Grades 6–8

Scope and Sequence

Grade 6

Imagine IM Grade 6 begins with an exploration of area and surface area—an invitation for students to engage with novel ideas that they can represent concretely and visually, and reason about in intuitive ways. Starting with geometry also creates opportunities to elicit close observation, sense- and connection-making, and the exchange of ideas—elements of a healthy learning community.

The next two units introduce ratios and rates, concepts that are also new. Students learn to represent, make sense of, and solve problems about equivalent ratios, rates, unit rates, and percentages. The mathematical reasoning here constitutes major work of the grade.

In the two units that follow, students expand and deepen their prior knowledge of numbers and operations. In one unit, students explore division involving fractions, and work toward dividing a fraction by fraction. In the other, they learn to multiply and divide multi-digit, base-ten numbers, including decimals, using the standard algorithm for each operation. Building fluency with algorithms takes time and continues beyond the two units.

Next, students further their understanding of equations and expressions, including those with variables. Students consider ways to represent, justify, and generate equivalent expressions. They also use expressions and equations to describe the relationship between quantities.

From there, students are introduced to rational numbers. Students learn about negative numbers, and represent negative numbers on the number line and on the coordinate plane. They analyze and write inequalities that compare rational numbers.

Toward the end of the course, students examine data sets and distribution. They learn about statistical questions, categorical data, and numerical data. They also explore ways to describe the centers and the distribution of a data set.

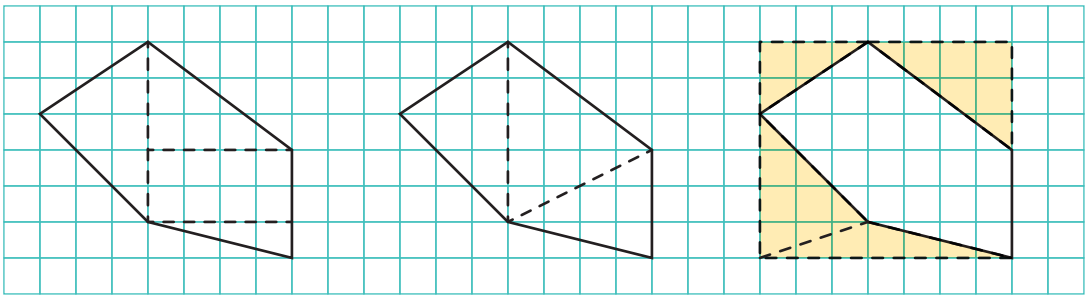
The final unit of the course is optional. The lessons provide students with additional opportunities to integrate and apply various ideas from the course to solve real-world and mathematical problems.

Unit 1: Area and Surface Area

In this unit, students reason about areas of polygons and surface areas of polyhedra, building on geometric understandings developed in earlier grades.

In grade 3, students found the area of rectangles with whole-number side lengths. They also found the area of rectilinear figures by decomposing them into non-overlapping rectangles and adding those areas. Students used a formula for the area of rectangles in grade 4 and found the area of rectangles with fractional side lengths in grade 5.

In this unit, students extend their reasoning about area to include shapes that are not composed of rectangles. They use strategies such as decomposing and rearranging to find areas of parallelograms and generalize their process as a formula. Their work with parallelograms then becomes the basis for finding the area of triangles. Students see that other polygons can be decomposed into triangles and use this knowledge to find areas of polygons.



Next, students calculate the surface areas of polyhedra with triangular and rectangular faces. They study, assemble, and draw nets of prisms and pyramids and use nets to determine surface areas. Students also learn to use exponents ^{2} and ^{3} to express surface areas and volumes of cubes and their units.

In many lessons, students engage in geometric work without a context. This design choice is made in recognition of the significant intellectual work of reasoning about area. Later in the unit, students have opportunities to apply their learning in context.

Students will draw on the work here to further study exponents later in grade 6 and to find volumes of prisms and pyramids in grade 7. Their understanding of "two figures that match up exactly" will support their work on congruence and rigid motions in grade 8.

A note about multiplication notation:

Students in grade 6 will be writing algebraic expressions and equations involving the letter x. Because x is easily confused with the "cross" notation for multiplication, ×, these materials use the "dot" notation for multiplication. Starting a few lessons into the unit, students will see, for instance, $2 \cdot 3$ instead of 2×3. The notation will be new to many students, so they will need explicit guidance in using it.

A note about tools:

Students are likely to need physical tools to support their reasoning. For instance, they may find that tracing paper is an excellent tool for verifying that figures "match up exactly." At all times in the unit, each student should have access to a geometry toolkit, which contains tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles. Access to the toolkit also enables students to practice selecting appropriate tools and using them strategically (MP5). In a digitally enhanced classroom, apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as comparing, explaining, and describing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Geometric patterns and shapes (Lesson 1).
- Strategies for finding areas of shapes (Lesson 3) and polygons (Lesson 11).
- The characteristics of prisms and pyramids (Lesson 13).
- The measurements and units of 1-, 2-, and 3-dimensional attributes (Lesson 16).
- Representations of area and volume (Lesson 17).

Explain

- How to find areas by composing (Lesson 3).
- Strategies used to find areas of parallelograms (Lesson 4) and triangles (Lesson 8).
- How to determine the area of a triangle using its base and height (Lesson 9).
- Strategies to find surface areas of polyhedra (Lesson 14).

Describe

- Observations about decomposition of parallelograms (Lesson 7).
- Information needed to find the surface area of rectangular prisms (Lesson 12).
- The features of polyhedra and their nets (Lesson 13).
- The features of polyhedra (Lesson 15).
- Relationships among features of a tent and the amount of fabric needed for the tent (Lesson 19).

In addition, students are expected to justify claims about the base, height, or area of shapes; generalize about the features of parallelograms and polygons; interpret relevant information for finding the surface area of rectangular prisms; and represent the measurements and units of 2- and 3-dimensional figures. Over the course of the unit, teachers can support students' mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
6.1.1	**area** **region** plane gap overlap	
6.1.2	compose decompose rearrange two-dimensional	
6.1.3	shaded strategy	
6.1.4	parallelogram opposite (sides or angles)	quadrilateral
6.1.5	**base (of a parallelogram or triangle)** **height** corresponding expression represent	
6.1.6	horizontal vertical	
6.1.7	identical	parallelogram
6.1.8	diagram	**base (of a parallelogram or triangle)** **height** **compose** **decompose** rearrange
6.1.9	**opposite vertex**	

Grade 6

lesson	new terminology	
	receptive	productive
6.1.10	**vertex** **edge**	
6.1.11	**polygon**	horizontal vertical
6.1.12	**face** **surface area**	**area** **region**
6.1.13	**polyhedron** **net** **prism** **pyramid** **base (of a prism or pyramid)** three-dimensional	**polygon** **vertex** **edge** **face**
6.1.15		**prism** **pyramid**
6.1.16	**volume** appropriate quantity	two-dimensional three-dimensional
6.1.17	**squared** **cubed** **exponent** edge length	
6.1.18	value (of an expression)	**squared** **cubed** **net**
6.1.19	estimate description	**surface area** **volume**

Section A: Reasoning to Find Area

- Lesson 1: Tiling the Plane
- Lesson 2: Finding Area by Decomposing and Rearranging
- Lesson 3: Reasoning to Find Area

Section B: Parallelograms

- Lesson 4: Parallelograms
- Lesson 5: Bases and Heights of Parallelograms
- Lesson 6: Area of Parallelograms

Section C: Triangles and Other Polygons

- Lesson 7: From Parallelograms to Triangles
- Lesson 8: Area of Triangles
- Lesson 9: Formula for the Area of a Triangle
- Lesson 10: Bases and Heights of Triangles
- Lesson 11: Polygons

Section D: Surface Area

- Lesson 12: What is Surface Area?
- Lesson 13: Polyhedra
- Lesson 14: Nets and Surface Area
- Lesson 15: More Nets, More Surface Area
- Lesson 16: Distinguishing Between Surface Area and Volume

Section E: Squares and Cubes

- Lesson 17: Squares and Cubes
- Lesson 18: Surface Area of a Cube

Section F: Let's Put it to Work

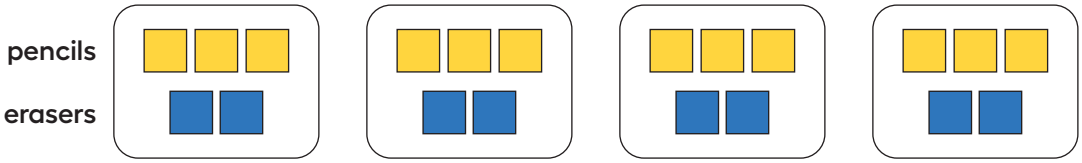
- Lesson 19: All about Tents

Grade 6

Unit 2: Introducing Ratios

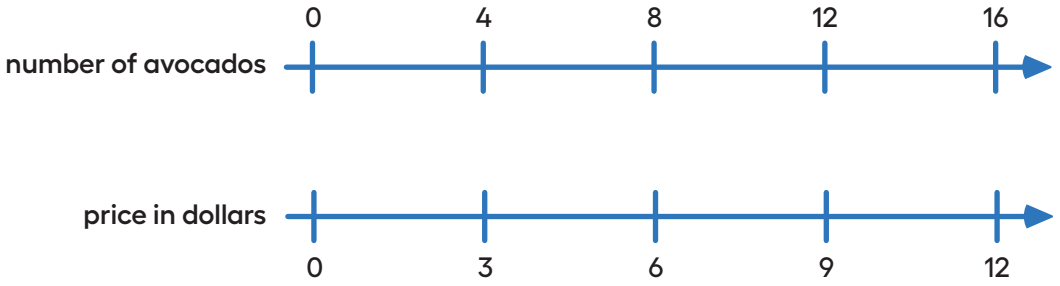
This unit introduces students to ratios and equivalent ratios. It builds on previous experiences students had with relating two quantities, such as converting measurements starting in grade 3, multiplicative comparison in grade 4, and interpreting multiplication as scaling in grade 5. The work prepares students to reason about unit rates and percentages in the next unit, proportional relationships in grade 7, and linear relationships in grade 8.

First, students learn that a ratio is an association between two quantities, for instance, "There are 3 pencils for every 2 erasers." Students use sentences, drawings, or discrete diagrams to represent ratios that describe collections of objects and recipes.



Next, students encounter equivalent ratios in terms of multiple batches of a recipe. "Equivalent" is first used to describe a perceivable sameness of two ratios, such as two mixtures of drink mix and water that taste the same, or two mixtures of yellow and blue paint that make the same shade of green. Later, "equivalent" acquires a more precise meaning: All ratios that are equivalent to $a : b$ can be made by multiplying both a and b by the same non-zero number (non-negative, for now).

Students then learn to use double number line diagrams and tables to represent and reason about equivalent ratios. These representations are more abstract than are discrete diagrams and offer greater flexibility. Use of tables here is a stepping stone toward use of tables to represent functional relationships in future courses. Students explore equivalent ratios in contexts such as constant speed and uniform pricing.



A note on using the terms quantity, ratio, rate, and proportion:

In these materials, a "quantity" is a measurement that can be specified by a number and a unit, for instance, 4 oranges, 4 centimeters, "my height in feet," or "my height" (with the understanding that a unit of measurement will need to be chosen).

Grade 6

The term "ratio" is used to mean an association between two or more quantities. In this unit, the fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called ratios, but the meanings of these fractions in contexts are very carefully developed. The word "per" is used with students in interpreting a unit rate in context, as in "$3 per ounce," and the phrase "at the same rate" is used to signify a situation characterized by equivalent ratios. In the next unit, the fractions $\frac{a}{b}$ and $\frac{b}{a}$ will be identified as "unit rates" for the ratio $a:b$. Students will learn then that if two ratios $a:b$ and $c:d$ are equivalent, then the unit rates $\frac{a}{b}$ and $\frac{c}{d}$ are equal.

The terms "proportion" and "proportional" are not used in grade 6. A "proportional relationship" is a collection of equivalent ratios, which will be studied in grade 7. In high school—after their study of ratios, rates, and proportional relationships—students can discard the term "unit rate" and refer to a to b, $a:b$, and $\frac{a}{b}$ all as "ratios."

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as interpreting, explaining, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Statements and notations describing ratios (Lesson 2).
- Different representations of ratios (Lessons 2 and 6).
- Situations involving equivalent ratios (Lesson 8).
- Situations with different rates (Lesson 9).
- Tables of equivalent ratios (Lessons 11 and 12).
- Questions about situations involving ratios (Lesson 17).

Explain

- Reasoning about equivalence (Lesson 4).
- Reasoning about equivalent rates (Lesson 10).
- Reasoning with reference to tables (Lesson 14).
- Reasoning with reference to tape diagrams (Lesson 15).

Compare

- Situations with and without equivalent ratios (Lesson 3).
- Representations of ratios (Lessons 6 and 13).
- Situations with different rates (Lessons 9 and 12).
- Situations with same rates and different rates (Lesson 10).
- Representations of ratio and rate situations (Lesson 16).

In addition, students are expected to describe and represent ratio associations, represent doubling and tripling of quantities in a ratio, represent equivalent ratios, justify whether ratios are or aren't equivalent and why information is needed to solve a ratio problem, generalize about equivalent ratios and about the usefulness of ratio representations, and critique representations of ratios.

Grade 6

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
6.2.1	**ratio** ___ to ___ ___ for every ___	
6.2.2	diagram	
6.2.3	recipe batch same taste equivalent	**ratio** ___ to ___ ___ for every ___
6.2.4	mixture same color check (an answer)	batch
6.2.5	**equivalent ratios**	
6.2.6	**double number line diagram** tick marks representation	diagram
6.2.7	**per**	
6.2.8	**unit price** how much for 1 at this rate	**double number line diagram**
6.2.9	**constant speed** meters per second	
6.2.10	**same rate**	**equivalent ratios**
6.2.11	table row column	
6.2.14	calculation	**per** table
6.2.15	**tape diagram** parts suppose	
6.2.16		**tape diagram**

Grade 6

Section A: What are Ratios?

- Lesson 1: Introducing Ratios and Ratio Language
- Lesson 2: Representing Ratios with Diagrams

Section B: Equivalent Ratios

- Lesson 3: Recipes
- Lesson 4: Color Mixtures
- Lesson 5: Defining Equivalent Ratios

Section C: Representing Equivalent Ratios

- Lesson 6: Introducing Double Number Line Diagrams
- Lesson 7: Creating Double Number Line Diagrams
- Lesson 8: How Much for One?
- Lesson 9: Constant Speed
- Lesson 10: Comparing Situations by Examining Ratios

Section D: Solving Ratio and Rate Problems

- Lesson 11: Representing Ratios with Tables
- Lesson 12: Navigating a Table of Equivalent Ratios
- Lesson 13: Tables and Double Number Line Diagrams
- Lesson 14: Solving Equivalent Ratio Problems

Section E: Part-part-whole Ratios

- Lesson 15: Part-Part-Whole Ratios
- Lesson 16: Solving More Ratio Problems

Section F: Let's Put it to Work

- Lesson 17: A Fermi Problem

Unit 3: Unit Rates and Percentages

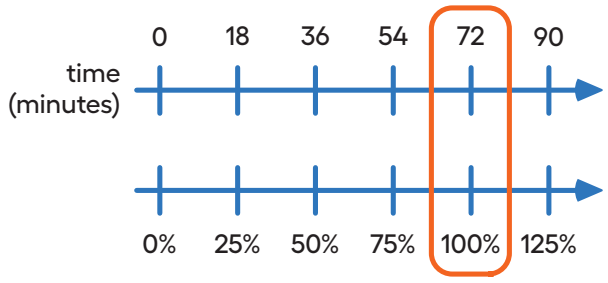
This unit develops students' understanding of unit rates and percentages. Students build on their experience with equivalent ratios and constant rates earlier in the course. They also build on knowledge of measurement and unit conversion in earlier grades. When learning about percentages, they draw on ideas about multiplicative comparison and equivalent fractions from grade 4 and multiplication of fractions from grade 5.

Students begin by recalling what they know about standard units of measurement—the attributes that they measure and their relative sizes. They use ratios and rates to reason about measurements and to convert between units of measurement.

Next, students learn about unit rates. They see that there are two unit rates—$\frac{a}{b}$ and $\frac{b}{a}$—associated with any ratio $a : b$ and interpret them in context. Students practice finding unit rates and using them to solve various problems.

Students then use their understanding of ratios and rates to make sense of percentages. Just as a unit rate can be interpreted in context as a rate per 1, a percentage can be interpreted in context as a rate per 100.

Throughout the unit, students can use familiar representations such as tables and double number line diagrams in their reasoning. Sometimes a particular representation is suggested to help students make connections or to make sense of a situation. At other times students decide which representations to use, if needed.



time (minutes)	percentage
90	125
18	25
72	100

In a later unit, students will write equations of the form $px = q$ to represent situations where the value corresponding to 100% is unknown and will solve such equations. In grade 7, students will rely on their knowledge of equivalent ratios and unit rates to make sense of proportional relationships and constants of proportionality. Their understanding of percentages will support them in reasoning about percent increase and decrease.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Unit rates in different contexts (Lesson 5).
- A context in which identifying a unit rate is helpful (Lesson 8).
- Situations involving constant speed (Lesson 9).
- Diagrams used to represent percentages (Lessons 11 and 12).
- Situations involving measurement, rate, and cost (Lesson 17).

Explain

- Reasoning for estimating and sorting measurements (Lesson 1).
- Reasoning about relative sizes of units of measurement (Lesson 2).
- Reasoning for comparing rates (Lessons 4 and 7).
- Reasoning about percentages (Lesson 11).
- Strategies for finding missing information involving percentages (Lesson 14).

Justify

- Reasoning about equivalent ratios and unit rates (Lesson 6).
- Reasoning about finding percentages (Lessons 15 and 16).
- Reasoning about costs and time (Lesson 17).

In addition, students have opportunities to generalize about equivalent ratios, unit rates, and percentages from multiple contexts and with reference to benchmark percentages, tape diagrams, and other mathematical representations. Students can also be expected to describe measurements and observations, describe and compare situations involving percentages, compare speeds, compare prices, and critique reasoning about costs and time.

The table shows the lessons in which new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear in the glossary are bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
6.3.2	order	
6.3.4	(good / better / best) deal rate per 1	**unit price** same speed
6.3.5	**unit rate**	
6.3.6	result	**unit rate**
6.3.7		meters per second (good / better / best) deal
6.3.8		at this rate
6.3.9	**pace**	**speed**
6.3.10	**percentage** ___% of	
6.3.11		tick marks
6.3.12	___% as much as	___% of
6.3.14		
6.3.15		**percentage**

Section A: Units of Measurement and Unit Conversion

- Lesson 1: Anchoring Units of Measurement
- Lesson 2: Measuring with Different-Size Units
- Lesson 3: Converting Units

Section B: Rates

- Lesson 4: Comparing Speeds and Prices
- Lesson 5: Interpreting Rates
- Lesson 6: Equivalent Ratios Have the Same Unit Rates
- Lesson 7: More Rate Comparisons
- Lesson 8: Solving Rate Problems
- Lesson 9: More about Constant Speed

Section C: Percentages

- Lesson 10: What Are Percentages?
- Lesson 11: Representing Percentages with Double Number Line Diagrams
- Lesson 12: Representing Percentages in Different Ways
- Lesson 13: Benchmark Percentages
- Lesson 14: Solving Percentage Problems
- Lesson 15: Finding This Percent of That
- Lesson 16: Finding the Percentage

Section D: Let's Put it to Work

- Lesson 17: Painting a Room

Unit 4: Dividing Fractions

This unit develops students' understanding of division of fractions by fractions. This work draws on students' prior knowledge of multiplication, division, and the relationship between the two. It also builds on concepts from grades 3 to 5 about multiplicative situations—equal-size groups, multiplicative comparison, and the area of a rectangle—and about fractions.

Students begin by exploring meanings of division and the relationship between the quantities in division situations. They recall that we can think of dividing as finding an unknown factor in a multiplication equation. In situations involving equal-size groups, division can be used to answer two questions: "How many groups?" and "How much in each group?"

Next, students investigate ways to answer those two questions. They reason about situations in which the size of a group is known but the number of groups is not (as in, "How many $\frac{2}{3}$s are in 4?") and in which the number of groups is know but the size is not (as in, "What is in each bottle if there are 14 liters in $3\frac{1}{2}$ bottles?"). They also explore division in situations involving multiplicative comparison.

Students then apply their insights to generalize the process of finding quotients. In reasoning repeatedly to find the value of expressions such as $6 \div \frac{1}{4}$, $6 \div 34$, and $6 \div a4$, students notice regularity: Dividing a number by a fraction $\frac{a}{b}$ is the same as multiplying that number by $\frac{b}{a}$.

Students go on to use this algorithm to solve problems about geometric figures that have fractional length, area, or volume measurements. They also apply the concepts from the unit to solve multi-step problems involving fractions in other contexts.

Throughout the unit, students interpret and create equations and diagrams to make sense of the relationship between known and unknown quantities.

A deeper understanding of multiplication, division, and ways to represent them will support students in reasoning about decimal operations as well as in writing and solving variable equations later in the course.

A note about diagrams:

Because tape diagrams are a flexible tool for illustrating and reasoning about division of fractions, they are the primary representation used in this unit. Students may, however, create other representations to support their reasoning.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as "interpreting," representing," "justifying," and "explaining." Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret and Represent

- Situations involving division (Lessons 2, 3, 9, 12, and 16).
- Situations involving measurement constraints (Lesson 17).

Justify

- Reasoning about division and diagrams (Lessons 4 and 5).
- Strategies for dividing numbers (Lesson 11).
- Reasoning about volume (Lesson 15).

Explain

- How to create and make sense of division diagrams (Lesson 6).
- How to represent division situations (Lesson 9).
- How to find missing lengths (Lesson 14).
- A plan for optimizing costs (Lesson 17).

In addition, students are expected to critique the reasoning of others about division situations and representations, and to make generalizations about division by comparing and connecting across division situations and across the representations used in reasoning about these situations. The *Lesson Syntheses* in Lessons 2 and 12 offer specific disciplinary language that may be especially helpful for supporting students in navigating the language of important ideas in this unit.

Grade 6

The table shows lessons in which new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
6.4.1	divisor dividend	quotient
6.4.2	equation interpretation equal-size	How many groups of ___? How many ___ in each group?
6.4.3	unknown	
6.4.4	whole	
6.4.5		whole
6.4.6		equal-size
6.4.7	times as ___ fraction of ___	
6.4.8	container section	unknown fraction of ___
6.4.10	observations	times as ___ numerator denominator
6.4.11	**reciprocal**	
6.4.13		gaps
6.4.14	packed	
6.4.17	assumption	packed

Section A: Making Sense of Division

- Lesson 1: Size of Divisor and Size of Quotient
- Lesson 2: Meanings of Division
- Lesson 3: Interpreting Division Situations

Section B: Meanings of Fraction Division

- Lesson 4: How Many Groups? (Part 1)
- Lesson 5: How Many Groups? (Part 2)
- Lesson 6: Using Diagrams to Find the Number of Groups
- Lesson 7: What Fraction of a Group?
- Lesson 8: How Much in Each Group? (Part 1)
- Lesson 9: How Much in Each Group? (Part 2)

Section C: Algorithm for Fraction Division

- Lesson 10: Dividing by Unit and Non-Unit Fractions
- Lesson 11: Using an Algorithm to Divide Fractions

Section D: Fractions in Lengths, Areas, and Volumes

- Lesson 12: Fractional Lengths
- Lesson 13: Rectangles with Fractional Side Lengths
- Lesson 14: Fractional Lengths in Triangles and Prisms
- Lesson 15: Volume of Prisms

Section E: Let's Put It to Work

- Lesson 16: Solving Problems Involving Fractions
- Lesson 17: Fitting Boxes into Boxes

Grade 6

Unit 5: Arithmetic in Base Ten

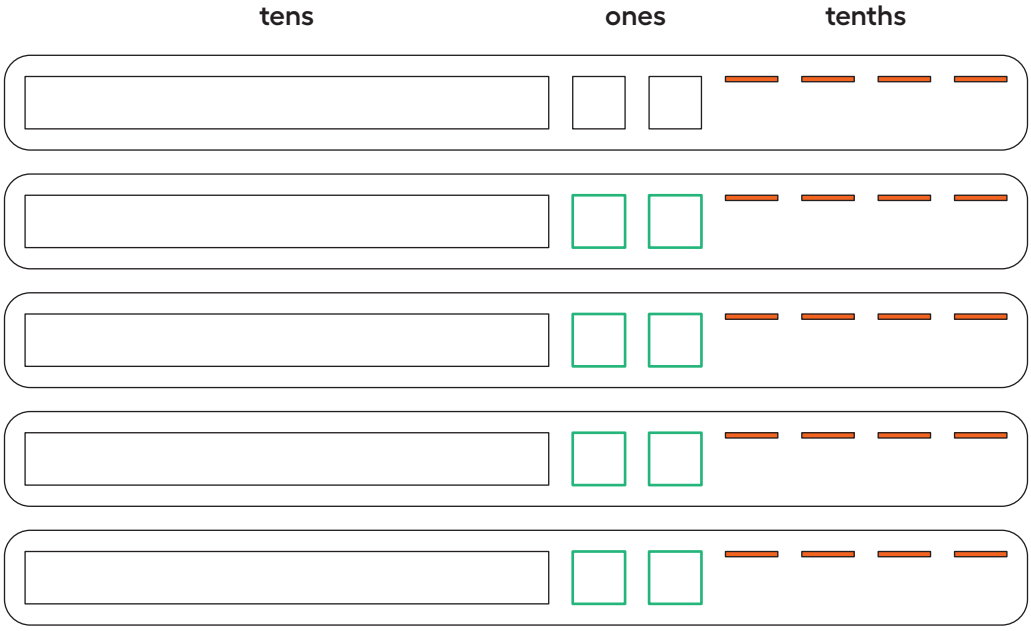
In this unit, students solidify their understanding of the base-ten number system, extend their use of the standard algorithms to add, subtract, and multiply decimals beyond tenths and hundredths, and learn to use algorithms to calculate quotients. The work here builds on what students learned in earlier grades about operations on whole numbers and decimals.

Students begin by exploring the use of decimals in a shopping context and by revisiting addition and subtraction of decimals, using both concrete representations and numerical calculations. The activities in the first section reinforce ideas about place value, properties of operations, the algorithms for adding and subtracting, and the relationship between addition and subtraction.

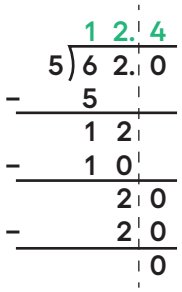
Next, students investigate various ways to find the product of two decimals: by using decimal fractions, writing equivalent expressions with whole numbers and unit decimals (such as 0.1 and 0.01), using diagrams and partial products, and reasoning about the relationship between a decimal and a related whole number. Students notice that the different methods of reasoning are governed by the same structure based on place value, which also underlies the standard algorithm for multiplication.

The next section focuses on division. Students have an opportunity to use base-ten blocks or diagrams to represent division of multi-digit numbers before exploring other numerical methods, such as using partial quotients and long division. Students progress through calculations of increasing complexity. They first divide whole numbers that give a whole-number quotient, and then divide whole numbers with a (terminating) decimal quotient. Next, they divide a decimal by a whole number, and finally a decimal by a decimal.

Mai's diagram for $62 \div 5$



Lin's calculation for 62 ÷ 5



In the last section, students apply the mathematics from the unit to solve problems in applied situations. These require students to interpret quantities and results in context, and to consider appropriate levels of precision in their work.

A note about materials:

Base-ten blocks and paper versions of them will be useful throughout the unit. Consider preparing commercially produced base-ten blocks, if available, or printing representations of base-ten units on card stock, cutting them out, and organizing them for easy reuse.

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as explaining, interpreting, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Explain

- Processes of estimating and finding costs (Lesson 1).
- Approaches to adding and subtracting decimals (Lesson 4).
- Reasoning about products and quotients involving powers of 10 (Lesson 5).
- Methods for multiplying decimals (Lesson 8).
- Reasoning about relationships among measurements (Lesson 15).

Interpret

- Representations of decimals (Lesson 2).
- Base-ten diagrams showing addition or subtraction of decimals (Lesson 3).
- Area diagrams showing products of decimals (Lesson 7).
- Base-ten diagrams representing division of a whole number or a decimal by a whole number (Lessons 9, 12).
- Calculations showing partial quotients or steps in long division (Lessons 10, 11, 12).

Compare

- Base-ten diagrams with numerical calculations (Lesson 4).
- Methods for multiplying decimals (Lesson 6).
- Methods for finding quotients (Lessons 10, 11, 12).
- Measurements of two- and three-dimensional objects (Lesson 15).

In addition, students are expected to describe decimal values to hundredths, generalize about multiplication by powers of 10 and about decimal measurements, critique approaches to operations on decimals, and justify strategies for finding sums, differences, products, and quotients.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
6.5.1	digits budget at least	
6.5.2	base-ten diagram compose vertical calculation	place value digits
6.5.3	decompose	
6.5.4	method	compose decompose
6.5.5	powers of 10	product decimal point
6.5.7	partial products	method
6.5.9	remainder	
6.5.10	partial quotients	divisor
6.5.11	**long division**	
6.5.12		remainder
6.5.13		**long division**
6.5.14	precision accuracy operation	

Section A: Exploring, Adding, and Subtracting Decimals

- Lesson 1: Using Decimals in a Shopping Context
- Lesson 2: Using Diagrams to Represent Addition and Subtraction
- Lesson 3: Adding and Subtracting Decimals with Few Non-Zero Digits
- Lesson 4: Adding and Subtracting Decimals with Many Non-Zero Digits

Section B: Multiplying Decimals

- Lesson 5: Using Fractions to Multiply Decimals
- Lesson 6: Methods for Multiplying Decimals
- Lesson 7: Using Diagrams to Represent Multiplication
- Lesson 8: Calculating Products of Decimals

Section C: Dividing Decimals

- Lesson 9: Using Base-Ten Diagrams to Divide
- Lesson 10: Using Partial Quotients
- Lesson 11: Using Long Division
- Lesson 12: Dividing Numbers that Result in a Decimal
- Lesson 13: Dividing a Decimal by a Decimal

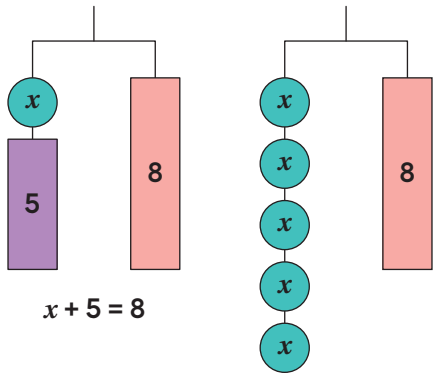
Section D: Let's Put It to Work

- Lesson 14: Solving Problems Involving Decimals
- Lesson 15: Making and Measuring Boxes

Unit 6: Expressions and Equations

In this unit, students apply their understanding of arithmetic to reason about algebraic expressions and equations.

In the first section, students work with equations of the form $x + p = q$ and $px = q$ where p and q are positive rational numbers. They use tape diagrams and hanger diagrams to reason about the meaning of equations, and to develop an understanding that to solve an equation is to find a value that would make the equation true. Students end the section by identifying, interpreting, and writing equations to represent and solve real-world problems.



In the second section, students write algebraic expressions and evaluate them for given values. They identify and write equivalent expressions, reasoning using diagrams, the distributive property, and other properties of operations.

The third section is all about exponents. Students write expressions with a whole-number exponent and a base that may be a whole number, a fraction, or a variable. They analyze such expressions for equivalence, as well as use the conventional order of operations to evaluate them. Students also identify solutions to simple exponential equations.

In the last two sections, students analyze real-world relationships between two quantities where one quantity depends on the other. They use tables, graphs, and equations to represent and reason about such relationships.

The work here prepares students to represent quantities and relationships involving all rational numbers in a later unit, as well as to solve equations that are more complex and work with proportional relationships in grade 7.

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, describing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- tape diagrams involving letters that stand for numbers (Lesson 1)
- the parts of an equation (Lesson 5)
- numerical expressions involving exponents (Lesson 12)
- different representations of the same relationship between quantities (Lesson 17)

Describe

- solutions to equations (Lesson 2)
- stories represented by given equations (Lesson 5)
- patterns of growth that can be represented using exponents (Lesson 12)
- relationships between independent and dependent variables using tables, graphs, and equations (Lesson 16)

Explain

- the meaning of a solution using hanger diagrams (Lesson 3)
- how to solve an equation (Lesson 4)
- how to use equations to solve problems involving percentages (Lesson 6)
- how to determine whether two expressions are equivalent, including with reference to diagrams (Lesson 8)
- strategies for determining whether expressions are equivalent (Lesson 14)

In addition, students are expected to compare descriptions of situations, expressions, equations, diagrams, tables, and graphs. They generalize about properties of operations and strategies for solving equations. Students also justify claims about equivalent expressions and justify reasoning when evaluating expressions.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

Grade 6

lesson	new terminology	
	receptive	**productive**
6.6.1	value (of a variable)	operation
6.6.2	**variable** **coefficient** **solution to an equation** true equation false equation	value (of a variable)
6.6.3	each side (of an equal sign) balanced hanger diagram	
6.6.4	solve (an equation)	each side (of an equal sign)
6.6.6		true equation false equation
6.6.7		
6.6.8	**equivalent expressions** commutative property	
6.6.9	distributive property area as a product area as a sum	
6.6.10	**term**	**equivalent expressions**
6.6.12	to the power	
6.6.13		to the power **exponent**
6.6.15		**solution to an equation**
6.6.16	**independent variable** **dependent variable** horizontal axis vertical axis	**variable** relationship
6.6.17	coordinates	
6.6.18	plot	

Section A: Equations in One Variable

- Lesson 1: Tape Diagrams and Equations
- Lesson 2: Truth and Equations
- Lesson 3: Staying in Balance
- Lesson 4: Practice Solving Equations
- Lesson 5: Represent Situations with Equations
- Lesson 6: Percentages and Equations

Section B: Equal and Equivalent

- Lesson 7: Write Expressions with Variables
- Lesson 8: Equal and Equivalent
- Lesson 9: The Distributive Property, Part 1
- Lesson 10: The Distributive Property, Part 2
- Lesson 11: The Distributive Property, Part 3

Section C: Expressions with Exponents

- Lesson 12: Meaning of Exponents
- Lesson 13: Expressions with Exponents
- Lesson 14: Evaluating Expressions with Exponents
- Lesson 15: Equivalent Exponential Expressions

Section D: Relationships Between Quantities

- Lesson 16: Two Related Quantities, Part 1
- Lesson 17: Two Related Quantities, Part 2
- Lesson 18: More Relationships

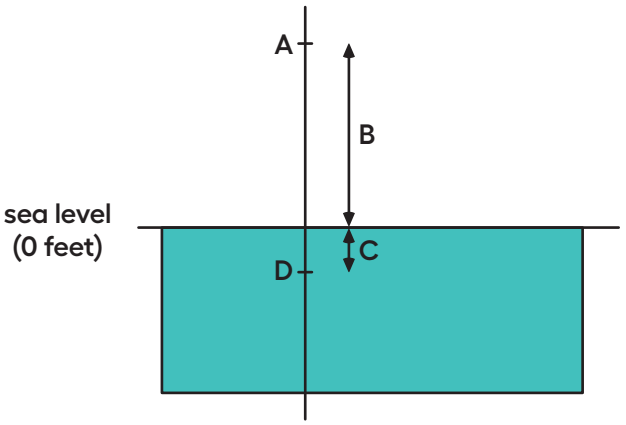
Section E: Let's Put it to Work

- Lesson 19: Tables, Equations, and Graphs, Oh My!

Unit 7: Rational Numbers

In this unit, students learn about negative numbers and ways to represent them on a number line and the coordinate plane. They write and graph simple inequalities in one variable and determine the greatest common factor and least common multiple of two whole numbers. In grade 7, students will perform arithmetic operations with signed numbers and write and solve more complex inequalities.

Students begin by considering situations involving temperature or elevation and interpreting what negative numbers mean in those contexts. They also plot points to represent positive and negative values and their opposites. Previously, when students worked only with nonnegative numbers, magnitude and order were indistinguishable. In this unit, when comparing two signed numbers, students learn to distinguish between the absolute value of a number (magnitude) and a number's relative positions on the number line (order).



A. 15 feet
B. |15| feet
C. |-4| feet
D. -4 feet

Next, students use the symbols < and > to compare two values. They represent an unknown value with constraints as an inequality, and they graph the solutions on a number line. Students consider the inclusion or exclusion of boundary values and interpret solutions based on the context. In these grade 6 materials, inequality symbols in are limited to < and >. However, in this unit students encounter situations that are best represented by both an inequality and an equation, such as "$x > 2$ or $x = 2$."

Then, students use ordered pairs to describe pairs of numbers that include negative numbers. In grade 5, they plotted pairs of positive numbers on the coordinate grid. Here, they plot pairs of rational numbers in all four quadrants of the coordinate plane. They interpret the meanings of plotted points in given contexts and use coordinates to calculate horizontal or vertical distances between two points.

The last section of the unit returns to whole numbers. Students are introduced to common factors and common multiples. They determine the greatest common factor or the least common multiple of two numbers. They identify how these new concepts are involved in real-world situations and use their understanding to solve related problems.

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, interpreting, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe and Interpret

- Situations involving negative numbers (Lesson 1).
- Features of a number line (Lessons 2, 4 and 6).
- Situations involving elevation (Lesson 7).
- Situations involving minimums and maximums (Lesson 8).
- Points on a coordinate plane (Lessons 11 and 14).
- Situations involving factors and multiples (Lesson 18).

Justify

- Reasoning about magnitude (Lesson 3).
- Reasoning about a situation involving negative numbers (Lesson 5).
- Reasoning about solutions to inequalities (Lesson 9).
- That all possible pairs of factors have been identified (Lesson 16).

Generalize

- The meaning of integers for a specific context (Lesson 5).
- Understanding of solutions to inequalities (Lesson 9).
- About the relationships between shapes (Lesson 10).
- About greatest common factors (Lesson 16).
- About least common multiples (Lesson 17).

In addition, students are expected to critique the reasoning of others, represent inequalities symbolically and in words, and explain how to order rational numbers and how to determine distances on the coordinate plane. Students also have opportunities to use language to compare magnitudes of positive and negative numbers, compare features of ordered pairs, and compare appropriate axes for different sets of coordinates.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
6.7.1	**positive number** **negative number** temperature degrees Celsius elevation sea level	number line below zero
6.7.2	**opposite (numbers)** **rational number** location distance (away) from zero	
6.7.3	**sign** **inequality** closer to 0 farther from 0	greater than less than

Grade 6

lesson	new terminology	
	receptive	**productive**
6.7.4	from least to greatest	temperature elevation sea level
6.7.5	positive change negative change context	
6.7.6	**absolute value**	**positive number** **negative number** distance (away) from zero
6.7.7		closer to 0 farther from 0
6.7.8	maximum minimum	
6.7.9	requirement **solution to an inequality**	
6.7.10	unbalanced hanger	inequality
6.7.11	**quadrant** **coordinate plane** x-coordinate y-coordinate	
6.7.12	(line) segment	axis
6.7.13	degrees Fahrenheit	degrees Celsius
6.7.14		**absolute value** x-coordinate y-coordinate
6.7.16	**common factor** **greatest common factor (GCF)**	factor
6.7.17	**common multiple** **least common multiple (LCM)**	multiple

Section A: Negative Numbers and Absolute Value

- Lesson 1: Positive and Negative Numbers
- Lesson 2: Points on the Number Line
- Lesson 3: Comparing Positive and Negative Numbers
- Lesson 4: Ordering Rational Numbers
- Lesson 5: Using Negative Numbers to Make Sense of Contexts
- Lesson 6: Absolute Value of Numbers
- Lesson 7: Comparing Numbers and Distance from Zero

Section B: Inequalities

- Lesson 8: Writing and Graphing Inequalities
- Lesson 9: Solutions of Inequalities
- Lesson 10: Interpreting Inequalities

Section C: The Coordinate Plane

- Lesson 11: Points in the Coordinate Plane
- Lesson 12: Constructing the Coordinate Plane
- Lesson 13: Interpreting Points in a Coordinate Plane
- Lesson 14: Distances in the Coordinate Plane
- Lesson 15: Shapes in the Coordinate Plane

Section D: Common Factors and Common Multiples

- Lesson 16: Common Factors
- Lesson 17: Common Multiples
- Lesson 18: Using Common Multiples and Common Factors

Section E: Let's Put It to Work

- Lesson 19: Drawing in the Coordinate Plane

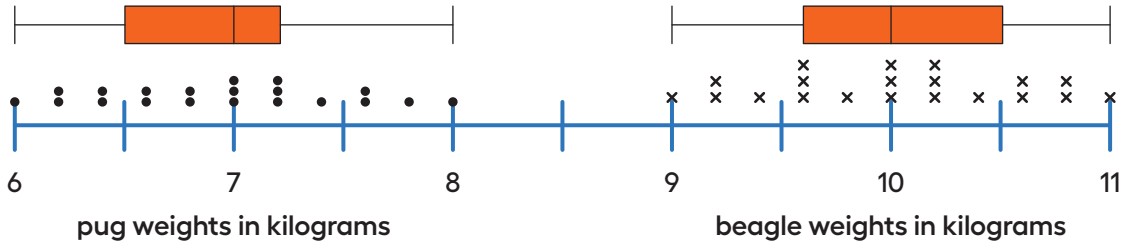
Grade 6

Unit 8: Data Sets and Distributions

In this unit, students learn about populations and study variables associated with a population. They begin by classifying questions as either statistical or non-statistical—based on whether variable data is necessary to answer the question. This leads to further investigation into variability and data displays, such as dot plots and histograms. As students visualize data, they begin to describe the distribution of data more precisely as they work with "mean" and "mean absolute deviation" (MAD).

After working with those statistics, students begin to recognize that some distributions are not well-suited to description by mean and MAD. Students are introduced to "median," "range," and "interquartile range" as additional measures of center and variability that can be used to describe distributions in some situations. That also leads to the "box plot" as an additional way to visualize data.

Note that the introduction of mean absolute deviation is used as an introductory model for understanding variability. Although standard deviation is more mathematically useful, its calculation and meaning may be difficult for students at this level without an understanding of normal distributions. In later courses, when student understanding of variability and their exposure to additional distributions is expanded, students will learn about standard deviation and evolve their understanding away from mean absolute deviation.



Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as justifying, representing, and interpreting. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Justify

- Reasoning for matching data sets to questions (Lesson 2).
- Reasoning about dot plots (Lesson 3).
- Reasoning about mean and median (Lesson 13).
- Reasoning about changes in mean and median (Lesson 14).
- Reasoning about which information is needed (Lesson 17).
- Which summaries and graphs best represent given data sets (Lesson 18).

Represent

- Data using dot plots (Lessons 3 and 4).
- Data using histograms (Lesson 7).
- Mean using bar graphs (Lesson 9).
- Data with five number summaries (Lesson 15).
- Data using box plots (Lesson 16).

Interpret

- Dot plots (Lessons 4 and 11).
- Histograms (Lessons 6 and 18).
- Mean of a data set (Lesson 9).
- Five-number summaries (Lesson 15).
- Box plots (Lesson 16).

In addition, students are expected to critique the reasoning of others, describe how quantities are measured, describe and compare features and distributions of data sets, generalize about means and distances in data sets, generalize categories for sorting data sets, and generalize about statistical questions. Students are also expected to use language to compare questions that produce numerical and categorical data, compare dot plots and histograms, and compare histograms and bar graphs.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
6.8.1	**numerical data** **categorical data** **dot plot**	
6.8.2	**statistical question** **variability**	
6.8.3	**distribution** **frequency**	bar graph
6.8.4	typical	
6.8.5	**center** **spread**	**variability**
6.8.6	**histogram** bins	**distribution** **center**
6.8.7		**statistical question** **spread**
6.8.8	symmetrical peak cluster unusual value	**numerical data** **categorical data** gap
6.8.9	**average** **mean** fair share	
6.8.10	**measure of center** balance point	
6.8.11	**mean absolute deviation (MAD)** measure of spread	symmetrical **mean**

Grade 6

lesson	new terminology	
	receptive	productive
6.8.12		**mean absolute deviation (MAD)** typical
6.8.13	**median**	**measure of center**
6.8.14		peak cluster unusual value
6.8.15	**range** **quartile** **interquartile range (IQR)** five-number summary	measure of spread minimum maximum
6.8.16	**box plot** whisker	**median** **interquartile range (IQR)**
6.8.17		**range** **quartile**
6.8.18		**dot plot** **histogram** **box plot**

Section A: Data, Variability, and Statistical Questions

- Lesson 1: Got Data?
- Lesson 2: Statistical Questions

Section B: Dot Plots and Histograms

- Lesson 3: Representing Data Graphically
- Lesson 4: Dot Plots
- Lesson 5: Using Dot Plots to Answer Statistical Questions
- Lesson 6: Interpreting Histograms
- Lesson 7: Using Histograms to Answer Statistical Questions
- Lesson 8: Describing Distributions on Histograms

Section C: Measures of Center and Variability

- Lesson 9: Mean
- Lesson 10: Finding and Interpreting the Mean as the Balance Point
- Lesson 11: Variability and MAD
- Lesson 12: Using Mean and MAD to Make Comparisons

Section D: Median and IQR

- Lesson 13: Median
- Lesson 14: Comparing Mean and Median
- Lesson 15: Quartiles and Interquartile Range
- Lesson 16: Box Plots
- Lesson 17: Using Box Plots

Section E: Let's Put it to Work

- Lesson 18: Using Data to Solve Problems

Unit 9: Putting it All Together

This optional unit consists of eleven lessons. The first section has four lessons about exploring our world. These lessons are independent of each other and explore working with estimation and decimals or large numbers. These lessons should be taught after Unit 3. The second section has five lessons about different systems of voting. These lessons build on each other and should be completed in order after Unit 3 has been taught. The final section has two lessons making connections between algebraic and geometric representations of topics in earlier units. This section should be taught after Unit 8 and the lessons should be completed in order.

All related standards in this unit have been addressed in prior units. These sections provide an *optional* opportunity for students to go more deeply and make connections between domains.

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes, such as "critiquing," "justifying, "and "comparing." Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- Reasoning about Fermi problems (Lesson 1).
- Claims about percentages (Lesson 5).
- Reasoning about the fairness of voting systems (Lessons 8 and 9).

Justify

- Reasoning about Fermi problems (Lesson 1).
- Reasoning about the fairness of voting systems (Lessons 6, 7, 8, and 9).

Compare

- Sources of energy (Lessons 2 and 3).
- Rectangles and fractions (Lesson 10).
- Voting systems (Lessons 6 and 7).

In addition, students are expected to interpret and represent characteristics of the world population, describe distributions of voters, and generalize about decomposition of area and numbers.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
6.9.2	kilowatt-hour (kWh)	
6.9.5	in favor majority	
6.9.6	plurality runoff	majority
6.9.8	in all fair	
6.9.10	mixed number	

Section A: Exploring Our World

- Lesson 1: Fermi Problems
- Lesson 2: Energy Flow
- Lesson 3: Making Paper
- Lesson 4: If Our Class Were the World

Section B: Voting

- Lesson 5: How Do We Choose?
- Lesson 6: More than Two Choices
- Lesson 7: Comparing Voting Systems
- Lesson 8: Picking Representatives
- Lesson 9: Designing Districts

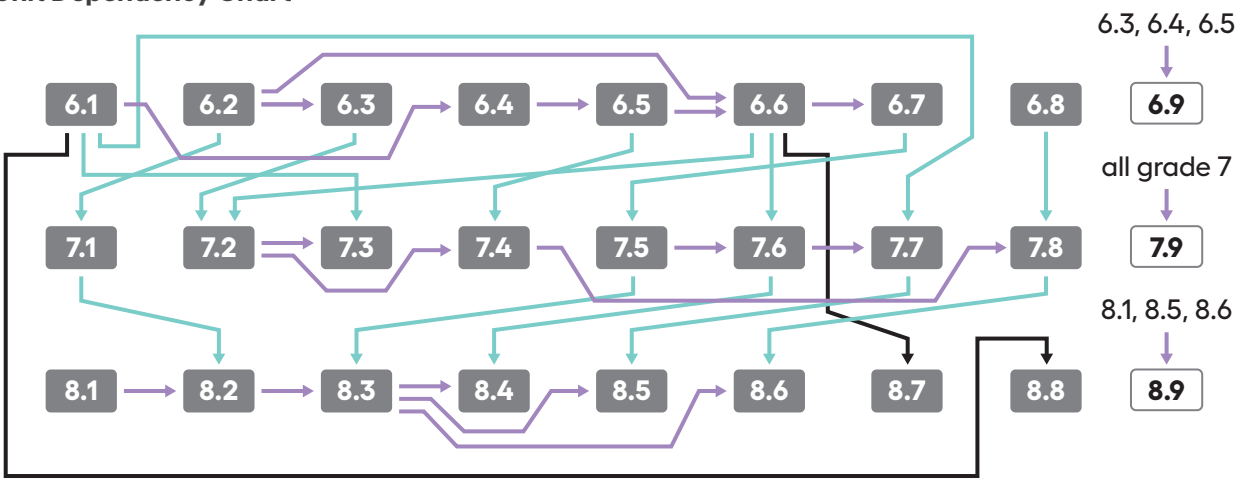
Section C: Making Connections

- Lesson 10: Rectangle Madness
- Lesson 11: Rectangle Fractions

Grade 6

Dependency Chart

Unit Dependency Chart



In the unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order of the units would have a negative effect on mathematical or pedagogical coherence. Examples:

- There is an arrow from 6.2 to 6.6. Students are expected to use their knowledge of contexts involving ratios (from 6.2) to write and solve equations representing such contexts (in 6.6).
- There is an arrow from 7.4 to 7.8. Students are expected to use their skills in representing percentages (from 7.4) when solving problems about probability (in 7.8).
- There is an arrow from 8.3 to 8.6. Students are expected to use their skills in writing and interpreting an equation that represents a line (from 8.3) to interpret the parameters in an equation that represents a line that fits a scatter plot (in 8.6).

Grade 7

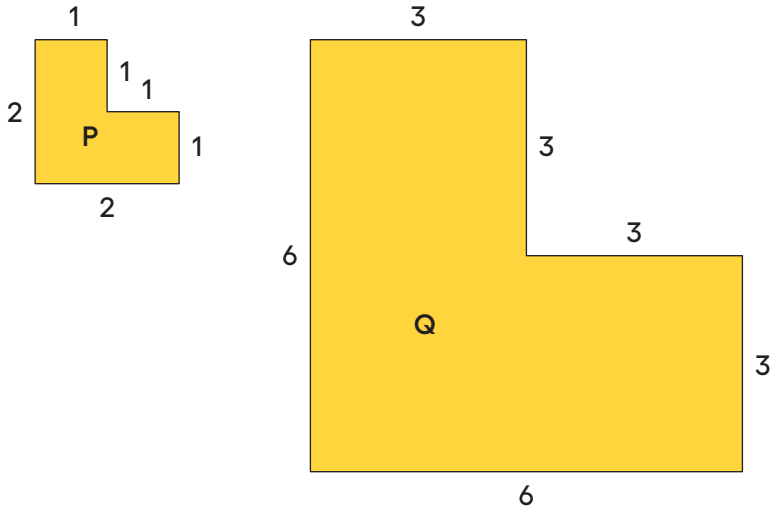
IM Grade 7 begins with students studying scale drawings, an engaging geometric topic that sets the stage for the subsequent work on proportional relationships in the following three units. Students also have opportunities to build fluency with IM Grade 6 arithmetic. They work with proportional relationships represented by tables, equations, and graphs. Geometry and proportional relationships are interwoven in the third unit, when the important proportional relationship between a circle's circumference and its diameter is studied. Then students work with percent increase and percent decrease.

By the fifth unit, on operations with rational numbers, students have had time to brush up on and solidify their understanding of, and skill in, IM Grade 6 arithmetic. At this point, the emphasis becomes the role of the properties of operations in determining the rules for operating with negative numbers. This is a natural lead-in to the work on solving equations and simplifying expressions in the next unit. Students then put their arithmetical and algebraic skills to work in the last two units: on angles, triangles, and prisms, and on probability and sampling.

The final unit of the course is optional. The lessons provide students with additional opportunities to integrate and apply various ideas from the course to solve real-world and mathematical problems.

Unit 1: Scale Drawings

In this unit, students study scaled copies of plane figures and scale drawings of real-world objects. Students learn that all lengths in a scaled copy are the result of multiplying the original lengths by a scale factor. Also, the angle measures in a scaled copy are the same as in the original figure.



This work builds on what students learned in previous grades about measuring lengths, areas, and angles. This unit provides a geometric context to preview the type of reasoning that students will use with proportional relationships later in grade 7. It also lays the foundation for grade 8 work on dilations and similarity.

Grade 7

Students begin the unit by looking at copies of a picture and describing what differentiates scaled and non-scaled copies. They calculate scale factors and draw scaled copies of figures. Note that the study of scaled copies is limited to pairs of figures that have the same orientation — in other words, they are not rotations or reflections of each other. In grade 8, students will extend their knowledge of scaled copies when they study translations, rotations, reflections, and dilations.

Next, students study scale drawings. They see that the principles and strategies that they used to reason about scaled copies of figures can also be used with scale drawings. They use scale drawings to calculate actual lengths and areas, and they create scale drawings.

A note about the geometry toolkit:

In the unit, several lesson plans suggest that each student have access to a *geometry toolkit*. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, a centimeter ruler, a protractor (clear protractors with no holes that show radial lines are recommended), and an index card to use as a straightedge or to mark right angles. Providing students with these toolkits gives opportunities for students to develop abilities to select appropriate tools and use them strategically to solve problems (MP5). Note that even students in a digitally enhanced classroom should have access to such tools. Applets and simulations should be considered additions to their toolkits, not replacements for physical tools.

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as representing, generalizing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent

- A scaled copy for a given scale factor (Lessons 3 and 5).
- Distances using different scales (Lesson 11).
- Relevant features of a classroom with a scale drawing (Lesson 13).

Generalize

- About corresponding distances and angles in scaled copies (Lesson 4).
- About scale factors greater than, less than, and equal to 1 (Lesson 5).
- About scale factors and area (Lessons 6 and 10).
- About scale factors with and without units (Lesson 12).

Explain

- How to use scale drawings to find actual distances (Lessons 7 and 11).
- How to use scale drawings to find actual distances, speed, and elapsed time (Lesson 8).
- How to use scale drawings to find actual areas (Lesson 12).

In addition, students are expected to describe features of scaled copies, justify and critique reasoning about scaled copies, and compare how different scales affect drawings. Over the course of the unit, teachers can support students' mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

Grade 7

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
7.1.1	**scaled copy** original polygon	
7.1.2	**corresponding** **scale factor** figure segment	
7.1.4	quadrilateral measurement distance	**corresponding** **scale factor** original
7.1.5	**reciprocal**	
7.1.6	area one-dimensional two-dimensional	squared
7.1.7	**scale drawing** **scale** represent actual three-dimensional	**scaled copy**
7.1.8	estimate travel constant speed	**scale**
7.1.9	floor plan	
7.1.10	appropriate dimension	actual represent
7.1.11	scale without units ___ to ___	**scale drawing**
7.1.12	equivalent scales	___ to ___

Section A: Scaled Copies

- Lesson 1: What Are Scaled Copies?
- Lesson 2: Corresponding Parts and Scale Factors
- Lesson 3: Making Scaled Copies
- Lesson 4: Scaled Relationships
- Lesson 5: The Size of the Scale Factor
- Lesson 6: Scaling and Area

Section B: Scale Drawings

- Lesson 7: Scale Drawings
- Lesson 8: Scale Drawings and Maps
- Lesson 9: Creating Scale Drawings
- Lesson 10: Changing Scales in Scale Drawings
- Lesson 11: Scales without Units
- Lesson 12: Units in Scale Drawings

Section C: Let's Put It to Work

- Lesson 13: Draw It to Scale

Grade 7

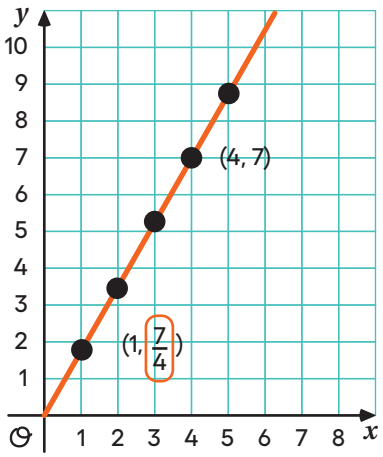
Unit 2: Introducing Proportional Relationships

In this unit, students develop the idea of a proportional relationship. They work with proportional relationships that are represented in tables, as equations, and on graphs. This builds on grade 6 work with equivalent ratios and helps prepare students for the study of linear functions in grade 8.

Students begin by looking at tables. In a table of equivalent ratios, a multiplicative relationship between a pair of rows is given by a scale factor, while the multiplicative relationship between the columns is given by a unit rate. Students learn that the relationship between pairs of values in the two columns is called a *proportional relationship*, and the unit rate that describes this relationship is called a *constant of proportionality*.

Next, students use equations of the form $y = kx$ to represent proportional relationships and solve problems. They determine whether given tables and equations could represent a proportional relationship.

Then students investigate graphs of proportional relationships. They recognize that the graph of a proportional relationship is a straight line through (0, 0). They interpret points on the graph, including the point (1, k). Here is an example of a graph, an equation, and a table that all represent the same proportional relationship.



$y = \frac{7}{4}x$

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

By the end of the unit, students should be comfortable working with common contexts associated with proportional relationships (such as constant speed, unit pricing, and measurement conversions) and be able to determine whether or not a relationship is proportional. In a later unit, students will apply proportional reasoning to solve multi-step problems and to calculate more complex rates.

A note on using the terms ratio, proportional relationship, and unit rate:

In these materials, the term "ratio" is used to mean a type of association between two or more quantities. A quantity is a measurement that can be specified by a number and a unit, for example 4 oranges, 4 centimeters, or "my height in feet." A *proportional relationship* is a collection of equivalent ratios.

A *unit rate* is the numerical part of a rate per 1 unit, for example, the 6 in 6 miles per hour. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called ratios. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are identified as "unit rates" for the ratio $a:b$. In high school—after the study of ratios, rates, and proportional relationships—students discard the term "unit rate" and start referring to a to b, $a:b$, and $\frac{a}{b}$ as "ratios."

Grade 7

In grades 6–8, students write rates without abbreviated units, for example as "3 miles per hour" or "3 miles in every 1 hour." Use of notation for derived units such as $\frac{\text{mi}}{\text{hr}}$ waits for high school—except for the special cases of area and volume.

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as comparing, interpreting, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Drink mixtures and figures (Lesson 1).
- Approaches to solving problems involving proportional relationships (Lesson 6).
- Proportional relationships with nonproportional relationships (Lesson 8).
- Tables, descriptions, and graphs representing the same situations (Lesson 10).
- Graphs of proportional relationships (Lesson 12).

Interpret

- Representations showing equivalent ratios (Lesson 1).
- Tables showing equivalent ratios (Lesson 2).
- Situations involving proportional relationships (Lesson 6 and 9).
- How a graph represents features of a situation (Lesson 11).

Generalize

- About proportional relationships (Lesson 4).
- About equations that represent proportional relationships (Lesson 5).
- About how a constant of proportionality is represented by graphs and tables (Lesson 13).

In addition, students are expected to describe proportional relationships and constants of proportionality, explain how to determine whether or not a relationship is proportional and how to compare and represent situations with different constants of proportionality, justify whether or not a relationship is proportional, and represent proportional and nonproportional relationships in multiple ways.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
7.2.1	equivalent ratios	
7.2.2	**constant of proportionality** **proportional relationship** value	equivalent ratios row column
7.2.3	___ is proportional to ___ relate constant	**reciprocal** per

Grade 7

lesson	new terminology	
	receptive	**productive**
7.2.4	equation quotient	___ is proportional to ___
7.2.5	steady situation	
7.2.6		equation quotient
7.2.7		**constant of proportionality** **proportional relationship**
7.2.8		constant
7.2.10	**origin** coordinate plane plot	
7.2.11	quantity axes coordinates	
7.2.13	x-coordinate y-coordinate	**origin**
7.2.14		axes
7.2.15	reasonable	

Section A: Representing Proportional Relationships with Tables

- Lesson 1: One of These Things Is Not Like the Others
- Lesson 2: Introducing Proportional Relationships with Tables
- Lesson 3: More about Constant of Proportionality

Section B: Representing Proportional Relationships with Equations

- Lesson 4: Proportional Relationships and Equations
- Lesson 5: Two Equations for Each Relationship
- Lesson 6: Writing Equations to Represent Relationships

Section C: Comparing Proportional and Nonproportional Relationships

- Lesson 7: Comparing Relationships with Tables
- Lesson 8: Comparing Relationships with Equations
- Lesson 9: Solving Problems about Proportional Relationships

Section D: Representing Proportional Relationships with Graphs

- Lesson 10: Introducing Graphs of Proportional Relationships
- Lesson 11: Interpreting Graphs of Proportional Relationships
- Lesson 12: Using Graphs to Compare Relationships
- Lesson 13: Two Graphs for Each Relationship

Section E: Let's Put it to Work

- Lesson 14: Four Representations
- Lesson 15: Using Water Efficiently

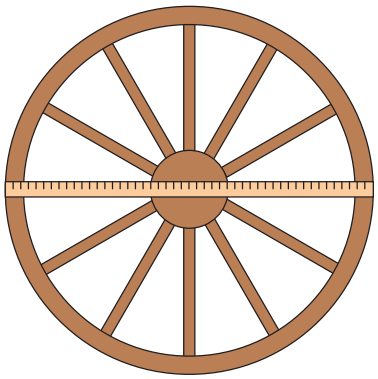
Unit 3: Measuring Circles

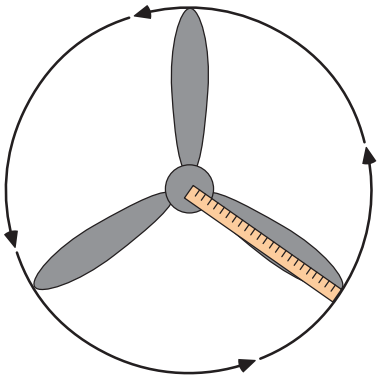
In this unit, students apply their knowledge of proportional relationships to the context of measuring circles. They learn the relationships between radius, diameter, circumference, and area of circles and use these relationships to solve problems. This builds on students' work from previous grades with perimeter and area of polygons. Students will build on this work in grade 8 when they study the volume of spheres, cylinders, and cones.

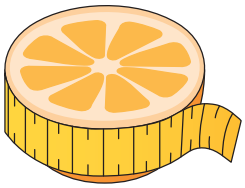
The unit begins with activities designed to help build up students' vocabulary for describing circles more precisely. The terms center, radius, diameter, and circumference are introduced. Then, students investigate the relationship between circumference and diameter and see that it is a proportional relationship. They apply this relationship to solve problems.

Next, students explore the area of circular regions. They see an informal derivation that shows where the formula $A = \pi r^2$ comes from and then use this formula to solve problems. Finally, students solve problems that require deciding whether the situation relates to the circumference or area of a circle.

The first section of this unit, in which students recognize and apply proportional relationships involving circumference, serves as a bridge between the foundational work with proportional relationships in the previous unit and the more advanced applications in the following unit. The remaining sections of this unit, which deal with the area of circles, are preparation for the continued geometry work students will do later in this course.







A note on using the term circle:

Strictly speaking, a circle is one-dimensional. It is the boundary of a two-dimensional region, rather than the region itself. The circular region is called a "disk." Because students are not yet expected to make this distinction, these materials refer to both disks and the boundaries of disks as "circles," using illustrations to eliminate ambiguity.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as generalizing, justifying, and interpreting. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Generalize

- About categories for sorting circles (Lesson 2).
- About the relationship between circumference and diameter (Lesson 3).

- About circumference and rotation (Lesson 5).
- About the relationship between the radius and the area of a circle (Lesson 8).

Justify

- Reasoning about circumference and perimeter (Lesson 4).
- Estimates for the areas of circles (Lesson 7).
- Reasoning about areas of curved figures (Lesson 9).
- Reasoning about the cost of stained glass windows (Lesson 11).

Interpret

- Situations involving circles (Lessons 5 and 8).
- Floor plans and maps (Lesson 6).
- Situations involving circumference and area (Lesson 10).

In addition, students are expected to critique reasoning about circles and circle measurements, explain reasoning, including about different approximations of pi, and describe features of graphs and of deconstructed circles.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
7.3.1	relationship perimeter	
7.3.2	**radius** **diameter** **circumference** center (of a circle)	**circle**
7.3.3	**pi**	
7.3.4	half-circle rotation approximation	
7.3.5		**diameter** **circumference** **pi** travel
7.3.6		approximate estimate
7.3.7	**area of a circle**	
7.3.8	**squared** formula	**radius** **area of a circle**
7.3.9	in terms of π	
7.3.10		**squared** center (of a circle) formula

lesson	new terminology	
	receptive	productive
7.3.11	design	

Section A: Circumference of a Circle

- Lesson 1: How Well Can You Measure?
- Lesson 2: Exploring Circles
- Lesson 3: Exploring Circumference
- Lesson 4: Applying Circumference
- Lesson 5: Circumference and Wheels

Section B: Area of a Circle

- Lesson 6: Estimating Areas
- Lesson 7: Exploring the Area of a Circle
- Lesson 8: Relating Area to Circumference
- Lesson 9: Applying Area of Circles

Section C: Let's Put it to Work

- Lesson 10: Distinguishing Circumference and Area
- Lesson 11: Stained-Glass Windows

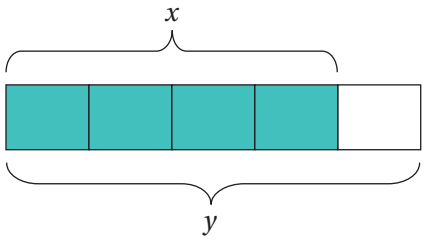
Unit 4: Proportional Relationships and Percentages

In this unit, students deepen their understanding of proportional relationships and percentages. They solve multi-step problems and work with situations that involve fractional amounts. This builds on the work students did in grade 6 with ratios, rates, and percentages as well as previous units in grade 7 with proportional relationships. Students will build on this work in high school with exponential functions representing compounded percent increase and decrease.

Students begin the unit by revisiting constant rates, but this time the given values are fractional amounts. To determine the unit rate for the situation, students must compute the quotient of two fractions. Students also make sense of situations where an increase or decrease is expressed as a fraction of the initial amount. They create diagrams and apply the distributive property to generate expressions that represent these situations. They also use long division to write fractions as decimals, including their first introduction to repeating decimals.

Next, students make sense of situations where an increase or decrease is expressed as a percentage of the initial amount. They continue creating diagrams and writing equations to represent the situations. They solve for any one of the three quantities—the initial amount, the final amount, or the percentage of the change—given the other two quantities. They also reason about fractional percentages.

Then, students apply percent increase and decrease to solve problems in a variety of real-world situations, such as tax, tip, interest, markup, discount, depreciation, and commission. Lastly, students make sense of situations where the difference between a correct measurement and an incorrect measurement is expressed as a percentage of the correct amount.



$y = x + \frac{1}{4}x$

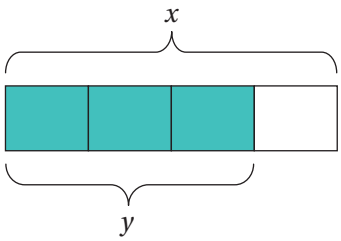
$y = \left(1 + \frac{1}{4}\right)x$

$y = \frac{5}{4}x$

$y = (1 + 0.25)x$

$y = 1.25x$

"a 25% increase"



$y = x - \frac{1}{4}x$

$y = \left(1 - \frac{1}{4}\right)x$

$y = \frac{3}{4}x$

$y = (1 - 0.25)x$

$y = 0.75x$

"a 25% decrease"

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and representing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Situations involving constant speed (Lesson 2).
- Concrete problems involving percent increase and decrease (Lesson 7).
- Problems involving sales tax and tip (Lesson 10).
- Concrete situations involving percent error (Lesson 14).

Explain

- How to solve concrete and abstract problems involving an amount plus (or minus) a fraction of that amount (Lesson 4).
- How to solve percent change problems (Lesson 6).
- Strategies for solving percent problems with fractional percentages (Lesson 9).
- How to measure lengths and interpret measurement error (Lesson 13).
- Strategies for solving percent error problems (Lesson 14).

Represent

- Situations involving percent increase and decrease (Lessons 8 and 15).
- Situations from the news involving percent change (Lesson 16).

Grade 7

In addition, students are expected to compare measurements, scale factors, and decimal and fraction representations, compare representations of an increase (or decrease) of an amount by a fraction or decimal, generalize about using constants of proportionality to solve problems efficiently and about relationships with percent increase and decrease, and justify why specific information is needed to solve percent change problems.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
7.4.1	percentage	
7.4.2		**unit rate**
7.4.4	(a fraction) more than (a fraction) less than initial/original amount final/new amount	**tape diagram** distributive property
7.4.5	**repeating decimal** **long division** decimal representation	
7.4.6	**percent increase** **percent decrease**	(a fraction) more than (a fraction) less than
7.4.7	discount	initial/original amount final/new amount
7.4.10	sales tax tax rate tip	**percent increase**
7.4.11	interest commission markup markdown	**percent decrease**
7.4.12		percentage discount
7.4.13	**measurement error**	
7.4.14	**percent error**	

Section A: Proportional Relationships with Fractions

- Lesson 1: Lots of Flags
- Lesson 2: Ratios and Rates with Fractions
- Lesson 3: Revisiting Proportional Relationships
- Lesson 4: More than That, Less than That
- Lesson 5: Say It with Decimals

Section B: Percent Increase and Decrease

- Lesson 6: Increasing and Decreasing
- Lesson 7: One Hundred Percent
- Lesson 8: Percent Increase and Decrease with Equations
- Lesson 9: Part of a Percent

Section C: Applying Percentages

- Lesson 10: Tax and Tip
- Lesson 11: Percentage Contexts
- Lesson 12: Solving Multi-step Percentage Problems
- Lesson 13: Measurement Error
- Lesson 14: Percent Error

Section D: Let's Put It to Work

- Lesson 15: Changes on the Earth
- Lesson 16: Posing Percentage Problems

Unit 5: Rational Number Arithmetic

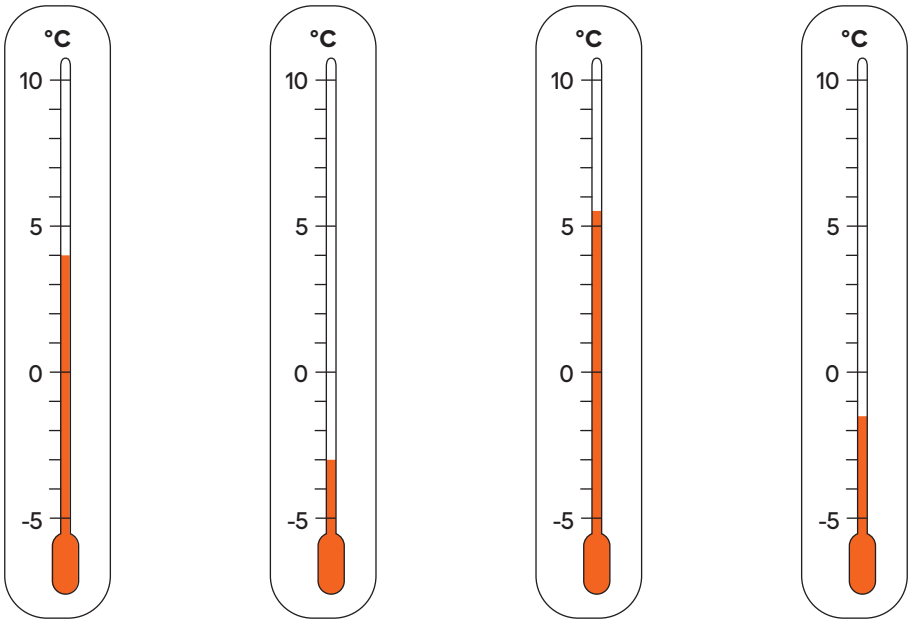
In this unit, students perform operations on rational numbers, which are all numbers that can be written as a positive or negative fraction. This builds on grade 6 work with interpreting, comparing, and plotting rational numbers. It prepares students for a later unit when they will solve equations of the form $px + q = r$ or $p(x + q) = r$, where p, q, and r are rational numbers.

Students begin by revisiting how signed numbers are used to represent quantities above and below a reference point, such as measurements of temperature and elevation. They use tables and number line diagrams to represent changes in temperature or elevation. They extend addition and subtraction from fractions to all rational numbers. And they see that $a - b$ is equivalent to $a + (-b)$.

Next, students examine multiplication and division. They work with constant velocity, which is a signed number that indicates direction and speed. This allows products of signed numbers to be interpreted in terms of position, direction of movement, and time before or after a specific point. Students use the relationship between multiplication and division to understand how division extends to rational numbers.

Then students work with expressions that use the four operations on rational numbers. They also solve problems that involve interpreting negative numbers in context. They solve linear equations of the form $p + x = q$ or $px = q$, where p and q are rational numbers. The focus of these lessons is representing situations with equations and what it means for a number to be a solution for an equation, rather than methods for solving equations. Such methods are the focus of a later unit.

Grade 7



A note on using the terms expression, equation, and signed number:

In these materials, an *expression* is built from numbers, variables, operation symbols (+, −, ·, ÷), parentheses, and exponents. (Exponents—in particular, negative exponents—are not a focus of this unit. Students work with integer exponents in grade 8 and noninteger exponents in high school.) An *equation* is a statement that two expressions are equal, thus it always has an equal sign. *Signed numbers* include all rational numbers, written as decimals or in the form $\frac{a}{b}$.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, representing, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Situations involving signed numbers (throughout unit).
- Tables with signed numbers (Lesson 3).
- Bank statements with signed numbers (Lesson 4).

Represent

- Addition of signed numbers on a number line (Lesson 2).
- Situations involving signed numbers (Lessons 3 and 11).
- Changes in elevation (Lesson 6).
- Position, speed, and direction (Lesson 8).

Generalize

- About subtracting and adding signed numbers (Lesson 5).
- About differences and magnitude (Lesson 6).
- About multiplying negative numbers (Lesson 9).
- About additive and multiplicative inverses (Lesson 15).

In addition, students are expected to justify reasoning about distances on a number line and about negative numbers, account balances, and debt. Students are also expected to explain how to determine changes in temperature, how to find information using inverses, and how to model situations involving signed numbers.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
7.5.1	degrees Celsius vertical elevation sea level	**positive number** **negative number**
7.5.2	signed numbers	temperature number line
7.5.3	**opposite** sum expression	
7.5.4	**deposit** **withdrawal** account balance debt	
7.5.6	difference	distance
7.5.7		absolute value x-coordinate y-coordinate
7.5.8	velocity	
7.5.11	**solution (to an equation)** factor	
7.5.13	rational number variable	sum difference
7.5.15	additive inverse multiplicative inverse	**opposite** **solution (to an equation)**
7.5.17		increase decrease

Section A: Adding and Subtracting Rational Numbers

- Lesson 1: Interpreting Negative Numbers
- Lesson 2: Changing Temperatures
- Lesson 3: Changing Elevation
- Lesson 4: Money and Debts
- Lesson 5: Representing Subtraction
- Lesson 6: Finding Differences
- Lesson 7: Adding and Subtracting to Solve Problems

Section B: Multiplying and Dividing Rational Numbers

- Lesson 8: Multiplying Rational Numbers (Part 1)
- Lesson 9: Multiplying Rational Numbers (Part 2)
- Lesson 10: Multiply!
- Lesson 11: Dividing Rational Numbers
- Lesson 12: Negative Rates

Section C: Four Operations with Rational Numbers

- Lesson 13: Expressions with Rational Numbers
- Lesson 14: Solving Problems with Rational Numbers
- Lesson 15: Solving Equations with Rational Numbers
- Lesson 16: Representing Contexts with Equations

Section D: Let's Put It to Work

- Lesson 17: The Stock Market

Unit 6: Expressions, Equations, and Inequalities

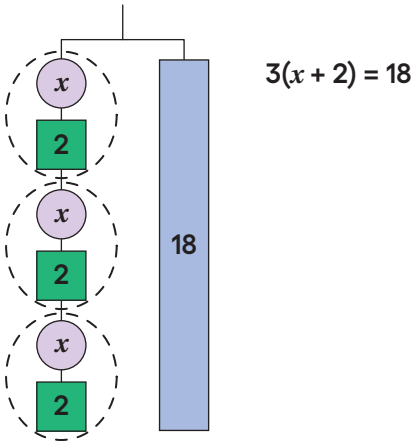
In this unit, students deepen their algebraic reasoning as they write and solve equations of the forms $px + q = r$ and $p(x + q) = r$ and inequalities of the forms $px + q > r$ and $p(x + q) < r$. Students also work with equivalent expressions that are more complex than what they have seen previously. This builds on grade 6 work with equations of the form $p + x = q$ or $px = q$ and with simpler equivalent expressions. Students will build on this work in grade 8 when they solve equations that have a variable on both sides of the equal sign and when they work with systems of equations.

Students begin the unit by making sense of situations that involve both multiplication and addition. They represent such situations with tape diagrams and with equations. They see that different diagrams and equations can represent the same situation, and they use diagrams to find solutions to equations.

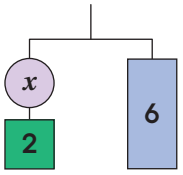
Next, students consider hanger diagrams as another way to represent equations. The diagrams help students understand solving equations in terms of "doing the same thing to each side of the equation." Students examine different pathways for solving the same equation and consider whether one method is more efficient than another.

Then students apply what they have learned about equations to inequalities. They write inequalities to represent situations and solve inequalities by reasoning about the related equation. The inequality symbols ≥ and ≤ are introduced.

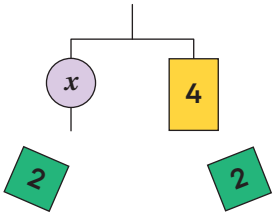
Lastly, students work with equivalent linear expressions that are more complex due to having more terms, more parentheses, and negative rational numbers. Students use properties of operations to justify why the expressions are equivalent.



$3(x + 2) = 18$



$x + 2 = 6$



$x = 4$

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, explaining, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Stories with corresponding tape diagrams (Lesson 2).
- Tape diagrams with corresponding equations (Lesson 3).
- Hanger diagrams and equations (Lesson 7).
- Solution pathways (especially Lesson 10).
- Descriptions of situations with corresponding inequalities (Lesson 16).

Explain

- Strategies for using hanger diagrams to solve equations (Lesson 8).
- Different strategies for solving equations (Lesson 9) and inequalities (Lesson 14).
- Reasoning about situations, tape diagrams, and equations (Lesson 12).
- Strategies for identifying and writing equivalent expressions (Lesson 21).

Justify

- Reasoning about inequalities (Lesson 13).
- Reasoning about solutions to inequalities (Lesson 15).
- The need for specific information in order to write and solve inequalities (Lesson 17).
- Reasoning about the distributive property (Lesson 19).
- Whether different sequences of calculations give the same result (Lesson 22).

In addition, students are expected to interpret solutions to equations, interpret and represent nonproportional situations with constant rates of change, represent nonproportional situations using tape diagrams, describe the structure of equations and tape diagrams, critique reasoning of peers about expressions and corresponding diagrams, critique reasoning about solving equations, critique reasoning about equivalent expressions, and generalize about solving equations and about when expressions are equivalent.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
7.6.2	unknown amount	
7.6.3	**equivalent expressions** commutative (property)	
7.6.4		unknown amount relationship
7.6.6		**variable**
7.6.7	balanced hanger each side (of an equation)	
7.6.8		**equivalent expression** each side (of an equation)
7.6.9		operation solve
7.6.10	distribute substitute	
7.6.13	inequality less than or equal to greater than or equal to open/closed circle	less than greater than

Grade 7

lesson	new terminology	
	receptive	**productive**
7.6.14	**solution to an inequality** boundary direction (of an inequality)	less than or equal to greater than or equal to substitute
7.6.15		open/closed circle
7.6.16		**solution to an inequality**
7.6.17		inequality
7.6.18	**term**	
7.6.19	**factor (an expression)** **expand (an expression)**	
7.6.20	combine like terms	**term** commutative (property)
7.6.21		distribute

Section A: Representing Situations of the Form $px + q = r$ and $p(x + q) = r$

- Lesson 1: Relationships between Quantities
- Lesson 2: Reasoning about Contexts with Tape Diagrams
- Lesson 3: Reasoning about Equations with Tape Diagrams
- Lesson 4: Reasoning about Equations and Tape Diagrams (Part 1)
- Lesson 5: Reasoning about Equations and Tape Diagrams (Part 2)
- Lesson 6: Distinguishing between Two Types of Situations

Section B: Solving Equations of the Form $px + q = r$ and $p(x + q) = r$ and Problems That Lead to Those Equations

- Lesson 7: Reasoning about Solving Equations (Part 1)
- Lesson 8: Reasoning about Solving Equations (Part 2)
- Lesson 9: Dealing with Negative Numbers
- Lesson 10: Different Options for Solving One Equation
- Lesson 11: Using Equations to Solve Problems
- Lesson 12: Solving Problems about Percent Increase or Decrease

Section C: Inequalities

- Lesson 13: Reintroducing Inequalities
- Lesson 14: Finding Solutions to Inequalities in Context
- Lesson 15: Efficiently Solving Inequalities
- Lesson 16: Interpreting Inequalities
- Lesson 17: Modeling with Inequalities

Section D: Writing Equivalent Expressions

- Lesson 18: Subtraction in Equivalent Expressions
- Lesson 19: Expanding and Factoring
- Lesson 20: Combining Like Terms (Part 1)
- Lesson 21: Combining Like Terms (Part 2)

Section E: Let's Put it to Work

- Lesson 22: Applications of Expressions

Grade 7

Unit 7: Angles, Triangles, and Prisms

In this unit, students investigate whether sets of angle and side length measurements determine unique triangles or multiple triangles, or fail to determine triangles. Students also study and apply angle relationships, learning to understand and use the terms "complementary," "supplementary," "vertical angles," and "unique" (MP6). The work gives them practice working with rational numbers and equations for angle relationships. Students analyze and describe cross-sections of prisms, pyramids, and polyhedra. They understand and use the formula for the volume of a right rectangular prism, and solve problems involving area, surface area, and volume (MP1, MP4). Students should have access to their geometry toolkits so that they have an opportunity to select and use appropriate tools strategically (MP5).

Note: It is not expected that students memorize which conditions result in a unique triangle, an impossible-to-create triangle, or multiple possible triangles. Understanding that, for example, side-side-side (SSS) information results in zero or exactly one triangle will be explored in high school geometry. At this level, students should attempt to draw triangles with the given information and notice that there is only one way to do it (or that it is impossible to do).

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as critiquing, explaining, interpreting, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- Reasoning about measuring angles (Lesson 1).
- Reasoning about decomposition of prisms (Lesson 13).
- Reasoning about surface area of prisms (Lesson 14).

Explain

- How to measure angles (Lesson 2).
- How to find unknown angle measurements (Lessons 4 and 5).
- How to find the volume of prisms (Lessons 12 and 13).
- How to find the surface area of prisms (Lesson 14).

Interpret

- Situations involving intersecting lines in order to form a conjecture (Lesson 3).
- Which information is relevant to answer questions (Lesson 4).
- Equations representing angle measurements (Lesson 5).
- Situations involving volume and surface area (Lesson 15 and 16).

Justify

- Whether or not shapes are identical copies (Lesson 6).
- Whether or not measurements determine identical copies (Lesson 9).
- Whether or not measurements determine unique triangles (Lesson 10).

In addition, students are expected to use language to compare angle measurements, compare triangles in a set, compare cross sections of figures, describe characteristics of pattern blocks, describe positioning and movement of side lengths and angles, and describe cross sections of prisms and pyramids. Students also have opportunities to generalize about patterns of angle measurements, about categories for unique triangles, and about categories for cross sections.

Grade 7

The table shows lessons in which new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	**productive**
7.7.1	**straight angle** **adjacent angles** degree	**right angle**
7.7.2	**supplementary** **complementary** angle measure protractor	**measurement error** degrees
7.7.3	**vertical angles** intersect vertex (of an angle)	
7.7.4		**supplementary** **vertical angles**
7.7.5	perpendicular	**complementary**
7.7.6	identical copy condition	angle measurement side length quadrilateral
7.7.7	compass different triangle	intersect identical copy segment
7.7.8		condition different triangle
7.7.9	unique triangle parallel	
7.7.10		protractor compass
7.7.11	**cross section** **base (of a prism or pyramid)** vertex (of a pyramid) face	**prism** **pyramid** perpendicular parallel
7.7.12		**volume** **cross section** **base (of a prism or pyramid)**
7.7.14		face perimeter
7.7.15		**surface area**

Section A: Angle Relationships

- Lesson 1: Relationships of Angles
- Lesson 2: Adjacent Angles
- Lesson 3: Nonadjacent Angles
- Lesson 4: Solving for Unknown Angles
- Lesson 5: Using Equations to Solve for Unknown Angles

Section B: Drawing Polygons with Given Conditions

- Lesson 6: Building Polygons (Part 1)
- Lesson 7: Building Polygons (Part 2)
- Lesson 8: Triangles with 3 Common Measures
- Lesson 9: Drawing Triangles (Part 1)
- Lesson 10: Drawing Triangles (Part 2)

Section C: Solid Geometry

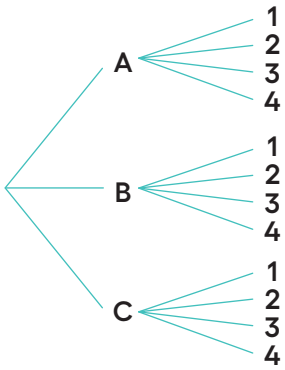
- Lesson 11: Slicing Solids
- Lesson 12: Volume of Right Prisms
- Lesson 13: Decomposing Bases for Area
- Lesson 14: Surface Area of Right Prisms
- Lesson 15: Distinguishing Volume and Surface Area
- Lesson 16: Applying Volume and Surface Area

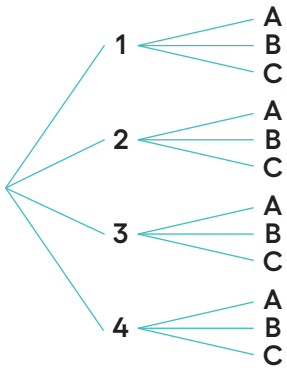
Section D: Let's Put It to Work

- Lesson 17: Building Prisms

Unit 8: Probability and Sampling

In this unit, students work with probability and sampling. They use their understanding of basic chance experiments to quantify how likely events are to happen and develop a working understanding of probability. Then they design and use simulations to further understand probability as the frequency of the event occurring when repeating an experiment many times. Students represent sample spaces using tables, tree diagrams, and lists, and use the number of outcomes in a sample space to calculate an expected probability.





Next, students examine different ways to collect data from samples within a population to understand why random selection is useful. Then students generate samples and estimate information about the population from sample data. Finally, students compare two groups by examining the measures of center and measures of variability calculated from sample data representing each group.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as describing, explaining, justifying, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Grade 7

Describe

- Observations and predictions during a game (Lesson 1).
- Patterns observed in repeated experiments (Lesson 4).
- Chance experiments to model situations (Lessons 6 and 7).
- A simulation used to model a situation (Lesson 10).
- Observations about data sets (Lessons 11 and 17).

Explain

- Predictions (Lesson 2).
- How to determine which events are more likely (Lesson 3).
- Possible differences in experimental and theoretical probability (Lesson 5).
- How to use simulations to estimate probability (Lesson 7).
- How to use a simulation to answer questions about the situation (Lesson 10).

Justify

- Whether situations are surprising and possible (Lesson 4).
- Which samples are or are not representative of a larger population (Lesson 13).
- Which samples correspond with each show, which show is most appropriate for a commercial, and whether a movie is eligible for an award (Lesson 15).
- Reasoning about samples and populations (Lesson 16).
- Whether or not differences between samples are meaningful (Lesson 18, 19, and 20).

Compare

- Sample spaces and probability of outcomes for different spinners (Lesson 5).
- Methods for writing sample spaces (Lesson 8).
- Heights of two groups (Lesson 11).
- Measures of center with samples (Lesson 13).
- Sampling methods (Lesson 14).
- Populations based on samples (Lessons 18 and 20).

In addition, students are expected to critique predictions about the mean of random samples and generalize about sample spaces, predictions, sampling, and fairness. Students also have opportunities to use language to represent data from repeated experiments, represent probabilities and sample spaces, and interpret situations involving sample spaces, probability, and populations.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

Grade 7

lesson	new terminology	
	receptive	**productive**
7.8.1		more likely less likely
7.8.2	**event** **chance experiment** **outcome** equally likely as not	likely unlikely impossible certain
7.8.3	**probability** **random** **sample space**	**outcome**
7.8.5	**simulation**	**probability** **random**
7.8.7		**event** **simulation**
7.8.8	tree (diagram)	**sample space**
7.8.9		tree (diagram)
7.8.11	mean absolute deviation (MAD) distribution very different overlap	mean median dot plot
7.8.12	**population** **sample** survey	mean absolute deviation (MAD)
7.8.13	**representative** measure of center	distribution center (of a distribution) spread
7.8.14	random sample	
7.8.15	interquartile range (IQR) measure of variability box plot	**population** **sample** random sample symmetric
7.8.16	**proportion**	**representative**
7.8.17		interquartile range (IQR) measure of variability
7.8.18	meaningful difference	overlap measure of center
7.8.20		meaningful difference

Section A: Probabilities of Single-Step Events

- Lesson 1: Mystery Bags
- Lesson 2: Chance Experiments
- Lesson 3: What Are Probabilities?
- Lesson 4: Estimating Probabilities Through Repeated Experiments
- Lesson 5: More Estimating Probabilities
- Lesson 6: Estimating Probabilities Using Simulation

Section B: Probabilities of Multi-step Events

- Lesson 7: Simulating Multi-step Experiments
- Lesson 8: Keeping Track of All Possible Outcomes
- Lesson 9: Multi-step Experiments
- Lesson 10: Designing Simulations

Section C: Sampling

- Lesson 11: Comparing Groups
- Lesson 12: Larger Populations
- Lesson 13: What Makes a Good Sample?
- Lesson 14: Sampling in a Fair Way

Section D: Using Samples

- Lesson 15: Estimating Population Measures of Center
- Lesson 16: Estimating Population Proportions
- Lesson 17: More about Sampling Variability
- Lesson 18: Comparing Populations Using Samples
- Lesson 19: Comparing Populations with Friends

Section E: Let's Put It to Work

- Lesson 20: Memory Test

Unit 9: Putting it All Together

In this optional unit, students use concepts and skills from previous units to solve problems. The first section focuses on calculating or estimating quantities associated with running a restaurant. The second section explores a variety of different contexts, such as population density, Fermi problems, measurement error, and deforestation. In the last section, students build a trundle wheel and design a five-kilometer race course.

All related standards in this unit have been addressed in prior units. These sections provide an optional opportunity for students to go more deeply and make connections between domains.

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as justifying, representing, and critiquing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Justify

- Choices and predictions in the context of running a restaurant (Lesson 1).
- Reasoning about length, area, and volume in the context of a restaurant (Lesson 3).
- Reasoning about the forested area on a map (Lesson 8).

Represent

- Costs of ingredients in a spreadsheet (Lesson 1).
- Situations using expressions and equations (Lesson 6).
- A map of a designed race course (Lesson 12).

Critique

- Peer reasoning about calculations of age, heart beats, and hairs (Lesson 5).
- Peer reasoning about percent error in length, area, and volume measurement (Lesson 7).
- Peer methods of measuring distance (Lesson 9).

In addition, students are also expected to describe methods for measuring distance, including how to build and use a trundle wheel, and compare advantages and disadvantages of different methods.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

Grade 7

lesson	new terminology	
	receptive	productive
7.9.1	spreadsheet cell formula serving	
7.9.2	profit expense	
7.9.4	population density	
7.9.7		**percent error**
7.9.8	forested land deforestation reforestation	
7.9.10	trundle wheel	
7.9.12		trundle wheel

Section A: Running a Restaurant

- Lesson 1: Cost of a Meal
- Lesson 2: Costs of Running a Restaurant
- Lesson 3: Restaurant Floor Plan

Section B: Making Connections

- Lesson 4: How Crowded Is This Neighborhood?
- Lesson 5: Fermi Problems
- Lesson 6: More Expressions and Equations
- Lesson 7: Measurement Error
- Lesson 8: Deforestation at Scale

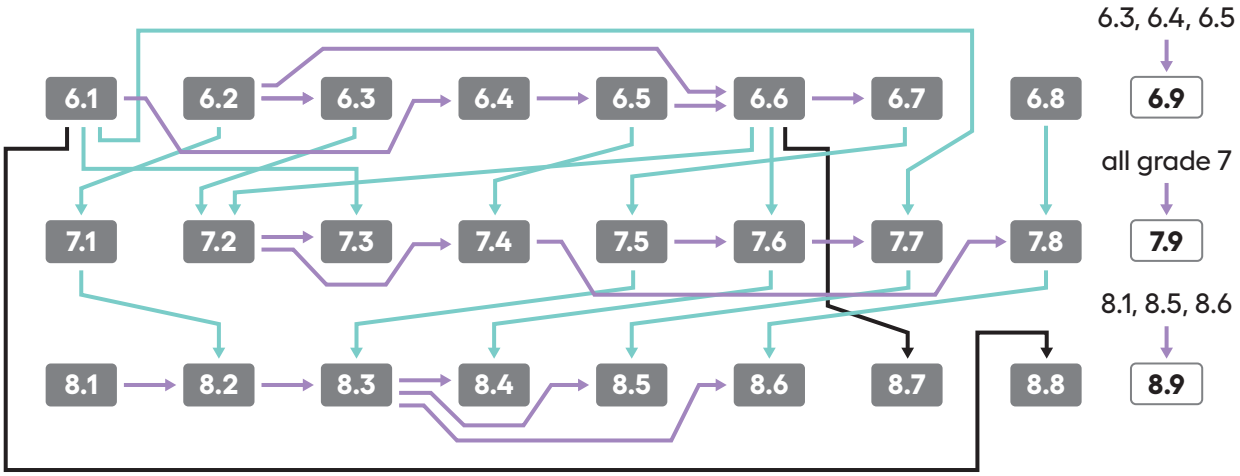
Section C: Designing a Course

- Lesson 9: Measuring Long Distances over Uneven Terrain
- Lesson 10: Building a Trundle Wheel
- Lesson 11: Using a Trundle Wheel to Measure Distances
- Lesson 12: Designing a 5K Course

Grade 7

Dependency Chart

Unit Dependency Chart



In the unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order of the units would have a negative effect on mathematical or pedagogical coherence. Examples:

- There is an arrow from 6.2 to 6.6. Students are expected to use their knowledge of contexts involving ratios (from 6.2) to write and solve equations representing such contexts (in 6.6).
- There is an arrow from 7.4 to 7.8. Students are expected to use their skills in representing percentages (from 7.4) when solving problems about probability (in 7.8).
- There is an arrow from 8.3 to 8.6. Students are expected to use their skills in writing and interpreting an equation that represents a line (from 8.3) to interpret the parameters in an equation that represents a line that fits a scatter plot (in 8.6).

Grade 8

IM Grade 8 begins with transformational geometry. Students study rigid transformations and congruence, and then dilations and similarity. This provides background for understanding the slope of a line in the coordinate plane.

Next, students build on their understanding of proportional relationships, from IM Grade 7, to study linear relationships. They use equations, tables, and graphs to represent linear relationships, and make connections across these representations. Students expand their ability to work with linear equations in one and two variables, extending their understanding of a solution to an equation in one or two variables to comprehend a solution to a system of equations in two variables. They learn that linear relationships are an example of a special kind of relationship called a *function*. Students apply their understanding of linear relationships and functions to contexts involving data with variability.

The course ends the year with students extending their understanding of exponents to include all integers, and in the process codifying the properties of exponents. They learn about orders of magnitude and scientific notation in order to represent and compute with very large and very small quantities. They encounter irrational numbers for the first time and informally extend the rational-number system to the real-number system, motivated by their work with the Pythagorean Theorem.

The final unit of the course is optional. The lessons provide students with additional opportunities to integrate and apply various ideas from the course to solve real-world and mathematical problems.

Unit 1: Rigid Transformations and Congruence

In this unit, students explore translations, rotations, and reflections of plane figures in order to understand the structure of rigid transformations. They use the properties of rigid transformations to formally define what it means for shapes to be congruent.

In earlier grades, students studied geometric measurement to find angle measures and side lengths of two-dimensional figures as well as applied area and perimeter formulas for polygons including rectangles, parallelograms, and triangles. In this unit, students build on this work as they identify corresponding congruent angles and side lengths of figures and their images under rigid transformations. In an upcoming unit, students will explore dilations and similar figures in the plane.

In the first section, students begin with an informal exploration of transformations in the plane, then increase their precision of language to describe translations, rotations, and reflections with formal descriptions, including coordinates (MP6).

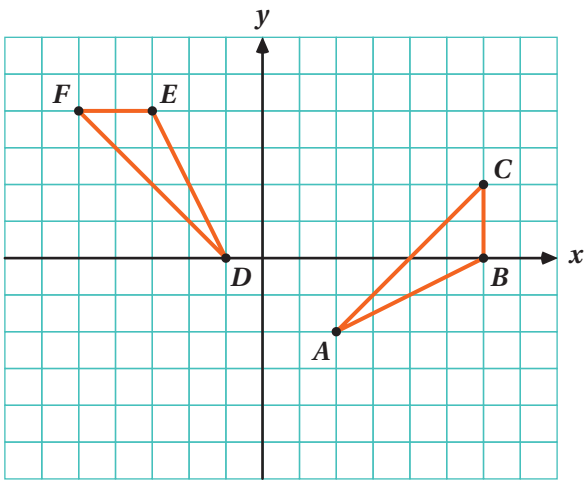
Then, students identify corresponding parts of figures and conclude that angles and distances are preserved under rigid transformations. Students use this property to reason about plane figures, including parallel lines cut by a transversal.

Students then learn the formal definition of congruent and use this definition to show that corresponding parts of congruent figures are also congruent. Finally, students apply their understanding of congruence and rigid motions to justify that the sum of the interior angles in a triangle must be 180°.

The lessons in this unit ask students to work on geometric figures that are not set in a real-world context. Students have opportunities to engage in real-world applications in the culminating lesson of the unit where they examine tessellations and other symmetric designs.

Grade 8

In this unit, students reason about congruence and justify properties of figures using rigid transformations, but they are not required to create a formal proof. They will prove these and other geometric properties more formally in later courses.



Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as describing, generalizing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Movements of figures (Lessons 1 and 2).
- Observations about transforming parallel lines (Lesson 9).
- Transformations using corresponding points, line segments, and angles (Lesson 10).
- Observations about angle measurements (Lesson 16).
- Transformations found in tessellations and in designs with rotational symmetry (Lesson 17).

Generalize

- About categories for movement (Lesson 2).
- About rotating line segments 180° (Lesson 8).
- About the relationship between vertical angles (Lesson 9).
- About transformations and congruence (Lesson 12).
- About corresponding segments and length (Lesson 13).
- About alternate interior angles (Lesson 14).
- About the sum of angles in a triangle (Lesson 16).

Justify

- Whether or not rigid transformations could produce an image (Lesson 7).
- Whether or not shapes are congruent (Lesson 11).
- Whether or not polygons are congruent (Lesson 12).
- Whether or not ovals are congruent (Lesson 13).
- Whether or not triangles can be created from given angle measurements (Lesson 15).

In addition, students are expected to explain and interpret directions for transforming figures and apply transformations to find specific images. Students are also asked to use language to compare rotations of a line segment and compare perimeters and areas of rectangles. Over the course of the unit, teachers can support students' mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

lesson	new terminology	
	receptive	**productive**
8.1.1	**vertex** plane measure direction	slide turn
8.1.2	**clockwise** **counterclockwise** **reflection** **rotation** **translation**	opposite
8.1.3	**image** angle of rotation center (of rotation) line of reflection	**vertex**
8.1.4	**transformation** **sequence of transformations** distance	**clockwise** **counterclockwise** reflect rotate translate
8.1.5	**coordinate plane** point segment coordinates x-axis y-axis	
8.1.6	polygon	angle of rotation center (of rotation) line of reflection
8.1.7	**rigid transformation** **corresponding** measurements preserve	**reflection** **rotation** **translation** measure point
8.1.8	midpoint	segment
8.1.9	**vertical angles** parallel intersect	distance

lesson	new terminology	
	receptive	productive
8.1.10		**image** **rigid transformation** midpoint parallel
8.1.11	**congruent** perimeter area	
8.1.12		**right angle** x-axis y-axis area
8.1.13		**corresponding**
8.1.14	**alternate interior angles** **transversal**	**vertical angles** **congruent** supplementary angles
8.1.15	**straight angle**	
8.1.16		**alternate interior angles** **transversal** **straight angle**
8.1.17	**tessellation** symmetry	

Section A: Rigid Transformations

- Lesson 1: Moving in the Plane
- Lesson 2: Naming the Moves
- Lesson 3: Grid Moves
- Lesson 4: Making the Moves
- Lesson 5: Coordinate Moves
- Lesson 6: Describing Transformations

Section B: Properties of Rigid Transformations

- Lesson 7: No Bending or Stretching
- Lesson 8: Rotation Patterns
- Lesson 9: Moves in Parallel
- Lesson 10: Composing Figures

Section C: Congruence

- Lesson 11: What Is the Same?
- Lesson 12: Congruent Polygons
- Lesson 13: Congruence

Section D: Angles in a Triangle

- Lesson 14: Alternate Interior Angles
- Lesson 15: Adding the Angles in a Triangle
- Lesson 16: Parallel Lines and the Angles in a Triangle

Section E: Let's Put It to Work

- Lesson 17: Rotate and Tessellate

Unit 2: Dilations, Similarity, and Introducing Slope

In this unit students learn what makes figures similar and justify claims of similarity. They are introduced to the slope of a line and use properties of similar triangles to write equations that can describe all points (x, y) on a given line.

In prior grades, students learned about the relationship between scale factors and scaled copies. Students expand on this in the first section where they learn about dilations as a new transformation that creates scaled copies.

Grade 8

In the next section, students connect dilations to earlier work with rigid transformations as they explain why two figures are similar by describing a sequence of translations, reflections, rotations, and dilations that take one figure to the other. They discover that angle measures in similar figures are preserved, which can be used to justify that two triangles are similar if they share two (or three) angle measures. Students also find that the quotients of corresponding side lengths in similar figures are equal. This along with the fact that side lengths in similar figures are all multiplied by the same scale factor allows students to calculate unknown lengths in similar figures.

In the following section, students use the similarity of slope triangles to understand why any two distinct points on a line determine the same slope. Using these same properties of similar triangles, students practice writing equations for a given line, though students are not expected at this time to write equations in the form $y = mx + b$.

The lessons in this unit ask students to work on geometric figures that are not set in a real-world context, as those tasks are sometimes contrived and hinder rather than help understanding. Students do have opportunities to tackle real-world applications in the culminating activity of the unit where students examine shadows cast by objects.

In this unit, several lesson plans suggest that each student have access to a *geometry toolkit*. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to develop their abilities to select appropriate tools and use them strategically to solve problems (MP5). Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as describing, explaining, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Observations about scaled rectangles (Lesson 1).
- Observations about dilated points, circles, and polygons (Lesson 2).
- Sequences of transformations (Lesson 6).
- Observations about side lengths in similar triangles (Lesson 9).

Explain

- How to apply dilations to find specific images (Lesson 5).
- How to determine whether triangles are congruent, similar, or neither (Lesson 8).
- Strategies for finding missing side lengths (Lesson 9).
- How to apply dilations to find specific images of points (Lesson 12).
- Reasoning for a conjecture (Lesson 13).

Represent

- Dilations using given scale factors and coordinates (Lesson 4).
- Figures using specific transformations (Lesson 6).
- Graphs of lines using equations (Lesson 12).

In addition, students are expected to use language to interpret directions for dilating figures and for creating triangles; compare dilated polygons and methods for determining similarity; critique reasoning about angles, sides, and similarity; justify whether polygons are similar; and generalize about points on a line and similar triangles.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the lesson in which it was first introduced.

lesson	new terminology	
	receptive	productive
8.2.1	scale factor scaled copy scaling	
8.2.2	**dilation** center of dilation dilate	
8.2.4		center of dilation scale factor
8.2.6	**similar**	dilate
8.2.7		**dilation**
8.2.9	quotient	
8.2.10		**slope** slope triangle
8.2.11	similarity x-coordinate y-coordinate equation of a line	quotient
8.2.13	estimate approximate/approximately	

Section A: Dilations

- Lesson 1: Projecting and Scaling
- Lesson 2: Circular Grid
- Lesson 3: Dilations with No Grid
- Lesson 4: Dilations on a Square Grid
- Lesson 5: More Dilations

Section B: Similarity

- Lesson 6: Similarity
- Lesson 7: Similar Polygons
- Lesson 8: Similar Triangles
- Lesson 9: Side Length Quotients in Similar Triangles

Section C: Slope

- Lesson 10: Meet Slope
- Lesson 11: Writing Equations for Lines
- Lesson 12: Using Equations for Lines

Section D: Let's Put It to Work

- Lesson 13: The Shadow Knows

Grade 8

Unit 3: Linear Relationships

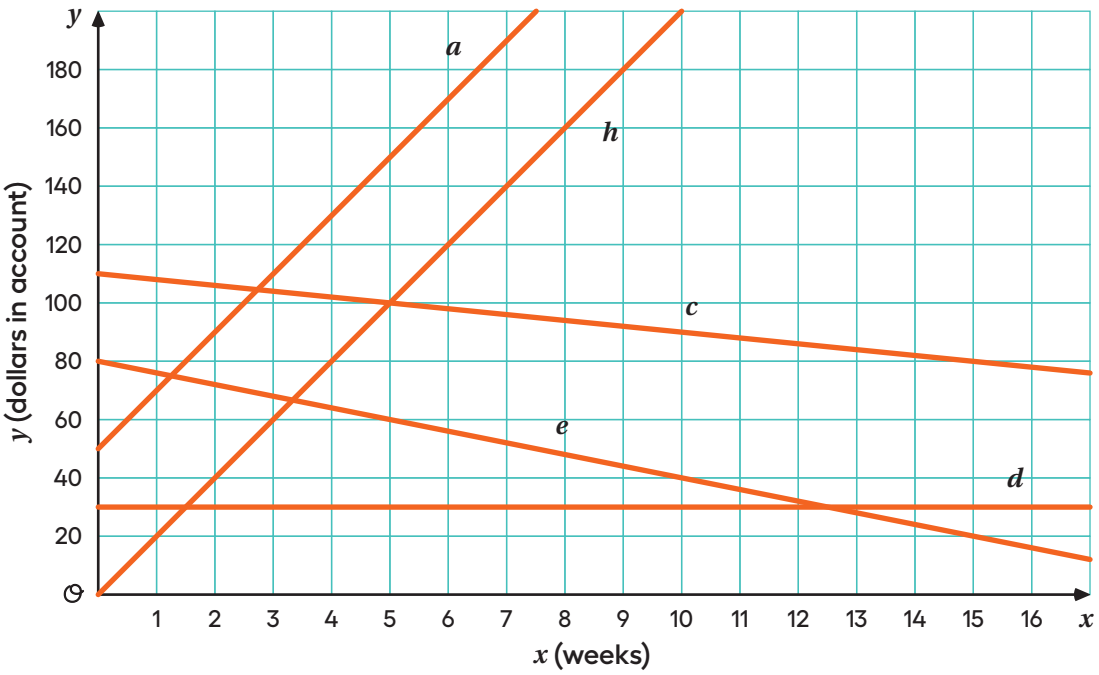
This unit introduces students to nonproportional linear relationships by building on earlier work with rates and proportional relationships from grade 7, and on earlier grade 8 work around similarity and slope.

The unit begins by revisiting different representations of proportional relationships. Students create graphs, tables, and equations in order to interpret the constant of proportionality in a context. They see the constant of proportionality between two variables as the rate of change of one variable with respect to the other.

Next, students analyze a relationship that is linear but not proportional. In this context, students see that the rate of change has a numerical value that is the same as the slope of the line that represents the relationship. Students also view the graph of a line in the coordinate plane as the vertical translation of a proportional relationship.

In the following section, students are introduced to lines with non-positive slopes and vertical intercepts. They consider situations represented by linear relationships with negative rates of change and establish a way to compute the slope of a line from any two distinct points on the line. Students also write equations of horizontal and vertical lines.

In the last section, students consider what it means for a pair of values to be a solution to an equation and the correspondence between coordinates of points on a graph and solutions of an equation.



Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as representing, generalizing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent

- Situations involving proportional relationships (Lesson 1).
- Constants of proportionality in different ways (Lesson 3).
- Slope using expressions (Lesson 10).
- Linear relationships using graphs, tables, equations, and verbal descriptions (Lesson 5).
- Situations using negative slopes and slopes of zero (Lesson 9).
- Situations by graphing lines and writing equations (Lesson 13).
- Situations involving linear relationships (Lesson 15).

Generalize

- Categories for graphs (Lesson 2).
- About equations and linear relationships (Lesson 7).
- In order to make predictions about the slope of lines (Lesson 10).

Explain

- How to graph proportional relationships (Lesson 3).
- How to use a graph to determine information about a linear situation (Lessons 5 and 6).
- How to graph linear relationships (Lesson 10 and 11).
- How slope relates to changes in a situation (Lesson 11).

In addition, students are expected to describe observations about the equation of a translated line. Students will also have opportunities to use language to interpret situations involving proportional relationships, interpret graphs using different scales, interpret slopes and intercepts of linear graphs, justify reasoning about linear relationships, justify correspondences between different representations, and justify which equations correspond to graphs of horizontal and vertical lines.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

lesson	new terminology	
	receptive	**productive**
8.3.1	represent scale label	constant of proportionality
8.3.2	equation	
8.3.3	**rate of change**	equation
8.3.5	**linear relationship** constant rate **rate of change**	slope
8.3.6	**vertical intercept** y-intercept	
8.3.7	initial (value or amount)	constant rate
8.3.8	relate	
8.3.9	horizontal intercept x-intercept	

lesson	new terminology	
	receptive	productive
8.3.10		**rate of change** **vertical intercept** y-intercept
8.3.12	constraint	horizontal line vertical line
8.3.13	**solution to an equation with two variables** variable combination set of solutions	

Section A: Proportional Relationships

Lesson 1: Understanding Proportional Relationships
Lesson 2: Graphs of Proportional Relationships
Lesson 3: Representing Proportional Relationships
Lesson 4: Comparing Proportional Relationships

Section B: Representing Linear Relationships

Lesson 5: Introduction to Linear Relationships
Lesson 6: More Linear Relationships
Lesson 7: Representations of Linear Relationships
Lesson 8: Translating to $y = mx + b$

Section C: Finding Slopes

Lesson 9: Slopes Don't Have to Be Positive
Lesson 10: Calculating Slope
Lesson 11: Line Designs
Lesson 12: Equations of All Kinds of Lines

Section D: Linear Equations

Lesson 13: Solutions to Linear Equations
Lesson 14: More Solutions to Linear Equations

Section E: Let's Put it to Work

Lesson 15: Using Linear Relations to Solve Problems

Unit 4: Linear Equations and Linear Systems

In this unit, students work with writing equivalent equations and use reasoning to solve equations for a variable. Then students solve systems of linear equations using graphic and algebraic methods.

The unit begins with a focus on moves that can be done to write equivalent equations. At first, students use hanger diagrams as an intuitive representation of equality and represent their reasoning by labeling arrows that connect equivalent representations. With the reintroduction of negative values, students move away from hanger diagrams to algebraic equations and writing equivalent equations with the intention of solving for a variable.

Next, students examine the conditions under which equations could have 0, 1, or infinite solutions as a transition to thinking about similar situations involving systems of equations. Students finish the unit by examining systems of equations graphically and then finding solutions algebraically. They build on their understanding that the line representing an equation with 2 variables is made up of coordinate pairs that make the equation true. They find that the intersection of 2 lines is the point that makes both equations for the system true. Students also recognize when systems have no solution or infinite solutions based on the graphs and the slope and intercept.

Grade 8

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as critiquing, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- Strategies for writing equivalent equations (Lesson 1).
- Reasoning about maintaining balance in equations (Lesson 3).
- Solutions of linear equations (Lessons 4 and 5).
- Reasoning about structures of systems of equations (Lesson 14).
- Explanations of solutions (Lesson 16).

Justify

- Strategies for writing equivalent equations (Lessons 1 and 5).
- Predictions about maintaining balance (Lesson 2).
- Predictions about solutions of linear equations (Lesson 6).

Generalize

- About the structures of equations that have one, infinite, and no solutions (Lessons 7 and 8).
- About the structures of systems of equations (Lessons 14 and 15).

In addition, students are expected to use language to represent and interpret situations involving systems of linear equations, compare solutions of linear equations, and describe graphs of systems of linear equations.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

lesson	new terminology	
	receptive	productive
8.4.1	representation equivalent expression	
8.4.2	expression	
8.4.3	solution to an equation distribute	
8.4.4	substitute	equation
8.4.5	term like terms distributive property factor	
8.4.6		term like terms distribute common denominator

lesson	new terminology	
	receptive	**productive**
8.4.7	no solution (only) one solution	
8.4.8	**constant term** coefficient linear equation infinitely many solutions	expression variable
8.4.11	ordered pair	
8.4.12	**system of equations** solution to a system of equations	
8.4.13	substitution	substitute no solution (only) one solution infinitely many solutions
8.4.14	algebraically	
8.4.15		**system of equations** substitution

Section A: Equivalent Equations

- Lesson 1: Writing Equivalent Equations
- Lesson 2: Keeping the Equation Balanced
- Lesson 3: Balanced Moves
- Lesson 4: More Balanced Moves
- Lesson 5: Solving Any Linear Equation
- Lesson 6: Strategic Solving

Section B: Linear Equations in One Variable

- Lesson 7: All, Some, or No Solutions
- Lesson 8: How Many Solutions?
- Lesson 9: When Are They the Same?

Section C: Systems of Linear Equations

- Lesson 10: On or Off the Line?
- Lesson 11: On Both of the Lines
- Lesson 12: Systems of Equations
- Lesson 13: Solving Systems of Equations
- Lesson 14: Solving More Systems
- Lesson 15: Writing Systems of Equations

Section D: Let's Put It to Work

- Lesson 16: Solving Problems with Systems of Equations

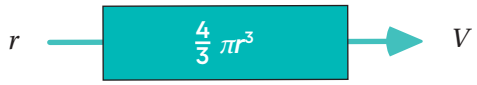
Unit 5: Functions and Volume

In this unit, students are introduced to the concept of a function as a relationship between "inputs" and "outputs" in which each allowable input determines exactly one output.

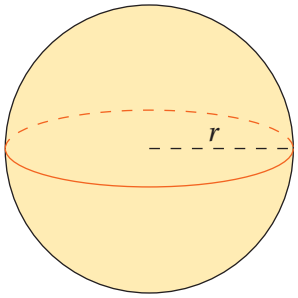
In the first three sections of the unit, students work with relationships that are familiar from previous grades or units (perimeter formulas, proportional relationships, linear relationships), expressing them as functions. They study the different ways functions can be represented, making connections between the representations and interpreting what they mean in context. Linear functions are a focus of the third section, and students will continue to work with linear functions in a later unit to model data. The use of function notation is left for a future course.

In the remaining three sections of the unit, students build on their knowledge of the formula for the volume of a right rectangular prism from grade 7, learning formulas for volumes of cylinders, cones, and spheres. Students express functional relationships described by these formulas as equations, focusing on situations involving proportional relationships. They use these relationships to reason

about how the volume of a figure changes as one of its dimensions changes, transforming algebraic expressions to get the information they need. In future courses, students will continue this thinking as they study nonlinear relationships and question how, for example, the volume of a sphere changes as the radius increases.



r	V
0	0
2	$\frac{32}{3}\pi$
6	288π
r	$\frac{4}{3}\pi r^3$



Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as generalizing, justifying, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Generalize

- About what happens to inputs for each rule (Lesson 1).
- About dimensions of cylinders (Lesson 14).
- About the relationship between the volumes of cylinders and cones (Lesson 15).
- About dimensions of cones (Lesson 16).
- About volumes of spheres, cones, and cylinders as functions of their radii (Lesson 21).

Justify

- Claims about what can be determined from given information (Lesson 2).
- Claims about volumes of cubes and spheres based on graphs (Lesson 7).
- Claims about approximately linear relationships (Lesson 10).
- Reasoning about the volumes of spheres and cones (Lesson 21).

Compare

- Different representations of functions (Lesson 3).
- Features of graphs, equations, and situations (Lesson 4).
- Features of a situation with features of a graph (Lesson 6).
- Temperatures shown on a graph with different temperatures given in a table (Lesson 7).
- The volumes of cones with the volumes of cylinders (Lesson 16).
- Methods for finding and approximating the volume of a sphere as function of its radius (Lesson 20).

In addition, students are expected to interpret representations of volume functions of cylinders, cones, and spheres and expected to describe the following: quantities in a situation, volume measurements and features of three-dimensional figures, the effects of varying dimensions of rectangular prisms and cones on their volumes, approximately linear relationships. Students are also expected to use language to represent relationships between volume and variable side length of a rectangular prism and relationships between volume and variable height of a cylinder, explain

and represent how height and volume of cylinders are related, and explain reasoning about finding the volume of a cylinder and about the relationship between volumes of hemispheres and volumes of boxes, cylinders, and cones.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

lesson	new terminology	
	receptive	productive
8.5.1	input output	
8.5.2	**function**	input output depends on
8.5.3	**independent variable** **dependent variable** radius	
8.5.5	prediction	
8.5.7	volume cube	
8.5.8	functional relationship linear function	**function**
8.5.9	mathematical model	prediction
8.5.10	piecewise linear function	linear function constant rate
8.5.11	cylinder three-dimensional	radius
8.5.12	cone sphere dimension	cylinder cube cubic centimeter rectangular prism
8.5.13	base (of a cylinder or cone) approximation for π	volume
8.5.14		base (of a cylinder or cone)
8.5.16		cone
8.5.19	hemisphere	
8.5.20		sphere
8.5.21	spherical	
8.5.22	approximate range	

Section A: Inputs and Outputs

- Lesson 1: Inputs and Outputs
- Lesson 2: Introduction to Functions

Section B: Representing and Interpreting Functions

- Lesson 3: Equations for Functions
- Lesson 4: Tables, Equations, and Graphs of Functions
- Lesson 5: More Graphs of Functions
- Lesson 6: Even More Graphs of Functions
- Lesson 7: Connecting Representations of Functions

Section C: Linear Functions and Rates of Change

- Lesson 8: Linear Functions
- Lesson 9: Linear Models
- Lesson 10: Piecewise Linear Functions

Section D: Cylinders and Cones

- Lesson 11: Filling Containers
- Lesson 12: How Much Will Fit?
- Lesson 13: The Volume of a Cylinder
- Lesson 14: Finding Cylinder Dimensions
- Lesson 15: The Volume of a Cone
- Lesson 16: Finding Cone Dimensions

Section E: Dimensions and Spheres

- Lesson 17: Scaling One Dimension
- Lesson 18: Scaling Two Dimensions
- Lesson 19: Estimating a Hemisphere
- Lesson 20: The Volume of a Sphere
- Lesson 21: Cylinders, Cones, and Spheres

Section F: Let's Put It to Work

- Lesson 22: Volume as a Function of . . .

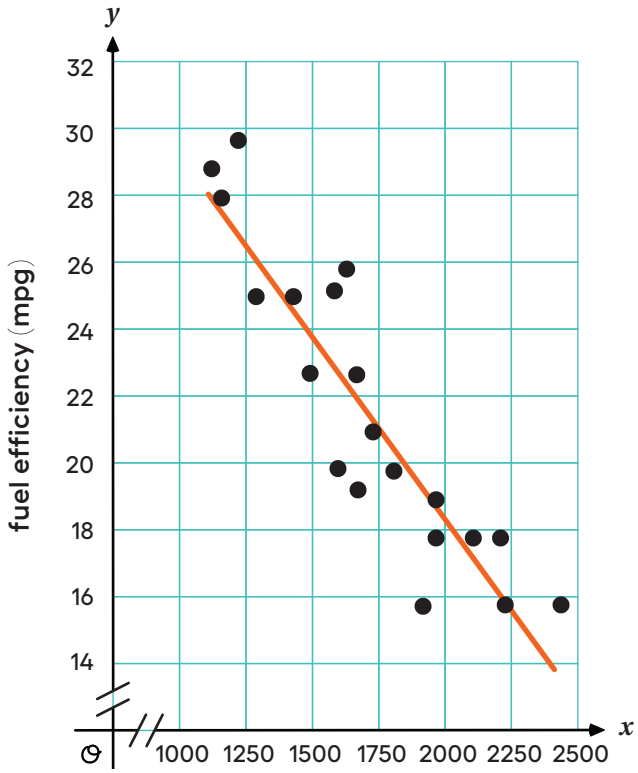
Unit 6: Associations in Data

In this unit, students analyze bivariate data. They will use scatter plots and fitted lines to analyze numerical data, and two-way tables, bar graphs, and segmented bar graphs to analyze categorical data. Students advance their understanding of lines by examining slopes in the context of data. They will revisit these data analysis topics in a later course in more depth. At this level, students should be able to construct and interpret points on a scatter plot, informally fit linear models to data, interpret a given linear model in the context of data, and generally recognize patterns of association using relative frequencies in a two-way table.

In prior grades, students analyzed data collected about one variable using dot plots, histograms, and box plots. This unit expands on that by considering the possible influence of a second variable on measurements about individuals.
In the first section, students are introduced to scatter plots and are reminded how to interpret points on a graph using a context. They also begin to recognize general trends in data.

In the second section, students look more closely at associations in data by informally drawing lines that model the general trend of the data. They also classify associations as positive, negative, linear, and non-linear by looking at the shape of the data in a scatter plot.

In the third section, students look at categorical data using two-way tables and relative frequencies. They then informally look at the relative frequencies to notice whether the variables are associated or not.



The unit ends with a lesson in which students collect and analyze numerical data using a scatter plot, then categorize the data based on a threshold and analyze the categories based on a two-way table.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as explaining, representing, and interpreting. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Explain

- How to estimate using available data (Lesson 1).
- How to use tables and scatter plots to make estimates and predictions (Lesson 3).
- The meaning of slope for a situation (Lesson 6).
- How to use lines to show associations, identify outliers, and answer questions (Lesson 8).

Represent

- Data in organized ways (Lesson 1).
- Data using two-way tables, bar graphs, and segmented bar graphs (Lessons 9 and 10).
- Data using scatter plots (Lesson 11).

Interpret

- Situations and graphs involving bivariate data (Lesson 2).
- Tables and scatter plots of bivariate data (Lesson 3).
- Tables, scatter plots, equations, and situations involving bivariate data (Lesson 4).

In addition, students are expected to compare different representations of the same situation, describe and compare features of scatter plots, justify whether or not lines are good fits for a situation, and justify associations between bivariate data. Students also have opportunities to use language to generalize about what makes a line fit a data set well and about categories for sorting scatter plots.

This table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

lesson	new terminology	
	receptive	productive
8.6.1	**scatter plot**	
8.6.2	data display attribute	numerical data categorical data
8.6.4	**outlier** predict overpredict underpredict linear model	
8.6.5	**positive association** **negative association**	
8.6.6	linear association nonlinear association no association fitted line	
8.6.7	cluster	
8.6.8		**independent variable** **dependent variable** **positive association** **negative association** linear association
8.6.9	**segmented bar graph** **relative frequency** **two-way (frequency) table**	
8.6.11		**scatter plot** **outlier** cluster

Grade 8

Section A: Does This Predict That?

- Lesson 1: Organizing Data
- Lesson 2: Plotting Data
- Lesson 3: What a Point in a Scatter Plot Means

Section B: Associations in Numerical Data

- Lesson 4: Fitting a Line to Data
- Lesson 5: Describing Trends in Scatter Plots
- Lesson 6: The Slope of a Fitted Line
- Lesson 7: Observing More Patterns in Scatter Plots
- Lesson 8: Analyzing Bivariate Data

Section C: Associations in Categorical Data

- Lesson 9: Looking for Associations
- Lesson 10: Using Data Displays to Find Associations

Section D: Let's Put It to Work

- Lesson 11: Gone In 30 Seconds

Unit 7: Exponents and Scientific Notation

In this unit, students deepen their understanding of exponents, powers of 10, and place value before being introduced to scientific notation. They build on work done in a previous course where students focused on whole-number exponents with whole-number, fraction, decimal, or variable bases, but did not formulate rules regarding the use of exponents.

Students begin this unit by identifying patterns that emerge when multiplying and dividing powers of 10, and when raising powers of 10 to another power. Students generalize these patterns to develop exponent rules. They extend these rules to see why 10^0 must be equal to 1 and to understand what negative exponents mean.

Next, students determine that the rules developed for powers of 10 also work with other bases, as long as the bases in both expressions are the same. They observe a new rule that applies when multiplying bases that are different if the exponents are the same.

$a^n \cdot a^m = a^{n+m}$	$(a^n)^m = a^{n \cdot m}$
$\frac{a^n}{a^m} = a^{n-m}$	$a^0 = 1$
$a^{-n} = \frac{1}{a^n}$	$a^n \cdot b^n = (a \cdot b)^n$

In the next section, students return to working with powers of 10 as they use multiples of powers of 10 to describe magnitudes of very large and very small quantities, such as the distance from Earth to the sun in kilometers or the mass of a proton in grams. Students plot these large and small values on number lines labeled using exponents and see how these numbers can be expressed in different ways — for example as $75 \cdot 10^5$ or $7.5 \cdot 10^6$.

After building a foundation connecting powers of 10 with place value, students are finally introduced to scientific notation as a specific and useful way of writing numbers as a power of 10. They compute sums, differences, products, and quotients of numbers written in scientific notation to make additive and multiplicative comparisons, estimate quantities, and make measurement conversions.

Grade 8

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as critiquing, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- Reasoning about powers of powers (Lesson 3).
- Reasoning about zero exponents (Lesson 4).
- Applications of exponent rules (Lesson 7).
- Reasoning about scientific notation (Lesson 15).

Represent

- Situations using exponents (Lesson 1).
- Large and small numbers using number lines, exponents, and decimals (Lesson 9–11).
- Situations comparing quantities expressed in scientific notation (Lesson 14).

Justify

- Reasoning about multiplying powers of 10 (Lesson 2).
- Reasoning about powers of powers (Lesson 3).
- Reasoning about dividing powers of 10 (Lesson 4).
- Whether or not expressions are equivalent to exponential expressions (Lesson 6).
- Reasoning about situations comparing powers of 10 (Lesson 12).

In addition, students are expected to use language to generalize reasoning about repeated multiplication and generalize about patterns when multiplying different bases and exponents; describe how negative powers of 10 affect placement of decimals; and interpret situations comparing quantities expressed in scientific notation. Students also have opportunities to compare correspondences between exponential expressions and base-ten diagrams; compare expressions in scientific notation to other expressions; explain how to simplify expressions with negative powers of 10; and explain how to place and order large numbers on a number line.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

Grade 8

lesson	new terminology	
	receptive	productive
8.7.1	**exponent** **base (of an exponent)** power factor reciprocal	repeated multiplication
8.7.2	powers of 10	
8.7.3	power of powers	
8.7.4	expanded positive exponent zero exponent	
8.7.5	negative exponent	positive exponent
8.7.6		**exponent** **base (of an exponent)** power zero exponent
8.7.7	evaluate	factor power of powers negative exponent
8.7.8	square (of a number)	
8.7.9	billion trillion multiple of	
8.7.10	integer	
8.7.12		multiple of
8.7.13	**scientific notation**	integer
8.7.14		powers of 10 billion trillion
8.7.15		**scientific notation**

Section A: Exponent Rules

- Lesson 1: Exponent Review
- Lesson 2: Multiplying Powers of 10
- Lesson 3: Powers of Powers of 10
- Lesson 4: Dividing Powers of 10
- Lesson 5: Negative Exponents with Powers of 10

Section B: More Exponent Rules

- Lesson 6: What About Other Bases?
- Lesson 7: Practice with Rational Bases
- Lesson 8: Combining Bases

Section C: Large and Small Numbers

- Lesson 9: Describing Large and Small Numbers Using Powers of 10
- Lesson 10: Representing Large Numbers on the Number Line
- Lesson 11: Representing Small Numbers on the Number Line
- Lesson 12: Applications of Arithmetic with Powers of 10

Grade 8

Section D: Scientific Notation

- Lesson 13: Definition of Scientific Notation
- Lesson 14: Estimating with Scientific Notation
- Lesson 15: Adding and Subtracting with Scientific Notation

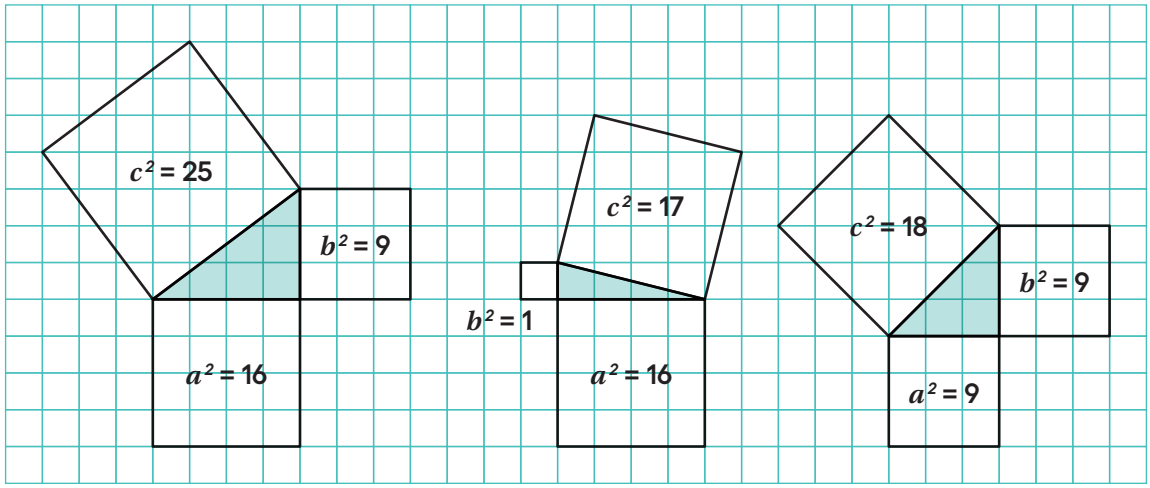
Section E: Let's Put it To Work

- Lesson 16: Is a Smartphone Smart Enough to Go to the Moon?

Unit 8: Pythagorean Theorem and Irrational Numbers

This unit introduces students to irrational numbers with a focus on connecting geometric and algebraic representations of square roots, cube roots, and the Pythagorean Theorem.

In the first section, students extend work from grade 6, composing and decomposing shapes to find the areas of tilted squares. They see "square root of n" and $\sqrt{n}$ to mean the side length of a square with area n square units, and understand that finding the solution to equations of the form $x^2 = n$ means determining which values of x make the equation true. Students learn and use definitions for "rational number" and "irrational number," learn (without proof) that $\sqrt{2}$ is irrational, and they plot square roots on the number line.



In the second section, students continue using tilted squares as they investigate relationships between side lengths of right and non-right triangles. Students are encouraged to notice patterns among the triangles before being shown geometric and algebraic proofs of the Pythagorean Theorem. They use the Pythagorean Theorem and its converse to solve problems in two and three dimensions, for example, to determine lengths of diagonals of rectangles and right rectangular prisms, and to estimate distances between points in the coordinate plane.

In the third section, students see that "cube root of n" and $\sqrt[3]{n}$ mean the side length of a cube with volume n cubic units. They also represent a cube root as a decimal approximation and as a point on the number line.

In the fourth section, students consider the decimal expansions of rational and irrational numbers. They learn how to rewrite fractions as a repeating decimal, how to rewrite a repeating decimal as a fraction, and reinforce their understanding that irrational numbers have a place on the number line even if they cannot be written as a fraction of integers.

Grade 8

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as explaining, justifying, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Explain

- Strategies for finding area (Lesson 1).
- Strategies for approximating and finding square roots (Lesson 5).
- Strategies for finding triangle side lengths (Lesson 7).
- Predictions about situations involving right triangles and strategies to verify (Lesson 11).
- Strategies for finding distances between points on a coordinate plane (Lesson 13).
- Strategies for approximating the value of cube roots (Lesson 15).

Justify

- Which squares have side lengths in a given range (Lesson 2).
- Ordering of irrational numbers (Lesson 6).
- Ordering of hypotenuse lengths (Lesson 10).

Compare

- Rational and irrational numbers (Lesson 4).
- Lengths of diagonals in rectangular prisms (Lesson 11).
- Strategies for approximating irrational numbers (Lesson 17).

In addition, students will do the following: Use language to generalize about area of squares, square roots, and approximations of side lengths and about the distance between any two coordinate pairs, critique reasoning about square root approximations and critique a strategy to represent repeating decimal expansions as fractions, describe observations about the relationships between triangle side lengths and describe hypotenuses and side lengths for given triangles, interpret diagrams involving squares and right triangles, interpret equations and approximations for the value of square and cube roots, and represent relationships between side lengths and areas.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

lesson	new terminology	
	receptive	productive
8.8.2	**square root**	square (of a number)
8.8.4	**irrational number** square root symbol	**rational number**
8.8.5	diagonal decimal approximation	
8.8.6		**square root** square root symbol

Grade 8

lesson	new terminology	
	receptive	productive
8.8.7	**Pythagorean Theorem** **hypotenuse** **legs**	right triangle
8.8.10	converse of the Pythagorean Theorem	**Pythagorean Theorem**
8.8.11	edge length	**hypotenuse** **legs**
8.8.14	**cube root**	
8.8.15		**cube root** edge length
8.8.16	decimal representation finite decimal expansion	
8.8.17	**repeating decimal** infinite decimal expansion	**irrational number**

Section A: Side Lengths and Areas of Squares

- Lesson 1: The Areas of Squares
- Lesson 2: Side Lengths and Areas
- Lesson 3: Square Roots
- Lesson 4: Rational and Irrational Numbers
- Lesson 5: Square Roots on the Number Line
- Lesson 6: Reasoning about Square Roots

Section B: The Pythagorean Theorem

- Lesson 7: Finding Side Lengths of Triangles
- Lesson 8: A Proof of the Pythagorean Theorem
- Lesson 9: Finding Unknown Side Lengths
- Lesson 10: The Converse
- Lesson 11: Applications of the Pythagorean Theorem
- Lesson 12: More Applications of the Pythagorean Theorem
- Lesson 13: Finding Distances in the Coordinate Plane

Section C: Side Lengths and Volumes of Cubes

- Lesson 14: Edge Lengths and Volumes
- Lesson 15: Cube Roots

Section D: Decimal Representation of Rational and Irrational Numbers

- Lesson 16: Decimal Representations of Rational Numbers
- Lesson 17: Infinite Decimal Expansions

Section E: Let's Put It to Work

- Lesson 18: When Is the Same Size Not the Same Size?

Unit 9: Putting It All Together

In these optional lessons, students solve complex problems. In the first several lessons, they consider tessellations of the plane, understanding and using the terms "tessellation" and "regular tessellation" in their work, as well as using properties of shapes (for example, the sum of the interior angles of a quadrilateral is 360 degrees) and transformations to make inferences about regular tessellations. These lessons need to come after unit 8.1 has been done. In the later lessons, they investigate factors that impact predicting the temperature. In particular, they use scatter plots and lines of best fit to model the association between temperature and latitude. These lessons need to come after units 8.5 and 8.6 have been done.

All related standards in this unit have been addressed in prior units. These sections provide an *optional* opportunity for students to go more deeply and make connections between domains.

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes such as describing, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Tessellations (Lesson 1)
- Associations in bivariate data (Lesson 5).

Represent

- The relationship between latitude and weather (Lesson 5).

Justify

- Claims about shapes that can and cannot be used to produce regular tessellations (Lesson 2).

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

lesson	new terminology	
	receptive	productive
8.9.1		**tessellation** pattern
8.9.2	regular tessellation	regular polygon
8.9.6		mathematical model

Section A: Tessellations

- Lesson 1: Tessellations of the Plane
- Lesson 2: Regular Tessellations
- Lesson 3: Tessellating Polygons

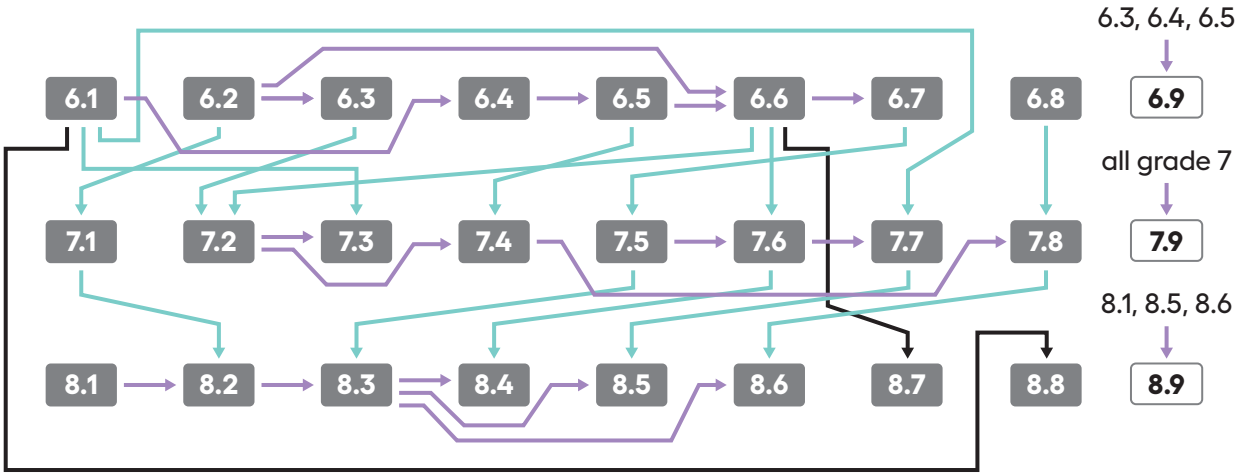
Section B: Predicting the Temperature

- Lesson 4: What Influences Temperature?
- Lesson 5: Plotting the Temperature
- Lesson 6: Using and Interpreting a Mathematical Model

Grade 8

Dependency Chart

Unit Dependency Chart



In the unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order of the units would have a negative effect on mathematical or pedagogical coherence. Examples:

- There is an arrow from 6.2 to 6.6. Students are expected to use their knowledge of contexts involving ratios (from 6.2) to write and solve equations representing such contexts (in 6.6).
- There is an arrow from 7.4 to 7.8. Students are expected to use their skills in representing percentages (from 7.4) when solving problems about probability (in 7.8).
- There is an arrow from 8.3 to 8.6. Students are expected to use their skills in writing and interpreting an equation that represents a line (from 8.3) to interpret the parameters in an equation that represents a line that fits a scatter plot (in 8.6).

Pacing Guide

Pacing Guide

The number of days includes two assessment days per unit. The upper bound of the range includes optional lessons.

	Grade 6	Grade 7	Grade 8
week 1	Unit 1 Area and Surface Area (20–22 days) (MA) Optional Lessons: 16, 19	Unit 1 Scale Drawings (12–15 days) Optional Lessons: 6, 8, 13	Unit 1 Rigid Transformations and Congruence (20 days) (MA) Optional Lessons: none
week 2			
week 3			
week 4		Unit 2 Introducing Proportional Relationships (16–17 days) Optional Lessons: 14	
week 5	Unit 2 Introducing Ratios (19 days) Optional Lessons: none		Unit 2 Dilations, Similarity, and Introducing Slope (15 days) Optional Lessons: none
week 6			
week 7			
week 8		Unit 3 Measuring Circles (11–13 days) Optional Lessons: 5, 11	Unit 3 Linear Relationships (16–17 days) Optional Lessons: 11
week 9	Unit 3 Unit Rates and Percentages (17–19 days) Optional Lessons: 9, 16		
week 10		Unit 4 Proportional Relationships and Percentages (17–18 days) Optional Lesson: 15	
week 11			Unit 4 Linear Equations and Linear Systems (18 days) Optional Lessons: none
week 12			
week 13	Unit 4 Dividing Fractions (16–20 days) (MA) Optional Lessons: 3, 4, 9, 16	Unit 5 Rational Number Arithmetic (18–19 days) Optional Lesson: 10	
week 14			
week 15			Unit 5 Functions and Volume (23–25 days) (MA) Optional Lessons: 18, 22
week 16		Unit 6 Expressions, Equations, and Inequalities (25 days) (MA) Optional Lessons: none	
week 17	Unit 5 Arithmetic in Base Ten (15–18 days) (MA) Optional Lessons: 2, 9, 15		
week 18			
week 19			Unit 6 Associations in Data (12–13 days) Optional Lesson: 11
week 20			
week 21	Unit 6 Expressions and Equations (20–22 days) (MA) Optional Lessons: 11, 18	Unit 7 Angles, Triangles, and Prisms. (18–19 days) Optional Lesson: 17	Unit 7 Exponents and Scientific Notation (18 days) Optional Lessons: none
week 22			
week 23			
week 24			
week 25	Unit 7 Rational Numbers (20–21 days) Optional Lesson: 19	Unit 8 Probability and Sampling (21–23 days) (MA) Optional Lessons: 17, 20	Unit 8 Pythagorean Theorem and Irrational Numbers (20 days) Optional Lessons: none
week 26			
week 27			
week 28			
week 29	Unit 8 Data Sets and Distributions (20–21 days) (MA) Optional Lesson: 18	Unit 9 Putting It All Together (0–12 days) Optional Lessons: all	Unit 9 Putting It All Together (0–6 days) Optional Lessons: all
week 30			
week 31			
week 32			
week 33	Unit 9 Putting It All Together (0–11 days) Optional Lessons: all		
week 34			
week 35			

(MA) = Unit has Mid-Unit Assessment

Total number of days for each course = Lessons + Assessments – Optional Lessons

Grade 6 = 147 Days Grade 7 = 138 Days Grade 8 = 142 Days