how do they relate to each other? • How are fractions compared? • How can fractions be used with addition and subtraction? • What are angles, and how are they measured? • How are geometric shapes and solids measured and categorized? • How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decisions?
Course Enduring Understandings:	• Numbers are factors and multiples of other numbers. • Mathematical problems can be written as mathematical expressions and vs. • Commutative property for addition and multiplication - The order of addends or factors does

	not change the result. (i.e. $10 + 3 = 3 + 10$ or $10 \times 3 = 3 \times 10$) • Associative property for addition and multiplication the order in which addends are added and factors are multiplied to not change the result. i.e. $(2 + 3) + 4 = 2 + (3 + 4)$, and $(2 \times 3) \times 4 = 2 \times (3 \times 4)$. • The distributive property uses partial products to simplify multiplication problems, e.g., $15 \times 12 = (10 + 5) \times (10 + 2) = 100 + 20 + 50 + 10 = 180$.) • Our number system is structured around multiples of ten. • The expanded form shows the place value structure as multiples of ten. e.g. or $1,472 = 1,000 + 400 + 70 + 2$ or, (1 thousand) + (4 hundreds) + (7 tens) + (2 units) or, $1,472 = (1 \times 1,000) + (4 \times 100) + (7 \times 10) + (2 \times 1)$, or $(1 \times 10 \times 10 \times 10) + (4 \times 10 \times 10) + (7 \times 10) + (2 \times 1)$ • Numbers can be composed and decomposed to make estimation and mental computation easier. • Angles can be measured and are additive. • A fraction is a division problem and the denominator is the divisor. • Fractions can be compared to determine relative amounts.
Duration:	1 year
Course Materials/Resources:	Bridges 2nd ed.
FPS Course Academic Expectation(s):	☒ Exploring and Understanding (EU) ☐ Synthesizing and Evaluating (SE) ☐ Creating and Constructing (CC) ☒ Conveying Ideas (CI) ☐ Collaborating Strategically (CS) ☐ Using Communication Tools (UCT)
At a Glance	Unit 1: Multiplicative Thinking (Bridges Unit 1 and August/September Number Corner) Unit 2: Multi-digit Multiplication and Early Division (Bridges Unit 2 and October Number Corner)

	Unit 3: Fractions and Decimals (Bridges Unit 3 and November/December Number Corner) Unit 4: Addition, Subtraction and Measurement (Bridges Unit 4 and January Number Corner) Unit 5: Geometry and Measurement (Bridges Unit 5 and February Number Corner) Unit 6: Multiplication, Division, Data and Fractions (Bridges Unit 6 and March Number Corner) Unit 7: Reviewing and Extending Fractions, Decimals and Multi-Digit Multiplications (Bridges Unit 7 and April, May/June Number Corner)
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Unit 1: Multiplicative Thinking	Unit 1 begins the year with a study of multiplication and division, focusing in particular on models, strategies and multiplicative comparisons. In Module 1, students use open number lines, arrays, and ratio tables. They also solve multiplication and division story problems and participate in their first math forum. In Module 2, they use the area model to investigate factors and multiples and prime and composite numbers. They also review strategies for finding single-digit multiplication facts. Module 3 has them working with factors and products as well as multiplicative comparisons and equations. Module 4 extends the idea of multiplicative comparison into the arena of measurement, as students develop deeper understandings of the relative sizes of metric units for length, mass, and liquid volume.
Standard(s):	Operations and Algebraic Thinking Use the four operations with whole numbers to solve problems. 4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. Gain familiarity with factors and multiples. 4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. Measurement and Data Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. 4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm;

	kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i>
Essential Question(s):	• How do benchmark numbers help to solve problems involving measurement? • How are mathematical problems written as expressions and how do you know if two equivalent expressions are equivalent? • How do partial products and partial factors make it easier to do mental computations? • What are prime and composite numbers? • How can using all the prime number factors and the associative property help you find all the other factors of a whole number?
Enduring Understanding(s):	• Benchmark numbers help to flexibly and efficiently multiply and divide numbers. • Whole numbers are factors and multiples of other numbers. • Mathematical problems can be written as mathematical expressions and vs. • Commutative property for addition and multiplication - The order of addends or factors does not change the result. (i.e. $10 + 3 = 3 + 10$ or $10 \times 3 = 3 \times 10$) • Associative property for addition and multiplication the order in which addends are added and factors are multiplied to not change the result. i.e. $(2 + 3) + 4 = 2 + (3 + 4)$, and $(2 \times 3) \times 4 = 2 \times (3 \times 4)$. • The distributive property uses partial products to simplify multiplication problems, e.g., $15 \times 12 = (10 + 5) \times (10 + 2) = 100 + 20 + 50 + 10 = 180$.) • Our number system is structured around multiples of ten. • The expanded form shows the place value structure as multiples of ten. e.g. or $1,472 = 1,000 + 400 + 70 + 2$ or, (1 thousand) + (4 hundreds) + (7 tens) + (2 units) or, $1,472 = (1 \times 1,000) + (4 \times 100) + (7 \times 10) + (2 \times 1)$, or $(1 \times 10 \times 10 \times 10) + (4 \times 10 \times 10) + (7 \times 10) + (2 \times 1)$ • Addition, subtraction, multiplication, and division by multiples of ten make estimation and computation easier.
Learning Goal(s):	Content: (Students will know...)

Students will be able to use their learning to:

- using multiplication and division within 100 to solve story problems in situations involving equal groups, arrays, and measurement quantities.
- determining the unknown whole number in a multiplication or division equation.
- applying properties of operations as strategies to multiply and divide.
- fluently multiplying and dividing within 100.
- factors of numbers, factor pairs, and whether numbers are prime or composite.
- multiplication equations as statements of multiplicative comparison.
- story problems involving multiplicative comparison.
- multiplication strategies:
 - zero (x0)
 - one (x1)
 - doubles (x2)
 - doubles plus one (x3)
 - double doubles (x4)
 - double double double (x8)
 - ten facts (x10)
 - half-ten facts (x5)
 - half-ten plus one (x6)
 - ten minus one (x9)
- multiplicative strategies: doubling/halving, partial products
- models:
 - open number line
 - array or area model
 - ratio table

Skills: (Students will be able to...)

- look for and generate number patterns that follow a given rule.
- use properties of operations to model mathematical thinking when solving multiplication problems.

- develop and use models to represent their thinking and use the models as tools for solving problems.
- construct viable arguments to explain and defend reasoning to others and respond critically to others' reasoning.
- interpreting products and quotients of whole numbers.
- determine the unknown whole number in a multiplication or division equation relating three whole numbers.
- solve division problems by finding an unknown factor (e.g., Solve $32 \div 8$ by finding the number that makes 32 when multiplied by 8).
- fluently multiply with products to 100 using strategies.
- explain patterns among basic multiplication facts by referring to properties of the operation.
- multiply whole numbers from 1–9 by multiples of 10 from 10–90 using strategies based on place value and properties of operations.
- find the area of a rectangle by multiplying its side lengths.
- make a comparison statement to match a multiplication equation; write a multiplication equation to represent a verbal statement of a multiplicative comparison.
- solve story problems involving a multiplicative comparison using multiplication or division.
- solve multi-step story problems involving only whole numbers, using addition , multiplication, and division.
- find all factor pairs for a whole number between 1 and 100.
- demonstrate an understanding that a whole number is a multiple of each of its factors.
- determine whether a whole number between 1 and 100 is a multiple of a given 1-digit number.
- determine whether a whole number between 1 and 100 is prime or composite.

Unit 2: Multi-Digit Multiplication and Early Division	This unit takes an applied and visual approach to multi-digit multiplication and early division with remainders. Students deepen their understandings of multiplication and division, solve and pose a wide variety of story problems, and apply number sense to developing useful models and mental strategies for multiplying and dividing with an increasing degree of efficiency. They move from building multiplication arrays to sketching them. They also build ratio tables and use them to make generalizations about the effects of multiplying by 10, 100, and 1,000. Students solve a variety of multiplication story problems and work together to compile and compare the strategies they have been practicing. Students solve division problems that require them to make sense of remainders in a variety of contexts. Students practice with multiplication facts, multi-digit multiplication, and division with remainders. Number sense is deepened while students focus on the numbers rather than the manipulation of digits.
Standard(s):	Operations and Algebraic Thinking Use the four operations with whole numbers to solve problems. 4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. Gain familiarity with factors and multiples. 4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. Number and Operations in Base Ten Generalize place value understanding for multi-digit whole numbers.

	4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. <i>For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.</i> 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. Measurement and Data Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. 4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</i>
Essential Question(s):	• How can you decompose a number into parts to make it a “mentally friendlier” number to calculate? • How can a mathematical problem be interpreted as expressions? • How do partial products and partial factors make it easier to do mental computations? • What are the different ways symbols are used to show division? • How do factors and products show the relationship between multiplication and division? • How do partial products and partial factors make it easier to estimate and do mental computations with multiplication and division?
Enduring Understanding(s):	• Factors and products show the relationship between multiplication and division. • Commutative property applies to addition and multiplication. • Associative property applies to addition and multiplication.

	• The distributive property uses partial products and can simplify multiplication problems. • Addition, subtraction, multiplication, and division using benchmark numbers make estimation and computation more efficient.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) • the operations of multiplication and division and how they relate to one another. • proficiency with basic multiplication and division facts. • estimation helps to decide whether a solution is reasonable. • solid place value understanding. (To comprehend that 4×270 is actually 4×200 plus 4×70, you have to know that 270 is 2 hundreds, 7 tens, and 0 ones. Furthermore, to apply one's knowledge of 3×5 to a larger problem such as 30×50, you have to understand that 30 and 50 are 10 times the size of 3 and 5 and that as a result, their product is 100 times the size of the product of 3×5.) • the rectangular array (area model). • to solve problems that involve multi-digit multiplication and division, as well as opportunities to generate and share a variety of strategies for doing so. • to apply properties of operations • strategies: ○ four partial products ○ two partial products ○ doubleline/halving along place value lines ○ ratio tables • models: ○ array or area model ○ the ratio table ○ base ten area pieces (note: Nudge students to more efficient and sophisticated models like grid paper and open arrays or to mental images of area) Skills: (Students will be able to...)

- multiply using the commutative and associative properties.
- apply the area formula for a rectangle to solve a problem.
- make a comparison statement to match a multiplication equation.
- solve story problems involving a multiplicative comparison using multiplication or division.
- solve multi-step story problems involving only whole numbers, using multiplication.
- demonstrate an understanding that a whole number is a multiple of each of its factors.
- determine whether a whole number between 1 and 100 is prime or composite.
- demonstrate an understanding that in a multi-digit number, each digit represents ten times what it represents in the place to its right.
- multiply 2- and 3-digit whole numbers by a 1-digit whole number using strategies based on place value and the properties of operations.
- multiply two 2-digit numbers using strategies based on place value and the properties of operations.
- use an equation or a rectangular array to explain strategies for multiplying with multi-digit numbers.
- divide a 2- or 3-digit number by a 1-digit number, using strategies based on place value, the properties of operations, or the relationship between multiplication and division
- use a rectangular array to explain strategies for dividing a multidigit number by a 1-digit number.
- apply the area formula for a rectangle to solve a problem.
- use models to help reason quantitatively and abstractly about numbers which develops deeper understandings of place value and the ability to calculate fluently with large numbers.
- use appropriate tools such as base ten area pieces to solve multi-digit multiplication problems.
- examine patterns to develop a stronger sense of place value and an ability to multiply by powers of ten with fluency and understanding.
- create and use array models to help look for and make use of structure. The structures evident in an array provide a visual justification for application of the properties of multiplication.

Unit 3: Fractions	The purpose of this unit is to develop an understanding of fraction equivalence and operations with fractions. Unit 3 takes an applied and visual approach to fractions and decimals. Students develop the understanding that properties of operations with whole numbers also apply to operations with fractions. They build a foundational understanding of fractions and how they relate to decimals. Students make the connection between fractions and division. When making comparisons, the size of the whole matters. Estimation strategies using benchmark numbers deepens the understanding of fractions and decimals.
Standard(s):	Number and Operations—Fractions Extend understanding of fraction equivalence and ordering. 4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. 4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

	d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. 4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of $1/b$. <i>For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.</i> b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.) c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. <i>For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i> Understand decimal notation for fractions, and compare decimal fractions. 4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. <i>For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.</i> 4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram. 4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.
Essential Question(s):	• How is a fraction like division? • How do benchmark fractions help you to estimate and solve problems? • What strategies could we use to add and subtract fractions?

	• In what ways can two fractions be equivalent? • What does “remainder” mean in the context of the division problem?, e.g., an integer, a left over, or a fractional part, or do you need to adjust your answer? Example: There are 36 people for 10 person vans, how many vans do you need? • How do you know when a fraction is larger or smaller than another? • When can $\frac{1}{2}$ be a smaller quantity than $\frac{1}{4}$? • What is a mixed number? • What are fractions on a number line or ruler? • How are fractions and decimals similar? • What are benchmark decimals and how do they relate to benchmark fractions? • How do you know which fraction or decimal is larger or smaller?
Enduring Understanding(s):	• Benchmark fractions are helpful for estimating and computing. • The more the whole is divided into equal parts, the smaller the parts, e.g. 8ths are smaller than 4ths • The denominator is the number of equal parts and the numerator is the number of equal parts being considered. • Equivalent fractions are different representations of the same quantity. • A fraction with the same numerator and denominator is equal to one whole. • Fractions are relations - the size of the whole (unit) is important, and when comparing fractions the size of the whole matters. • Fractional parts can be represented using different models, e.g., as set model (or groups), area model, or linear model • Benchmark numbers like $\frac{1}{2}$ or $\frac{1}{4}$ help to flexibly and efficiently estimate sums and differences with fractions. • Algebraic properties that apply to whole numbers also apply to fractional numbers. • A fraction is a division problem and the denominator is the divisor. • Benchmark decimals like 0.25, or 0.5 are helpful for estimating and computing. • Decimals are base-ten fractions, and there are many ways to represent an equivalent quantity, e.g., $\frac{1}{2}$, $\frac{5}{10}$, 0.5, 0.50. • Fractions can be applied to a multiplicative context.

Learning Goal(s):

Students will be able to use their learning to:

Content: (Students will know...)

- equivalent fractions can be created by dividing a model into various parts, or multiplying both the numerator and denominator by the same number
- fractions with unlike numerators can be compared by finding common denominators, although at this grade level, much of the work focuses on visual fraction models rather than algorithms
- comparisons of fractions or decimals are valid only when the two fractions or the two decimals refer to the same whole. Half a mini pizza is probably not greater than one-fourth of a giant pizza. Likewise, 5/10 of an apple is probably less than 25/100 of a watermelon.
- a fraction with a numerator of 1, such as $\frac{1}{6}$, is called a unit fraction. A common fraction, or a fraction with a numerator greater than 1, such as $\frac{4}{6}$, is the sum of unit fractions with the same denominator: $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$, or $4 \times \frac{1}{6}$
- fractions can be decomposed into a sum of fractions with the same denominator in more than one way. For example, $\frac{4}{6}$ also equals $\frac{2}{6} + \frac{2}{6}$ and $\frac{3}{6} + \frac{1}{6}$.
- multiplying a fraction by a whole number involves repeated addition. For example $3 \times \frac{1}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$, or $\frac{3}{6}$. By extension, $4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$, or $\frac{8}{3}$, which can also be expressed as $2 \frac{2}{3}$.
- fractions with denominator 10 can be expressed as equivalent fractions with denominator 100. This technique can be used to add tenths and hundreds: $\frac{2}{10} + \frac{34}{100} = \frac{20}{100} + \frac{34}{100} = \frac{54}{100}$.
- fractions with denominator 10 or 100 can be written in decimal notation. For example, $\frac{4}{10}$ can be written as 0.4, and $\frac{76}{100}$ can be written as 0.76.
- models:
 - folded paper strips
 - egg caron
 - geoboard
 - base ten area pieces
- strategies:
 - equivalent ratios and proportional reasoning
 - changing the number of divisions in a model

Skills: (Students will be able to...)

- use a visual model to explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$
- recognize equivalent fractions.
- generate a fraction equivalent to fraction a/b by multiplying the numerator (a) and denominator (b) by the same number.
- compare two fractions with different numerators and different denominators using the symbols $>$, $=$, and $<$, and explain why one fraction must be greater than or less than another fraction
- demonstrate an understanding that a comparison of fractions is valid only when they refer to the same whole.
- write an equation showing a fraction a/b as the sum of a number of the unit fraction $1/b$.
- express a fraction as the sum of other fractions with the same denominator in more than one way.
- add and subtract fractions and mixed numbers with like denominators.
- solve story problems involving addition and subtraction of fractions and mixed numbers referring to the same whole and with like denominators.
- solve story problems that involve multiplying a fraction by a whole number.
- express a fraction with denominator 10 as an equivalent fraction with denominator 100.
- add a fraction with denominator 10 to a fraction with denominator 100 by rewriting the first fraction as an equivalent fraction with denominator 100.
- write fractions with denominator 10 or 100 in decimal notation.
- compare two decimal numbers with digits to the hundredths place using the symbols $>$, $=$, and $<$ to record the comparison.
- develop and use a variety of models to represent fractional parts and use them as tools for solving problems.
- reason quantitatively and abstractly to recognize that fractions and decimals can represent a equivalent quantities differently.
- use base ten pieces as a model for decimal numbers and justify comparisons of those numbers.
- attend to precision while using models to consider the relationships among fractions and decimals.

- build foundational understanding of fractions as operators.

Unit 4: Addition, Subtraction and Measurement	In this unit, a strand of numeric exploration and investigation that was launched in Grade 1 and developed throughout Grades 2 and 3 comes to a logical conclusion as students are introduced to the standard, or traditional, algorithms for multi-digit addition and subtraction. This unit extends student understanding of addition and subtraction using benchmark numbers. Students estimate and make mental computation problems simpler by identifying ‘mentally friendlier’ numbers and use the commutative and associative as strategies. This unit focuses on place value to 1,000,000 and multi-digit addition and subtraction strategies. Students apply place value understanding to round multi-digit whole numbers to any place. Students explore benchmarks and relative sizes for length, time, liquid volume, mass, and weight. Students use ratio tables to convert units within the same measuring system, and apply some of the place value and multi-digit computation skills they’ve been practicing to solve story problems related to measurement.
Standard(s):	Number and Operations in Base Ten Generalize place value understanding for multi-digit whole numbers. 4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division. 4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. 4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place. Use place value understanding and properties of operations to perform multi-digit arithmetic. 4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm. Measurement and Data 4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a

	larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i>
Essential Question(s):	• What pattern do you notice in our number system when you multiply and divide by multiples of ten, 10, 100, 1,000, 10,000...? • How do benchmark numbers help you solve problems? • How does computation with double digit numbers apply to multi-digit numbers? • How do you know if equivalent expressions are equivalent?
Enduring Understanding(s):	• Benchmark numbers help to flexibly and efficiently add & subtract numbers. • Commutative property for addition. • Associative property for addition. • Our number system is structured around multiples and factors of ten. • The expanded form shows the place value structure as multiples of ten. e.g. $4,236 = (4 \text{ thousands}) + (2 \text{ hundreds}) + (3 \text{ tens}) + (6 \text{ units})$ or $1,472 = 1,000 + 400 + 70 + 2$ • Equivalent quantities can be represented differently. e.g. $1,472 =$ - o $(1 \times 1,000) = (4 \times 100) + (7 \times 10) + (2 \times 1)$, or - o $(1 \times 10 \times 10 \times 10) + (4 \times 10 \times 10) + (7 \times 10) + (2 \times 1)$ • Addition, subtraction, multiplication, and division by multiples of ten make estimation and mental computation easier.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) • numbers can be written in a variety of ways, including base ten numerals, number names, and in expanded form. For example, 15,675 can also be written as fifteen thousand, six hundred seventy-five or $10,000 + 5,000 + 600 + 70 + 5$. • multi-digit numbers can be rounded finely or more broadly, depending on the degree of accuracy required. • rounding and computational estimation go together. • estimation and mental math are necessary strategies when approaching an arithmetic problem

- there are standard algorithms for multi-digit addition and subtraction that work in a consistent and reliable manner for all combinations.
- there are units of measure other than those studied in previous grades for dealing with length and distance, weight and mass, liquid volume, and time. Units newly introduced at this grade level and featured in Unit 4 include kilometers, pounds and ounces, milliliters, and seconds.
- within one system of measurement, larger units can be converted to smaller units and vice versa, and these conversions can be displayed in two-column tables.
- to recognize problems for which finding the difference is more efficient, problems for which removing (taking away) is more efficient, and problems for which neither is really better, the constant difference strategy might be helpful, and when the standard algorithm is most efficient.
- models:
 - base ten area pieces
 - open number line
 - ratio tables
- strategies:
 - give & take for addition
 - constant difference for subtraction
 - estimation
 - mental computation
 - standard algorithm

Skills: (Students will be able to...)

- think critically about when best to deploy the several methods they have for adding and subtracting multi-digit numbers, and make the decision based on the numbers themselves.
- look for and make use of structure to understand that our number system is structured around multiples of ten.
- make sense of problems involving place value and magnitude and reason abstractly when comparing, ordering, and finding equivalence of larger numbers
- construct viable arguments and critique the reasoning of others when composing and decomposing numbers to estimate and make mental computations within 1000000.

- use tools and clear and precise language to solve problems in a variety of ways and defend thinking and respond to the thinking of others.
- find the value of an unknown in an equation.
- fluently add and subtract multi-digit whole numbers, using an algorithm or another strategy.
- demonstrate an understanding that in a multi-digit number, each digit represents ten times what it represents in the place to its right.
- read and write multi-digit whole numbers represented with base ten numerals, number names, and expanded form.
- compare pairs of multi-digit numbers; use $,$ $=$, and $<$ symbols to record comparisons.
- round multi-digit whole numbers to the nearest ten, hundred, thousand, ten thousand, hundred thousand, million.
- use the standard algorithm with fluency to add and subtract multi-digit whole numbers.
- identify the relative sizes of centimeters, meters, and kilometers; ounces and pounds; milliliters and liters; and seconds, minutes, and hours.
- record equivalent measurements in different units from the same system of measurement using a 2-column table.
- express a measurement in a larger unit in terms of a smaller unit within the same system of measurement (e.g., convert from km to m, l to ml, kg to g, lbs to oz).
- solve story problems involving intervals of time, distance, liquid volume, and mass using addition, subtraction, multiplication and division of whole numbers.
- solve story problems that involve expressing measurements given in a larger unit in terms of a smaller unit within the same system of measurement.

Unit 5: Geometry and Measurement	In this unit, students are formally introduced to a host of new geometric concepts, including angles and angle measure, parallel and perpendicular lines, and reflective symmetry The purpose of this unit is to develop the understanding that an angle measure is a measure of rotation and that angle measure is additive. Spatial reasoning using geometrical models to identify polygons and visualize spatial relationships is developed. Students describe, analyze, compare, and classify two-dimensional shapes. Students classify shapes by properties of their lines and angles.
Standard(s):	Geometric measurement: understand concepts of angle and measure angles. 4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. 4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. 4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. Geometry Draw and identify lines and angles, and classify shapes by properties of their lines and angles. 4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

	4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Essential Question(s):	• What are angles, and how are they measured? • How are polygons similar and different? • How does knowing the attributes of different polygons help to understand relationships among them? • What attributes can you use to sort and classify the polygons? • How do you know if two polygons are congruent or similar? • How do you know if a polygon is symmetrical? • What are lines, rays, and how do they relate to angles?
Enduring Understanding(s):	• Polygons can be composed and decomposed into other shapes. • Attributes of polygons enable us to categorize and classify them. • Polygons are two-dimensional figures. • Some polygons have more than one line of symmetry. • Two-dimensional figures can be classified by the presence/absence of parallel or perpendicular line or angles. • Right angles can be classified in their own category. • Angle measures are additive. • Angles are measured by identifying the fraction of the circular arc between the points where the two rays forming the angle intersect the circle whose center is at the endpoints of those rays
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) • basic elements: points, rays, line segments, lines, and degrees of rotation. • an angle as a geometric figure formed where two rays share a common endpoint.

- angle types (right, straight, obtuse, and acute), as well as angles of specified measure.
- parallel and perpendicular lines.
- reflective or line symmetry.
- angles of a polygon are the interior angles.
- Angles are measured in degrees.
- the sum of the measures of the interior angles of a triangle is 180 degrees.
- a protractor measures angles in degrees.
- a protractor can be used to sketch an angle.
- models:
 - rulers, protractors
 - pattern blocks
 - geoboards
 - colored tiles
 - clock face

Skills: (Students will be able to...)

- develop and use models to understand and represent the concepts of angle measurements.
- use appropriate tools strategically to measure and name angles and to recognize angle measure as additive.
- reason abstractly and quantitatively to classify polygons by angles and sides.
- use models and clear and precise language to classify polygons by their specific attributes.
- apply the area and perimeter formulas for a rectangle to solve a problem.
- identify an angle as a geometric figure formed where two rays share a common endpoint.
- use a protractor to measure angles in whole degrees, and sketch an angle of a specified measure.
- express the measure of an angle as the sum of the angle measures of the non-overlapping parts into which it has been decomposed.
- demonstrate an understanding that angle measure is additive.
- solve problems involving finding the unknown angle in a diagram, using addition and subtraction.
- draw lines, line segments, angles (right, acute, obtuse), and perpendicular and parallel lines.

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| | <ul style="list-style-type: none">• identify points, lines, line segments, rays, and angles (right, acute, obtuse), parallel lines, and perpendicular lines in 2-D figures.• classify 2-D figures based on the presence or absence of parallel lines, perpendicular lines, and angles of a specified size.• identify right triangles.• identify and draw lines of symmetry; identify figures with line symmetry. |
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Unit 6: Multiplication, Division, Area, and Fractions	The instruction in Unit 6 is designed to help students understand, in ways that are both deep and robust, the many connections between multiplication and division. Students discover that the models they have been using and strategies they have developed for multi-digit multiplication work equally well for division. They learn to divide numbers into the thousands by 1-digit numbers, using strategies based on the relationship between multiplication and division, as well as on place value, and the properties of operations. Estimation is an integral and integrated part of the problem solving process. Connections to fractional parts of a whole are reinforced involving division problems with remainders. Students extend their understanding of the area and perimeter of rectangles, focusing on the role division plays in area and perimeter problems. Students find unknown dimensions for area and perimeter problems.
Standard(s):	Operations and Algebraic Thinking Use the four operations with whole numbers to solve problems. 4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. Generalize place value understanding for multi-digit whole numbers. 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations—Fractions

Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Represent and interpret data.

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit ($1/2$, $1/4$, $1/8$).

Solve problems involving addition and subtraction

of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Generalize place value understanding for multi-digit whole numbers.

Essential Question(s):	• How does decomposing numbers make mental computation more efficient? • How do benchmark numbers help to estimate and solve problems? • How do partial products and partial factors make it easier to do mental computations? • How do partial products and partial factors make it easier to estimate? • How can “remainders” be interpreted in the context of a division problem? • What is the connection between division, fractions, and decimals?
Enduring Understanding(s):	• The distributive property using 2- and 3-digit numbers may be represented using the array model. • Factors and products show the relationship between multiplication and division. • Commutative property for addition and multiplication • Associative property for addition and multiplication • Distributive property of multiplication over addition and over subtraction. • Partial sums, partial products and partial factors help to simplify problems when estimating and mentally computing. • Addition, subtraction, multiplication, and division by multiples of ten make estimation and computation easier. • Division is sharing (partitive) or grouping (quotative), e.g., 12 cookies shared by 3 people, or 12 cookies in bags of 3. • Multiplication and division are inverse relations. • The use of equivalent expressions and substitutions develops flexibility in approaching and solving problems.

Learning Goal(s):

Students will be able to use their learning to:

Content: (Students will know...)

- language of division (dividend, divisor, quotient), the language of multiplication (factor, product) and the language of arrays (area, dimensions) when discussing strategies.
- as the numbers get larger, counting one by one or in small chunks is limiting, they know to think in fewer, bigger chunks.
- remainders are interpreted in their contexts. For example, \$81.00 shared by 5 people results in \$16.20 for each person. When sharing continuous items, such as cookies or brownies, students learn that the leftovers can be split into fractional parts.
- when division involves discrete objects, such as balloons or fish, they learn that the answer is often rounded up or down. For example, students will generally opt to leave the leftover balloon whole, and round the answer down to 16 when the situation involves dividing 81 balloons evenly to decorate 5 tables.
- arrays and ratio tables are used to model and solve multi-digit multiplication problems, both can also be used to model and solve division problems.
- In using an array to solve a division combination, the dividend is the area and the divisor is one of the dimensions.
- division helps to find a dimension when you have area or perimeter
- a line plot helps organize data to find the mean, median, and mode of the data.
- a division expression can be written as a fraction and can also be written as a decimal.
- strategies for multi-digit numbers - multiplication:
 - four partial products
 - two partial products
 - over strategy
 - five is half of ten
 - doubling and halving
- strategies for multi-digit numbers - division
 - multiply to divide
 - partial quotients
 - over division
 - equivalent ratios

- models
 - array model
 - ratio table

Skills: (Students will be able to...)

- use the standard algorithm with fluency to add and subtract multi-digit whole numbers.
- multiply a 1 or 2-digit whole number by a 1 or 2-digit whole number using strategies based on place value and the properties of operations.
- multiply 2- and 3-digit whole numbers by 1-digit whole numbers using strategies based on place value and the properties of operations.
- multiply two 2-digit numbers using strategies based on place value and the properties of operations.
- use equations and rectangular arrays to explain strategies for multiplying with multi-digit numbers.
- divide a 2-, 3-, or 4-digit number by a 1-digit number, with or without a remainder, using strategies based on place value, the properties of operations, or the relationship between multiplication and division.
- use equations and rectangular arrays to explain strategies for dividing a multi-digit number by a 1-digit number.
- apply the area or perimeter formulas for a rectangle to solve a problem.
- develop and use models to represent problem situations in a variety of ways.
- investigate problems and use properties of operations and models to efficiently solve multiplication and division problems, including partial products and partial quotients.
- interpret remainders of division problems and construct viable arguments to explain thinking and respond to others.
- distinguish between partitive and quotative division problems.
- make sense of and solve problems that involve repeated reasoning to develop generalized methods for division.

**Unit 7:
Reviewing and
extending Fractions,
Decimals, and
Multi-Digit
Multiplication**

Unit 7 reviews and extends skills and concepts in several areas that are foundational to the major work of fourth grade. In the first two modules, students refine their skills at recognizing and generating equivalent fractions, as well as comparing fractions with unlike denominators using visual models, benchmarks such as one half, and rewriting to share common denominators. In the latter half of the unit, students review some of the strategies they have developed for multi-digit multiplication over the year and explore the standard multiplication algorithm.

Standard(s):	Operations and Algebraic Thinking Use the four operations with whole numbers to solve problems. 4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. Generalize place value understanding for multi-digit whole numbers. 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area model Number and Operations—Fractions Extend understanding of fraction equivalence and ordering. 4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. Understand decimal notation for fractions, and compare decimal fractions. 4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. <i>For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.</i>
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	4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. 4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model. Measurement and Data Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. 4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</i>
Essential Question(s):	• What strategies are most efficient for multiplying a three-digit number by a one-digit number or two two-digit numbers? • How can you determine if a final answer is reasonable after completing multiple calculation steps? • How do you compare and order fractions when they have different numerators and denominators? • In what ways can multiplication and division strategies be used to design and evaluate a model?
Enduring Understanding(s):	• Multi-digit multiplication and division are most effectively solved by applying strategies based on place value and properties of operations, such as partial products and area models. • Multi-step story problems can be represented using equations with letters standing for unknown quantities, allowing for more structured problem-solving. • Mental computation and rounding are essential for assessing whether the solution to a complex multi-step problem is reasonable. • Fractions with different numerators and denominators can be compared and related by generating equivalent fractions through multiplication. • Mathematical models and computational skills are tools that can be applied to solve contextual

	problems in engineering and science
Learning Goal(s): <i>Students will be able to use their learning to:</i>	Content: (Students will know...) ● recognize and generate equivalent fractions ● bar models and number lines can be used to represent, compare and order fractions with unlike denominators ● decimal strips can be used to represent, compare, and order fractions with denominators of 10 and 100. ● finding a common denominator requires the use of factors and multiples ● common denominators make it possible to compare, combine or subtract two fractions with unlike denominators ● when the standard algorithm is best used and when other strategies make more sense ● models: Fractions and Decimals ○ bar model ○ geoboard ○ decimal strip ● models: multi-digit multiplication ○ two partial products (two-part arrays) ○ four partial products (four-part arrays) Skills: (Students will be able to...) ● solve multi-step story problems involving only whole numbers, using addition, subtraction, multiplication, and division. ● select and write equations with a letter standing for an unknown quantity to represent a multi-step story problem. ● assess the reasonableness of answers to multi-step story problems using mental computation, rounding, and other estimation strategies. ● multiply 2- or 3-digit whole numbers by 1-digit whole numbers using strategies based on place value and the properties of operations.

- multiply two 2-digit numbers using strategies based on place value and the properties of operations.
- use equations, rectangular arrays, or an area model to explain strategies for multiplying with multi-digit numbers.
- use a visual model to explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$
- recognize equivalent fractions.
- generate a fraction equivalent to fraction a/b by multiplying the numerator (a) and denominator (b) by the same number.
- compare two fractions with different numerators and different denominators.
- use the symbols $>$, $=$, and $<$, to record comparisons of two fractions with different numerators and different denominators.
- explain why one fraction must be greater than or less than another fraction.
- express a fraction with denominator 10 as an equivalent fraction with denominator 100.
- add a fraction with denominator 10 to a fraction with denominator 100 by rewriting the first fraction as an equivalent fraction with denominator 100.
- write fractions with denominator 10 or 100 in decimal notation.
- compare two decimal numbers with digits to the hundredths place.
- use the symbols $>$, $=$, and $<$ to record comparisons of two decimal numbers with digits to the hundredths place.
- explain why one decimal number must be greater than or less than another decimal number.
- apply the area formula for a rectangle to solve a problem.