Grades K–5

Scope and Sequence

Kindergarten

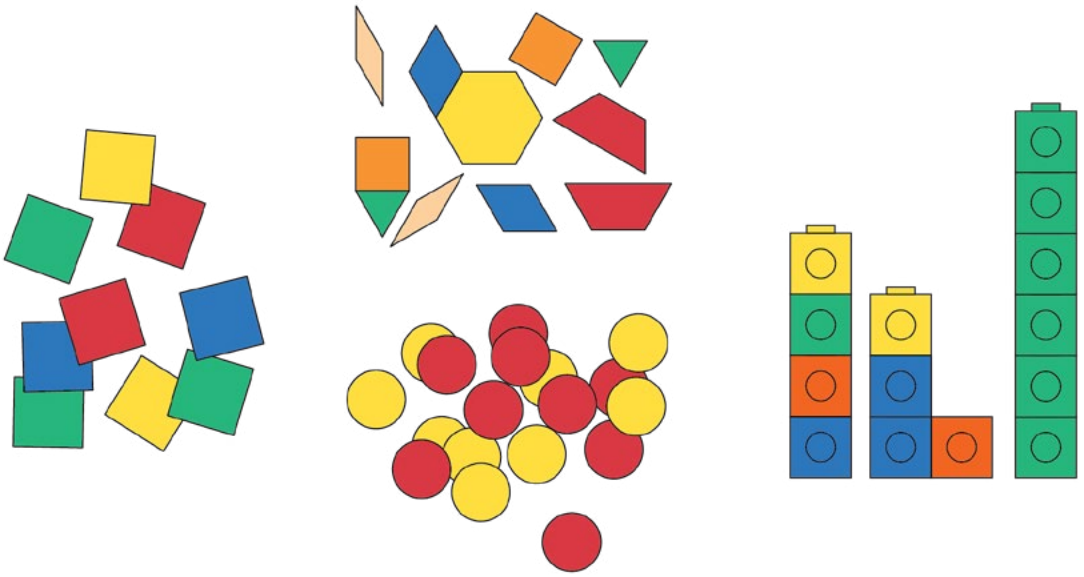
The big ideas in IM Kindergarten include: representing and comparing whole numbers, initially with sets of objects; understanding and applying addition and subtraction; and describing shapes and space. In IM Kindergarten, more time is devoted to numbers than to other topics. The materials, particularly units that focus on addition and subtraction, include problem types, such as Add To, Take From, Put Together or Take Apart, Compare, Result Unknown, and so on. These problem types are based on common addition and subtraction situations, as outlined in Table 1 of the "Mathematics Glossary" section of the *Common Core State Standards* (NGA & CCSSO).

Unit 1: Math in Our World

In this unit, students explore mathematical tools and notice numbers and quantities around them. Teachers gather information about students' counting skills and understanding of number concepts.

Students enter kindergarten with a range of counting experiences, concepts, and skills. So, this unit is designed to be accessible to all learners regardless of their prior experience. In the first three sections, activities do not require counting, though students may choose to count. Students also have opportunities to work with math tools and topics related to geometry, measurement, and data through a variety of centers.

In the last section, students count collections of objects and groups of people to answer, "how many?" questions. These questions reinforce the idea that counting is a way to tell how many objects there are. Counting up to 10 objects will support students in the next unit, which will focus more deeply on numbers 1–10.



The content of this unit is designed to establish the structures and routines for centers, to create norms for classroom learning, and to begin building a mathematical community. In the first section, lessons are shorter to give students time to learn these routines and norms and to develop a mathematical community.

At different points throughout the unit, consider asking individual students to count a small group of objects. As the student works, observe the skills or understandings in the Checklist provided at the beginning of each section and in the Unit 1 Sections A–D Checkpoint document in the teacher resource packet. The end-of-unit assessment (a one-on-one interview) is another opportunity to find out what students know and can do. This assessment is not necessary for students who have demonstrated the skills on the checklist throughout the unit.

Section A: Explore Our Math Tools

- Lesson 1: Explore Connecting Cubes
- Lesson 2: Explore Pattern Blocks
- Lesson 3: Explore 2-Color Counters and 5-Frames
- Lesson 4: Explore Geoblocks
- Lesson 5: Explore Math Tools

Section B: Recognize Quantities

- Lesson 6: Look for Small Groups
- Lesson 7: Classroom Scavenger Hunt
- Lesson 8: Different Groups, Same Quantity
- Lesson 9: Create Picture Books

Section C: Are There Enough?

- Lesson 10: Are There Enough?
- Lesson 11: Get Enough

Section D: Counting Collections

- Lesson 12: How Many Are There? (Part 1)
- Lesson 13: How Many Are There? (Part 2)
- Lesson 14: Answer "How Many?" Questions
- Lesson 15: Explain How You Counted
- Lesson 16: Represent Our Collections
- Lesson 17: Model with Math Tools

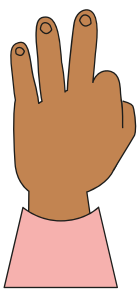
Unit 2: Numbers 1–10

In this unit, students continue to develop counting concepts and skills, including comparing groups of objects and images, and representing quantities with objects, pictures, and numbers.

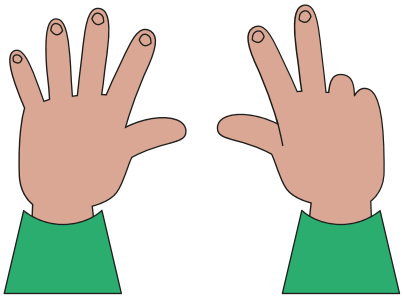
Previously, students learned structures and routines for activities and centers. They counted up to 10 objects to answer "how many?" questions. They also answered "are there enough?" questions—the basis of comparing quantities.

Here, students rely on familiar activity structures to build counting skills and an understanding of the connection between quantities and numbers. Students first count groups of objects. Then they count groups of images. Objects and images appear in different arrangements, such as lines, arrays, number cube patterns, and on 5-frames. This helps build an understanding that changing the arrangement doesn't change the quantity.

Use of fingers and 5-frames to represent numbers are emphasized to help students see the structure of numbers 6 – 10 as 5 + *n*. Fingers are also always available and help with counting.



These fingers show 3.



These fingers show 8.

Students also compare numbers of objects and images. To describe the comparisons, students start by using the terms "fewer" and "more." Later, when comparing written numbers, the term "less" is introduced. In general, "less" is used to compare numerals, and "fewer" is used to compare groups of objects. Students may use these terms interchangeably at first, but they will develop proficiency with the distinction over time.

Section A: Count and Compare Groups of Objects

- Lesson 1: Fingers as a Math Tool
- Lesson 2: Count and Arrange
- Lesson 3: Groups That Look Very Different
- Lesson 4: Groups That Look Alike
- Lesson 5: Make Groups of More, Fewer, or the Same
- Lesson 6: Use "More," "Fewer," or "the Same Number" to Describe Groups
- Lesson 7: Write Numbers 1–5
- Lesson 8: Write Numbers 6–10

Section B: Count and Compare Groups of Images

- Lesson 9: Count Images in Different Arrangements
- Lesson 10: Compare Matching Images
- Lesson 11: More, Fewer, or the Same
- Lesson 12: Find More, Fewer, or the Same
- Lesson 13: Create Groups of Images

Section C: Connect Quantities and Numbers

- Lesson 14: Connect Quantities and Numbers
- Lesson 15: Numbers in Many Ways
- Lesson 16: Count Out Objects
- Lesson 17: Draw Groups of Things
- Lesson 18: Write Numbers to Represent Quantities

Section D: Compare Numbers

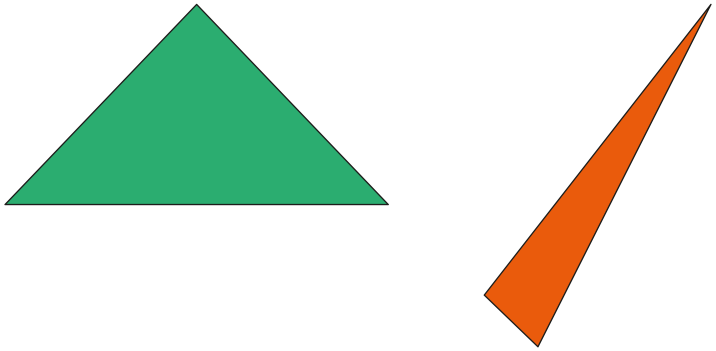
- Lesson 19: Order Towers and Numbers
- Lesson 20: 1 More or 1 Less with Towers and Numbers
- Lesson 21: Compare Numbers and Images
- Lesson 22: Represent and Compare Numbers
- Lesson 23: Compare Numbers
- Lesson 24: Set the Table

Unit 3: Flat Shapes All around Us

This unit introduces students to the foundational concepts of geometry, with a focus on familiar flat (two-dimensional) shapes.

Students may initially associate names of shapes with everyday objects. For example, a rectangle is a shape that looks like a door. Students need to see and interact with many examples of a shape to accurately relate objects in their environment to the geometric term.

For instance, students may say that only one of the two shapes is a triangle—the isosceles triangle sitting on its base—because they have seen examples like it referred to as triangles. They may not consider a scalene triangle sitting on a vertex as a part of the same shape category because, in their experience, a shape like it hasn't been associated with the term "triangle."



Students explore differences in shapes and use informal language to describe, compare, and sort them. Circle, triangle, rectangle, and square are four shapes that students study and name here. (They will not describe what defines each shape until grade 1.) Students also learn a key idea, that congruent shapes are still "the same" even if they are in different orientations.

Later in the unit, students use pattern blocks to make larger shapes. They reinforce their counting and comparison skills as they count and compare the pattern blocks used to create larger shapes. Students also use positional words (above, below, next to, beside) to describe the shapes they compose.

Section A: Exploring Shapes in Our Environment

- Lesson 1: What We Know about Shapes
- Lesson 2: Match Shapes
- Lesson 3: Describe and Compare Shapes
- Lesson 4: Describe, Compare, and Sort Shapes
- Lesson 5: Circles and Triangles
- Lesson 6: Rectangles and Squares
- Lesson 7: Build with Straws
- Lesson 8: Draw Shapes
- Lesson 9: Shapes Are Everywhere

Section B: Making Shapes

- Lesson 10: Put Together Pattern Blocks
- Lesson 11: Same Shapes
- Lesson 12: More than 1 Way to Make a Shape
- Lesson 13: Describe and Match Shapes
- Lesson 14: Shapes in Art
- Lesson 15: Animal Shape Stamp Art

Unit 4: Understanding Addition and Subtraction

In this unit, students develop their understanding of addition and subtraction as they represent and solve story problems.

Previously, students developed their counting skills. Students learn addition as an extension of counting though joining two groups and counting to find the total. Students also extend their counting through subtraction. They count to find and remove objects within a collection and then count what remains. (The word "total" is used instead of "sum" to avoid confusion with the word "some" or part of a whole.)

Students then represent and solve Add To, Result Unknown and Take From, Result Unknown story problems. Students represent the problems in different ways, by acting them out, drawing, using numbers, or using objects. Connecting cubes and two-color counters should be made accessible in all lesson activities, including cool-downs, for students that want to use them throughout the unit.

Students are also introduced to expressions, a symbolic way to represent addition and subtraction. Initially, the teacher records the process of adding and subtracting using words such as "5 and 3" or "4 take away 1." Later, students see that "5 and 3" and "4 take away 1" can be expressed by 5 + 3 and 4 – 1, respectively. They learn these expressions are read as "5 plus 3" and "4 minus 1." Students are not expected to read expressions out loud or to use precise language at this point.

Later in the unit, students connect expressions to pictures and story problems. They find the value of addition and subtraction expressions within 10.

In a future unit, students will compose and decompose numbers up to 10 and solve other types of addition and subtraction problems.

Section A: Count to Add and Subtract

- Lesson 1: Count 2 Groups of Objects
- Lesson 2: Count 2 Groups of Images
- Lesson 3: Count 2 Groups of Scattered Images
- Lesson 4: Add with Objects
- Lesson 5: Subtract with Objects

Section B: Represent and Solve Story Problems

- Lesson 6: Tell and Act Out Stories
- Lesson 7: Use Objects to Represent Stories
- Lesson 8: Represent and Solve Story Problems
- Lesson 9: Solve Story Problems
- Lesson 10: Compare Drawings
- Lesson 11: Drawings to Represent Story Problems
- Lesson 12: Compare Addition and Subtraction Story Problems
- Lesson 13: Create Story Problems

Section C: Addition and Subtraction Expressions

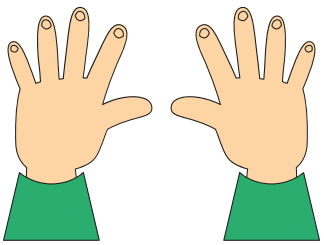
- Lesson 14: Expressions and Story Problems
- Lesson 15: Expressions and Drawings
- Lesson 16: Find the Value of Expressions
- Lesson 17: Add 0 and 1
- Lesson 18: Tell Story Problems for Expressions

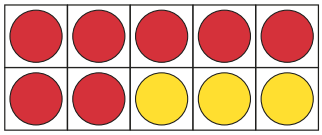
Unit 5: Composing and Decomposing Numbers to 10

In this unit, students explore different ways to compose and decompose numbers within 10 and to represent the compositions and decompositions.

Previously, students counted and compared groups and images of up to 10 objects. Students solved addition and subtraction story problems and wrote expressions to represent the problems. In this unit, students use those experiences to compose and decompose numbers within 10. (The terms “make” or “break apart” are used with students.)

Special attention is given to composing and decomposing 10, as it is the basis of place value in our number system. To support their reasoning, students use their fingers and a 10-frame—created by putting together two 5-frames. They use these tools to think about pairs of numbers that make 10.





Symbolic notation develops slowly across the units. Students first complete expressions that represent numbers being composed and decomposed. They also practice writing numbers without handwriting lines.

Later, students encounter equations of the form 5 = 3 + 2. Teachers read this equation as "5 is 3 plus 2." Note that the equations are written with the total on the left side of the equal sign and the addends on the right. Aside from representing composition and decomposition, this notation helps students see that the equal sign means that "both sides have the same value," rather than "the answer comes next." In a later unit, students will see equations with the addends on the left side.

The work in this unit prepares students to make sense of teen numbers in the next unit and lays the groundwork for students to develop fluency with addition and subtraction facts within 10 in grade 1. (For example, to find the sum of 9 + 5, they can decompose 5 into 1 + 4 and find 9 + 1 + 4 or 10 + 4 .) Much of the addition and subtraction work in future grades also hinges on the idea of composing and decomposing numbers, 10 in particular.

Section A: Make and Break Apart Numbers to 9

Lesson 1: Make 2 Parts
Lesson 2: Make and Break Apart Pattern Block Designs
Lesson 3: Snap the Cubes
Lesson 4: Find All the Ways

Section B: More Types of Story Problems

Lesson 5: Put Together
Lesson 6: Red and Yellow Apples
Lesson 7: Solve Both Addends In Unknown Story Problems
Lesson 8: More than One Way
Lesson 9: Story Problems

Section C: Make and Break Apart 10

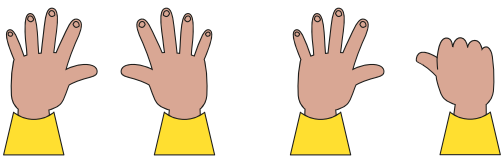
Lesson 10: Introduce the 10-Frame
Lesson 11: Equations That Show 10
Lesson 12: How Many Are Missing?
Lesson 13: Make 10
Lesson 14: Towers of 10
Lesson 15: Packing Up Fruit

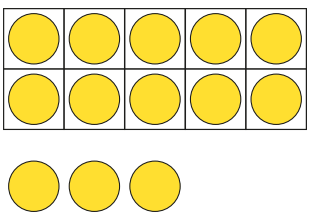
Unit 6: Numbers 0–20

In this unit, students count and represent collections of objects and images within 20. They apply previously developed counting concepts, such as one-to-one correspondence, keeping track of what has been counted, and conservation of numbers, to larger numbers.

Previously, students counted, composed, and decomposed numbers up to 10. They used counters, connecting cubes, 5-frames, 10-frames, drawings, their fingers, and other tools. They also wrote expressions to record compositions and decompositions.

Here, students use the 10-frame to organize groups of 11–19 objects and images. This tool encourages students to see teen numbers as 10 and some more, emphasizing the 10 + *n* structure of the numbers 11–19. Students use this structure as they represent teen numbers with their fingers, objects, drawings, expressions, and equations. Students see equations with the addends written first, such as 10 + 6 = 16. It is important to note that students are not expected to think of 10 ones as a unit called "a ten" or refer to single units as "ones" until Grade 1.





Kindergarten

Throughout the unit, students practice tracing and writing numbers 11–20. It is common for students at this stage to write numbers backwards, so the emphasis is on writing a number that is recognizable to others. Reversing the order of the digits of teen numbers is also expected, due to how teen numbers are said in English. Repeatedly seeing the number 1 written first to represent teen numbers helps students recognize the structure of these numbers.

When tracing and writing numbers, students should write on a flat surface while sitting in a chair with feet flat on the floor. Number writing practice can also happen in other parts of the day and can be done using a variety of writing tools (crayons, colored pencils, markers, and so on) for increased engagement. Students can practice creating numbers with dough, tracing numbers in sand, or forming numbers with pipe cleaners.

Section A: Count Groups of 11–20 Objects

- Lesson 1: Count Larger Collections of Objects
- Lesson 2: Keep Track of Objects
- Lesson 3: Count Carefully
- Lesson 4: Does the Number Change?

Section B: 10 and Some More

- Lesson 5: How Many Fingers? How Many Dots?
- Lesson 6: Fingers and 10-Frames
- Lesson 7: Make Numbers with 10 and Some More (Part 1)
- Lesson 8: Make Numbers with 10 and Some More (Part 2)
- Lesson 9: Expressions and Equations
- Lesson 10: Complete Equations

Section C: Count Groups of 11–20 Images

- Lesson 11: Count Images (Part 1)
- Lesson 12: Count Images (Part 2)
- Lesson 13: Design a Garden

Unit 7: Solid Shapes All around Us

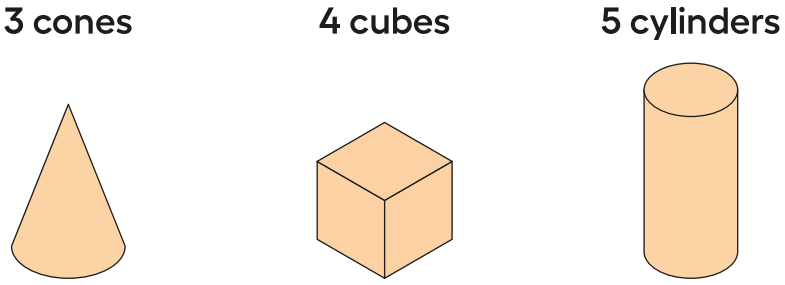
In this unit, students explore solid shapes while reinforcing their knowledge of counting, number writing and comparison, and flat shapes. They compose figures with pattern blocks and continue to count up to 20 objects, write and compare numbers, and solve story problems.

In an earlier unit, students investigated two-dimensional shapes. They named shapes (circle, triangle, rectangle, and square) and described the ways the shapes are different. Students used pattern blocks to build larger shapes and used positional words (above, below, next to, beside) along the way.

Here, students distinguish between flat and solid shapes before focusing on solid shapes. They consider the weights and capacities of solid objects and identify solid shapes around them.

Geoblocks, connecting cubes, and everyday objects are used throughout the unit. Standard geoblock sets do not include cylinders, spheres, and cones. When these shapes are required, "solid shapes" are indicated as required materials. If solid shapes are not available, students can work with everyday items that represent each shape.

The mathematical names **cube**, **cone**, **sphere**, and **cylinder** are introduced in this unit; however, students are not expected to use the names of solid shapes. Students can and are encouraged to continue to use their own language to describe and identify solid shapes.



How many shapes did you use all together?

The work here prepares students to identify defining attributes of shapes and to use flat and solid shapes to create composite shapes in grade 1.

Section A: Put Together and Count with Flat Shapes

- Lesson 1: Build Shapes
- Lesson 2: More or Fewer Pattern Blocks
- Lesson 3: Questions and Stories about Shapes
- Lesson 4: Pattern Block Puzzles and Equations
- Lesson 5: Story Problems about Shapes
- Lesson 6: Make and Break Apart 10 with Pattern Blocks

Section B: Describe, Compare, and Create Solid Shapes

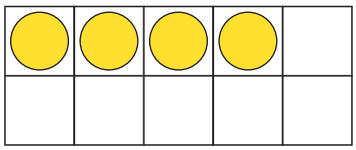
- Lesson 7: Flat and Solid Shapes
- Lesson 8: Compare Weights
- Lesson 9: Compare Capacities
- Lesson 10: Identify and Describe Solid Shapes
- Lesson 11: Compare and Sort Solid Shapes
- Lesson 12: Build Solid Shapes
- Lesson 13: Describe Solid Shapes around Us
- Lesson 14: Build with Solid Shapes
- Lesson 15: Build and Count with Solid Shapes
- Lesson 16: Represent the Classroom with Shapes

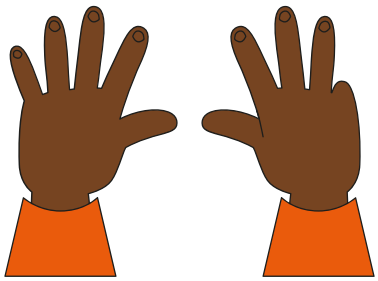
Unit 8: Putting It All Together

In this unit, students apply their learning from the year, revisiting the major work and fluency goals of the grade.

Section A focuses on the concepts of counting and comparing. Section B highlights the presence of math in students' school community. Section C enables students to practice composing and decomposing numbers within 5, as well as adding and subtracting within 5. Section D focuses on composing and decomposing 10.

The sections in this unit are standalone sections, not required to be completed in order. The goal is to offer ample opportunities for students to integrate the knowledge they have gained and to practice skills related to the expected fluencies of the grade.





10 = 8 + 2

The content here lays the foundation for grade 1, where students add and subtract fluently within 10 and count and compare larger quantities. Students also learn about "ten" as a unit, which is the basis for understanding place value in the base-ten system.

Section A: Counting and Comparing

- Lesson 1: Sort, Count, and Compare Groups of Objects
- Lesson 2: Count and Compare Collections
- Lesson 3: Count to Add and Subtract
- Lesson 4: 1 More and 1 Less
- Lesson 5: Order Numbers 1–20

Section B: Math in Our School

- Lesson 6: Create Number Books (Part 1)
- Lesson 7: Create Number Books (Part 2)
- Lesson 8: Find Someone Who, Find Something That
- Lesson 9: Where's the Math?
- Lesson 10: Tell Stories about Our School
- Lesson 11: Share Story Problems

Section C: Fluency within 5

- Lesson 12: Make Dot Images
- Lesson 13: Dominoes to 5
- Lesson 14: Sort and Color Expressions and Images within 5
- Lesson 15: Addition and Subtraction Expressions within 5
- Lesson 16: Parts to Make 5

Section D: All about 10

- Lesson 17: Make and Break Apart 10
- Lesson 18: All the Ways to Make 10
- Lesson 19: Find the Number That Makes 10
- Lesson 20: More or Fewer than 10?
- Lesson 21: Make and Break Apart Numbers 11–19

Grade 1

The big ideas in IM Grade 1 include: developing understanding of addition and subtraction, and strategies for addition and subtraction within 20; developing understanding of whole-number relationships and place value, including grouping tens and ones; developing understanding of linear measurement and measuring lengths as iterating length units; and reasoning about attributes of, and composing and decomposing, geometric shapes. The materials, particularly units that focus on addition and subtraction, include problem types, such as Add To, Take From, Put Together or Take Apart, Compare, Result Unknown, and so on. These problem types are based on common addition and subtraction situations, as outlined in Table 1 of the "Mathematics Glossary" section of the *Common Core State Standards* (NGA & CCSSO, 2010).

Unit 1: Adding, Subtracting, and Working with Data

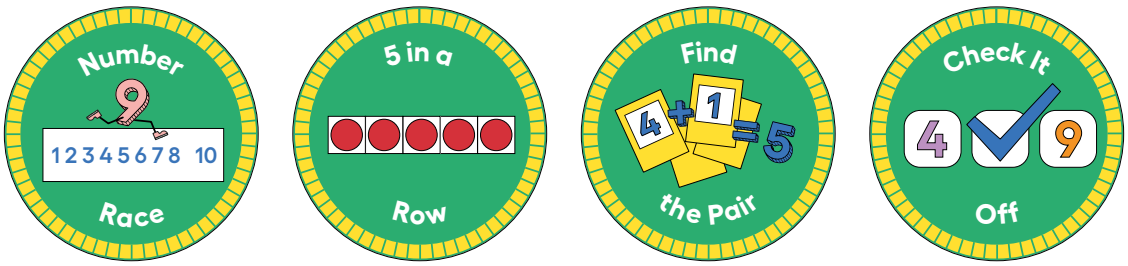
In this unit, students deepen their understanding of addition and subtraction within 10. They also extend what they know about organizing objects into categories and representing the quantities.

In kindergarten, students solved addition and subtraction word problems within 10 using objects and drawings. They learned about Put-Together, Result-Unknown problems and worked toward fluency with sums and differences within 5.

The activities in this unit reinforce these understandings and initiate the year-long work of developing fluency with sums and differences within 10. Some problems involve finding sums greater than 10, a skill to be honed throughout the course and with the support of tools, such as connecting cubes.

Students also build on the work of kindergarten as they engage with data. Previously, students sorted objects into given categories, such as by size or shape. Here, students use drawings, symbols, tally marks, and numbers to represent categorical data. They go further by choosing their own categories, interpreting representations with up to three categories, and asking and answering questions about the data.

This opening unit offers teachers opportunities to introduce mathematical routines and structures for centers, and to develop a shared understanding of what it means to do math and to be a part of a mathematical community.



Section A: Add and Subtract within 10

- Lesson 1: Count and Add
- Lesson 2: Explore Expressions and Sums
- Lesson 3: Add 1 or 2
- Lesson 4: More Work with 1 and 2
- Lesson 5: Explore Addition and Subtraction
- Lesson 6: Center Day 1

Section B: Show Us Your Data

- Lesson 7: Sort Math Tools
- Lesson 8: Sort and Count Shape Cards
- Lesson 9: Which is Your Favorite?
- Lesson 10: Center Day 2

Section C: What Does the Data Tell Us?

- Lesson 11: Class Pet Surveys
- Lesson 12: How Many?
- Lesson 13: Questions about Data
- Lesson 14: Center Day 3
- Lesson 15: Free-Time Activities

Grade 1

Unit 2: Addition and Subtraction Story Problems

In this unit, students learn to solve new types of addition and subtraction story problems. As students make sense of the problems and share the ways they solve them, they deepen their understanding of addition, subtraction, and the relationship between these operations. These new problem types also elicit computation strategies, such as counting on, that students will use throughout the year as they add and subtract within 20 and develop fluency within 10.

In kindergarten, students solved a limited number of story problem types within 10. They made sense of story problems by acting them out with objects and drawings. As they compared different ways to represent and solve these problems, including the use of expressions, students developed an understanding of addition as adding to or putting together and subtraction as taking from or taking apart.

Here, students are introduced to three of the new problem types for grade 1:

- Add To, Change Unknown
- Put Together/Take Apart, Addend Unknown
- Compare, Difference Unknown

Each of these problem types involves an unknown addend. Still, they all provide unique opportunities for students to learn about the relationship between addition and subtraction as they make sense of the actions or relationships in the problems.

Throughout the unit, it is important to maintain a focus on sense-making as students share and compare the different ways they represent and solve problems. It is recommended to read the story problems aloud to all students during this unit to ensure access to the mathematics. Students will continue to use objects and drawings during the unit and throughout the year to make sense of problems and show their thinking.

In the next unit, students will solve addition and subtraction problems within 20 and work with equations with a symbol for the unknown in all positions. They will also further develop their fluency within 10.

Section A: Add To and Take From Story Problems

- Lesson 1: Represent Story Problems
- Lesson 2: Solve Story Problems
- Lesson 3: A Change Is Coming
- Lesson 4: A Change Unknown
- Lesson 5: Center Day 1

Section B: Put Together/Take Apart Story Problems

- Lesson 6: Problems about Pets
- Lesson 7: One Part Red, One Part Yellow
- Lesson 8: One Part Red, How Many Yellow?
- Lesson 9: What Is the Unknown?
- Lesson 10: Center Day 2

Section C: Compare Story Problems

- Lesson 11: Are There Enough?
- Lesson 12: How Many More? How Many Fewer?
- Lesson 13: Compare Data
- Lesson 14: Compare with Addition or Subtraction
- Lesson 15: Different Types of Story Problems
- Lesson 16: Center Day 3

Section D: All Kinds of Story Problems

- Lesson 17: Equations with Unknowns
- Lesson 18: Addition and Subtraction Equations
- Lesson 19: How Do the Stories Compare?
- Lesson 20: Story Problems and Equations
- Lesson 21: Make Sense of Equations
- Lesson 22: Center Day 4
- Lesson 23: Story Problem Information

Grade 1

Unit 3: Adding and Subtracting within 20

In this unit, students develop an understanding of 10 ones as a unit called "a ten" and use the structure of 10 + *n* to add and subtract within 20.

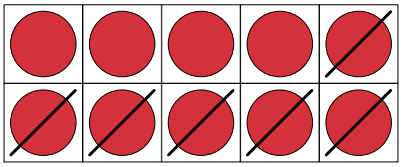
In kindergarten, students composed and decomposed the numbers 11–19 into 10 ones and some more ones. In a previous unit, students solved story problems of all types with unknown values in all positions and numbers within 10. They used the relationship between addition and subtraction, drawings and equations, and various tools (10-frames, connecting cubes, two-color counters) to represent the quantities in the problems. They learned that the values represented by the numbers or expressions on each side of an equation are equal.

Here, students decompose and recompose addends to find the sum of two or three numbers. For example, to find the value of 9 + 6, they may decompose 6 into 1 and 5, compose the 1 and 9 into 10, and find 10 + 5.

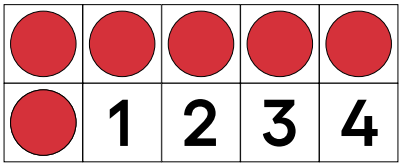
Subtraction work occurs throughout the unit and becomes the focus in the last section. Students consider taking away and counting on as methods for subtracting. They understand subtraction as an unknown-addend problem and use their knowledge of addition to find the difference of two numbers.

For instance, students may reason about the value of 10 − 6 by:

Taking away 6 from 10.



Counting on to 10, starting from 6.



Using an addition fact, 6 + 4 = 10.

Students solve story problems throughout the unit and learn two new problem types—Add To, Start Unknown and Take From, Change Unknown. Students compare the structure of different types of story problems as they practice adding and subtracting within 20.

Section A: Develop Fluency with Addition and Subtraction within 10

- Lesson 1: Sums I Know
- Lesson 2: Relate Counting to Addition
- Lesson 3: Are They Equal?
- Lesson 4: Sums of 10
- Lesson 5: Find the Difference
- Lesson 6: Story Problems within 10
- Lesson 7: Center Day 1

Section B: Add and Subtract Using Ten as a Unit

- Lesson 8: Ten as a Unit
- Lesson 9: Addition with a Ten
- Lesson 10: Addition and Subtraction with a Ten
- Lesson 11: Add to a Teen Number
- Lesson 12: Subtract from a Teen Number
- Lesson 13: More Story Problems with Teen Numbers
- Lesson 14: Center Day 2

Section C: Add within 20

- Lesson 15: Solve Story Problems with Three Numbers
- Lesson 16: Add 3 Numbers
- Lesson 17: Make 10 to Add
- Lesson 18: Patterns in Addition
- Lesson 19: Methods for Addition within 20
- Lesson 20: A Trip to the Zoo
- Lesson 21: Center Day 3

Section D: Subtract within 20

- Lesson 22: Subtract from Teen Numbers
- Lesson 23: Use a Ten to Subtract
- Lesson 24: Relate Counting to Addition and Subtraction
- Lesson 25: How Do You Want to Subtract?
- Lesson 26: What's the Story?
- Lesson 27: Center Day 4
- Lesson 28: The Garden

Grade 1

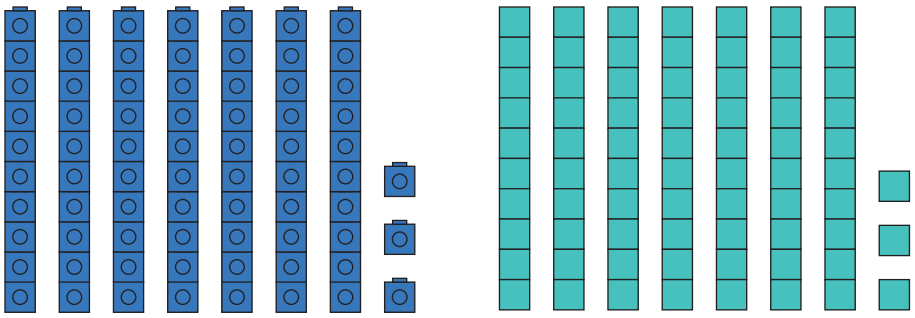
Unit 4: Numbers to 99

This unit develops students' understanding of the structure of numbers in base ten, allowing them to see that the two digits of a two-digit number represent how many tens and ones there are.

In a previous unit, students counted forward by one and ten within 100 in the *Choral Count* routine. They learned that 10 ones make a unit called a "ten" and that a "teen number" is a ten and some ones.

As students count and group quantities, they generalize the structure of two-digit numbers in terms of the number of tens and ones. This understanding enables students to transition from counting by one to counting by ten and then counting on. For example, to count to 73, students may count 7 tens and then count on—71, 72, 73.

Students interpret and use multiple representations of two-digit numbers: connecting cubes, base-ten diagrams, words, and expressions. Connecting cubes in towers of 10 and singles are used instead of base-ten blocks, so units of ten can be physically composed and decomposed with the cubes. Base-ten blocks will be introduced in grade 2. Here are some representations for 73:



7 tens and 3 ones

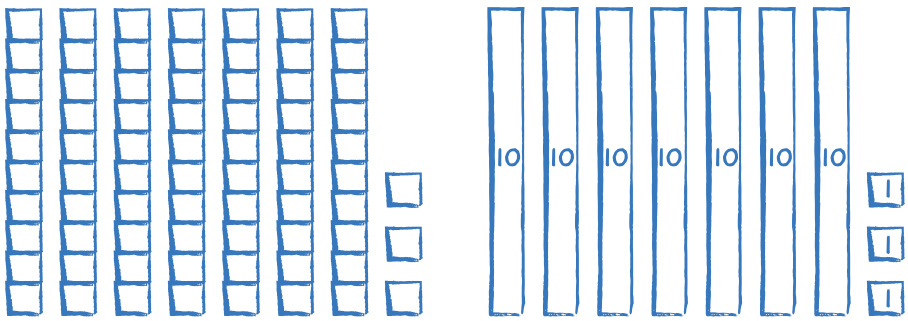
3 ones and 7 tens

70 + 3

63 + 10

60 + 13

Students also represent two-digit numbers with drawings. They may start by drawing towers of ten and showing each unit of one within each ten. Later, students simplify their drawings to show rectangles for tens and small squares for ones. Encourage students to use the drawings that make sense to them. Students that use abstract drawings should express how many ones each ten represents.



Students should have access to connecting cubes—towers of 10 and singles —in all lessons to help students make sense of base-ten representations. Some students may also benefit from access to double 10-frames and two-color counters. Students should be encouraged to work toward using connecting cubes in towers of 10 and singles.

Later in the unit, students use the value of the digits to compare two-digit numbers. Students learn to use comparison symbols (<, >) to record their comparisons. The unit concludes with opportunities for students to explore different ways of using tens and ones to represent two-digit numbers.

Section A: Units of Ten

- Lesson 1: Count Large Collections
- Lesson 2: Match Representations of Tens
- Lesson 3: Addition and Subtraction with Tens
- Lesson 4: More Addition and Subtraction with Tens
- Lesson 5: Center Day 1

Section B: Tens and Ones

- Lesson 6: Count Larger Collections
- Lesson 7: Numbers with Tens and Ones
- Lesson 8: Different Representations of Tens and Ones
- Lesson 9: Show Me Your Number
- Lesson 10: Write 2-Digit Numbers
- Lesson 11: Add Tens to 2-Digit Numbers
- Lesson 12: Mentally Add and Subtract Tens
- Lesson 13: Center Day 2

Section C: Compare Numbers to 99

- Lesson 14: Let's Compare
- Lesson 15: Greater Than, Less Than
- Lesson 16: Write Comparisons with Symbols
- Lesson 17: Compare and Order Numbers
- Lesson 18: Center Day 3

Section D: Different Ways to Make a Number

- Lesson 19: Make 2-Digit Numbers
- Lesson 20: Make 2-Digit Numbers in Different Ways
- Lesson 21: Compare 2-Digit Numbers Shown in Different Ways
- Lesson 22: Center Day 4
- Lesson 23: What Happens When You Estimate?

Unit 5: Adding within 100

In this unit, students add within 100, using place value and properties of operations in their reasoning.

In a previous unit, students composed, decomposed, and compared numbers within 100. They reasoned about units of tens and ones and represented numbers with connecting cubes, base-ten drawings, expressions, and equations in different ways (for example, 65 = 60 + 5 and 65 = 50 + 15). In this unit, students build on these understandings of place value to find sums.

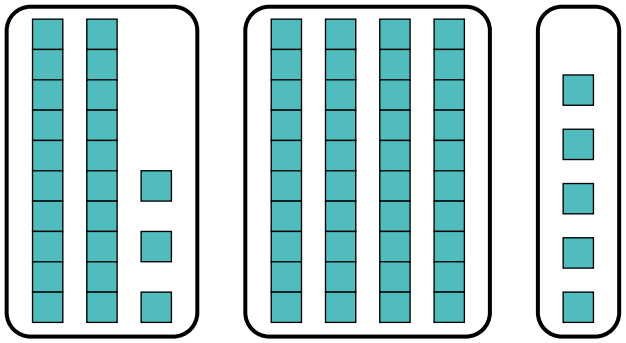
Students begin by adding a two-digit number with another two-digit number or with a one-digit number where it is not necessary to compose a new ten. Then they observe cases in which adding some ones together requires composing a new ten.

Two broad methods for finding sums are explored: adding on by place (adding on tens, then ones), and adding units by place (combining tens with tens and ones with ones).

They also compare methods from earlier work, such as counting on and making use of known sums, including sums of 10.

23 + 45

Add on tens, then add on ones:



Students make sense of methods for adding (especially as it relates to composing a ten when adding ones and ones). They work with a variety of representations—connecting cubes in towers of 10 and singles, base-ten drawings, expressions, and equations. They also use different representations to share their thinking with others.

Expressions and equations are presented horizontally to encourage students to make sense of the numbers and ways of adding rather than apply an algorithm. Eventually, students write equations to represent their thinking. For example, to find the sum of 52 + 46, they might write:

52 + 40 = 92
92 + 6 = 98

or

50 + 40 = 90
2 + 6 = 8
90 + 8 = 98

Students are not expected to write or use equations in any specific way. Even in activities that focus on interpreting and writing equations, students should have continued access to drawings and other tools. Provide access to connecting cubes in towers of 10 and singles throughout the unit.

Section A: Add without Making a Ten

- Lesson 1: Add Tens or Ones
- Lesson 2: How Did You Add?
- Lesson 3: Add It, Explain It
- Lesson 4: Center Day 1

Section B: Make a Ten: Add 1- and 2-Digit Numbers

- Lesson 5: Make a Ten
- Lesson 6: Make a Ten and Make Sense of Equations
- Lesson 7: Does It Make a New Ten?
- Lesson 8: Center Day 2

Section C: Make a Ten: Add within 100

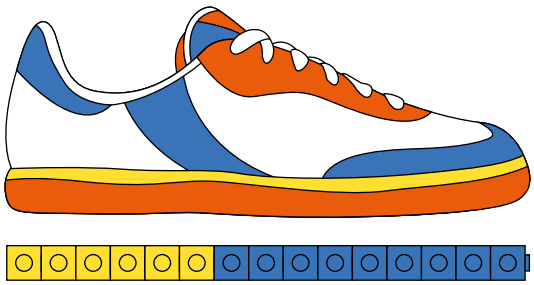
- Lesson 9: Add 2 Two-Digit Numbers
- Lesson 10: Tens and Tens, Ones and Ones
- Lesson 11: How Did You Do That?
- Lesson 12: Add It Up
- Lesson 13: Center Day 3
- Lesson 14: Food Drive

Grade 1

Unit 6: Length Measurements within 120 Units

In this unit, students extend their knowledge of linear measurement while continuing to develop their understanding of operations, algebraic thinking, and place value.

In kindergarten, students identified attributes of objects that can be compared, such as length, weight, and capacity. In this unit, students compare the lengths of objects by lining them up at their endpoints, and explore ways to compare lengths of two objects that cannot be lined up.



From there, students transition to the idea of iterating length units, or using the same length unit, to measure the lengths of objects and to communicate measurements clearly.

Students begin by using the length of a connecting cube as a unit of measurement. Because connecting cubes snap together, students can focus on counting the length of the cubes without worrying about any gaps or overlaps in the units.

Later, students measure with length units that don't connect together, such as paper clips and centimeter cubes (small cubes). Throughout the unit, students do not use formal units of length, and therefore centimeter cubes are referred to as small cubes. Students develop precision as they make sure that there are no gaps or overlap in the units used to measure.

Students measure some lengths by iterating small units, yielding measurements of over 100 length units. Students consider how to count and represent these larger groups of units—up to 120—with a written number. They use familiar representations (connecting cubes and base-ten drawings) to recognize 100 as 10 tens, but do not consider the unit of a hundred until grade 2.

Later in the unit, students solve problems in various contexts, including measurement. They revisit Compare, Difference Unknown story problems and learn to solve Compare, Bigger Unknown and Smaller Unknown problems about lengths. Next, students are introduced to a new problem type: Take From, Start Unknown. They practice solving all story problems types with unknowns in all positions.

Section A: From Direct to Indirect Comparisons

- Lesson 1: Compare Lengths
- Lesson 2: Compare the Lengths of Objects Indirectly
- Lesson 3: Choose Objects to Compare Length Indirectly
- Lesson 4: Center Day 1

Section B: Measure by Iterating up to 120 Length Units

- Lesson 5: Measure with Connecting Cubes
- Lesson 6: Measure with Paper Clips
- Lesson 7: Measure Length with Different Length Units
- Lesson 8: Groups up to 110
- Lesson 9: Write Numbers to 120
- Lesson 10: Center Day 2

Section C: All Kinds of Story Problems

- Lesson 11: How Long Are Our Shoes?
- Lesson 12: Compare Measurements
- Lesson 13: Solve Take From Story Problems
- Lesson 14: Which Equation Matches?
- Lesson 15: Write Equations for Story Problems
- Lesson 16: Center Day 3
- Lesson 17: Class Quilt

Grade 1

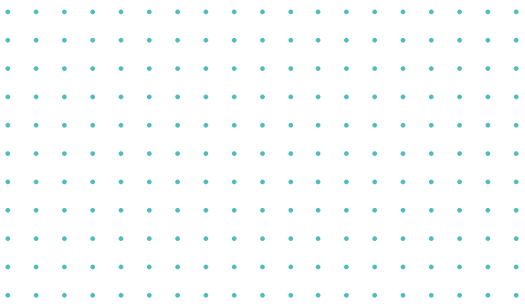
Unit 7: Geometry and Time

In this unit, students focus on geometry and time. They expand their knowledge of two- and three-dimensional shapes, partition shapes into halves and fourths, and tell time to the hour and half hour. Center activities and warm-ups continue to enable students to solidify their work with adding and subtracting within 20 and adding within 100.

In kindergarten, students learned about flat and solid shapes. They named, described, built, and compared shapes. They learned the names of some flat shapes (triangle, circle, square, and rectangle) and some solid shapes (cube, sphere, cylinder, and cone).

Here, students extend those experiences as they work with shape cards, pattern blocks, geoblocks, and solid shapes. They develop increasingly precise vocabulary as they use defining attributes ("squares have four equal-length sides") rather than nondefining attributes ("the square is blue") to describe why a specific shape belongs to a given category. Students should focus on manipulating, comparing, and composing shapes and using their own language, rather than learning the formal definitions of shapes.

Draw 3 shapes that are not rectangles.



How do you know these are not rectangles?

Next, students transition to thinking about how to partition shapes into halves and fourths or quarters. These experiences allow students to learn the language of fractions. They come to understand that each piece gets smaller as the number of equal pieces increases.

In the last section, students tell time to the hour and the half hour. They learn about the hour and minute hands and what an analog clock looks like when the hour hand moves from one hour to the next. The experience of partitioning circles helps students make sense of time on the clock. Students see that a clock shows half hours when the minute hand has moved halfway around the clock from the hour, and the time can be written as "half past" or "___:30."

Section A: Flat and Solid Shapes

- Lesson 1: Shapes That Are Solid
- Lesson 2: Build Shapes from Other Shapes
- Lesson 3: Shapes That Are Flat
- Lesson 4: Draw Flat Shapes
- Lesson 5: Some Triangles, All Triangles
- Lesson 6: Rectangles and Squares
- Lesson 7: Put Together Flat Shapes
- Lesson 8: Center Day 1

Section B: Halves and Quarters

- Lesson 9: Equal Pieces
- Lesson 10: One of the Pieces, All of the Pieces
- Lesson 11: A Bigger Piece
- Lesson 12: Center Day 2

Section C: Tell Time in Hours and Half Hours

- Lesson 13: It's Time to Learn about Clocks
- Lesson 14: Half of the Clock
- Lesson 15: Write Times
- Lesson 16: Hard Times
- Lesson 17: Center Day 3
- Lesson 18: Halves and Fourths in Our World

Grade 1

Unit 8: Putting It All Together

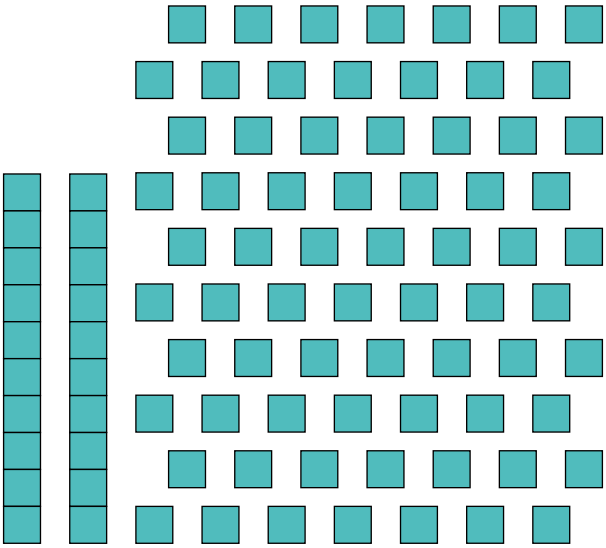
In this unit, students revisit major work and fluency goals of the grade, applying their learning from the year.

In *Section A*, students add and subtract within 20, concurrently working toward the goal of adding and subtracting fluently within 10. In *Section B*, they practice solving story problems of familiar types (those introduced in earlier units). In *Section C*, students count and represent numbers within 120.

Each of these topics is critical for students' readiness for the work in grade 2, in which students will expand their understanding of place value and add and subtract within 100.

What number is shown?

Record an estimate that is too low, too high, and about right.



The sections in this unit are standalone sections, not required to be completed in order. The goal is to offer ample opportunities for students to integrate the knowledge they have gained and to practice skills related to the expected fluencies of the grade.

Section A: Add and Subtract within 20

- Lesson 1: Addition Fluency within 10
- Lesson 2: Relate Addition and Subtraction
- Lesson 3: Add and Subtract within 20

Section B: Story Problems

- Lesson 4: Change Unknown Story Problems
- Lesson 5: Put Together and Take Apart Story Problems
- Lesson 6: Compare Story Problems

Section C: Numbers to 120

- Lesson 7: Count Large Collections
- Lesson 8: Show Me All the Ways
- Lesson 9: Solve Number Riddles
- Lesson 10: Write Number Riddles

Grade 2

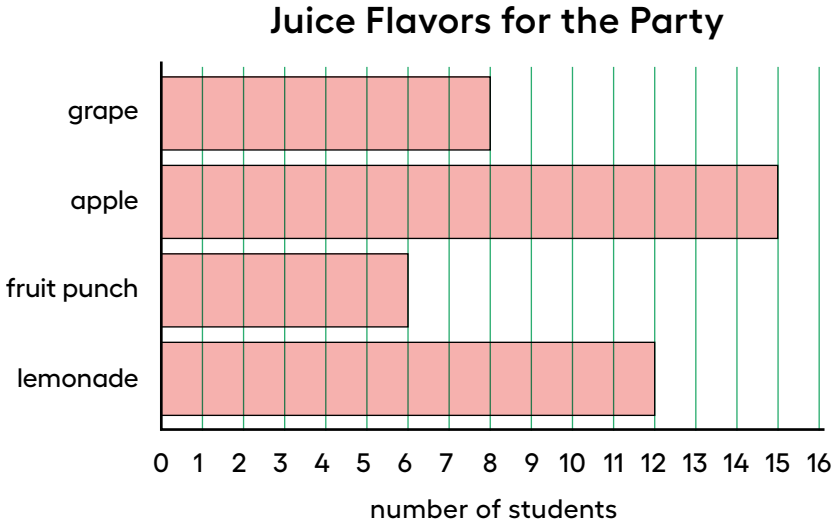
The big ideas in IM Grade 2 include: extending understanding of the base-ten number system; building fluency with addition and subtraction; using standard units of measure; and describing and analyzing shapes. The materials, particularly units that focus on addition and subtraction, include problem types such as Add To, Take From, Put Together or Take Apart, Compare, Result Unknown, and so on. These problem types are based on common addition and subtraction situations, as outlined in Table 1 of the "Mathematics Glossary" section of the *Common Core State Standards* (NGA & CCSSO).

Unit 1: Adding, Subtracting, and Working with Data

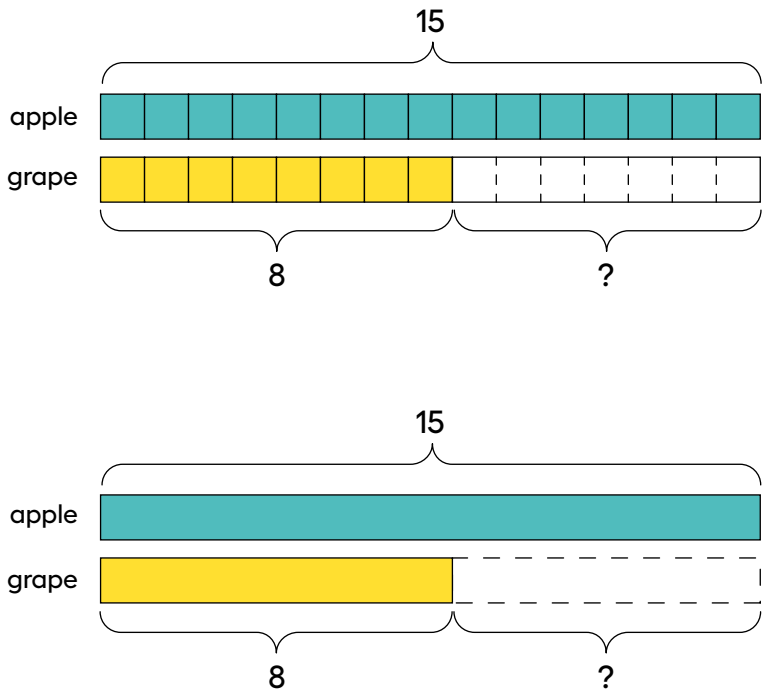
In this unit, students begin the year-long work to develop fluency with sums and differences within 20, building on concepts of addition and subtraction from grade 1. They learn new ways to represent and solve problems involving addition, subtraction, and categorical data.

In grade 1, students added and subtracted within 20 using strategies based on properties of addition and place value. They developed fluency with sums and differences within 10. Students also gained experience in collecting, organizing, and representing categorical data.

In this unit, students are introduced to picture graphs and bar graphs as a way to represent categorical data. They ask and answer questions about situations described by the data. The structure of the bar graphs paves the way for a new representation, the tape diagram.



Grade 2



Students learn that tape diagrams can be used to represent and make sense of problems involving the comparison of two quantities. The diagrams also help to deepen students' understanding of the relationship between addition and subtraction.

This opening unit also offers opportunities to introduce mathematical routines and structures for centers, and to develop a shared understanding of what it means to do math and to be a part of a mathematical community.

Section A: Add and Subtract within 20

- Lesson 1: Add and Subtract within 10
- Lesson 2: Relate Addition and Subtraction within 10
- Lesson 3: Relate Addition and Subtraction within 20
- Lesson 4: Add and Subtract Your Way
- Lesson 5: Add within 50
- Lesson 6: Center Day 1

Section B: Ways to Represent Data

- Lesson 7: Collect and Represent Data
- Lesson 8: Interpret Picture Graphs
- Lesson 9: Interpret Bar Graphs
- Lesson 10: Represent Data Using Picture Graphs and Bar Graphs
- Lesson 11: Questions about Data
- Lesson 12: Center Day 2

Section C: Diagrams to Compare

- Lesson 13: Use Bar Graphs to Compare
- Lesson 14: Use Diagrams to Compare
- Lesson 15: Diagrams with All Kinds of Compare Problems
- Lesson 16: Solve All Kinds of Compare Problems
- Lesson 17: Center Day 3
- Lesson 18: Class Surveys

Grade 2

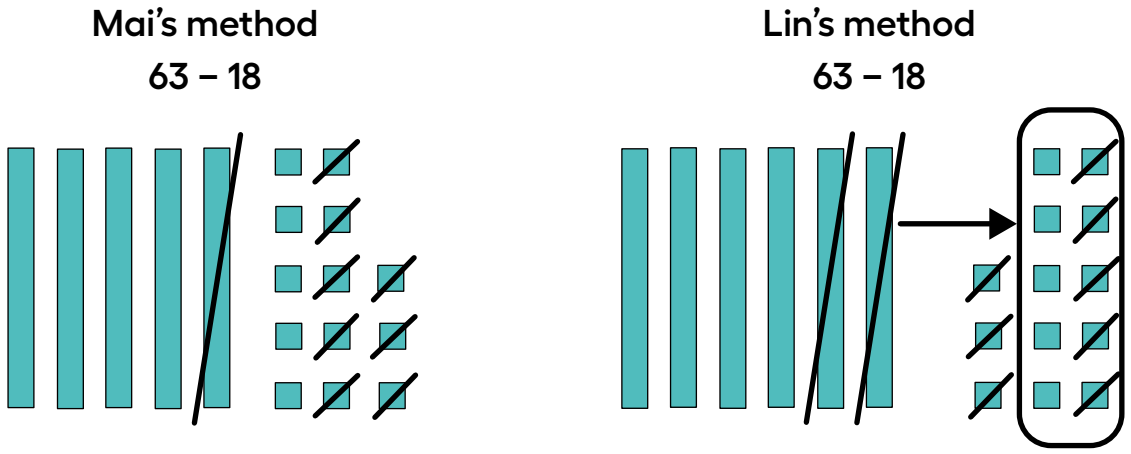
Unit 2: Adding and Subtracting within 100

Previously, students added and subtracted numbers within 100 using strategies they learned in grade 1, such as counting on and counting back, and with the support of tools, such as connecting cubes. In this unit, students add and subtract within 100 using strategies based on place value, the properties of operations, and the relationship between addition and subtraction.

Students begin by using any strategy to find the value of sums and differences that do not involve composing or decomposing a ten. They are then introduced to base-ten blocks as a tool to represent addition and subtraction and move towards strategies that involve composing and decomposing tens.

Students develop their understanding of grouping by place value, and begin to subtract one- and two-digit numbers from two-digit numbers by decomposing a ten as needed. They apply properties of operations and practice reasoning flexibly as they arrange numbers to facilitate addition or subtraction.

For example, students compare Mai's and Lin's methods for finding the value of 63 – 18.



At the end of the unit, students apply their knowledge of addition and subtraction within 100 to solve one- and two-step story problems of all types, with unknowns in all positions. To support reasoning about place value when adding and subtracting, students may choose to use connecting cubes, base-ten blocks, tape diagrams, or other representations learned in earlier units and grades.

Section A: Add and Subtract

- Lesson 1: Add and Subtract to Compare
- Lesson 2: Find the Unknown Addend
- Lesson 3: Add or Subtract to Solve Story Problems
- Lesson 4: Center Day 1

Section B: Decompose to Subtract

- Lesson 5: Subtract Your Way
- Lesson 6: Compare Methods for Subtraction
- Lesson 7: Subtract 2 Digits
- Lesson 8: Different Ways to Decompose
- Lesson 9: Add and Subtract within 100
- Lesson 10: Center Day 2

Section C: Represent and Solve Story Problems

- Lesson 11: How Do You Solve Story Problems?
- Lesson 12: Story Problems and Diagrams
- Lesson 13: Story Problems and Equations
- Lesson 14: Solve It Your Way
- Lesson 15: Does My Estimation Make Sense?
- Lesson 16: Center Day 3
- Lesson 17: Our Store's Inventory

Grade 2

Unit 3: Measuring Length

This unit introduces students to standard units of lengths in the metric and customary systems.

In grade 1, students expressed the lengths of objects in terms of multiple copies of a shorter object laid end to end without gaps or overlaps. The length of the shorter object serves as the unit of measurement.

In this unit, students learn about standard units of length: centimeters, meters, inches, and feet. They examine how different measuring tools represent length units, learn how to use measurement tools, and measure and estimate the lengths of objects. Along the way, students notice that the length of the same object can be described with different measurements and relate this to differences in the size of the unit used to measure.

Throughout the unit, students solve one- and two-step story problems involving addition and subtraction of lengths. To make sense of and solve these problems, they use previously learned strategies for adding and subtracting within 100, including strategies based on place value.

To close the unit, students learn that line plots can be used to represent numerical data. They create and interpret line plots that show measurement data and use them to answer questions about the data.

Students relate the structure of a line plot to the tools they use to measure lengths. This prepares students for the work in the next unit, where they interpret numbers on the number line as lengths from 0. The number line is an essential representation that will be used in future grades and throughout students' mathematical experiences.

Section A: Metric Measurement

- Lesson 1: Standard Units of Measure
- Lesson 2: Measure in Centimeters
- Lesson 3: Create and Use a Ruler
- Lesson 4: Measure and Estimate in Centimeters
- Lesson 5: Measure in Meters
- Lesson 6: Compare Reptile Lengths in Story Problems
- Lesson 7: Center Day 1

Section B: Customary Measurement

- Lesson 8: What Is an Inch?
- Lesson 9: From Feet to Inches
- Lesson 10: Measure with a Torn Tape
- Lesson 11: Saree Silk Stories: Necklaces and Bracelets
- Lesson 12: Saree Silk Stories: Friendship Bracelets
- Lesson 13: Center Day 2

Section C: Line Plots

- Lesson 14: What Is a Line Plot?
- Lesson 15: Create Line Plots
- Lesson 16: Interpret Measurement Data
- Lesson 17: Center Day 3
- Lesson 18: Make a Yardstick

Grade 2

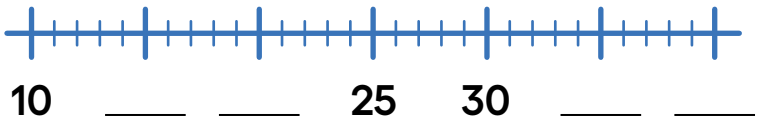
Unit 4: Addition and Subtraction on the Number Line

In this unit, students are introduced to the number line, an essential representation that will be used throughout students' K–12 mathematical experience. They learn to use number lines to represent whole numbers, sums, and differences.

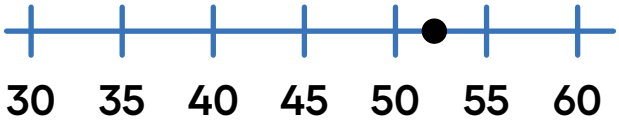
In a previous unit, students learned to measure length with rulers. Here, they see that the tick marks and numbers on the number line are like those on a ruler: both show equally spaced numbers that represent lengths from 0.

Students use this understanding of structure to locate and compare numbers on number lines and to estimate numbers represented by points on number lines.

Locate and label 17 on the number line.



What number could this be? ____



Students then learn conventions for representing addition and subtraction on a number line: using arrows pointing to the right for adding and arrows pointing to the left for subtracting. Students also use number lines to represent addition and subtraction methods discussed in *Number Talks*, such as counting on, counting back by place, and decomposing a number to get to a ten. The reasoning here deepens students' understanding of the relationship between addition and subtraction.

The number lines in this unit show a tick mark for every whole number in the given range, though not all may be labeled with the numeral. As students become more comfortable with this representation, they may draw number lines that show only the numbers needed to solve the problems, which is acceptable.

Section A: The Structure of the Number Line

- Lesson 1: Whole Numbers on the Number Line
- Lesson 2: Features of a Number Line
- Lesson 3: Unlabeled Tick Marks
- Lesson 4: Compare Numbers on a Number Line
- Lesson 5: Estimate on a Number Line
- Lesson 6: Center Day 1

Section B: Add and Subtract on a Number Line

- Lesson 7: Addition and Subtraction on the Number Line
- Lesson 8: Equations on a Number Line
- Lesson 9: The Difference between Numbers
- Lesson 10: Place Value and the Number Line
- Lesson 11: Different Ways to Add and Subtract
- Lesson 12: Equations with Unknowns
- Lesson 13: Represent Story Problems
- Lesson 14: Center Day 2
- Lesson 15: Tall Enough to Ride

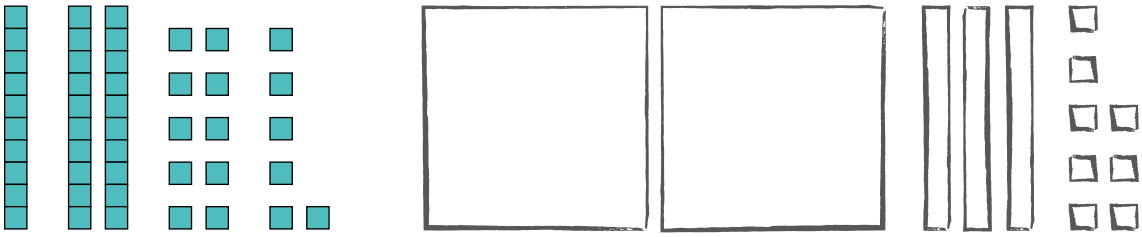
Grade 2

Unit 5: Numbers to 1,000

In this unit, students extend their knowledge of the units in the base-ten system to include hundreds.

In grade 1, students learned that a ten is a unit made up of 10 ones, and two-digit numbers are formed using units of tens and ones. In this unit, students learn that a hundred is a unit made up of 10 tens, and three-digit numbers are formed using units of hundreds, tens, and ones.

To make sense of numbers in different ways and to build flexibility in reasoning with them, students work with a variety of representations: base-ten blocks, base-ten diagrams or drawings, number lines, expressions, and equations.



At the start of the unit, students express a quantity in terms of the number of units represented by base-ten blocks (3 hundreds, 14 tens, 22 ones). They practice composing larger units from smaller units and representing the value using the fewest number of each unit (4 hundreds, 6 tens, 2 ones). They connect the number of units to three-digit numerals (462).

Next, students make sense of three-digit numbers on the number line. In a previous unit, students learned about the structure of the number line by representing whole numbers within 100 as lengths from 0. They get a sense of the relative distance of whole numbers within 1,000 from 0. Students learn to count to 1,000 by skip-counting on a number line by 10 and 100. They also locate, compare, and order three-digit numbers on a number line.

Throughout the unit, the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 are referred to as multiples of 100. The same is true for multiples of 10. “Multiple” is not a word that students are expected to understand or use in grade 2. Students can describe the numbers as a number of tens or hundreds, such as “20 tens” or “3 hundreds.”

Section A: The Values of Three Digits

- Lesson 1: How Do We Compose a Hundred?
- Lesson 2: Make Hundreds
- Lesson 3: Compose 3-Digit Numbers
- Lesson 4: Write 3-Digit Numbers
- Lesson 5: Expanded Form of Numbers
- Lesson 6: Represent Numbers in Different Ways
- Lesson 7: Center Day 1

Section B: Compare and Order Numbers within 1,000

- Lesson 8: Three-Digit Numbers on the Number Line
- Lesson 9: Compare Numbers on the Number Line
- Lesson 10: Place Value Comparisons (Part 1)
- Lesson 11: Place Value Comparisons (Part 2)
- Lesson 12: Order Numbers
- Lesson 13: Center Day 2
- Lesson 14: Around the School

Grade 2

Unit 6: Geometry, Time, and Money

In this unit, students transition from place value and numbers to geometry, time, and money.

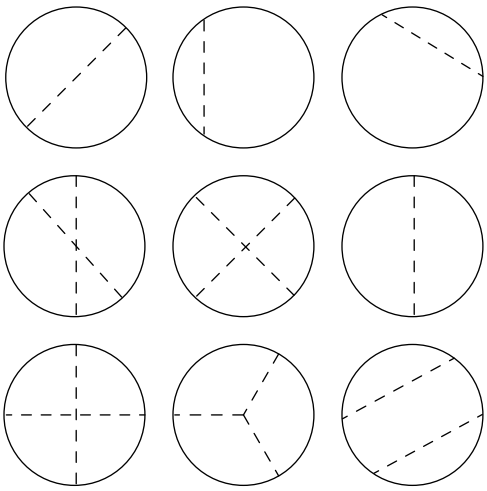
In grade 1, students distinguished between defining and nondefining attributes of shapes, including triangles, rectangles, trapezoids, and circles. Here, they continue to look at attributes of a variety of shapes and see that shapes can be identified by the numbers of sides and vertices (corners). Students then study three-dimensional (solid) shapes, and identify the two-dimensional (flat) shapes that make up the faces of these solid shapes.

Next, students look at ways to partition shapes and create equal shares. They extend their knowledge of halves and fourths (or quarters) from grade 1 to now include thirds.

Students compose larger shapes from smaller equal-size shapes and partition shapes into two, three, and four equal pieces.

As they develop the language of fractions, students also recognize that a whole can be described as two halves, three thirds, or four fourths, and that equal-size pieces of the same whole need not have the same shape.

Which circles are not examples of circles partitioned into halves, thirds, or fourths?



Later, students use their understanding of halves and fourths (or quarters) to tell time. In grade 1, they learned to tell time to the half hour. Here, they relate a quarter of a circle to the features of an analog clock. They use "quarter past" and "quarter till" to describe time, and skip-count to tell time in 5-minute intervals. They also learn to associate the notations "a.m." and "p.m." with their daily activities.

To continue to build fluency with addition and subtraction within 100, students conclude the unit with a money context. They skip-count, count on from the greatest value, and group like coins, and then add or subtract to find the value of a set of coins. Students also solve one- and two-step story problems involving sets of dollars and different coins, and use the symbols $ and ¢.

Section A: Attributes of Shapes

- Lesson 1: Identify and Sort Shapes
- Lesson 2: Draw Shapes
- Lesson 3: Specific Side Lengths
- Lesson 4: Solid Shapes
- Lesson 5: Center Day 1

Section B: Halves, Thirds, and Fourths

- Lesson 6: Compose and Decompose Shapes
- Lesson 7: Make Halves, Thirds, and Fourths
- Lesson 8: Are All Pieces Created Equal?
- Lesson 9: You Took the Whole Thing!
- Lesson 10: Center Day 2

Section C: Time on the Clock

- Lesson 11: Tell Time with Halves and Quarters
- Lesson 12: Count by 5 to Tell Time
- Lesson 13: Is It a.m. or p.m.?
- Lesson 14: Center Day 3

Section D: The Value of Money

- Lesson 15: Identify Pennies, Nickels, and Dimes
- Lesson 16: Identify Quarters
- Lesson 17: Let's Make a Dollar
- Lesson 18: Money Problems
- Lesson 19: More Money Problems
- Lesson 20: Does My Answer Make Sense?
- Lesson 21: Center Day 4
- Lesson 22: Spending Money

Unit 7: Adding and Subtracting within 1,000

In this unit, students add and subtract within 1,000, with and without composing and decomposing a base-ten unit.

Previously, students added and subtracted within 100, using methods such as counting on, counting back, and composing or decomposing a ten. Here, they apply the methods they know and their understanding of place value and three-digit numbers to find sums and differences within 1,000.

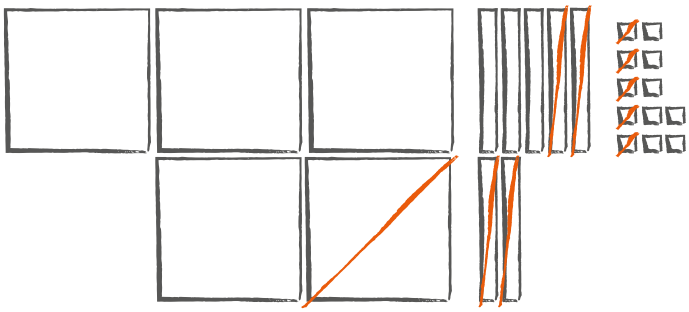
Initially, students add and subtract, without composing or decomposing a ten or a hundred. Instead, they rely on methods based on the relationship between addition and subtraction and the properties of operations. They make sense of sums and differences, using counting sequences, number relationships, and representations (number lines, base-ten blocks, base-ten diagrams, and equations).

As the unit progresses, students work with numbers that prompt them to compose and decompose one or more units, eliciting strategies based on place value. When adding and subtracting by place, students first compose or decompose only a ten, then either a ten or a hundred, and finally both a ten and a hundred. They also make sense of and connect different ways to represent place-value strategies. For example, students make sense of a written method for subtracting 145 from 582 by connecting it to a base-ten diagram and their experiences with base-ten blocks.

How do Jada's equations match Lin's diagram?

Finish Jada's work to find 582 – 145.

Lin's diagram



Jada's equations

500 – 100 =

70
~~80~~ – 40 =

12
~~2~~ – 5 =

Students learn to recognize when composition or decomposition is a useful strategy for adding or subtracting by place. In the later half of the unit, they encounter lessons that encourage them to think flexibly and to use strategies that make sense to them, based on number relationships, properties of operations, and the relationship between addition and subtraction.

Section A: Add and Subtract within 1,000, without Composition or Decomposition

- Lesson 1: Compare, Count On, and Count Back
- Lesson 2: Add and Subtract with Tens and Hundreds
- Lesson 3: Count On or Count Back to Subtract
- Lesson 4: Add and Subtract 3-Digit Numbers in Different Ways
- Lesson 5: Center Day 1

Section B: Add within 1,000, Using Place-Value Strategies

- Lesson 6: Use a Ten to Add within 1,000
- Lesson 7: Compose a Greater Unit
- Lesson 8: Compose Tens and Hundreds to Add
- Lesson 9: Add 3-Digit Numbers
- Lesson 10: Add within 1,000
- Lesson 11: Center Day 2

Section C: Subtract within 1,000, Using Place-Value Strategies

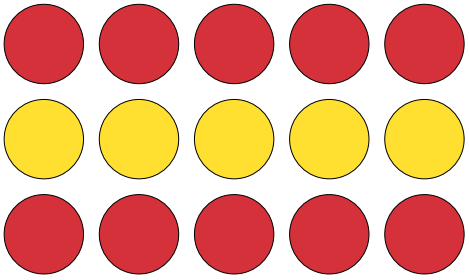
- Lesson 12: Decompose to Subtract
- Lesson 13: Decompose Tens or Hundreds
- Lesson 14: Think Before You Subtract
- Lesson 15: Decompose a Ten and a Hundred to Subtract
- Lesson 16: Subtract within 1,000
- Lesson 17: Does It Make Sense?
- Lesson 18: Center Day 3
- Lesson 19: Book Drive

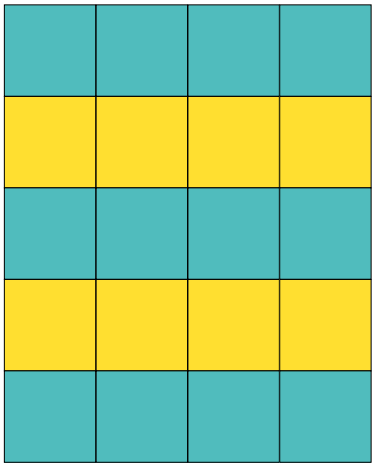
Unit 8: Equal Groups

In this unit, students develop an understanding of equal groups, building on their experiences with skip-counting and with finding the sums of equal addends. The work here serves as the foundation for multiplication and division in grade 3 and beyond.

Students begin by analyzing even and odd numbers of objects. They learn that any even number can be split into 2 equal groups or into groups of 2, with no objects left over. Students use visual patterns to identify whether numbers of objects are even or odd.

Next, students learn about rectangular arrays. They describe arrays using mathematical terms, such as“rows” and “columns.” Students see the total number of objects as a sum of the objects in each row and as a sum of the objects in each column, which they express by writing equations with equal addends. Students also recognize that there are many ways of seeing the equal groups in an array.





Later, students transition from working with arrays that contain discrete objects to equal-size squares within a rectangle. Students build rectangular arrays using inch tiles and partition rectangles into rows and columns of equal-size squares. The work here sets the stage for the concept of area in grade 3.

Section A: Odd and Even

- Lesson 1: Can You Share?
- Lesson 2: Partners Make Pairs
- Lesson 3: Is It Odd or Even?
- Lesson 4: Decompose Even and Odd Numbers
- Lesson 5: Patterns with Even and Odd Numbers
- Lesson 6: Center Day 1

Section B: Rectangular Arrays

- Lesson 7: What Is an Array?
- Lesson 8: Count Columns and Objects in Columns
- Lesson 9: A Sum of Equal Addends
- Lesson 10: Write Expressions and Equations to Represent Arrays
- Lesson 11: Arrays and Rectangles
- Lesson 12: Partition Rectangles into Squares
- Lesson 13: Center Day 2
- Lesson 14: A Band Concert

Unit 9: Putting It All Together

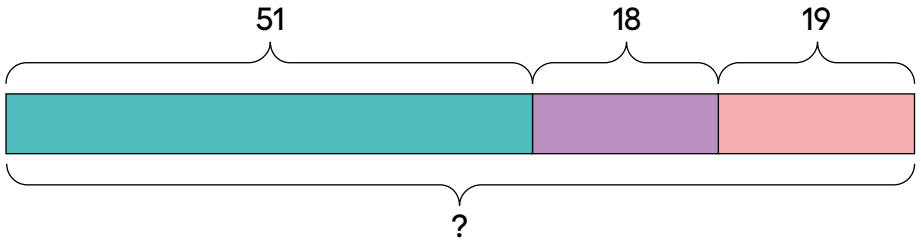
In this unit, students revisit major work and fluency goals of the grade, applying their learning from the year.

The first section gives students a chance to solidify their fluency with addition and subtraction within 20. The second section, students apply methods they used with smaller numbers to add and subtract numbers within 100. Students also revisit numbers within 1,000: composing and decomposing three-digit numbers in different ways and using methods based on place value to find their sums and differences.

In the final section, students interpret, solve, and write story problems involving numbers within 100, which further develops their fluency with addition and subtraction of two-digit numbers. Students work with all problem types with the unknown in all positions.

Clare picked 51 apples. Lin picked 18 apples. Andre picked 19 apples.

Here is the work a student shows to answer to a question about the apples.



51 + 19 = 70

70 + 18 = 88

What is the question?

The sections in this unit are standalone sections, not required to be completed in order. The goal is to offer ample opportunities for students to integrate the knowledge they have gained and to practice skills related to the expected fluencies of the grade.

Grade 2

Section A: Fluency within 20 and Measurement

- Lesson 1: Sums and Differences within 20
- Lesson 2: Fluency Flip
- Lesson 3: Measure on a Map
- Lesson 4: Measure and Plot

Section B: Numbers to 1,000

- Lesson 5: Compose and Decompose Numbers within 1,000
- Lesson 6: Represent Numbers with Expressions
- Lesson 7: Add and Subtract within 1,000
- Lesson 8: Add and Subtract within 100

Section C: Create and Solve Story Problems

- Lesson 9: Sort the Story Problems
- Lesson 10: What's the Question?
- Lesson 11: All about Tape Diagrams
- Lesson 12: What's the Story?
- Lesson 13: Let's Solve Our Story Problems

Grade 3

The big ideas in IM Grade 3 include: developing understanding of multiplication and division, and strategies for multiplication and division within 100; developing understanding of fractions, especially unit fractions (fractions with numerator 1); developing understanding of the structure of rectangular arrays and of area; and describing and analyzing two-dimensional shapes.

Unit 1: Introducing Multiplication

In this unit, students interpret and represent data on scaled picture graphs and scaled bar graphs. Then they learn the concept of multiplication.

This is the first of four units that focus on multiplication. In this unit, students explore scaled picture graphs and bar graphs as an entry point for learning about equal-size groups and multiplication.

In grade 2, students analyzed picture graphs in which one picture represented one object and bar graphs that were scaled by single units. Here, students encounter picture graphs in which each picture represents more than one object and bar graphs that are scaled by 2, 5, or 10 units. The idea that one picture can represent multiple objects helps to introduce the idea of equal-size groups.

Students learn that multiplication can mean finding the total number of objects in a groups of b objects each, and can be represented by $a \times b$ They then relate the idea of equal groups and the expression $a \times b$ to the rows and columns of an array. In working with arrays, students begin to notice the commutative property of multiplication.

In all cases, students make sense of the meaning of multiplication expressions before finding their value and before writing equations that relate two factors and a product.

Later in the unit, students see situations in which the total number of objects is known but either the number of groups or the size of each group is not known. Problems with a missing factor offer students a preview to division.

Throughout the unit, students should have access to connecting cubes or counters, as they may choose to use such tools to represent and solve problems.

Section A: Interpret and Represent Data in Scaled Graphs

- Lesson 1: Make Sense of Data
- Lesson 2: Represent Data and Solve Problems
- Lesson 3: Scaled Picture Graphs
- Lesson 4: Create Scaled Picture Graphs
- Lesson 5: Represent Data in Scaled Bar Graphs
- Lesson 6: Choose a Scale
- Lesson 7: Answer Questions about Scaled Bar Graphs
- Lesson 8: More Questions about Scaled Bar Graphs

Section B: From Graphs to Multiplication

- Lesson 9: Multiplication for Equal Groups
- Lesson 10: Situations, Drawings, and Diagrams, Oh My!
- Lesson 11: Multiplication Expressions
- Lesson 12: Represent and Solve Multiplication Problems
- Lesson 13: Multiplication Equations
- Lesson 14: Write and Solve Equations with Unknowns
- Lesson 15: More Factors, More Problems

Section C: Represent Multiplication with Arrays and the Commutative Property

- Lesson 16: Arrange Objects into Arrays
- Lesson 17: Match and Draw Arrays
- Lesson 18: Represent Arrays with Expressions
- Lesson 19: Solve Problems Involving Arrays
- Lesson 20: The Commutative Property
- Lesson 21: Game Night Seating Plan

Grade 3

Unit 2: Area and Multiplication

In this unit, students encounter the concept of area, relate the area of a rectangle to multiplication, and solve problems involving area.

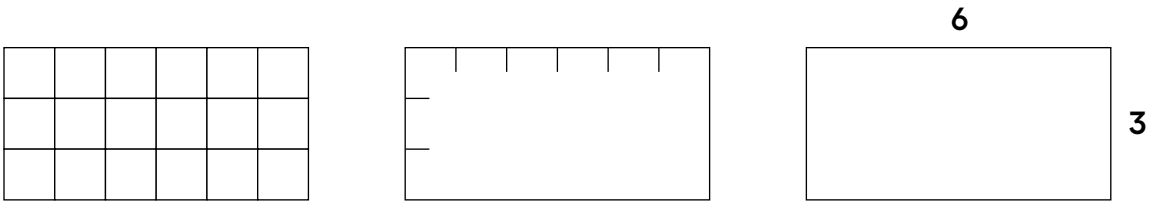
In grade 2, students explored attributes of shapes, such as number of sides, number of vertices, and lengths of sides. They measured and compared lengths (including side lengths of shapes).

In this unit, students make sense of another attribute of shapes: a measure of how much space a shape covers. They begin informally, by comparing two shapes and deciding which one covers more space. Later, they compare more precisely by tiling shapes with pattern blocks and square tiles. Students learn that the area of a flat figure is the number of square units that cover it without gaps or overlaps.

Students then focus on the area of a rectangle. They notice that a rectangle tiled with squares forms an array, with the rows and the columns as equal-size groups. This observation allows them to connect the area of a rectangle to multiplication—as a product of the number of rows and the number of squares per row.

To transition from counting to multiplying side lengths, students reason about area, using increasingly more abstract representations. They begin with tiled or gridded rectangles, move to partially gridded rectangles or those with marked sides, and end with rectangles labeled with their side lengths.

$$6 \times 3 = 18$$



Students also learn some standard units of area—square inch, square centimeter, square foot, and square meter—and solve real-world problems involving the areas of rectangles.

Later in the unit, students find the area and the unknown side lengths of figures composed of non-overlapping rectangles. This work includes cases with two non-overlapping rectangles that share one side, which lays the groundwork for understanding the distributive property of multiplication in a later unit.

Section A: Concepts of Area Measurement

- Lesson 1: What Is Area?
- Lesson 2: How Do We Measure Area?
- Lesson 3: Tile Rectangles
- Lesson 4: Area of a Rectangle

Section B: Relate Area to Multiplication

- Lesson 5: Represent Products as Areas
- Lesson 6: Different Square Units (Part 1)
- Lesson 7: Different Square Units (Part 2)
- Lesson 8: Area of a Rectangle without a Grid
- Lesson 9: Measure to Find the Area
- Lesson 10: Solve Area Problems
- Lesson 11: Area and the Multiplication Table

Section C: Find the Area of a Figure Composed of Rectangles

- Lesson 12: Area and Addition
- Lesson 13: Find the Area of a Figure
- Lesson 14: Find the Area of a Figure with Unknown Side Lengths
- Lesson 15: New Room

Grade 3

Unit 3: Wrapping Up Addition and Subtraction within 1,000

In this unit, students work toward the goal of fluently adding and subtracting within 1,000. They use mental math strategies developed in grade 2, and learn algorithms based on place value.

In grade 2, students added and subtracted within 1,000, using strategies based on place value, properties of operations, and the relationship between addition and subtraction. When students combine hundreds, tens, and ones, they use place-value understanding. When they decompose numbers to add or subtract, they rely on the commutative and associative properties. When students count up to subtract, they use the relationship between addition and subtraction.

To move toward fluency, students learn a few different algorithms that work with any numbers and are generalizable to greater numbers and decimals. Students work with a variety of algorithms, starting with those that show expanded form, and moving toward algorithms that are more streamlined and closer to the standard algorithm.

$$\begin{array}{rr} & 300 + 30 + 7 \\ + & 200 + 30 + 6 \\ \hline & 500 + 60 + 13 \end{array} \qquad \begin{array}{rr} & \overset{60}{\cancel{70}} + \overset{13}{\cancel{3}} \\ - & 200 + 30 + 6 \\ \hline & 300 + 30 + 7 \end{array}$$

Students explore various algorithms but are not required to use a specific one. They should, however, move from the strategy-based work of grade 2 to algorithm-based work, to set the stage for using the standard algorithm in grade 4. If students begin the unit with knowledge of the standard algorithm, it is still important for them to make sense of the place-value basis of the algorithm.

Understanding of place value also comes into play as students round numbers to the nearest multiples of 10 and of 100. Students do not need to know a formal definition of "multiples" until grade 4. At this point, it is enough to recognize that a multiple of 10 is a number called out when counting by 10, or the total in a whole-number of tens (such as 8 tens). Likewise, a multiple of 100 is a number called out when counting by 100, or the total in a whole-number of hundreds (such as 6 hundreds). Students use rounding to estimate answers to two-step problems and to determine if answers are reasonable.

Section A: Add within 1,000

- Lesson 1: Represent Numbers in Different Ways
- Lesson 2: Addition and Subtraction Situations
- Lesson 3: Add Your Way
- Lesson 4: Introduction to Addition Algorithms
- Lesson 5: Another Addition Algorithm
- Lesson 6: Use Strategies and Algorithms to Add

Section B: Subtract within 1,000

- Lesson 7: Subtract Your Way
- Lesson 8: Subtraction Algorithms (Part 1)
- Lesson 9: Subtraction Algorithms (Part 2)
- Lesson 10: Subtraction Algorithms (Part 3)
- Lesson 11: Analyze Subtraction Algorithms
- Lesson 12: Subtract Strategically

Section C: Round within 1,000

- Lesson 13: Multiples of 100
- Lesson 14: Nearest Multiples of 10 and of 100
- Lesson 15: Round to the Nearest Ten and the Nearest Hundred
- Lesson 16: Round and Round Again

Section D: Solve Two-Step Problems

- Lesson 17: Does It Make Sense?
- Lesson 18: Diagrams and Equations for Word Problems
- Lesson 19: Situations and Equations
- Lesson 20: More Practice to Represent and Solve
- Lesson 21: Classroom Supplies

Unit 4: Relating Multiplication to Division

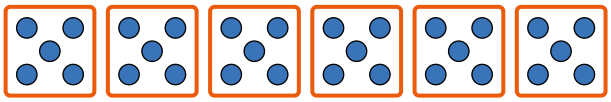
This unit introduces students to the concept of division and its relationship to multiplication.

Previously, students learned that multiplication can be understood in terms of equal-size groups. The expression 5 × 2 can represent the total number of objects when there are 5 groups of 2 objects, or when there are 2 groups of 5 objects.

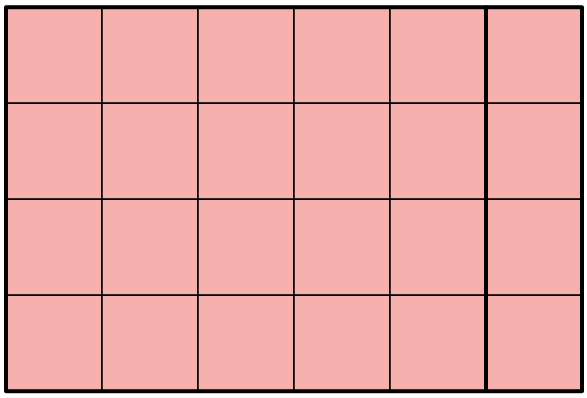
Here, students make sense of division also in terms of equal-size groups. For instance, the expression 30 ÷ 5 can represent putting 30 objects into 5 equal groups, or putting 30 objects into groups of 5. Students see that, in general, dividing can mean finding the size of each group, or finding the number of equal groups.

30 objects put into 5 equal groups

30 objects put into groups of 5



Students use the relationship between multiplication and division to develop fluency with single-digit multiplication and division facts. They continue to reason about products of two numbers in terms of the area of rectangles whose side lengths represent the factors, decomposing side lengths and applying properties of operations along the way.



As they multiply numbers greater than 10, students see that it is helpful to decompose the two-digit factor into tens and ones and distribute the multiplication. For instance, to find the value of 26 × 3, they can decompose the 26 into 20 and 6, and then multiply each by 3.

Toward the end of the unit, students solve two-step problems that involve all four operations. In some situations, students work with expressions that use parentheses to indicate which operation is completed first (for example: 276 + (45 ÷ 5) = ?).

Section A: What Is Division?

- Lesson 1: How Many Groups?
- Lesson 2: How Many in Each Group?
- Lesson 3: Division Situation Drawings
- Lesson 4: Interpret Division Expressions
- Lesson 5: Write Division Expressions

Section B: Relate Multiplication and Division

- Lesson 6: Division as an Unknown Factor
- Lesson 7: Relate Multiplication and Division
- Lesson 8: Relate Quotients to Familiar Products
- Lesson 9: Patterns in the Multiplication Table
- Lesson 10: Explore Multiplication Strategies with Rectangles
- Lesson 11: Multiplication Strategies on Ungridded Rectangles

Section C: Multiplying Greater Numbers

- Lesson 12: Multiply Multiples of 10
- Lesson 13: Solve Problems with Equal Groups
- Lesson 14: Ways to Represent Multiplication of Teen Numbers
- Lesson 15: Equal Groups, Greater Numbers
- Lesson 16: Multiply Numbers Greater than 20
- Lesson 17: Use the 4 Operations to Solve Problems

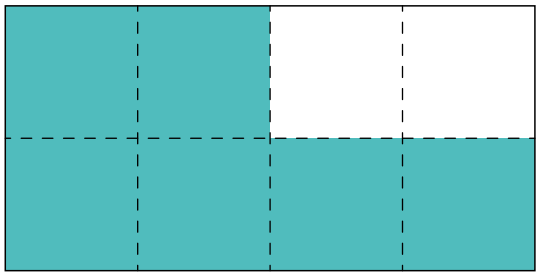
Section D: Dividing Greater Numbers

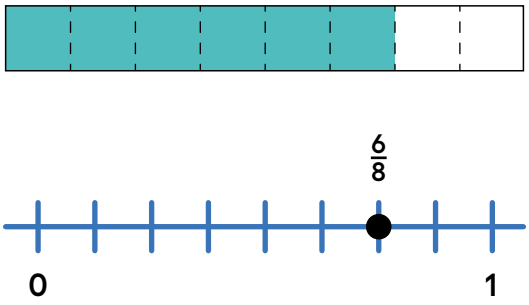
- Lesson 18: Greater Numbers in Equal Groups
- Lesson 19: Ways to Divide Greater Numbers
- Lesson 20: Strategies for Dividing
- Lesson 21: Solve Problems Using the Four Operations
- Lesson 22: School Community Garden

Unit 5: Fractions as Numbers

In this unit, students make sense of fractions as numbers, using various diagrams to represent and reason about fractions, compare their sizes, and relate them to whole numbers. The denominators of the fractions explored here are limited to 2, 3, 4, 6, and 8.

In grade 2, students partitioned circles and rectangles into equal parts and used the language "halves," "thirds," and "fourths." Students begin this unit in a similar way, by reasoning about the sizes of shaded parts in shapes. Next, they create fraction strips by folding strips of paper into equal parts, and later represent the strips as tape diagrams.

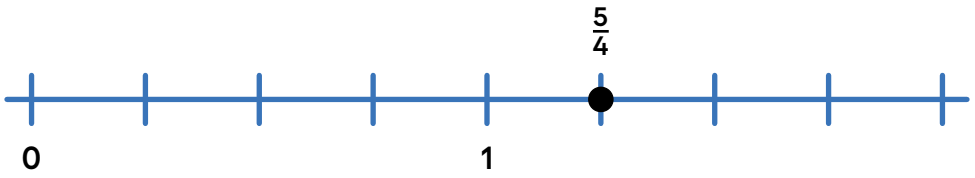




Using fraction strips and tape diagrams to represent fractions prepare students to think about fractions more abstractly as lengths and locations on the number line. This work builds on students' prior experience with representing whole numbers on the number line.

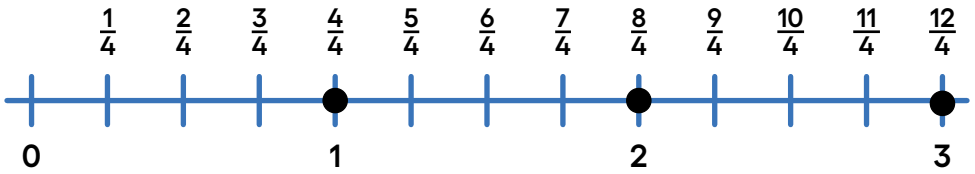
Grade 3

In each representation, students take care to identify 1 whole. This helps them reason about the size of the parts and whether a fraction is less than or greater than 1. (Fractions greater than 1 are not treated as special cases.)



Students then use these representations to learn about equivalent fractions and to compare fractions.

They see that fractions are equivalent if they are the same size or at the same location on the number line, and that some fractions are the same size as whole numbers.



$3 = \frac{12}{4}$

Later in the unit, students compare fractions with the same denominator and those with the same numerator. They recognize that as the numerator gets larger, more parts are counted, and as the denominator gets larger, the size of each part that makes up the whole gets smaller.

Section A: Introduction to Fractions

- Lesson 1: Name the Parts
- Lesson 2: Name Parts as Fractions
- Lesson 3: Non-unit Fractions
- Lesson 4: Build Fractions from Unit Fractions

Section B: Fractions on the Number Line

- Lesson 5: To the Number Line
- Lesson 6: Locate Unit Fractions on the Number Line
- Lesson 7: Non-unit Fractions on the Number Line
- Lesson 8: Fractions and Whole Numbers
- Lesson 9: All Kinds of Numbers on the Number Line

Section C: Equivalent Fractions

- Lesson 10: Equivalent Fractions
- Lesson 11: Generate Equivalent Fractions
- Lesson 12: Equivalent Fractions on a Number Line
- Lesson 13: Whole Numbers and Fractions

Section D: Fraction Comparisons

- Lesson 14: How Do You Compare Fractions?
- Lesson 15: Compare Fractions with the Same Denominator
- Lesson 16: Compare Fractions with the Same Numerator
- Lesson 17: Compare Fractions
- Lesson 18: Plan a Fun Run

Grade 3

Unit 6: Measuring Length, Time, Liquid Volume, and Weight

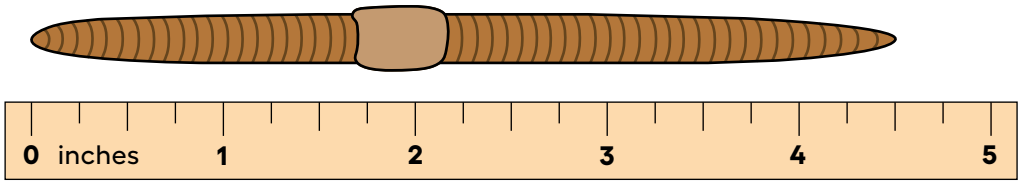
In this unit, students measure length, weight, liquid volume, and time. They begin with a study of length measurement, building on their recent work with fractions.

In grade 2, students measured lengths using informal and formal units to the nearest whole number. They also plotted such length data on line plots. Here, students explore length measurements in halves and fourths of an inch. They use a ruler to collect measurements and then display the data on line plots, learning about mixed numbers and revisiting equivalent fractions along the way.

Kiran says that the worm is $4\frac{2}{4}$ inches long.

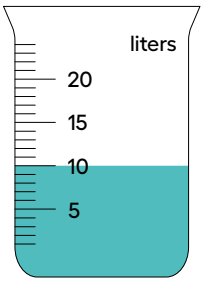
Jada says that the worm is $4\frac{1}{2}$ inches long.

Use the ruler to explain how both of their measurements are correct.



Next, students learn about standard units for measuring weight (kilograms and grams) and liquid volume (liters). To build a sense of the weight of 1 gram or 1 kilogram, students hold common objects, such as paper clips and bottles of water.

To gain familiarity with liters, students measure the volume of a container by filling it with water by the liter and estimate the volume of everyday containers, such as pots, tubs, and buckets. They then use the scale on measurement tools to measure and represent the volume of liquids.



From there, students move on to measure time. In grade 2, they told and wrote time to the nearest 5 minutes. Now, they tell time to the minute, using the relationship between the hour hand and the minute hand to make sense of times such as 3:57 p.m.

In the final section of the unit, students make sense of and solve problems related to all three measurements. The work here allows students to continue to develop their fluency with addition and subtraction within 1,000 and understanding of properties of operations. It also prompts them to use the relationship between multiplication and division to solve problems.

Section A: Measurement Data on Line Plots

- Lesson 1: Measure in Halves of an Inch
- Lesson 2: Measure in Fourths of an Inch
- Lesson 3: Measure in Halves and Fourths of an Inch
- Lesson 4: Interpret Measurement Data on Line Plots
- Lesson 5: Represent Measurement Data on Line Plots

Section B: Weight and Liquid Volume

- Lesson 6: Estimate and Measure Weight
- Lesson 7: Introduction to Liquid Volume
- Lesson 8: Estimate and Measure Liquid Volume

Section C: Problems Involving Time

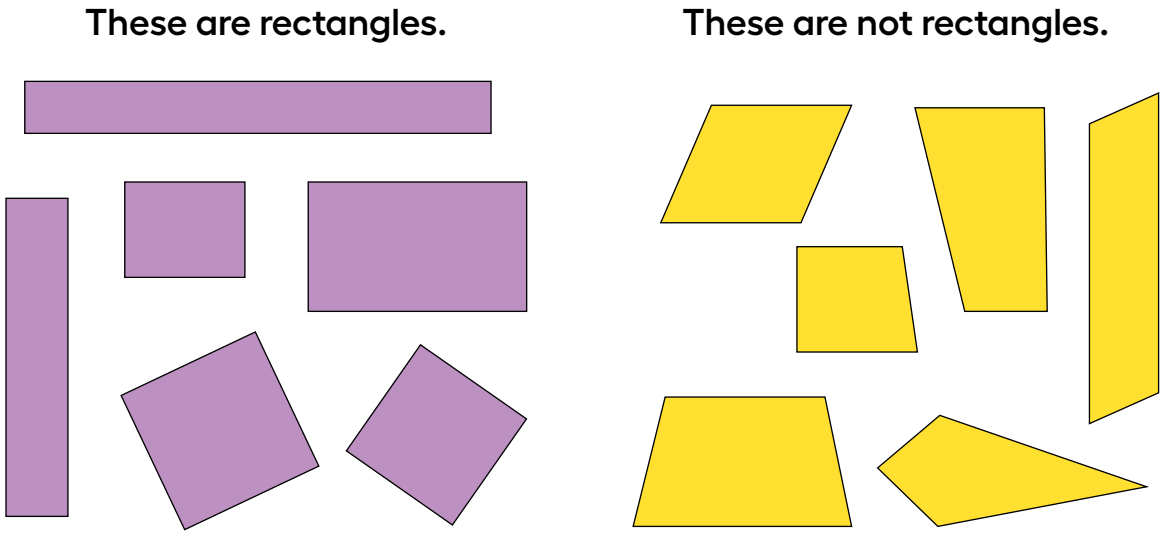
- Lesson 9: Time to the Nearest Minute
- Lesson 10: Solve Problems Involving Time (Part 1)
- Lesson 11: Solve Problems Involving Time (Part 2)

- **Section D: Measurement Problems in Context**
- Lesson 12: Ways to Represent Measurement Situations
- Lesson 13: Problems with Missing Information
- Lesson 14: What Makes Sense in the Problem?
- Lesson 15: Ways to Solve Problems and Show Solutions
- Lesson 16: Design a Game

Unit 7: Two-Dimensional Shapes and Perimeter

In this unit, students reason about attributes of two-dimensional shapes and learn about perimeter.

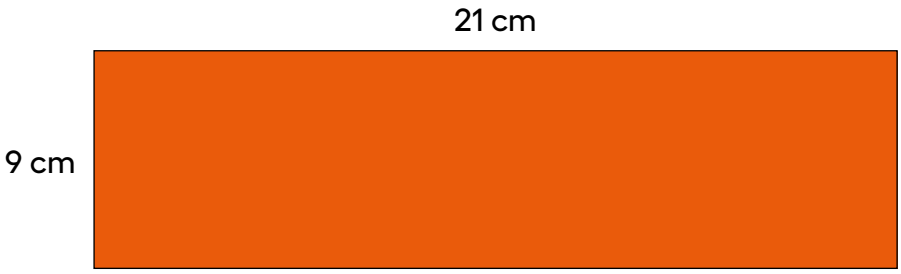
Students learn to describe, compare, and sort two-dimensional shapes in earlier grades. In this unit, students continue to develop language that is increasingly more precise to describe and categorize shapes. Students learn to classify broader categories of shapes (quadrilaterals and triangles) into more specific subcategories based on their attributes. For instance, they study examples and non-examples of rhombuses, rectangles, and squares, to recognize their specific attributes.



Students also expand their knowledge about attributes that can be measured.

Previously, they learned the meaning of area and found the area of rectangles and figures composed of rectangles. In this unit, students learn the meaning of perimeter and find the perimeter of shapes. They consider geometric attributes of shapes (such as opposite sides having the same length) that can help them find perimeter.

Find the perimeter of this rectangle.



As the lessons progress, they consider situations that involve perimeter, and then those that involve both perimeter and area. These lessons aim to distinguish the two attributes (which are commonly confused) and reinforce that perimeter measures length or distance (in length units) and area measures the amount of space covered by a shape (in square units).

At the end of the unit, students solve problems in a variety of contexts. They apply what they learn about geometric attributes of shapes, perimeter, and area, to design a park, and a West African wax print pattern. They then solve problems within the context of their design.

Section A: Reason with Shapes

- Lesson 1: What Attributes Do You See?
- Lesson 2: Attributes of Triangles and Quadrilaterals
- Lesson 3: Attributes that Define Shapes
- Lesson 4: Attributes of Rectangles, Rhombuses, and Squares
- Lesson 5: Attributes of Other Quadrilaterals

Section B: What is Perimeter?

- Lesson 6: Distance around Shapes
- Lesson 7: Same Perimeter, Different Shapes
- Lesson 8: Find the Perimeter
- Lesson 9: Perimeter Problems

Section C: Expanding on Perimeter

- Lesson 10: Problem Solving with Perimeter and Area
- Lesson 11: Rectangles with the Same Perimeter
- Lesson 12: Rectangles with the Same Area

Section D: Design with Perimeter and Area

- Lesson 13: Shapes and Play
- Lesson 14: Wax Prints
- Lesson 15: A Space for Chickens

Grade 3

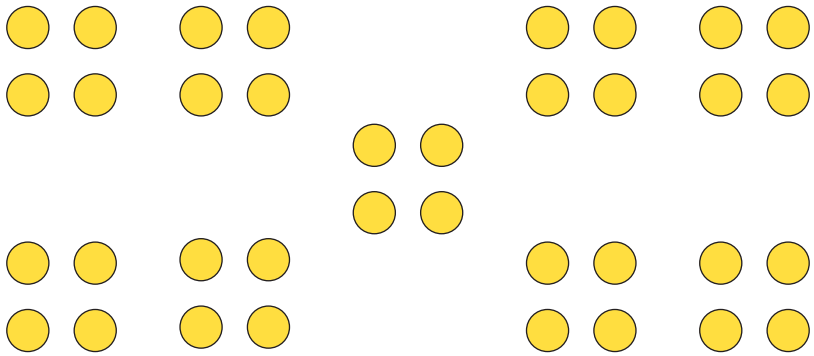
Unit 8: Putting It All Together

In this unit, students revisit major work and fluency goals of the grade, applying their learning from the year.

In Section A, students reinforce what they learned about fractions, their sizes, and their locations on the number line. In Section B, students deepen their understanding of perimeter, area, and scaled graphs by solving problems about measurement and data. Two of the lessons invite students to design a tiny house that meets certain conditions and to calculate the cost for furnishing it.

Section C enables students to work toward multiplication and division fluency goals through games. In Section D, students review major work of the grade as they create activities in the format of the Warm-up routines they have encountered throughout the year (*Notice and Wonder*, *Estimation Exploration*, *Number Talk*, and *How Many Do You See?*).

How many do you see? How do you see them?



The concepts and skills strengthened in this unit prepare students for major work in grade 4: comparing, adding, and subtracting fractions, multiplying and dividing within 1,000, and using the standard algorithm to add and subtract multi-digit numbers within 1 million.

The sections in this unit are standalone sections, with no requirement to be completed in order. Within each section, many lessons also can be completed independently of those preceding them. The goal is to offer ample opportunities for students to integrate the knowledge they have gained and to practice skills related to the expected fluencies of the grade.

Section A: Fraction Fun

- Lesson 1: Estimation Explorations with Fractions
- Lesson 2: Create Your Own Number Line
- Lesson 3: Fractions Round Table

Section B: Measurement and Data

- Lesson 4: Tiny House: Design and Solve
- Lesson 5: Tiny House: Cost
- Lesson 6: Survey the Class, Survey the School
- Lesson 7: Graph and Answer

Section C: Multiplication and Division Games

- Lesson 8: Multiplication Center Day
- Lesson 9: Multiplication Game Day
- Lesson 10: Multiplication and Division
- Lesson 11: Division Game Day

Section D: Create and Design

- Lesson 12: Notice and Wonder
- Lesson 13: How Many Do You See?
- Lesson 14: Estimation Exploration
- Lesson 15: Number Talk

Grade 4

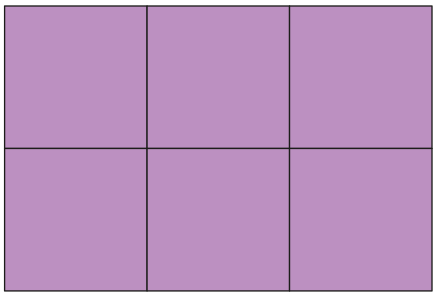
The big ideas in IM Grade 4 include: developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; understanding that geometric figures can be analyzed and classified, based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Unit 1: Factors and Multiples

In this unit, students extend their knowledge of multiplication, division, and the area of a rectangle to deepen their understanding of factors and to learn about multiples.

In grade 3, students learned that they can multiply the two side lengths of a rectangle to find its area, and divide the area by one side length to find the other side length.

To represent these ideas, students used area diagrams, wrote expressions and equations, and learned the terms "factors" and "products."



In this unit, students return to the concept of area to make sense of factors and multiples of numbers. Students find as many pairs of whole-number side lengths as they can given a rectangle with a specific area. They make sense of those side lengths as factor pairs of the whole-number area, and the area as a multiple of each side length.

Students also learn that a number can be classified as prime or composite based on the number of factor pairs it has.

Throughout the unit, students encounter various contexts related to school, gatherings, and celebrations. They are intended to invite conversations about students' lives and experiences. Consider them as opportunities to learn about students as individuals, to foster a positive learning community, and to shape each lesson based on insights about students.

Section A: Understand Factors and Multiples

- Lesson 1: Multiples of a Number
- Lesson 2: Factor Pairs
- Lesson 3: Prime and Composite Numbers
- Lesson 4: Multiplication Practice

Section B: Find Factor Pairs and Multiples

- Lesson 5: More Multiples
- Lesson 6: The Locker Problem
- Lesson 7: Find Factors and Multiples
- Lesson 8: Mondrian's Art

Grade 4

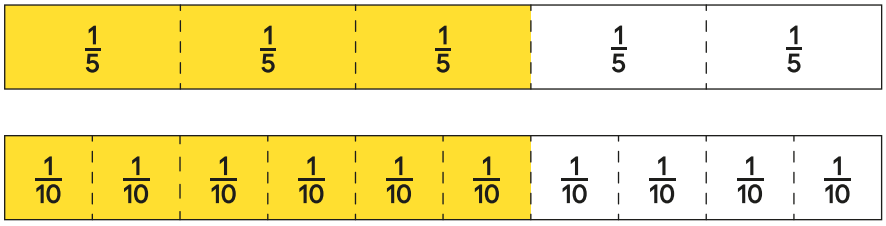
Unit 2: Fraction Equivalence and Comparison

In this unit, students extend their prior understanding of equivalent fractions and comparison of fractions.

In grade 3, students partitioned shapes into parts with equal area and expressed the area of each part as a unit fraction. They learned that any unit fraction $\frac{1}{b}$ results from a 1 partitioned into b equal parts. Students used unit fractions to build non-unit fractions, including fractions greater than 1, and represented them on fraction strips and tape diagrams. The denominators of these fractions were limited to 2, 3, 4, 6, and 8. Students also worked with fractions on a number line, establishing the idea of fractions as numbers and equivalent fractions as the same point on the number line.

Here, students follow a similar progression of representations. They use fraction strips, tape diagrams, and number lines to make sense of the size of fractions, generate equivalent fractions, and compare and order fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Students generalize that a fraction $\frac{a}{b}$ is equivalent to fraction $\frac{(n \times a)}{(n \times b)}$ because each unit fraction is being broken into n times as many equal parts, making the size of the part n times as small $\frac{1}{(n \times b)}$ and the number of parts in the whole n times as many $(n \times a)$. For example, we can see $\frac{3}{5}$ is equivalent to $\frac{6}{10}$ because when each fifth is partitioned into 2 parts, there are 2×3 or 6 shaded parts, twice as many as before, and the size of each part is half as small, $\frac{1}{(2 \times 5)}$ or $\frac{1}{10}$.



As the unit progresses, students use equivalent fractions and benchmarks, such as $\frac{1}{2}$ and 1, to reason about the relative location of fractions on a number line and to compare and order fractions.

Section A: Size and Location of Fractions

- Lesson 1: Representations of Fractions (Part 1)
- Lesson 2: Representations of Fractions (Part 2)
- Lesson 3: Same Denominator or Numerator
- Lesson 4: Same Size, Related Sizes
- Lesson 5: Fractions on Number Lines
- Lesson 6: Relate Fractions to Benchmarks

Section B: Equivalent Fractions

- Lesson 7: Equivalent Fractions
- Lesson 8: Equivalent Fractions on the Number Line
- Lesson 9: Explain Equivalence
- Lesson 10: Use Multiples to Find Equivalent Fractions
- Lesson 11: Use Factors to Find Equivalent Fractions

Section C: Fraction Comparison

- Lesson 12: Ways to Compare Fractions
- Lesson 13: Use Equivalent Fractions to Compare
- Lesson 14: Fraction Comparison Problems
- Lesson 15: Use Common Denominators to Compare
- Lesson 16: Compare and Order Fractions
- Lesson 17: Paper Clip Games

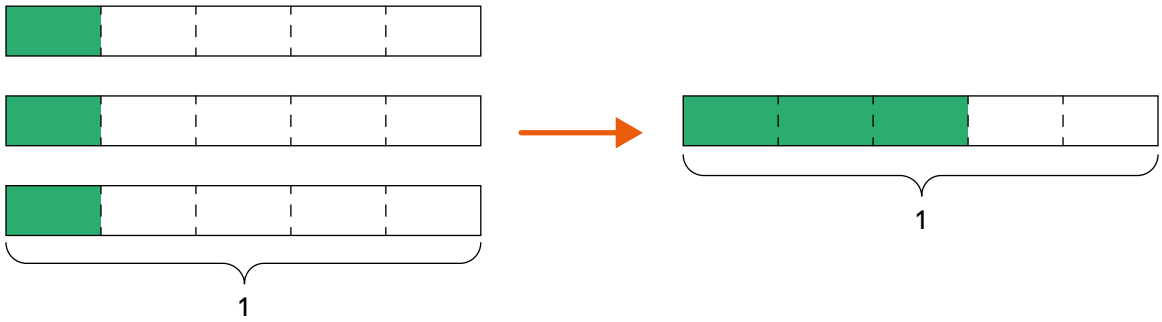
Unit 3: Extending Operations to Fractions

In this unit, students deepen their understanding of how fractions can be composed and decomposed, and they learn about operations on fractions.

In grade 3, students partitioned a whole into equal parts and identified one of the parts as a unit fraction. They learned that non-unit fractions and whole numbers are composed of unit fractions. They used visual fraction models, including tape diagrams and number lines, to represent and compare fractions. In a previous unit, students extended that work and reasoned about fraction equivalence.

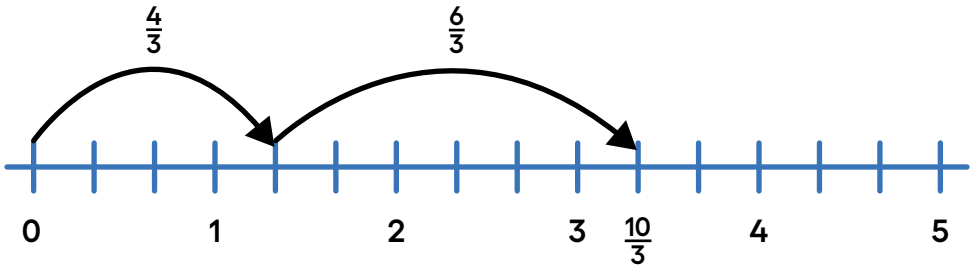
Here students multiply fractions by whole numbers, add and subtract fractions with the same denominator, and add tenths and hundredths. They rely on familiar concepts and representations to do so. For instance, students had represented multiplication on a tape diagram, with equal-size groups and a whole number in each group. Here they use a tape diagram that shows a fraction in each group.

$$3 \times \frac{1}{5} = \frac{3}{5}$$

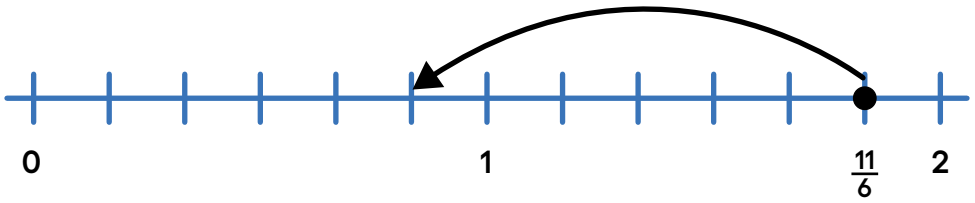


In earlier grades, students used number lines to represent addition and subtraction of whole numbers. Here they use number lines to represent the decomposition of fractions into sums, and to reason about addition and subtraction of fractions with the same denominator, including mixed numbers.

$$\frac{4}{3} + \frac{6}{3} = \frac{10}{3}$$



$$\frac{11}{6} - \frac{7}{6} = \frac{4}{6}$$



Students then apply these skills in the context of measurement and data. They analyze line plots showing fractional lengths and find sums and differences to answer questions about the data.

Lastly students use fraction equivalence to find sums of tenths and hundredths. For instance, to find $\frac{3}{10} + \frac{15}{100}$, they reason that $\frac{3}{10}$ is equivalent to $\frac{30}{100}$, so the sum is $\frac{30}{100} + \frac{15}{100}$, which is $\frac{45}{100}$.

Section A: Equal Groups of Fractions

- Lesson 1: Equal Groups of Unit Fractions
- Lesson 2: Representations of Equal Groups of Fractions
- Lesson 3: Patterns in Multiplication
- Lesson 4: Equal Groups of Non-unit Fractions
- Lesson 5: Equivalent Multiplication Expressions
- Lesson 6: Problems with Equal Groups of Fractions

Section B: Addition and Subtraction of Fractions

- Lesson 7: Fractions as Sums
- Lesson 8: Addition of Fractions
- Lesson 9: Differences of Fractions
- Lesson 10: The Numbers in Subtraction
- Lesson 11: Subtract Fractions Flexibly
- Lesson 12: Sums and Differences of Fractions
- Lesson 13: Fractional Measurements on Line Plots
- Lesson 14: Problems about Fractional Measurement Data

Section C: Addition of Tenths and Hundredths

- Lesson 15: An Assortment of Fractions
- Lesson 16: Add Tenths and Hundredths Together
- Lesson 17: Sums of Tenths and Hundredths
- Lesson 18: A Lot of Fractions to Add
- Lesson 19: Flexible with Fractions
- Lesson 20: Sticky Notes

Grade 4

Unit 4: From Hundredths to Hundred-Thousands

In this unit, students learn to express both small and large numbers in base ten, extending their understanding to include numbers from hundredths to hundred-thousands.

In previous units, students compared, added, subtracted, and wrote equivalent fractions for tenths and hundredths. In this unit, students take a closer look at the relationship between tenths and hundredths and learn to express them in decimal notation. Students analyze and represent fractions on square grids of 100 where the entire grid represents 1. They reason about the size of tenths and hundredths written as decimals, locate decimals on a number line, and compare and order decimals.

Students then explore large numbers. They begin by using base-ten blocks and diagrams to build, read, write, and represent whole numbers beyond 1,000. Students see that ten-thousands are related to thousands in the same way that thousands are related to hundreds, and hundreds are to tens, and tens are to ones.

As they make sense of this structure (MP7), students see that the value of the digit in a place represents ten times the value of the same digit in the place to its right.

Students reason about the size of multi-digit numbers and locate them on number lines. To do so, they need to consider the value of the digits. Students compare, round, and order numbers through 1,000,000. They also use place-value reasoning to add and subtract numbers within 1,000,000 using the standard algorithm.

Throughout the unit, students relate these concepts to real-world contexts and use what they have learned to determine the reasonableness of their responses.

Section A: Decimals with Tenths and Hundredths

- Lesson 1: Decimal Numbers
- Lesson 2: Equivalent Decimals
- Lesson 3: Decimals on Number Lines
- Lesson 4: Compare and Order Decimals
- Lesson 5: Compare and Order Decimals

Section B: Place Value Relationships through 1,000,000

- Lesson 6: How Much is 10,000?
- Lesson 7: Numbers Within 100,000
- Lesson 8: Beyond 100,000
- Lesson 9: Same Digit, Different Value
- Lesson 10: Ten Times As Much
- Lesson 11: Large Numbers on a Number Line

Section C: Compare, Order, and Round

- Lesson 12: Compare Multi-Digit Numbers
- Lesson 13: Order Multi-Digit Numbers
- Lesson 14: Multiples of 10,000 and 100,000
- Lesson 15: The Nearest Multiples of 1,000, 10,000, and 100,000
- Lesson 16: Round Numbers
- Lesson 17: Apply Rounding

Section D: Add and Subtract

- Lesson 18: Standard Algorithm to Add and Subtract
- Lesson 19: Compose and Decompose to Add and Subtract
- Lesson 20: Add and Subtract within 1,000,000
- Lesson 21: Zeros in the Standard Algorithm
- Lesson 22: Solve Problems Involving Large Numbers
- Lesson 23: Bees are Buzzing

Grade 4

Unit 5: Multiplicative Comparison and Measurement

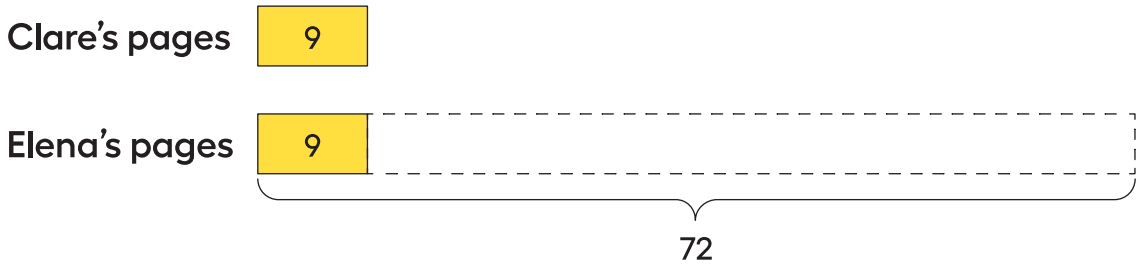
In this unit, students make sense of multiplication as a way to compare quantities. They use this understanding to solve problems about measurement.

In earlier grades, students related two quantities and made an additive comparison, where the key question was "How many more?" Here they make a multiplicative comparison, in which the underlying question is "How many times as many?" For example, if Mai has 3 cubes and Tyler has 18 cubes, we can say that Tyler has 6 times as many cubes as Mai does.

Initially, students reason, using concrete manipulatives and discrete images. Later, they reason more abstractly, using tape diagrams and equations. Comparative language, such as "_____ times as many (or much) as _____" is emphasized, offering students opportunities to attend to precision as they communicate mathematically (MP6).

Write a multiplication equation to compare the pages read by Elena and Clare.

Use a symbol to represent the unknown.



Next, students use the idea and language of multiplicative relationships to learn about various units of length, mass, capacity, and time, and to convert from larger units to smaller units, within the same system of measurement. For example, they describe 1 kilometer as 1,000 times as long as a meter. Students then use their new knowledge to solve measurement problems.

Elena's disc went 3 times as far as Clare's did.
Andre's disc went 4 times as far as Tyler's did.
How far did Elena and Tyler throw the disc?

student	distance
Han	17 yards
Lin	$51\frac{1}{2}$ feet
Clare	$21\frac{1}{3}$ feet
Andre	22 yards 2 feet
Elena	
Tyler	

Section A: Multiplicative Comparison

- Lesson 1: Times as Many
- Lesson 2: Interpret Representations of Multiplicative Comparison
- Lesson 3: Solve Multiplicative Comparison Problems
- Lesson 4: Solve Multiplicative Comparison Problems with Large Numbers
- Lesson 5: One- and Two-Step Comparison Problems
- Lesson 6: Ten Times as Many

Section B: Measurement Conversion

- Lesson 7: Meters and Centimeters
- Lesson 8: Meters and Kilometers
- Lesson 9: Grams and Kilograms, Liters, and Milliliters
- Lesson 10: Multi-Step Measurement Problems
- Lesson 11: Pounds and Ounces
- Lesson 12: Hours, Minutes, and Seconds
- Lesson 13: Multi-Step Measurement Problems with Fractions

Section C: Let's Put It to Work

- Lesson 14: Weight and Capacity Measurements
- Lesson 15: Length Measurements
- Lesson 16: Compare Perimeters of Rectangles
- Lesson 17: More Perimeter Problems
- Lesson 18: Two Truths and a Lie

Unit 6: Multiplying and Dividing Multi-digit Numbers

In this unit, students extend their knowledge of multiplication and division to find products and quotients of multi-digit numbers.

In IM Grade 3, students learned that they could find the value of a product by decomposing one factor into smaller parts, finding partial products, and then combining them. To support this reasoning, they used base-ten diagrams (decomposing two-digit factors into tens and ones) and area diagrams (decomposing one side length into smaller numbers). In this unit, students use those understandings to multiply up to four digits by single-digit numbers, and to multiply a pair of two-digit numbers.

Students begin by generating geometric and numerical patterns that follow a given rule. Students describe features of the patterns that are not explicit in the rule and use ideas and language related to multiplication and multiplicative relationships (such as factors, multiples, double) to explain what they notice. As they generate and analyze patterns, they deepen their understanding of properties of operations.

Next, students reason about products of multi-digit numbers. They transition from using base-ten diagrams to using algorithms to record partial products.

Students learn that they can multiply the factors by place value, one digit at a time, and then organize the partial products vertically. Here are two ways to show partial products for 3,419 × 8.

$$\begin{array}{r} 3,419 \\ \times \quad 8 \\ \hline 72 \\ 80 \\ 3,200 \\ +\ 24,000 \\ \hline \end{array} \qquad \begin{array}{r} 3,419 \\ \times \quad 8 \\ \hline 24,000 \\ 3,200 \\ 80 \\ +\ 72 \\ \hline \end{array}$$

Later in the unit, students divide dividends up to four-digit by single-digit divisors. Students see that it helps to decompose a dividend into smaller numbers and find partial quotients, just as it helps to decompose factors and find partial products.

They also recognize that sometimes it is most productive to decompose a dividend by place value. For instance, to find 465 ÷ 5, we can divide each 400, 60, and 5 by 5.

$$\begin{aligned} 400 \div 5 &= 80 \\ 60 \div 5 &= 12 \\ 5 \div 5 &= 1 \\ \hline 465 \div 5 &= 93 \end{aligned}$$

Students encounter various ways to record the division process, including an algorithm that records partial quotients in a vertical arrangement.

$$\begin{array}{rr|l} & \boxed{93} & \\ & 1 & \\ & 12 & \\ & 80 & \\ \hline 5 & 465 & \\ - & 400 & 5 \times 80 \\ \hline & 65 & \\ - & 60 & 5 \times 12 \\ \hline & 5 & \\ - & 5 & 5 \times 1 \\ \hline & 0 & \end{array}$$

At the end of the unit, students apply their expanded knowledge of operations to solve multi-step problems about measurement in various contexts—calendar days, distance, and population.

Section A: Features of Patterns

- Lesson 1: Patterns that Grow
- Lesson 2: Patterns that Repeat
- Lesson 3: Numerical Patterns
- Lesson 4: More Numerical Patterns

Section B: Multi-Digit Multiplication

- Lesson 5: Products Beyond 100
- Lesson 6: Multiply Two-Digit Numbers and One-Digit Numbers
- Lesson 7: Multiply Three- and Four-Digit Numbers by One-Digit Numbers
- Lesson 8: Multiply 2 Two-Digit Numbers
- Lesson 9: Recording Partial Products: One-Digit and Three- or Four-Digit Factors
- Lesson 10: Using Algorithms with Partial Products: 2 Two-Digit Numbers
- Lesson 11: Partial Products and the Standard Algorithm
- Lesson 12: Solve Problems Involving Multiplication

Section C: Multi-Digit Division

- Lesson 13: Situations Involving Equal-Size Groups
- Lesson 14: Situations Involving Area
- Lesson 15: Base-Ten Blocks to Divide
- Lesson 16: Base-Ten Diagrams to Represent Division
- Lesson 17: An Algorithm with Partial Quotients
- Lesson 18: Use an Algorithm with Partial Quotients
- Lesson 19: Divide with Remainders
- Lesson 20: Interpret Remainders in Division Situations
- Lesson 21: Problems with Remainders

Section D: Let's Put It to Work: Problem Solving with Large Numbers

- Lesson 22: Different Ways to Solve Problems
- Lesson 23: Problems about Perimeter and Area
- Lesson 24: Solve Problems with Many Operations
- Lesson 25: Assess the Reasonableness of Solutions
- Lesson 26: Paper Flower Decorations

Unit 7: Angles and Angle Measurement

In this unit, students deepen and refine their understanding of geometric figures and measurement.

In earlier grades, students learned about two-dimensional shapes and their attributes, which they described informally early on but with increasing precision over time. Here, students formalize their intuitive knowledge about geometric features and draw them. They identify and define some building blocks of geometry (points, lines, rays, and line segments), and develop concepts and language to more precisely describe and reason about other geometric figures.

Jada says Figure A shows an angle, but Figure B does not. Do you agree?

Students analyze cases where lines intersect and where they don't (for example, parallel lines). They learn that an angle is a figure composed of two rays that share the same starting point.

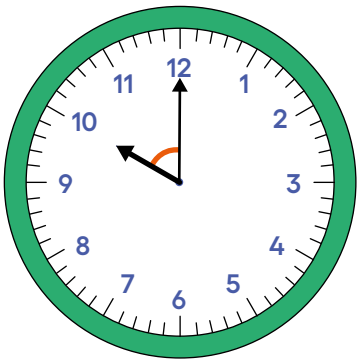
Later, students compare the sizes of angles and consider ways to quantify the comparison. They learn that angles can be measured in terms of the amount of turn one ray makes relative to another ray that shares the same vertex.

Students learn that a 1-degree angle is $\frac{1}{360}$ of a full turn or full circle and can be used to measure angles. They use a protractor to measure angles in whole-number degrees.

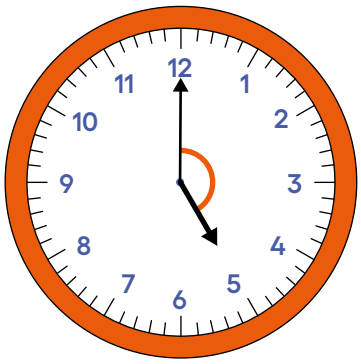
Students also learn that angles are additive. When an angle is composed of multiple non-overlapping parts, the measure of the whole is the sum of the angle measures of the parts. These insights enable students to classify angles (as acute, obtuse, right, or straight) and to solve problems about unknown angle measurements in concrete and abstract contexts.

How many degrees is each marked angle on the clock? Show your reasoning.

A



B



Section A: Points, Lines, Segments, Rays, and Angles

- Lesson 1: How Would You Describe These Figures?
- Lesson 2: Points, Lines, Rays, and Segments
- Lesson 3: Two or More Lines
- Lesson 4: Points and Lines All Around
- Lesson 5: What Is an Angle?

Section B: The Size of an Angle

- Lesson 6: Compare and Describe Angles
- Lesson 7: The Size of an Angle on a Clock
- Lesson 8: The Size of An Angle, in Degrees
- Lesson 9: Use a Protractor to Measure Angles
- Lesson 10: Angle Measurement and Perpendicular Lines
- Lesson 11: Use a Protractor to Draw Angles

Section C: Angle Analysis

- Lesson 12: Types of Angles
- Lesson 13: Find Angle Measurements
- Lesson 14: Reasoning about Angles (Part 1)
- Lesson 15: Reasoning about Angles (Part 2)
- Lesson 16: Angles, Streets, and Steps

Unit 8: Properties of Two-dimensional Shapes

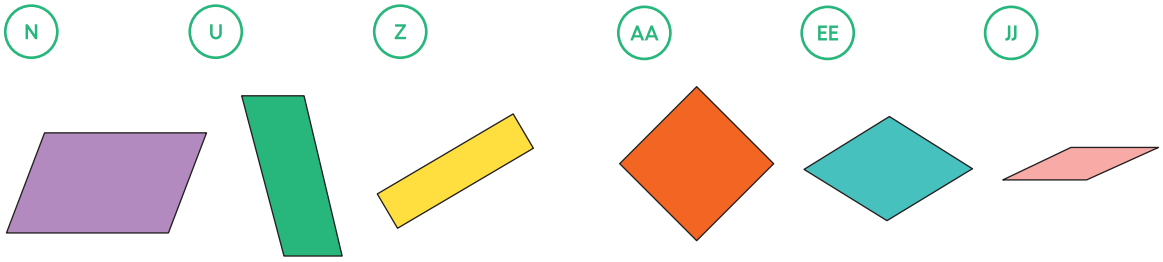
In this unit, students deepen their understanding of the attributes and measurement of two-dimensional figures.

Prior to this unit, students learned about some building blocks of geometry—points, lines, rays, segments, and angles. Students identified parallel and intersecting lines, measured angles, and classified angles based on their measurement. In this unit, they apply those insights to describe and reason about characteristics of shapes.

In the first half of the unit, students analyze and categorize two-dimensional shapes—triangles and quadrilaterals—by their attributes. They classify two-dimensional shapes based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Students also learn about symmetry. They identify line-symmetric figures and draw lines of symmetry.

Quadrilaterals N, U, and Z are parallelograms.

Quadrilaterals AA, EE, and JJ are rhombuses.



Write 4–5 statements about the sides and angles of the quadrilaterals in each set.

Each statement must be true for all the shapes in the set.

The second half of the unit gives students opportunities to apply their understanding of geometric attributes to solve problems about measurements (side lengths, perimeters, and angles).

Included in this unit are three optional lessons that offer opportunities for students to strengthen and extend their understanding of symmetry and other attributes of two-dimensional figures.

Section A: Side Lengths, Angles, and Lines of Symmetry

- Lesson 1: Ways to Look at Figures
- Lesson 2: Ways to Look at Triangles
- Lesson 3: Ways to Look at Quadrilaterals
- Lesson 4: Symmetry in Figures (Part 1)
- Lesson 5: Symmetry in Figures (Part 2)
- Lesson 6: All Kinds of Attributes

Section B: Reason about Attributes to Solve Problems

- Lesson 7: Ways to Find Unknown Length (Part 1)
- Lesson 8: Ways to Find Unknown Length (Part 2)
- Lesson 9: Symmetry in Action
- Lesson 10: Ways to Find Angle Measurements
- Lesson 11: Symmetry in Sports

Grade 4

Unit 9: Putting It All Together

In this unit, students revisit major work and fluency goals of the grade, applying their learning from the year.

In Section A, students reinforce what they learn about comparing fractions, adding and subtracting fractions, and multiplying fractions and whole numbers. In Section B, they strengthen their ability to add and subtract multi-digit numbers fluently, using the standard algorithm. They also multiply and divide numbers by reasoning about place value and practice doing so strategically.

Here are the times of the runners for two teams.

Which team won the relay race?

runner	Diego's team, time (seconds)	Jada's team, time (seconds)
1	$10\frac{25}{100}$	$11\frac{9}{10}$
2	$11\frac{40}{100}$	$9\frac{8}{10}$
3	$9\frac{7}{10}$	$9\frac{84}{100}$
4	$10\frac{5}{100}$	$10\frac{60}{100}$

In Section C, students practice making sense of situations and solving problems that involve reasoning with multiplication and division, including multiplicative comparison and interpreting remainders. In the final section, students review major work of the grade as they create activities in the format of the Warm-up routines they have encountered throughout the year (*Estimation Exploration*, *Number Talk*, and *Which Three Go Together?*).

The sections in this unit stand alone and are not required to be completed in order. Within a section, lessons also can be completed selectively, without completing prior lessons. The goal is to offer ample opportunities for students to integrate the knowledge they have gained and to practice skills related to the expected fluencies of the grade.

Section A: Reason with Fractions

- Lesson 1: Add, Subtract, and Multiply Fractions
- Lesson 2: Sums and Differences of Fractions
- Lesson 3: Stories with Fractions

Section B: Whole-Number Operations

- Lesson 4: Another Look at the Standard Algorithm
- Lesson 5: Multiplication of Multi-digit Numbers
- Lesson 6: What's the Quotient?

Section C: Solve Problems with Multiplication and Division

- Lesson 7: Solve Multiplicative Comparison Problems
- Lesson 8: Solve Problems with Multiplication and Division
- Lesson 9: Create Word Problems

Section D: Creation and Design

- Lesson 10: Estimation Exploration
- Lesson 11: Which Three Go Together
- Lesson 12: Number Talk

Grade 5

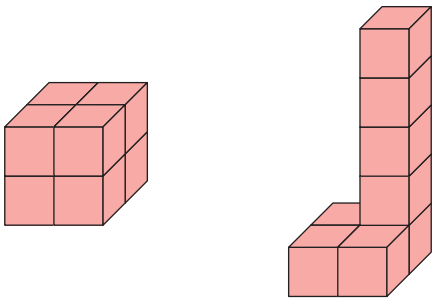
The big ideas in IM Grade 5 include: developing fluency with addition and subtraction of fractions, and developing understanding of multiplication and division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); extending understanding of division to two-digit divisors; developing understanding of operations with decimals to hundredths, and developing fluency with whole- number and decimal operations; and developing understanding of volume.

Unit 1: Finding Volume

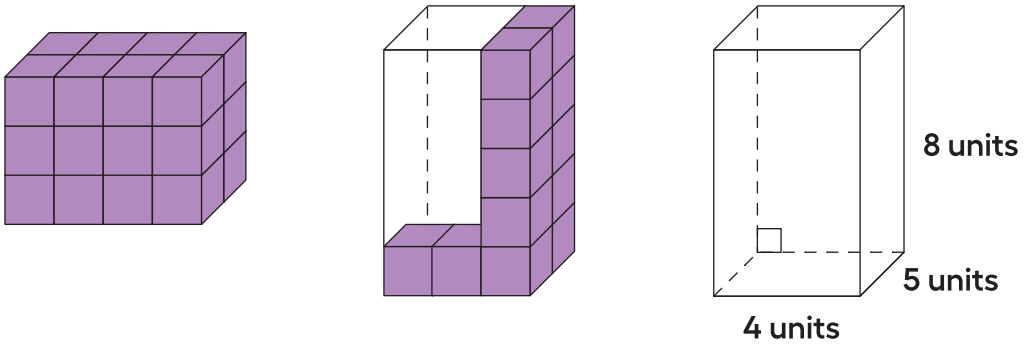
This unit introduces students to the concept of volume by building on their understanding of area and multiplication.

In grade 3, students learned that the area of a two-dimensional figure is the number of square units that cover it, without gaps or overlaps. Students first found areas by counting squares and began to intuit that area is additive. Later, they recognized the area of a rectangle as a product of its side lengths and found the areas of more-complex figures composed of rectangles.

Here, students learn that the volume of a solid figure is the number of unit cubes that fill it without gaps or overlaps. First, they measure volume by counting unit cubes and observe its additive nature. They also learn that different solid figures can have the same volume.

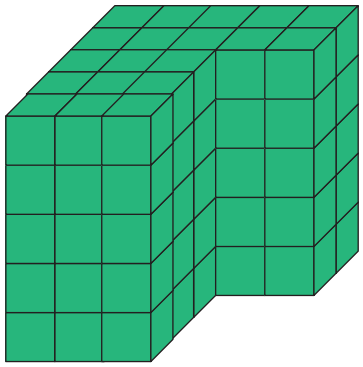


Next, students shift their focus to right rectangular prisms: building them using unit cubes, analyzing their structure, and finding their volumes. They write numerical expressions to represent their reasoning strategies and work with increasingly abstract representations of prisms.



Later, students generalize that the volume of a rectangular prism can be found by multiplying its side measurements (length × width × height), or by multiplying the area of the base by its height (area of the base × height). As they analyze, write, and evaluate different expressions that represent the volume of the same prism, students revisit familiar properties of operations from earlier grades.

Later in the unit, students apply these understandings to find the volume of a solid figure composed of two non-overlapping rectangular prisms and to solve real-world problems involving such figures. In doing so, they also progress from using cubes to using standard units to measure volume.



Section A: Unit Cubes and Volume

- Lesson 1: What Is Volume?
- Lesson 2: Measure Volume
- Lesson 3: Volume of a Prism Drawings
- Lesson 4: Use Layers to Determine Volume

Section B: Expressions for Finding Volume

- Lesson 5: Side Lengths of Rectangular Prisms
- Lesson 6: Expressions for Volume
- Lesson 7: Cubic Units of Measure

Section C: Volumes of Solid Figures

- Lesson 8: Figures Made of Prisms
- Lesson 9: Measure Figures Made from Prisms
- Lesson 10: Represent Volume with Expressions
- Lesson 11: All Kinds of Prisms
- Lesson 12: Tons and Tons of Garbage

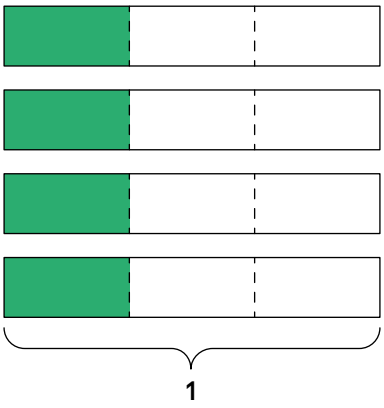
Unit 2: Fractions as Quotients and Fraction Multiplication

In this unit, students learn to interpret a fraction as a quotient and extend their understanding of multiplication of a whole number and a fraction.

In IM Grade 3, students made sense of multiplication and division of whole numbers in terms of equal-size groups. In IM Grade 4, they used multiplication to represent equal-size groups with a fractional amount in each group and to express comparison.

For instance, $4 \times \frac{1}{3}$ can represent "4 groups of $\frac{1}{3}$" or "4 times as much as $\frac{1}{3}$."

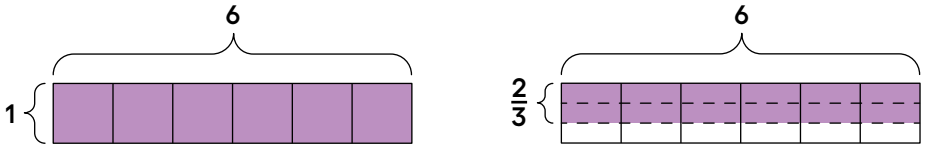
The amount in both situations can be represented by the shaded parts of a diagram like this:



Students learn that a fraction like $\frac{4}{3}$ can also represent:

- A division situation, where 4 objects are being shared equally by 3 people, or $4 \div 3$.
- A fraction of a group, in this case, $\frac{1}{3}$ of a group of 4 objects, or $\frac{1}{3} \times 4$.

Students also interpret the product of a whole number and a fraction in terms of the side lengths of a rectangle. The expression 6×1 represents the area of a rectangle that is 6 units by 1 unit. In the same way, $6 \times \frac{2}{3}$ represents a rectangle that is 6 units by $\frac{2}{3}$ unit.



The commutative and associative properties become evident as students connect different expressions to the same diagram. The distributive property is used as students multiply a whole number and a fraction written as a mixed number, for instance: $2 \times 3\frac{2}{5} = (2 \times 3) + \left(2 \times \frac{2}{5}\right)$.

Throughout this unit, it is assumed that the sharing is always equal sharing, whether explicitly stated or not. For example, in the situation above, 4 objects are being shared equally by 3 people.

Section A: Fractions as Quotients

- Lesson 1: Share Sandwiches
- Lesson 2: Share More Sandwiches
- Lesson 3: Interpret Equations
- Lesson 4: Division Situations
- Lesson 5: Relate Division and Fractions

Section B: Fractions of Whole Numbers

- Lesson 6: Relate Division and Multiplication
- Lesson 7: Divide to Multiply Unit Fractions
- Lesson 8: Divide to Multiply Non-Unit Fractions

Section C: Area and Fractional Side Lengths

- Lesson 9: Relate Area to Multiplication
- Lesson 10: Fractional Side Lengths Less than 1
- Lesson 11: Fractional Side Lengths Greater than 1
- Lesson 12: Decompose Area
- Lesson 13: Area and Properties of Operations
- Lesson 14: Area Situations
- Lesson 15: Multiply More Fractions
- Lesson 16: Estimate Products
- Lesson 17: Mosaic Pictures

Grade 5

Unit 3: Multiplying and Dividing Fractions

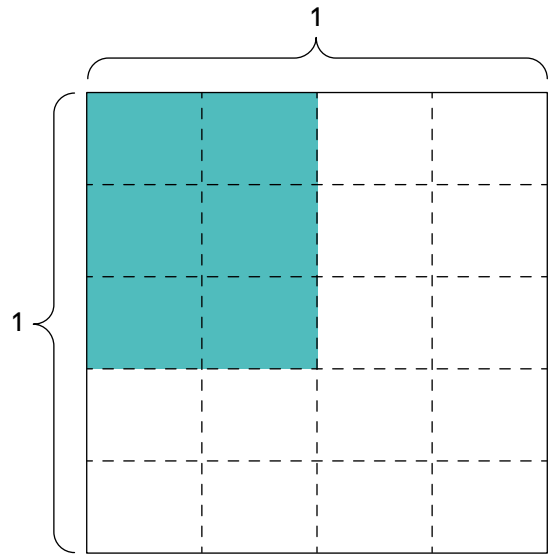
In this unit, students find the product of two fractions, divide a whole number by a unit fraction, and divide a unit fraction by a whole number.

Previously, students made sense of multiplication of a whole number and a fraction in terms of the side lengths and area of a rectangle. In this unit, students make sense of multiplication of two fractions the same way. Students interpret area diagrams with two unit fractions for their side lengths, a unit fraction and a non-unit fraction, and then two non-unit fractions.

Through repeated reasoning, students notice regularity in the value of the product (MP8). They generalize that it can be found by multiplying the numerators and multiplying the denominators of the factors:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

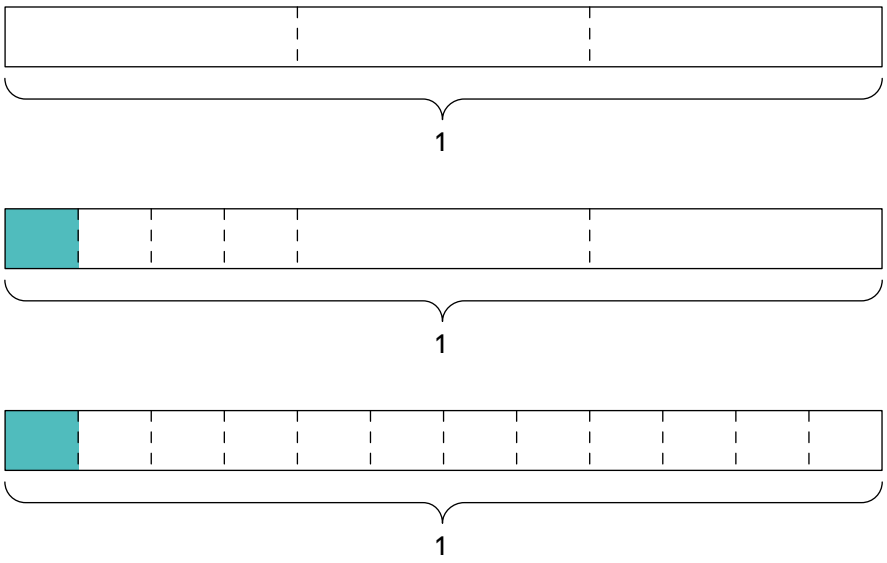
For example, $\frac{2}{4} \times \frac{3}{5}$ is $\frac{2 \times 3}{4 \times 5}$ because there are 4×5 equal parts in the whole square and 2×3 parts are shaded.



Next, students make sense of division situations and expressions that involve a whole number and a unit fraction. They recall that division can be understood in terms of finding the number of equal-size groups or finding the size of each group.

For instance, students interpret $\frac{1}{3} \div 4$ to mean finding the size of one part if $\frac{1}{3}$ is split into 4 equal parts, and $4 \div \frac{1}{3}$ to mean finding how many $\frac{1}{3}$s are in 4.

Students consider how changing the dividend or the divisor changes the value of the quotients and look for patterns (MP8). They use tape diagrams to represent and reason about division situations and expressions.



Later in the unit, students apply what they learned to solve problems. The relationship between multiplication and division is reinforced when they notice that both operations can be used to solve the same problem.

Section A: Fraction Multiplication

- Lesson 1: One Piece of One Part
- Lesson 2: Represent Unit Fraction Multiplication
- Lesson 3: Multiply Unit Fractions
- Lesson 4: Situations about Multiplying Fractions
- Lesson 5: Multiply a Unit Fraction by a Non-Unit Fraction
- Lesson 6: Multiply Fractions
- Lesson 7: Generalize Fraction Multiplication
- Lesson 8: Work with Mixed Numbers
- Lesson 9: Apply Fraction Multiplication

Section B: Fraction Division

- Lesson 10: Concepts of Division
- Lesson 11: Divide Unit Fractions by Whole Numbers
- Lesson 12: Represent Division of Unit Fractions by Whole Numbers
- Lesson 13: Divide Whole Numbers by Unit Fractions
- Lesson 14: Represent Division of Whole Numbers by Unit Fractions
- Lesson 15: Fraction Division Situations
- Lesson 16: Reason about Quotients

Section C: Problem Solving with Fractions

- Lesson 17: Fraction Multiplication and Division Situations
- Lesson 18: Represent Situations with Multiplication and Division
- Lesson 19: Fraction Games
- Lesson 20: Recipes and Fractions

Grade 5

Unit 4: Wrapping Up Multiplication and Division with Multi-digit Numbers

In this unit, students multiply multi-digit whole numbers, using the standard algorithm, and begin working toward end-of-grade expectations for fluency. They also find whole-number quotients, with up to four-digit dividends and two-digit divisors.

In IM Grade 4, students used strategies based on place value and the properties of operations to multiply a whole number of up to four digits by a one-digit whole number, and to multiply a pair of two-digit numbers. They decomposed the factors by place value, and used diagrams and algorithms using partial products to record their reasoning.

Here, students build on those strategies to make sense of the standard algorithm for multiplication. They recognize that it also is based on place value but records the partial products in a condensed way.

Han and Elena used different algorithms to find the value of 3×318.

Han

$$\begin{array}{r} 318 \\ \times \quad 3 \\ \hline 24 \\ 30 \\ +\ 900 \\ \hline 954 \end{array}$$

Elena

$$\begin{array}{r} 2 \\ 318 \\ \times \quad 3 \\ \hline 954 \end{array}$$

Explain to your partner what Han and Elena did. What does the 2 represent in Elena's algorithm?

In grade 4, students also found whole-number quotients, using place-value strategies and the relationship between multiplication and division. They decomposed dividends in various ways and found partial quotients. The numbers they encountered then were limited to four-digit dividends and one-digit divisors. In this unit, they extend that work to include two-digit divisors.

As they build their facility with multi-digit multiplication and division, students solve problems about area and volume and reinforce their understanding of these concepts.

Section A: Multi-digit Multiplication Using the Standard Algorithm

- Lesson 1: Estimate and Find Products
- Lesson 2: Partial Products in Diagrams
- Lesson 3: Partial Products in Algorithms
- Lesson 4: Standard Algorithm: One-digit and Multi-digit Numbers, with Composing
- Lesson 5: Standard Algorithm: Multi-digit Numbers, without Composing
- Lesson 6: Standard Algorithm: Multi-digit Numbers, with Composing
- Lesson 7: Build Multiplication Fluency
- Lesson 8: The Birds

Section B: Multi-digit Division Using Partial Quotients

- Lesson 9: World's Record Folk Dance
- Lesson 10: Different Partial Quotients
- Lesson 11: A Partial-Quotients Algorithm
- Lesson 12: Divide, Using Partial Quotients
- Lesson 13: Practice a Partial-Quotients Algorithm
- Lesson 14: Find Unknown Side Lengths
- Lesson 15: World's Record Noodle Soup
- Lesson 16: Fractions as Partial Quotients

Section C: Let's Put It to Work

- Lesson 17: A Lot of Milk
- Lesson 18: Trash Talk
- Lesson 19: Shipping Trash
- Lesson 20: Food-Waste Journal

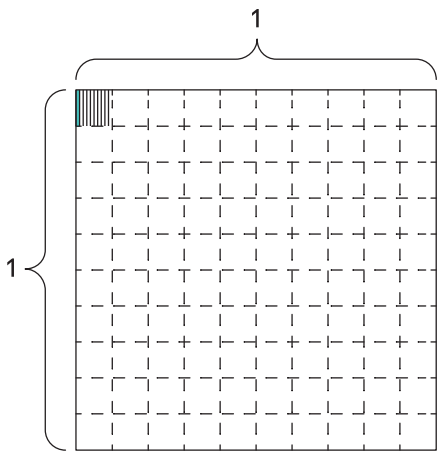
Grade 5

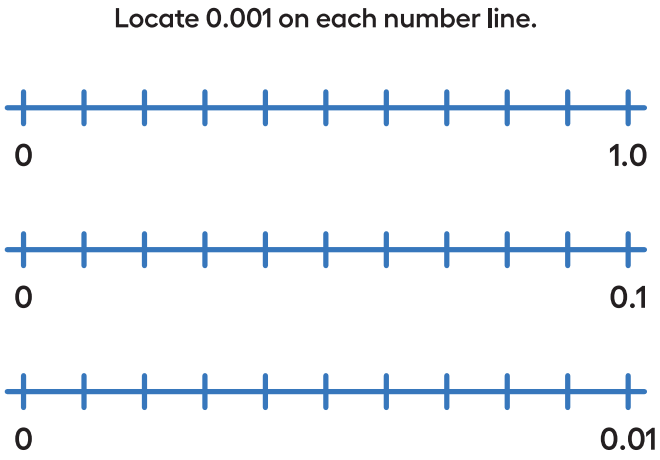
Unit 5: Place Value Patterns and Decimal Operations

In this unit, students expand their knowledge of decimals to read, write, compare, and round decimals to the thousandths. They also extend their understanding of place value and numbers in base ten by performing operations on decimals to the hundredth.

In IM Grade 4, students wrote fractions with denominators of 10 and 100 as decimals. They recognized that the notations 0.1 and $\frac{1}{10}$ express the same amount and are both called "one tenth." Students used hundredths grids and number lines to represent and compare tenths and hundredths.

Students rely on diagrams and their understanding of fractions to make sense of decimals to the thousandths. They see that "one thousandth" refers to the size of one part if a hundredth is partitioned into 10 equal parts, and that its decimal form is 0.001. Diagrams help students visualize the magnitude of each decimal place and compare decimals.





Students then apply their understanding of decimals and of whole-number operations to add, subtract, multiply, and divide decimal numbers to the hundredths, using strategies based on place value and the properties of operations.

Students see that the reasoning strategies and algorithms they used to operate on whole numbers are also applicable to decimals. For example, addition and subtraction can be done by attending to the place value of the digits in the numbers, and multiplication and division can still be understood in terms of equal-size groups.

In IM Grade 6, students will build on the work to reach the expectation to fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Section A: Numbers to Thousandths

- Lesson 1: What Is a Thousandth?
- Lesson 2: Thousandths on Diagrams and in Words
- Lesson 3: Thousandths in Expanded Form
- Lesson 4: Explore Place Value Relationships
- Lesson 5: Compare Decimals
- Lesson 6: Compare Decimals on the Number Line
- Lesson 7: Round Doubloons
- Lesson 8: Round Decimals
- Lesson 9: Order Decimals
- Lesson 10: Solve Problems with Decimals

Section B: Add and Subtract Decimals

- Lesson 11: Make Sense of Decimal Addition
- Lesson 12: Estimate and Add
- Lesson 13: Analyze Addition Mistakes
- Lesson 14: Make Sense of Decimal Subtraction
- Lesson 15: Estimate and Subtract
- Lesson 16: Addition and Subtraction

Section C: Multiply Decimals

- Lesson 17: Multiply Decimals and Whole Numbers
- Lesson 18: Use Whole Number Facts
- Lesson 19: Use Properties to Multiply Decimals
- Lesson 20: Products in the Hundredths Place
- Lesson 21: Multiply More Decimals

Section D: Divide Decimals

- Lesson 22: Divide Whole Numbers by 0.1 and 0.01
- Lesson 23: Divide Whole Numbers by Decimals
- Lesson 24: Divide Decimals by Whole Numbers
- Lesson 25: Divide Decimals by Decimals
- Lesson 26: Book Fair

Unit 6: More Decimal and Fraction Operations

In this unit, students deepen their understanding of place-value relationships of numbers in base ten, unit conversion, operations on fractions with unlike denominators, and multiplicative comparison. The work here builds on several important ideas from grade 4.

In grade 4, students learned the value of each digit in a whole number is 10 times the value of the same digit in a place to its right. Here, they extend that insight to include decimals to the thousandths. Students recognize that the value of each digit in a place (including decimal places) is $\frac{1}{10}$ the value of the same digit in the place to its left.

This idea is highlighted as students perform measurement conversions in metric units.

Previously, students learned to convert from a larger unit to a smaller unit. Here, they learn to convert from a smaller unit to a larger unit. They observe how the digits shift when multiplied or divided by a power of 10 and learn to use exponential notation for powers of 10 to represent large numbers.

L	mL
5	
6.3	
0.95	
10^2	
	800,000
	10^6
	65

Next, students turn their attention to fractions. In earlier grades, students made sense of equivalent fractions, added and subtracted fractions with the same denominator, and added tenths and hundredths. In this unit, they add and subtract fractions with different denominators. They see that the key is to find a common denominator and analyze different techniques for doing so.

Students then solve problems that involve measurement data (in halves, fourths, and eighths) that are displayed on line plots.

In the final section, students reason about the size of a product of fractions and the sizes of the factors. This work builds on the multiplicative comparison work in grade 4, in which students compared a whole number as "_____ times as many (or as much) as" another whole number. Here, students reason about products of a whole number and a fraction, without finding the value of each product. They use diagrams and expressions to support their reasoning.

Write >, <, or = in each blank to make true statements.

$\frac{4}{5} \times 851 \_\_\_\_ 851$

$\frac{99}{8} \times \frac{23}{22} \_\_\_\_ \frac{99}{8}$

$\frac{1}{4} \_\_\_\_ \frac{5}{5} \times \frac{1}{4}$

$\frac{100}{7} \times \frac{9}{13} \_\_\_\_ \frac{9}{13}$

Section A: Measurement Conversions and Powers of 10

- Lesson 1: Place-Value Patterns
- Lesson 2: Powers of 10
- Lesson 3: Metric Conversion and Multiplication by Powers of 10
- Lesson 4: Metric Conversion and Division by Powers of 10
- Lesson 5: Multi-step Conversion Problems: Metric Lengths
- Lesson 6: Multi-step Conversion Problems: Metric Liquid Volumes
- Lesson 7: Multi-step Conversion Problems: Customary Lengths

Section B: Add and Subtract Fractions with Unlike Denominators

- Lesson 8: Add and Subtract Fractions
- Lesson 9: Use Expressions with the Same Value
- Lesson 10: All Sorts of Denominators
- Lesson 11: Different Ways to Subtract
- Lesson 12: Solve Problems
- Lesson 13: Put It All Together: Add and Subtract Fractions
- Lesson 14: Representing Fractions on a Line Plot
- Lesson 15: Problem Solving with Line Plots

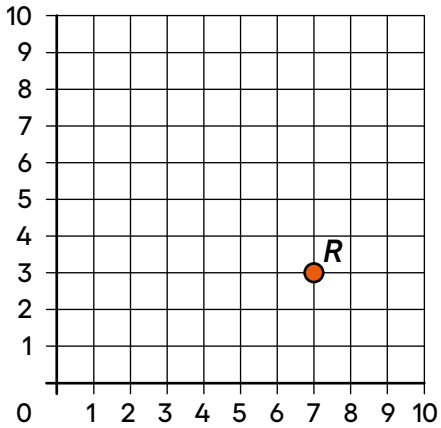
Section C: The Size of a Product

- Lesson 16: Compare Products
- Lesson 17: Interpret Diagrams
- Lesson 18: Compare without Multiplying
- Lesson 19: Compare to 1
- Lesson 20: Will It Always Work?
- Lesson 21: Weekend Investigation

Unit 7: Shapes on the Coordinate Grid

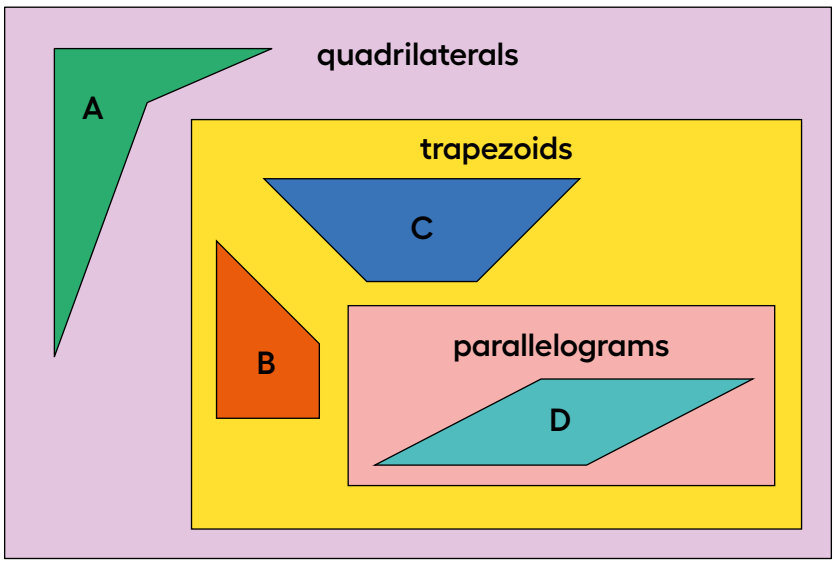
In this unit, students learn about the coordinate grid, deepen their knowledge of two-dimensional shapes, and use the coordinate grid to study relationships of pairs of numbers in various situations.

Students learn about grids that are numbered in two directions. They see that the structure of a coordinate grid allows us to precisely communicate the location of points and shapes.



Grade 5

Students also continue to study two-dimensional shapes and their attributes. In IM Grade 3, they classified triangles and quadrilaterals by the presence of right angles and sides of equal length. In IM Grade 4, they learned about angles and parallel and perpendicular lines, which allowed students to further distinguish shapes. In this unit, students use these insights to make sense of the hierarchy of shapes.



Later in the unit, students analyze and generate numerical patterns based on pairs of rules and graph pairs of numbers on the coordinate grid. They also interpret points on the coordinate grid in terms of situations, plot points to better understand the relationship between two sets of numbers, and use the coordinate grid to solve problems.

Section A: The Coordinate Grid

- Lesson 1: Explore the Coordinate Grid
- Lesson 2: Points on the Coordinate Grid
- Lesson 3: Plot More Points

Section B: The Hierarchy of Shapes

- Lesson 4: Sort Quadrilaterals
- Lesson 5: Trapezoids
- Lesson 6: Hierarchy of Quadrilaterals
- Lesson 7: Rectangles and Squares
- Lesson 8: Sort Triangles

Section C: Numerical Patterns

- Lesson 9: Generate Patterns
- Lesson 10: Interpret Relationships
- Lesson 11: Patterns and Ordered Pairs
- Lesson 12: Represent Problems on the Coordinate Grid
- Lesson 13: Perimeter and Area of Rectangles
- Lesson 14: Copies of Figures

Grade 5

Unit 8: Putting It All Together

In this unit, students revisit major work and fluency goals of the grade, applying their learning from the year.

In Section A, students deepen their understanding of the standard algorithm for multiplication and practice using it to find the value of products. They also revisit algorithms that use partial quotients to divide whole numbers. In Section B, students solve real-world problems about volume and have opportunities to model with mathematics.

The base of the Great Pyramid of Giza is a square.

Each side of the base is 230 meters long.

The pyramid is now 137 meters tall.

If the pyramid was shaped like a rectangular prism, what would be the volume of the prism?

Section C focuses on operations with decimals and fractions. In the final section, students review major work of the grade as they create activities in the format of the warm-up routines they have encountered throughout the year (*Notice and Wonder*, *Estimation Exploration*, *Number Talk*, *True or False?* and *Which Three Go Together?*).

The sections in this unit are standalone sections, not required to be completed in order. Within a section, lessons can also be completed selectively and without completing prior lessons. The goal is to offer ample opportunities for students to integrate the knowledge they have gained and to practice skills related to the expected fluencies of the grade.

Section A: Multiply and Divide Whole Numbers

- Lesson 1: Find the Greatest Product
- Lesson 2: More Multiplication
- Lesson 3: Factors as a Factor in Our Strategy Choice
- Lesson 4: Dive Back into Division
- Lesson 5: More Division

Section B: Apply Volume Concepts

- Lesson 6: Revisit Volume
- Lesson 7: Estimate the Volume of the World's Largest Wagon
- Lesson 8: Fill the World's Largest Wagon
- Lesson 9: Problem Solving with Volume: Water

Section C: Fraction and Decimal Operations

- Lesson 10: Here Comes the Sum
- Lesson 11: What's the Difference?
- Lesson 12: Decimal Game Day
- Lesson 13: Multiply Fractions Game Day

Section D: Creation and Design

- Lesson 14: Notice and Wonder
- Lesson 15: Estimation Exploration
- Lesson 16: Number Talk
- Lesson 17: True or False?
- Lesson 18: Which Three Go Together?

Pacing Guide

The number of days includes two assessment days per unit. The upper bound of the range includes optional lessons.

	Kindergarten	Grade 1	Grade 2
week 1	Unit 1 Math in Our World (18–19 days) Optional Lesson: 17	Unit 1 Adding, Subtracting, and Working with Data (16–17 days) Optional Lesson: 15	Unit 1 Adding, Subtracting, and Working with Data (16–20 days) Optional Lessons: 6, 12, 17, 18
week 2			
week 3			
week 4			
week 5	Unit 2 Numbers 1–10 (23–26 days) Optional Lessons: 7, 8, 24	Unit 2 Addition and Subtraction Story Problems (24–25 days) Optional Lesson: 23	Unit 2 Adding and Subtracting within 100 (15–19 days) Optional Lessons: 4, 10, 16, 17
week 6			
week 7			
week 8			
week 9			Unit 3 Measuring Length (16–20 days) Optional Lessons: 7, 13, 17, 18
week 10	Unit 3 Flat Shapes All Around Us (16–17 days) Optional Lesson: 15	Unit 3 Adding and Subtracting within 20 (29–30 days) Optional Lesson: 28	
week 11			
week 12			
week 13			Unit 4 Addition and Subtraction on the Number Line (14–17 days) Optional Lessons: 6, 14, 15
week 14	Unit 4 Understanding Addition and Subtraction (18–20 days) Optional Lesson: 13, 18		
week 15			
week 16			
week 17		Unit 4 Numbers to 99 (23–25 days) Optional Lessons: 12, 23	Unit 5 Numbers to 1,000 (13–16 days) Optional Lessons: 7, 13, 14
week 18	Unit 5 Composing and Decomposing Numbers to 10 (15–17 days) Optional Lessons: 4, 15		
week 19			
week 20			Unit 6 Geometry, Time, and Money (19–24 days) Optional Lessons: 5, 10, 14, 21, 22
week 21		Unit 5 Adding within 100 (15–16 days) Optional Lesson: 14	
week 22	Unit 6 Numbers 0–20 (13–15 days) Optional Lessons: 2, 13		
week 23			
week 24		Unit 6 Length Measurements within 120 Units (18–19 days) Optional: 17	
week 25	Unit 7 Solid Shapes All Around Us (17–18 days) Optional Lesson: 16		Unit 7 Adding and Subtracting within 1,000 (16–21 days) Optional Lessons: 5, 11, 17, 18, 19
week 26			
week 27			
week 28		Unit 7 Geometry and Time (19–20 days) Optional Lesson: 18	
week 29	Unit 8 Putting It All Together (17–23 days) Optional Lessons: 2, 4, 5, 17, 18, 19		Unit 8 Equal Groups (12–16 days) Optional Lessons: 5, 6, 13, 14
week 30			
week 31			
week 32		Unit 8 Putting It All Together (12 days) Optional Lessons: none	Unit 9 Putting It All Together (15 days) Optional Lessons: none
week 33			
week 34			

Pacing Guide

	Grade 3	Grade 4	Grade 5
week 1	Unit 1 Introducing Multiplication (22–23 days) Optional Lesson: 21	Unit 1 Factors and Multiples (8–10 days) Optional Lessons: 4, 8	Unit 1 Finding Volume (13–14 days) Optional Lesson: 12
week 2			
week 3		Unit 2 Fraction Equivalence and Comparison (18–19 days) Optional Lesson: 17	Unit 2 Fractions as Quotients and Fraction Multiplication (17–19 days) Optional Lessons: 16, 17
week 4			
week 5		Unit 3 Extending Operations to Fractions (20–22 days) Optional Lessons: 19, 20	Unit 3 Multiplying and Dividing Fractions (20–22 days) Optional Lessons: 10, 20
week 6	Unit 2 Area and Multiplication (16–17 days) Optional Lessons: 15		
week 7			
week 8		Unit 4 From Hundredths to Hundred-thousands (24–25 days) Optional Lesson: 23	Unit 4 Wrapping Up Multiplication and Division with Multi-Digit Numbers (20–22 days) Optional Lessons: 16, 20
week 9	Unit 3 Wrapping Up Addition and Subtraction within 1,000 (22–23 days) Optional Lesson: 21		
week 10			
week 11		Unit 5 Multiplicative Comparison and Measurement (19–20 days) Optional Lesson: 18	Unit 5 Place Value Patterns and Decimal Operations (26–28 days) Optional Lessons: 4, 26
week 12			
week 13		Unit 6 Multiplying and Dividing Multi-digit Numbers (27–28 days) Optional Lesson: 26	
week 14	Unit 4 Relating Multiplication to Division (23–24 days) Optional Lesson: 22		
week 15			Unit 6 More Decimal and Fraction Operations (21–23 days) Optional Lessons: 20, 21
week 16		Unit 7 Angles and Angle Measurement (17–18 days) Optional Lesson: 16	
week 17			
week 18			Unit 7 Shapes on the Coordinate Plane (15–16 days) Optional Lesson: 14
week 19	Unit 5 Fractions as Numbers (19–20 days) Optional Lesson: 18	Unit 8 Properties of Two-dimensional Shapes (9–13 days) Optional Lessons: 6, 9, 10, 11	
week 20			
week 21			
week 22		Unit 9 Putting It All Together (14 days) Optional Lessons: none	Unit 8 Putting It All Together (19–20 days) Optional Lesson: 9
week 23	Unit 6 Measuring Length, Time, Liquid Volume, and Weight (17–18 days) Optional Lesson: 16		
week 24			
week 25			
week 26			
week 27	Unit 7 Two-dimensional Shapes and Perimeter (16–17 days) Optional Lesson: 15		
week 28			
week 29			
week 30			
week 31	Unit 8 Putting It All Together (17 days) Optional Lessons: none		
week 32			
week 33			
week 34			