ESL
SCIENCE
BUSINESS
BILINGUAL
PRESCHOOL
MATHEMATICS
LIBRARY MEDIA
SOCIAL STUDIES
WORLD LANGUAGES
GIFTED & TALENTED
TECHNOLOGY EDUCATION
ENGLISH LANGUAGE ARTS
FINE & PERFORMING ARTS
FAMILY & CONSUMER SCIENCE
HEALTH & PHYSICAL EDUCATION

RAHWAY PUBLIC SCHOOLS

CURRICULUM & INSTRUCTION

Content Area: Mathematics

Course: Advanced Placement Statistics

Grade Level: 11-12

This curriculum is part of the Educational Program of Studies of the Rahway Public Schools.

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Program Supervisor of 7-12 Math & Science and 9-12 Business & Technology Education

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Subject/Course Title:
Advanced Placement Statistics
Grades 11-12

Date of Board Adoption:
August 25, 2026

RAHWAY PUBLIC SCHOOLS CURRICULUM

Advanced Placement Statistics: Grades 11-12

PACING GUIDE

Unit	Title	Pacing
1	Exploring One-Variable Data and Collecting Data	6 weeks
2	Probability, Random Variables, and Probability Distributions	5 weeks
3	Inference for Categorical Data: Proportions	6 weeks
4	Inference for Quantitative Data: Means	5 weeks
5	Regression Analysis	3 weeks

ACCOMMODATIONS

504 Accommodations:

- Provide scaffolded vocabulary and vocabulary lists.
- Provide extra visual and verbal cues and prompts.
- Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials.
- Provide links to audio files and utilize video clips.
- Provide graphic organizers and/or checklists.
- Provide modified rubrics.
- Provide a copy of teaching notes, especially any key terms, in advance.
- Allow additional time to complete assignments and/or assessments.
- Provide shorter writing assignments.
- Provide sentence starters.
- Utilize small group instruction.
- Utilize Think-Pair-Share structure.
- Check for understanding frequently.
- Have student restate information.
- Support auditory presentations with visuals.
- Weekly home-school communication tools (notebook, daily log, phone calls or email messages).
- Provide study sheets and teacher outlines prior to assessments.
- Quiet corner or room to calm down and relax when anxious.
- Reduction of distractions.
- Permit answers to be dictated.
- Hands-on activities.
- Use of manipulatives.
- Assign preferential seating.
- No penalty for spelling errors or sloppy handwriting.
- Follow a routine/schedule.
- Provide student with rest breaks.
- Use verbal and visual cues regarding directions and staying on task.
- Assist in maintaining agenda book.

IEP Accommodations:

- Provide scaffolded vocabulary and vocabulary lists.
- Differentiate reading levels of texts (e.g., Newsela).
- Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials.
- Provide extra visual and verbal cues and prompts.
- Provide links to audio files and utilize video clips.
- Provide graphic organizers and/or checklists.
- Provide modified rubrics.
- Provide a copy of teaching notes, especially any key terms, in advance.
- Provide students with additional information to supplement notes.
- Modify questioning techniques and provide a reduced number of questions or items on tests.
- Allow additional time to complete assignments and/or assessments.
- Provide shorter writing assignments.
- Provide sentence starters.
- Utilize small group instruction.
- Utilize Think-Pair-Share structure.
- Check for understanding frequently.
- Have student restate information.
- Support auditory presentations with visuals.
- Provide study sheets and teacher outlines prior to assessments.
- Use of manipulatives.
- Have students work with partners or in groups for reading, presentations, assignments, and analyses.
- Assign appropriate roles in collaborative work.
- Assign preferential seating.
- Follow a routine/schedule.

Gifted and Talented Accommodations:

- Differentiate reading levels of texts (e.g., Newsela).
- Offer students additional texts with higher lexile levels.
- Provide more challenging and/or more supplemental readings and/or activities to deepen understanding.
- Allow for independent reading, research, and projects.
- Accelerate or compact the curriculum.
- Offer higher-level thinking questions for deeper analysis.
- Offer more rigorous materials/tasks/prompts.
- Increase number and complexity of sources.
- Assign group research and presentations to teach the class.
- Assign/allow for leadership roles during collaborative work and in other learning activities.

ELL Accommodations:

- Provide extended time.
- Assign preferential seating.
- Assign peer buddy who the student can work with.
- Check for understanding frequently.
- Provide language feedback often (such as grammar errors, tenses, subject-verb agreements, etc…).
- Have student repeat directions.
- Make vocabulary words available during classwork and exams.
- Use study guides/checklists to organize information.
- Repeat directions.
- Increase one-on-one conferencing.
- Allow student to listen to an audio version of the text.
- Give directions in small, distinct steps.
- Allow copying from paper/book.
- Give student a copy of the class notes.

	● Provide written and oral instructions. ● Differentiate reading levels of texts (e.g., Newsela). ● Shorten assignments. ● Read directions aloud to student. ● Give oral clues or prompts. ● Record or type assignments. ● Adapt worksheets/packets. ● Create alternate assignments. ● Have student enter written assignments in criterion, where they can use the planning maps to help get them started and receive feedback after it is submitted. ● Allow student to resubmit assignments. ● Use small group instruction. ● Simplify language. ● Provide scaffolded vocabulary and vocabulary lists. ● Demonstrate concepts possibly through the use of visuals. ● Use manipulatives. ● Emphasize critical information by highlighting it for the student. ● Use graphic organizers. ● Pre-teach or pre-view vocabulary. ● Provide student with a list of prompts or sentence starters that they can use when completing a written assignment. ● Provide audio versions of the textbooks. ● Highlight textbooks/study guides. ● Use supplementary materials. ● Give assistance in note taking ● Use adapted/modified textbooks. ● Allow use of computer/word processor. ● Allow student to answer orally, give extended time (time-and-a-half). ● Allow tests to be given in a separate location (with the ESL teacher). ● Allow additional time to complete assignments and/or assessments. ● Read question to student to clarify. ● Provide a definition or synonym for words on a test that do not impact the validity of the exam. ● Modify the format of assessments. ● Shorten test length or require only selected test items. ● Create alternative assessments. ● On an exam other than a spelling test, don’t take points off for spelling errors.

UNIT 1 OVERVIEW

Content Area: Mathematics

Unit Title: Exploring One-Variable Data and Collecting Data

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: In each unit of the course, students engage with the principles and processes in the discipline of statistics. Starting in Unit 1, students identify components and formulate investigative questions of a statistical study, define and represent categorical and quantitative variables, describe and compare distributions of one-variable data, interpret statistical calculations to assess claims about individual data points or samples, and justify the use of important principles of sampling and experimental design. Students also learn to talk about data in real-world contexts. Variability in data may suggest certain conclusions about the data distribution, but not all variation is meaningful. Statistics allow us to develop shared understandings of uncertainty and variation. Additionally, students learn that, depending on how data are collected, they may or may not be able to generalize findings or establish evidence of causal relationships. For example, if random selection is not used to obtain a sample from a population, bias may result and statistics from the sample could not be assumed to generalize to the population. For data collected using well-designed experiments, statistically significant differences between or among experimental treatment groups are evidence that the treatments caused the effect.

Approximate Length of Unit: 6 weeks

LEARNING TARGETS

NJ Student Learning Standards:

S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

S.IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S.IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling

S.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S.IC.B.6 Evaluate reports based on data (e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed).

S.ID.A.1 Summarize, represent, and interpret data on a single count or measurement variable

S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S.ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
S.ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
S.ID.B.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
S.ID.C.9 Distinguish between correlation and causation.

College Board Standards:
1.1.A Identify components within a statistical study.
1.1.B Determine an investigative question within a statistical study.
1.2.A Identify observational units, variables, parameters, and statistics from a statistical study or data set.
1.2.B Identify types of variables.
1.2.C Identify types of quantitative variables.
1.3.A Construct categorical one-variable tabular representations.
1.3.B Describe categorical one-variable tabular representations with summary statistics.
1.4.A Construct categorical one-variable graphical representations.
1.4.B Justify a claim using categorical one-variable graphical representations.
1.4.C Compare multiple categorical one-variable tabular and graphical representations.
1.5.A Construct quantitative one-variable graphical representations.
1.6.A Describe distributions of quantitative one-variable graphical representations.
1.6.B Justify a claim using distributions of quantitative one-variable graphical representations.
1.7.A Calculate measures of center and position for quantitative data
1.7.B Calculate measures of variability for quantitative data.
1.7.C Calculate different units of measurement for summary statistics.
1.7.D Calculate outliers for quantitative data.
1.7.E Compare multiple quantitative one-variable summary statistics.
1.7.F Justify the selection of a summary statistic for describing quantitative data.
1.8.A Construct quantitative one-variable graphical representations of summary statistics.
1.8.B Describe quantitative one-variable graphical representations of summary statistics based on the relationship of the mean and the median.
1.9.A Compare multiple quantitative one-variable graphical representations.
1.9.B Compare multiple quantitative one-variable graphical representations of summary statistics.
1.9.C Justify a claim using multiple quantitative one-variable graphical representations.
1.9.D Calculate z-scores with population parameters.
1.9.E Compare z-scores as measures of relative position for distributions.
1.10.A Determine the components of an investigative question within a statistical study.
1.10.B Identify a census.
1.10.C Identify an experiment.
1.10.D Identify an observational study.
1.10.E Justify the appropriateness of generalizations for a statistical study.
1.11.A Identify a sampling method given a description of a study.
1.11.B Justify the appropriateness of a sampling method.
1.12.A Identify potential sources of bias in sampling methods.
1.13.A Identify elements of a well designed experiment.
1.13.B Identify experimental designs.
1.13.C Justify the appropriateness of a particular experimental design.
1.13.D Justify the appropriateness of the conclusions based on a well-designed experiment.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.

9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:

ELA

RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.

L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.

L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.

SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

HS-ETS1-4 Both physical models and computers can be used in various ways to aid in the engineering design process. Computers are useful for a variety of purposes, such as running simulations to test different ways of solving a problem or to see which one is most efficient or economical; and in making a persuasive presentation to a client about how a given design will meet his or her needs.

Unit Understandings:
Students will understand that…

- Given that variation may be random or not, conclusions are uncertain.
- Graphical representations and statistics allow us to identify and represent key features of data.
- The normal distribution can be used to represent some population distributions.
- The way we collect data influences what we can and cannot say about a population.
- Well-designed experiments can establish evidence of causal relationships.

Unit Essential Questions:

- What kind of question can be answered using data from websites?
- What kind of data is collected about me on social media?
- What methods should I use to collect the data to conduct future inference procedures?

Knowledge and Skills:
Students will know...

- A statistical study is a study in which data are collected from a sample to answer an investigative question about a larger population.
- Statistical studies are necessary when the population is too large or it is too difficult to collect data from every item or individual in the population.
- A datum (singular form of data) is a piece of information about an item or individual. A collection of data is called a data set.
- A population consists of all items or individuals of interest. The population size is represented by the symbol N.
- A sample selected for study is a subset of the population from which data are obtained. The number of items in the sample, called the sample size, is represented by the symbol n.
- Each component of a statistical study and the resulting calculations can be related to an aspect of the corresponding real-world context from which the components were derived. This identification of a statistical result with the corresponding contextual component is what is meant by "in context."
- An investigative question for a specific study should have a defined purpose and should not be changed based on the data analysis or results.
- An investigative question should be posed so that the required data can be collected and analyzed.
- An observational unit is an item or individual from which a datum is collected.
- A variable is a characteristic that may change from one observational unit to another
- Data collected on numerical and categorical variables measured on observational units, including photographs, sounds, videos, and text, can convey meaningful information
- A parameter is a numerical attribute or summary of the variable of interest for a population.
- A statistic is a numerical attribute or summary of the variable of interest for a sample. The value of a statistic from a certain sample is often not equal to the unknown value of the population parameter but may provide the basis for making inferences about the population parameter.
- A categorical variable, also called a qualitative variable, takes on values that are category names or group labels.
- A quantitative variable, also called a numerical variable, takes on numerical values for a measured or counted quantity and generally has units of measure.
- A discrete quantitative variable can take on a countable number of values. The number of values may be finite or countably infinite, as with the whole numbers.
- A continuous quantitative variable can take on an infinite number of possible values within a given interval. The number of values the variable can take on is measurable but not countable. This variable can take on all possible values between any pair of values.
- A frequency table shows the number of observational units in each category of a categorical variable.

- A relative frequency table shows the proportion of observational units in each category of a categorical variable
- Percentages, relative frequencies, and ratios all provide the same information as proportions.
- Counts and relative frequencies of categorical variables reveal information that can be used to justify claims about the variables in context.
- Bar charts, also called bar graphs, display frequencies (counts) or relative frequencies (proportions) for the categories of a single categorical variable. Each bar on a bar chart represents a category of the categorical variable of interest. The height or length of each bar corresponds to the frequency or relative frequency of the observational units in each category.
- Pie charts are used to display frequencies (counts) or relative frequencies (proportions) for categorical data. Each slice on a pie chart represents a category of the categorical variable of interest. The area of each slice, as a fraction of the total area, corresponds to the relative frequency of observational units falling within each category. The sum of the slices' areas together will equal 1, or 100% of the total area.
- Graphical representations of a categorical variable reveal information that can be used to justify claims about the variable in context.
- Frequency and relative frequency tables, bar charts, and pie charts can be used to compare two or more data sets in terms of the same categorical variable.
- Histograms, stem-and-leaf plots, and dotplots provide a visual representation of the distribution of the values of a quantitative variable. These graphs show the frequency or relative frequency of the quantitative variable values or intervals of values and maintain the natural ordering, smallest to largest, of the quantitative variable.
- A histogram places the observed values of the quantitative variable into ordered intervals, or bins, along the horizontal axis. Each bar represents an interval or bin, and the height of each bar shows the frequency or relative frequency of the observations within that interval. Altering the interval widths, or bin widths, can change the appearance of the histogram. Alternatively, a histogram can be constructed with bins on the vertical axis with bars appearing horizontally
- A stem-and-leaf plot splits each value of the quantitative variable into two parts: a "stem" (the first digit or digits) and a "leaf" (usually the single digit after the stem digit or digits). Both stems and leaves are ordered from smallest to largest.
- A dotplot represents each value of the quantitative variable by a dot. Each dot is placed above the horizontal or beside the vertical axis corresponding to the value of that observation, with nearly identical values stacked on top of each other.
- Descriptions of the distribution of one quantitative variable include shape, center, and variability (spread) as well as any unusual features such as outliers, gaps, or clusters in context.
- The shape of the distribution of one quantitative variable is skewed to the right (positively skewed) if the right tail (toward larger values) is longer than the left. The shape of the distribution is skewed to the left (negatively skewed) if the left tail (toward smaller values) is longer than the right. The shape of the distribution is approximately symmetric if the left half is approximately the mirror image of the right half.
- Distributions of one quantitative variable with one main peak are called unimodal. Distributions with two prominent peaks are called bimodal. A distribution in which each frequency or each relative frequency is approximately the same with no prominent peaks is approximately uniform.
- Outliers for one quantitative variable are data points that are unusually small or large relative to the rest of the data.
- A gap is a region in a distribution between two values in which there are no observed data.
- Clusters are concentrations of values usually separated by gaps.
- Graphical representations of a quantitative variable may reveal information that can be used to justify claims about the variable in context.
- Two commonly used measures of center in the distribution of a quantitative variable are the mean and median.

- The mean is the sum of all the data values divided by the number of values. For a sample, the mean is denoted by x-bar :$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, where x_i represents the i[th] data point in the sample and n represents the number of data values in the sample.
- The median is the middle value when the data set is ordered from smallest to largest and can be found with and without using technology. One common method for determining the median of a data set with an even number of values is to use the mean of the two middle values. A common method for determining the median of a data set with an odd number of values is to use the value in the middle of all the values.
- In an ordered data set, the smallest value is the minimum value, and the largest value is the maximum value.
- The first quartile, denoted by Q1, is the median value of the lower half of the ordered data set from the minimum value to the position of the median. Approximately 25% of the values in the data set are less than or equal to Q1. The third quartile, denoted by Q3, is the median value of the upper half of the ordered data set from the position of the median to the maximum value. Approximately 75% of the values in the data set are less than or equal to Q3. The second quartile, Q2, is also the median of the data set. Q1 and Q3 form the boundaries for the middle 50% of values in an ordered data set.
- The pth percentile is the value that has p% of the data less than or equal to it when the data set is ordered from smallest to largest. The first and third quartiles are the 25th and 75th percentiles, respectively.
- Three commonly used measures of variability (or spread) in the distribution of a quantitative variable are the range, interquartile range, and standard deviation.
- The interquartile range (IQR) is the difference between the third and first quartiles: Q3 −Q1.
- Standard deviation is a way to measure variability of the distribution of a quantitative variable. For a sample, the standard deviation is denoted by s: $s_x = \sqrt{\frac{1}{n-1}\sum (x_1 - \bar{x})^2}$. The square of the sample standard s^2 , is called the sample variance.
- There are many methods for determining potential outliers. Two methods frequently used are as follows: 1.An outlier is a value located more than 1.5 I × QR above the third quartile or more than 1.5 I × QR below the first quartile. 2.An outlier is a value located more than 2 standard deviations above, or below, the mean.
- Summary statistics can be used to compare features of two or more independent samples, including center, variability, shape, and outliers.
- The median and IQR are considered a resistant (or robust) measure of center and measure of variability, respectively, because outliers do not greatly (if at all) affect their values. Because outliers can affect their values greatly, the mean is considered a nonresistant (or non-robust) measure of center, and the range and standard deviation are considered nonresistant (or nonrobust) measures of variability.
- Summary statistics of a quantitative variable may reveal information that can be used to justify claims about the variable in context.
- A five-number summary is made up of the minimum data value, the first quartile (Q1), the median, the third quartile (Q3), and the maximum data value.
- A boxplot is a graphical representation of the five-number summary (minimum, first quartile, median, third quartile, maximum). The box represents the middle 50% of data, with a line at the median and the ends of the box corresponding to the quartiles. Lines (“whiskers”) that represent 25% of the data extend from the first quartile to the minimum and from the third quartile to the maximum. If there are outliers in the data, the whiskers extend to the most extreme data values that are not outliers, and outliers are usually denoted with an asterisk or other symbol.

- If a distribution is relatively symmetric, then the values of the mean and median are relatively close to each other. If a distribution is skewed right, then the value of the mean is usually larger than the median. If the distribution is skewed left, then the value of the mean is usually smaller than the median.
- Graphical representations of a quantitative variable can be used to compare important features between two or more distributions of the same quantitative variable. Histograms, back-to-back stem-and-leaf plots, and dotplots may be used to compare center, variability, shape, outliers, clusters, or gaps in two or more distributions. Boxplots may be used to compare center, variability, outliers, and skewness (or symmetry).
- A comparison of graphical representations for two or more distributions can include any of the numerical summaries (e.g., mean, standard deviation, etc.).
- Multiple quantitative one-variable graphical representations may reveal information that can be used to justify claims about the variable in context.
- A standardized score measures the number of standard deviations a data value falls above or below the mean.
- One example of a standardized score is a z-score, which is calculated as z -score = $(\frac{x_i - \mu}{\sigma})$. A z-score measures how many standard deviations a data value is above (positive z-score) or below (negative z-score) the mean. When the population mean and standard deviation are unknown, the sample mean and standard deviation may be used to determine a z-score.
- z-scores may be used to compare relative positions of individual values within a distribution or between distributions.
- The first component of an investigative question should guide the data collection process and should be phrased in terms of the variable(s) of interest in the study.
- The second component of an investigative question should guide the data analysis choice. In the case of a hypothesis test, the investigative question should make clear the parameter and the direction of the alternative hypothesis (i.e., not equal, greater than, less than, association, not independent). In the case of a confidence interval, the investigative question should make clear the parameter and the goal of estimation of that parameter within a range of potential values.
- The third component of an investigative question should indicate the type(s) of conclusion(s) applicable from the study. The investigative question should provide the population to which the conclusions will be applicable and, in the case of an experiment that uses random assignment, a cause-and-effect conclusion.
- A census consists of recording information from all items or individuals in a population.
- An experiment is a study in which a researcher assigns conditions, or treatments, to experimental units to explore an investigative question of interest about the population.
- The experimental unit is the observational unit to which the treatment is assigned. When experimental units consist of people, they are sometimes referred to as subjects or participants.
- An explanatory variable, or factor, is a variable whose different categories, or levels, are imposed on the experimental units. The different categories, or levels, of the explanatory variable are called treatments. When there is more than one explanatory variable, the combinations of the categories, or levels, of the explanatory variables are called treatments.
- A response variable is an outcome measured on each experimental unit after the treatment has been administered.
- An observational study is a study where treatments are not imposed. The researcher records the values of the variables of interest in order to explore an investigative question of interest.
- A prospective study is one in which the observational units of study are selected at a point in time, and data are gathered both at that time and into the future.
- A retrospective study is one in which the observational units of study are selected at a point in time and data from the past are gathered.

- A survey is an observational study in which the data are collected from humans using a standard set of questions.
- A confounding variable in an observational study provides an alternative explanation for the observed relationship between the explanatory and response variables determined in the study, thereby reducing the possibility of concluding a causal relationship between the explanatory and response variables of interest. To be a confounding variable, a variable must be associated with both the explanatory variable and the response variable.
- A sample is considered random when all observational units in the sample are selected from the population using some type of random mechanism, such as a random number generator.
- When observational units, or experimental units, in a sample are randomly selected from a population, it is appropriate to make generalizations about the entire population of individuals from which the sample was selected.
- A sample is not randomly selected when observational units are deliberately chosen or volunteer themselves to be in the sample.
- When observational units, or experimental units, in a sample are not randomly selected from a population, it is appropriate to make generalizations only about a population of individuals that are similar to those used in the study.
- Sampling without replacement is a sampling strategy in which an observational unit from a population can be selected only once. The observational unit is not returned to the population before subsequent selections of observational units are made, so there is no chance that the observational unit can be selected again.
- Sampling with replacement is a sampling strategy in which an observational unit from the population can be selected more than once. The observational unit is returned to the population before subsequent selections of observational units are made, so it is possible that the observational unit could be selected again
- In a simple random sample (SRS) of size n, every sample of the size n has the same chance of being selected. This method is the basis for many types of sampling mechanisms. There are several procedures to obtain a simple random sample; for example, using a random number generator or randomly selecting numbered slips of paper.
- A stratified random sample involves the division of all individuals in a population into non-overlapping groups, called strata, based on one or more shared attributes or characteristics (homogeneous grouping). Within each stratum a simple random sample is selected, and the selected individuals are combined to form one sample.
- A cluster random sample involves the division of a population into smaller groups, called clusters. Ideally, each cluster mirrors the heterogeneity of the population, with clusters similar to one another. A simple random sample of clusters is selected from the population to form the sample of clusters. Data are collected from all observational units in each of the selected clusters.
- A systematic random sample is a method in which sample members from a population are selected according to a random starting point and a fixed, periodic interval between successive sampling units.
- Each random sampling method has different characteristics that make it more appropriate for sampling populations depending on the question being investigated.
- Bias in a sampling method is a systematic error in the sampling procedure that results in a statistic being consistently larger or consistently smaller than the parameter the statistic is used to estimate.
- Voluntary response bias is a bias that may occur when a sample consists entirely of volunteers.
- Undercoverage bias may occur when the sampling method fails to include part of the population or a part of the population is less likely to be selected based on the sampling method.
- Nonresponse bias may occur because of a failure to obtain responses from some individuals chosen to be sampled. The respondents and nonrespondents could differ significantly in ways that are important for the study.

- Response bias may occur when responses to a survey or measurements of observational units tend to differ from the "true" value in one direction. Examples include questions that are confusing or leading (question wording bias) or self-reported responses.
- Nonrandom sampling methods (e.g., samples chosen by convenience or voluntary response) introduce potential bias because they do not use random chance to select the individuals.
- A well-designed experiment should include the following: 1.13.A.1.i Comparisons of at least two treatment groups, one of which could be a control group 1.13.A.1.ii Random assignment of treatments to experimental units 1.13.A.1.iii Replication 1.13.A.1.iv Direct control of potential extraneous sources of variation in the response.
- A control group is a collection of experimental units that are created for comparison. A control group may be given a treatment different from the treatment of interest to determine if the treatment of interest has an effect (e.g., a treatment with an inactive substance, a placebo, may be given).
- The placebo effect is the difference between the average response to a placebo and the average response to no treatment.
- In a single-blind, also called single-masked, experiment, participants do not know which treatment they are receiving, but members of the research team who interact with them know which treatment each participant is receiving, or vice versa.
- In a double-blind, also called double-masked, experiment, neither the participants nor the members of the research team who interact with them know which treatment each participant is receiving.
- An extraneous source of variation, also referred to as an extraneous variable, is a variable that is known (or believed) to affect the response but is not an explanatory variable being studied.
- The purpose of random assignment is to create treatment groups that are as similar as possible with respect to extraneous sources of variation. If random assignment is successful, the respective distributions of each extraneous variable will be approximately the same for all the treatment groups.
- A confounding variable in an experiment is a variable that is related to the explanatory variable in such a way that it is difficult to determine which variable, explanatory or confounding, is influencing the change in the response variable. However, in a well-designed experiment, the potential for confounding variables is reduced.
- Replication within an experiment means more than one experimental unit is assigned to each treatment.
- Direct control in an experiment means keeping the settings of certain potential extraneous sources of variation in the response variable the same from experimental unit to experimental unit.
- In a completely randomized design, treatments are assigned to experimental units completely at random. Often the number of experimental units assigned to each treatment will be the same, but the sample sizes in each treatment do not have to be the same.
- A blocking variable is a source of extraneous variation in the response variable. In a randomized block design, the experimental units are first grouped according to similar values of a blocking variable. These groups are called blocks. Units within the same block are homogeneous with respect to the blocking variable. After the blocks are formed, the treatments are randomly assigned to experimental units within each block so that all treatments occur within every block.
- The purpose of blocking is to separate the variation in the response caused by the blocking variable from the rest of the extraneous variation in the response. Blocking allows for more precise comparisons of the response across the treatments. Within a block, the treatments can be compared without having to worry about variation in the response caused by changes in the blocking variable.
- A matched pairs design is a randomized block design with only two treatments. Experimental units are arranged in pairs by matching on one or more extraneous sources of variation in the response variable. Each pair receives both treatments by randomly assigning one treatment to one

member of the pair and the other treatment to the second member of the pair. Alternatively, each experimental unit may get both treatments while the order of the treatments is randomized.
- One experimental design may be more appropriate than another experimental design based on the goals of the investigative study, the characteristics of the population, and the sample and variables involved.
- Using random assignment of treatments to experimental units allows for cause-and-effect conclusions between the explanatory and the response variables because the potential for confounding variables is reduced.
- Depending on the experimental unit, it may be unethical or difficult to randomly select experimental units to participate in an experiment. In that case, the study's experimental units are obtained from volunteers and will represent the population of experimental units similar to those who participated in the study.

Students will be able to...
- Determine a valid investigative question that requires a statistical investigation.
- Identify information to answer a question or solve a problem.
- Describe and compare tabular and graphical representations of data, as well as summary statistics.
- Construct tabular and graphical representations of data and distributions.
- Justify a claim based on statistical calculations and results.
- Calculate summary statistics, relative positions of points within a distribution, and predicted responses.
- Describe distributions and compare relative positions of points within a distribution.
- Justify an appropriate method for ethically gathering and representing data.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?
- End of Unit Common Assessment
- FRQ (Free Response Questions)
- Daily observation of student work
- Student/group presentations

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?
- Khan Academy assignments
- AP Classroom assignments
- **Gallery Walk:** Have students work in groups of four to construct a dotplot, stem-and-leaf plot, histogram, or boxplot for a set of student-generated data (e.g., time in minutes to get to school). After the gallery walk, discuss what information can be seen more easily in each graph (e.g., boxplots can easily show the IQR).
- **Match Mine:** Create a blank 3×3 grid as a gameboard and nine index cards depicting distributions of differing shapes, center, skewness, clustering, etc. Pair up students and place a folder between the partners so they cannot see each other's boards. Distribute a blank grid and a set of cards to each partner. Student A arranges the cards however they choose on their grid and then describes that arrangement so that Student B can attempt to match it on their own board.

When the pair thinks they have correctly made all matches, have them compare their arrangements to see how well they did.

- **Odd One Out:** After modeling an odd one out example, have students form groups of four and give each student a description of a statistical study. Explain that three of the studies are of the same type (observational or experimental) and one is different. Have students work together in their groups to determine which study is the odd one out and explain why.
- **Password-Style Games:** After completing the lessons on sampling and surveying, use the following nine terms in a password-style game: simple random sample, stratified random sample, cluster random sample, systematic random sample, bias, voluntary response bias, undercoverage bias, nonresponse bias, and response bias. Pair students and ask one student per pair to sit facing the back of the classroom. Project or write the terms on the board one at a time while the front-facing students give clues to their partners, who try to guess the terms. The winner is the pair that guesses the most terms correctly.

RESOURCES

Teacher Resources:

- **Textbook:** Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom
- Math Medic

Equipment Needed:

- Internet
- Laptop/Chromebook
- Document Camera
- Newline Board
- Graphing Calculator

UNIT 2 OVERVIEW

Content Area: Mathematics

Unit Title: Probability, Random Variables, and Probability Distributions

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: In this unit, students explore relationships in two-variable categorical data sets that will help them understand some of the rules of probability. Probabilistic reasoning allows statisticians to quantify the likelihood of random events over the long run and make statistical inferences. Simulations and concrete examples can help students understand the abstract definitions and calculations of probability. This unit builds on understandings of simulated or empirical data distributions and fundamental principles of probability to represent, interpret, and calculate parameters for theoretical probability distributions for discrete random variables. When students interpret probabilities and parameters associated with a probability distribution, they are expected to use appropriate units and relate to the context of the situation. This unit also introduces the standard normal distribution, including the empirical rule, sampling distribution of a statistic, and the central limit theorem (CLT).

Approximate Length of Unit: 5 weeks

LEARNING TARGETS

NJ Student Learning Standards:

S.ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S.ID.B.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related.

S.CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S.CP.A.2 Understand that two events *A* and *B* are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.A.3 Understand the conditional probability of *A* given *B* as , $\frac{P(A \text{ and } B)}{P(B)}$ and interpret independence of and as saying that the conditional probability of given is the same as the probability of , and the conditional probability of given is the same as the probability of .

S.CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

S.CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

S.CP.B.6 Find the conditional probability given as the fraction of 's outcomes that also belong to , and interpret the answer in terms of the model.

S.CP.B.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

S.CP.B.8 Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)\,P(B \mid A) = P(B)\,P(A \mid B)$ and interpret the answer in terms of the model.
S.CP.B.9 Use permutations and combinations to compute probabilities of compound events and solve problems.
S.MD.A.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
S.MD.A.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution
S.MD.A.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
S.MD.A.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
S.MD.B.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
S.MD.B.6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
S.MD.B.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

College Board Standards:
2.1.A Compare tabular and graphical representations for the relationship between two categorical variables.
2.1.B Justify a claim using tabular and graphical representations for the distributions of two categorical variables.
2.2.A Calculate summary statistics from two-way tables.
2.2.B Compare summary statistics for two categorical variables.
2.2.C Justify a claim using summary statistics for two categorical variables.
2.3.A Estimate probabilities using simulations.
2.4.A Calculate probabilities for events and their complements.
2.5.A Justify why two events are mutually exclusive (or disjoint) using joint probability.
2.6.A Calculate conditional probabilities.
2.7.A Calculate probabilities for independent events and for the union of two events.
2.8.A Construct a probability distribution for a discrete random variable.
2.9.A Calculate the mean and standard deviation for a discrete random variable.
2.9.B Interpret the mean and standard deviation for a discrete random variable.
2.10.A Justify why a random variable is or is not a binomial random variable.
2.10.B Calculate the mean and standard deviation for a binomial distribution.
2.10.C Interpret the mean, standard deviation, and probabilities for a binomial distribution.
2.10.D Estimate probabilities of binomial random variables using data from a simulation.
2.10.E Calculate probabilities for a binomial distribution.
2.11.A Describe a normal distribution.
2.11.B Calculate the mean and standard deviation for a normal distribution.
2.11.C Calculate percentages from a normal distribution using the empirical rule.
2.11.D Calculate the probability that a particular value lies within a given interval of a normal distribution.
2.11.E Calculate the associated intervals and areas of a normal distribution.
2.11.F Compare measures of relative position for distributions.
2.12.A Describe sampling distributions with simulations.

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.

9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:

ELA

RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.

L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.

L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.

SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

HS-ETS1-4 Both physical models and computers can be used in various ways to aid in the engineering design process. Computers are useful for a variety of purposes, such as running simulations to test different ways of solving a problem or to see which one is most efficient or economical; and in making a persuasive presentation to a client about how a given design will meet his or her needs.

Unit Understandings:

Students will understand that…

- Given that variation may be random or not, conclusions are uncertain.
- Graphical representations and statistics allow us to identify and represent
- Simulation allows us to anticipate patterns in data.
- The likelihood of a random event can be quantified.
- Probability distributions may be used to model variation in populations.
- Probabilistic reasoning allows us to anticipate patterns in data.

Unit Essential Questions:

- How might we represent the reading habits of people with and without an electronic reading device to help describe similarities and/or differences between the two groups?
- How can the weather be predicted?
- Why might the data we collect influence decisions being made?

Knowledge and Skills:

Students will know...

- A two-way table, also called a contingency table, can be used to summarize and compare data for two categorical variables. The entries in the cells of the table can be frequencies (i.e., counts) or relative frequencies (i.e., proportions).
- Side-by-side bar charts, segmented bar charts, and mosaic plots are examples of graphs used to display the relationship between two categorical variables. In these graphs, the frequency or relative frequency of each category, or level, of one of the categorical variables is displayed for each category of the other categorical variable.
- Graphical representations of two categorical variables can be used to compare the relationship of one categorical variable across the levels of the other categorical variable and determine whether the two variables are associated.
- Tabular and graphical representations for the distributions of two categorical variables may reveal information that can be used to justify claims about the variable in context.
- A joint relative frequency in a two-way table is a cell frequency divided by the total for the entire table.
- A marginal relative frequency in a two-way table is a row total divided by the total for the entire table or a column total divided by the total for the entire table.
- A conditional relative frequency is a relative frequency computed by restricting to a particular level, or category of interest. A conditional relative frequency can be a cell frequency in a row divided by the total for that row or it can be a cell frequency in a column divided by the total for that column.
- Summary statistics for two categorical variables can be used to compare distributions for evidence of association between the two variables.
- Summary statistics for two categorical variables may reveal information that can be used to justify claims about the variables in context.
- A random process generates results that are determined by chance.
- An outcome is the result of one trial of a random process.
- An event is a collection of outcomes.
- Simulation is a way to model random events such that simulated outcomes closely match real-world outcomes. All possible outcomes are associated with a value to be determined by chance. Record the counts of simulated outcomes and the count total.
- The probability of an outcome or event is its long-run relative frequency—that is, its relative frequency over a large number of trials.
- The relative frequency of an outcome or event determined from empirical data can be used to estimate the actual, or true, probability of that outcome or event.
- The law of large numbers states that for independent trials, as the number of trials increases, the long-run relative frequency of the outcome or event gets closer and closer to a single value.
- The sample space of a random process is the set of all possible nonoverlapping outcomes. The probability of the sample space is 1.
- If all outcomes in the sample space are equally likely, then the theoretical probability an event E will occur is the number of outcomes in event E divided by the total number of outcomes in the sample space . The probability of event E occurring is written as P (E) .
- The probability of an event is a number between 0 and 1, inclusive.

- The probability of the complement of an event E, which can be written as E' or E^c (i.e., the probability of "not E") is equal to 1- P (E).
- The probability that events A and B both will occur, sometimes called the joint probability, is the probability of the intersection of A and B. Joint probability is defined as P (A intersect B) or P (A ∩ B).
- Two events are mutually exclusive, or disjoint, if they cannot occur at the same time. This means that if two events are mutually exclusive, then P (A ∩ B) = 0.
- The probability that event A will occur given that event B has occurred is called a conditional probability and is written as P(A | B). Conditional probability is defined as $\frac{P(A \cap B)}{P(B)}$
- The general multiplication rule states that the probability that events A and B both will occur is equal to the probability that event A will occur multiplied by the conditional probability that event B will occur given that event A has occurred. The multiplication rule is defined as P(A ∩ B) = P(A)· P(B | A).
- Events A and B are independent if and only if knowing whether event A has occurred (or will occur) does not change the probability that event B will occur. When events A and B are independent, then $P(A \mid B) = P(A)$, $P(B \mid A) = P(B)$ and P(A ∩ B) = P(A)· P(B).
- The probability that event A or event B (or both) will occur is the probability of A union B. The probability of the union is defined as P (A ∪ B)= P(A) + P(B) - P(A ∩ B).
- A random variable is a variable whose values have numerical outcomes that result from a random phenomenon.
- A probability distribution for a discrete random variable shows the probability associated with every possible value of the random variable. The sum of the probabilities over all possible values of a discrete random variable is 1.
- A discrete probability distribution can be determined using the rules of probability or estimated with a simulation.
- A discrete probability distribution can be represented as a graph, table, or function showing the probabilities associated with values of a random variable.
- A cumulative probability distribution can be represented as a table or function and shows the probability of being less than or equal to each value of the discrete random variable.
- A numerical value measuring a characteristic of a probability distribution of a random variable, or a population, is a parameter. The value of a parameter is a single, fixed value.
- The expected value (or mean) of a probability distribution is a parameter and is denoted by E (X) or μ_x. For a discrete random variable X, the expected value is calculated as $\mu_x = \sum x_i \cdot P(x_i)$ where x_i is the possible value of the random variable and $P(x_i)$ is the probability of the possible value of the random variable. The expected value can be interpreted as the long-run average outcome of the random variable. The discrete random variable can only take on values that are countable or finite.
- The standard deviation of a probability distribution is a parameter represented by SD(X) or σ_x For a discrete random variable X, the standard deviation is calculated as $\sigma_x = \sqrt{\sum (x_i - \mu_x)^2 \cdot P(x_i)}$, where x_i is the possible value of the random variable, μ_x is the mean, and $P(x_i)$ is the probability of the possible value of the random variable. The standard deviation can be interpreted as the typical deviation of the values of the random variable from the mean value (or expected value) of the random variable over the long run. The square of the

standard deviation of a random variable is called the variance of the random variable and is denoted asV (X) or σ_x^2.

- The mean and standard deviation for the probability distribution of a discrete random variable should be interpreted in the context of a specific population.
- A binomial random variable, X, is a discrete random variable that counts the number of successes in repeated independent trials, n, that have only two possible outcomes (success or failure), with the probability of success p and the probability of failure 1− p.
- If a random variable is binomial, its mean, μ_x , is np and its standard deviation, σ_x , is $\sqrt{np(1-p)}$.
- The mean, standard deviation, and probabilities for a binomial distribution should be interpreted in context.
- A probability distribution can be constructed using the rules of probability or estimated with a simulation.
- The probability that a binomial random variable, X, has exactly x successes for n independent trials, when the probability of success is p, is calculated as

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

- A continuous random variable is a variable that can take on any value within a specified domain. Every interval within the domain has a probability associated with it.
- Many continuous random variables are wellmodeled by a normal distribution.
- A normal distribution can be described as a continuous, unimodal, bell-shaped, and symmetric curve.
- A normal curve can be used to model a distribution of data and a continuous random variable.
- The normal distribution, or the normal curve, is identified by two parameters, the mean, μ, and the standard deviation, σ . The smaller the standard deviation, the taller and more concentrated the normal curve is around its mean. The larger the standard deviation, the shorter and less concentrated the normal curve is around its mean.
- A standard normal distribution is a normal distribution with mean μ = 0 and standard deviation σ =1.
- The empirical rule can be used to estimate the area of a region under the graph of the normal distribution curve. For a normal distribution, approximately 68% of observations are within 1 standard deviation of the mean, approximately 95% of observations are within 2 standard deviations of the mean, and approximately 99.7% of observations are within 3 standard deviations of the mean. This is called the empirical rule, or the 68–95–99.7 rule.
- If the distribution of a random variable is approximately normal, the probability that the random variable takes on values within a particular interval of the random variable is determined by the area under the normal curve within that interval. The total probability or area under the normal curve is 1.
- The boundaries of an interval associated with a given area in a normal distribution can be determined using technology or using z-scores and a standard normal table.
- Intervals associated with a given area in a normal distribution can be determined by assigning appropriate inequalities to the boundaries of the intervals. To determine the intervals, p is defined as a number between 0 and 100, x_a is the lower bound, and x_b is the upper bound on a normal distribution.
- P ($X<x_a$) = $\frac{p}{100}$ means that the lowest p% of the values lie to the left of x_a.
- P ($x_a<X<x_b$) = $\frac{p}{100}$ means that the lowest p% of the values lies between $x_{a \text{ and }} x_b$.
- P ($X > x_b$) = $\frac{p}{100}$ means that the lowest p% of the values lie to the right of x_b.

- To determine the most extreme p% of values on both sides requires dividing the area associated with p% into two equal areas on either extreme of the distribution:
  $P(X<x_a) = \frac{1}{2}(\frac{p}{100})$ $P(X>x_b) = \frac{1}{2}(\frac{p}{100})$
  mean that half of the p% most extreme values lie to the left of x_a and half of the p% most extreme values lie to the right of x_b
- Percentiles and proportions may be used to compare relative positions of individual values within a normal distribution or between normal distributions.
- A sampling distribution of a statistic is the distribution of values of the statistic for all possible samples of a given size from a given population.
- The sampling distribution of a statistic can be simulated by repeatedly generating a large number of random samples from the population assuming known value(s) for the parameter(s). The value of the statistic is determined and recorded for each sample. The resulting distribution of the sample statistic values approximates the sampling distribution of the statistic.
- A randomization distribution is the distribution of a statistic generated by simulation from repeatedly randomly reallocating, or reassigning, the response values to treatment groups. The value of the statistic is determined and recorded for each reallocation, or reassignment. The resulting distribution of the statistical values approximates the sampling distribution of the statistic.
- The central limit theorem (CLT) states that the sampling distribution of a mean of a random sample has a shape that can be approximated by a normal distribution. The larger the sample is, the better the approximation will be.

Students will be able to...

- Describe and compare tabular and graphical representations of data, as well as summary statistics.
- Justify a claim based on statistical calculations and results.
- Calculate summary statistics, relative positions of points within a distribution, and predicted responses.
- Calculate and estimate expected counts, percentages, probabilities, and intervals.
- Construct tabular and graphical representations of data and distributions.
- Interpret statistical calculations and results to assess meaning or a claim.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment
- FRQ (Free Response Questions)
- Daily observation of student work
- Student/group presentations

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Khan Academy assignments
- AP Classroom assignments
- **Quiz-Quiz-Trade:** Give students a card containing a question with a two-way table and have them write the answer on the back. Then have them stand up and find a partner. One student

quizzes the other, and then they reverse roles. Have them switch cards, find a new partner, and repeat the process.

- **Think-Pair-Share:** Provide students with a set of five probability questions: one for each of the complement rules, a mutually exclusive event, a conditional probability, the general multiplication rule, and an independent event. Ask students to individually identify the formula needed to solve each problem without doing the final calculations.
- **Predict and Confirm:** Before performing a class simulation, have students discuss with their neighbor what values the distribution may take and how often they expect/predict those values to occur. For instance, what might the distribution look like for a binomial setting with p = 0 5. and n =16 when we perform 20 trials? Have students perform the simulation and compare the results to their predictions.
- **Reversing Interpretations:** Give pairs of students four pictures of normal distributions with various parts shaded. Have students create the question that could have resulted in the picture shown. For example, if a value of 15 is labeled and the distribution is shaded to the right of 15, students could write, "What is the probability that a value is more than 15?"

RESOURCES

Teacher Resources:

- **Textbook:** Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom
- Math Medic

Equipment Needed:

- Internet
- Laptop/Chromebook
- Document Camera
- Newline Board
- Graphing Calculator

UNIT 3 OVERVIEW

Content Area: Mathematics

Unit Title: Inference for Categorical Data: Proportions

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: This unit introduces sampling distributions and statistical inference. Students learn that sample statistics can be used to estimate corresponding population parameters and that measures of center (mean) and variability (standard deviation) for these sampling distributions can be determined directly from the population parameters when certain sampling criteria are met. For large enough samples from any population, these sampling distributions can be approximated by a normal distribution. Simulating sampling distributions helps students understand how the values of statistics vary in repeated random sampling from populations with known parameters. This unit also introduces statistical inference. Students analyze categorical data to make inferences about binomial population proportions. Provided that conditions are met, students use statistical inference to construct and interpret confidence intervals to estimate population proportions and perform significance tests to evaluate claims about population proportions. Students begin by learning inference procedures for one proportion and then examine inference methods for a difference between two proportions. They also interpret the two types of errors that can be made in a significance test, the probability of each error occurring, and possible consequences, in context. Students are also introduced to chi-square tests, which can be used when there are two or more categories. Students need to understand how to select between the following two tests: (1) the chi-square test for independence (for associations between categorical variables within a single population), and (2) the chi-square test for homogeneity (for comparing distributions of a categorical variable across populations or treatments). To integrate conceptual understanding, teachers can help students make connections among frequency tables, conditional probability, and calculating expected counts. The chi-square statistic is introduced to measure the distance between observed and expected counts relative to expected counts.

Approximate Length of Unit: 6 weeks

LEARNING TARGETS

NJ Student Learning Standards:

S.IC.A.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.MD.A.1: Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

College Board Standards:

3.1.A Justify why an estimator is or is not unbiased.

3.1.B Calculate estimates for a population parameter.

3.2.A Calculate the mean and standard deviation of a sampling distribution for a sample proportion.

3.2.B Justify the appropriateness of conditions for the sampling distribution of a sample proportion.

3.2.C Interpret the mean, standard deviation, and probabilities for a sampling distribution of a sample proportion.
3.3.A Identify an appropriate confidence interval procedure including the parameter for a population proportion.
3.3.B Justify the appropriateness of constructing a confidence interval for a population proportion by verifying conditions.
3.3.C Calculate an appropriate confidence interval for a population proportion.
3.3.D Calculate the standard error and margin of error of a sample statistic for a confidence interval for a population proportion, and estimate a given sample size from the margin of error.
3.4.A Interpret a confidence interval in context for a population proportion.
3.4.B Justify a claim based on a confidence interval for a population proportion.
3.4.C Identify the relationships among sample size, confidence interval width, confidence level, and margin of error for a population proportion.
3.5.A Identify an appropriate testing method for a population proportion including the parameter for the population proportion.
3.5.B Identify the null and alternative hypotheses for a population proportion.
3.5.C Justify the appropriateness of a hypothesis test for a population proportion by verifying conditions.
3.6.A Interpret the p-value of a hypothesis test for a population proportion.
3.7.A Calculate an appropriate test statistic and p-value for testing a hypothesis about a population proportion.
3.7.B Justify a claim about the population based on the results of a hypothesis test for a population proportion.
3.8.A Identify Type I and Type II errors.
3.8.B Calculate the probability of Type I and Type II errors.
3.8.C Identify the factors that affect the probability of errors in hypothesis testing.
3.8.D Interpret Type I and Type II errors.
3.9.A Calculate the mean and standard deviation of the sampling distribution for the difference between two sample proportions.
3.9.B Justify the appropriateness of conditions for the sampling distribution for the difference between two sample proportions.
3.9.C Interpret the mean, standard deviation, and probabilities for the sampling distribution for the difference between two sample proportions
3.10.A Identify an appropriate confidence interval procedure including the parameters for the difference between two population proportions.
3.10.B Justify the appropriateness of constructing a confidence interval for the difference between two population proportions by verifying conditions.
3.10.C Calculate an appropriate confidence interval for the difference between two population proportions.
3.10.D Calculate the standard error and margin of error for estimating the difference between two population proportions.
3.11.A Interpret a confidence interval in context for the difference between two population proportions.
3.11.B Justify a claim based on a confidence interval for the difference between two population proportions
3.12.A Identify an appropriate testing method for the difference between population proportions including the parameters.
3.12.B Identify the null and alternative hypotheses for the difference between population proportions.
3.12.C Justify the appropriateness of a hypothesis test for the difference between two population proportions by verifying conditions.
3.13.A Calculate an appropriate test statistic and p-value for testing a hypothesis for the difference between two population proportions.
3.13.B Interpret the p-value of a hypothesis test for the difference between two population proportions.
3.13.C Justify a claim about the populations based on the results of a hypothesis test for the difference between two population proportions.
3.14.A Describe chi-square distributions.
3.14.B Identify an appropriate testing method for comparing distributions in two-way tables of categorical data including the populations and variables.
3.14.C Identify the null and alternative hypotheses for a chi-square test for homogeneity or independence.

3.14.D Justify the appropriateness of a chi-square test for independence or homogeneity by verifying conditions.
3.15.A Calculate expected counts for two-way tables of categorical data.
3.15.B Calculate the appropriate test statistic and p-value for a chi-square test for homogeneity or independence.
3.15.C Interpret the p-value for the chi-square test for homogeneity or independence
3.15.D Justify a claim about the population(s) based on the results of a chi-square test for homogeneity or independence.

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.
L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
Science
HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

HS-ETS1-4 Both physical models and computers can be used in various ways to aid in the engineering design process. Computers are useful for a variety of purposes, such as running simulations to test different ways of solving a problem or to see which one is most efficient or economical; and in making a persuasive presentation to a client about how a given design will meet his or her needs.

Unit Understandings:

Students will understand that…

- Given that variation may be random or not, conclusions are uncertain..
- An interval of values should be used to estimate parameters, in order to account for uncertainty.
- The normal distribution may be used to model variation.
- Significance testing allows us to make decisions about hypotheses within a particular context.
- Probabilities of Type I and Type II errors influence inference.

Unit Essential Questions:

- How can we measure changing opinions in populations over time?
- When can we perform inference calculations using data from websites to answer everyday questions?
- How can we determine whether the difference between two groups is statistically significant?

Knowledge and Skills:

Students will know...

- When estimating a population parameter, an estimator is unbiased if, on average, the value of the estimator does not underestimate or overestimate the population parameter.
- A sample statistic is a point estimator of the corresponding population parameter and can be thought of as the estimate of the population parameter. For example, the sample proportion $\hat{p}$ is a point estimator for the population proportion p.
- For a population with population proportion p, when the sampled values are independent, the sampling distribution of a sample proportion $\hat{p}$ has a mean $\mu_p = p$ and a standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$.
- Sampling without replacement requires that two conditions be met: 3.2.B.1.i The randomization condition—the data should be collected using a random sample. 3.2.B.1.ii The 10% condition—the population size must be at least 10 times larger than the sample size ($n \leq 10\%N$) , where N is the size of the population and n is the sample size.
- The sampling distribution of the sample proportion $\hat{p}$ is approximately normal provided the sample size is large enough. To ensure the sample size is large enough, the following condition must be met: $np \geq 10$ and $n(1-p) \geq 10$, where np is the expected number of successes and n (1- p) is the expected number of failures.
- The mean, standard deviation, and probabilities for a sampling distribution of a sample proportion should be interpreted in the context of a specific population.
- Identify an appropriate confidence interval procedure including the parameter for a population proportion.
- Justify the appropriateness of constructing a confidence interval for a population proportion by verifying conditions.
- z^* denotes a critical value, such that z^* and z represent the boundaries enclosing the middle C% of the standard normal distribution, in which C% is an approximate confidence level with which the population proportion is estimated.
- An interval estimate can be constructed as point estimate (margin of error). For a population proportion, the one-sample z-interval estimate is

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- The margin of error of $\hat{p}$ is half the width of the confidence interval and is calculated as the critical value z^* times the standard error (SE) of $\hat{p}$, which equals $z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- The formula for the margin of error (MOE) can be rearranged to solve for n, $n = \left(\frac{z^*}{\text{MOE}} \right)^2 \hat{p}(1-\hat{p})$ the minimum sample needed to achieve a given margin of error. For this purpose, if $\hat{p}$ is not defined or unable to be calculated, use $\hat{p}$ = 0.5 in order to find the upper bound for the sample size that will result in a given margin of error.
- Because the confidence interval for a population proportion is calculated based on a sample from a population, the computed interval may or may not contain the value of the population proportion.
- The interpretation of the confidence level is that in repeated random sampling with the same sample size, approximately C% of confidence intervals calculated will capture the population proportion, with C representing the numerical value of the confidence level used.
- When interpreting a C% confidence interval for a population proportion, we say we are C% confident that the interval (a, b) contains the true value of the parameter for the population, where a represents the lower limit and b represents the upper limit. An interpretation of a confidence interval for a population proportion includes a reference to the parameter with details about the population it represents in the context of the study.
- A confidence interval for a population proportion provides a range of plausible values that may serve as convincing evidence to support a particular claim about the population proportion.
- For a given sample, increasing the confidence level will result in the following: 3.4.C.1.i The critical value will increase. 3.4.C.1.ii The margin of error will increase. 3.4.C.1.iii The width of the confidence interval will increase.
- Increasing the sample size decreases the standard error. Thus, when all other things remain the same, the width of the confidence interval for a population proportion tends to decrease as the sample size increases. For a confidence interval for a population proportion with a given confidence level, the width of the interval is approximately proportional to $\frac{1}{\sqrt{n}}$.
- A hypothesis test is a statistical inference procedure that is used to make a decision about the value of a population parameter. The appropriate hypothesis testing procedure is a one-sample z-test for a population proportion.
- The parameter for a hypothesis test for a population proportion should reference the population parameter, the response variable, and the population in context.
- In the hypothesis testing procedure, the null hypothesis, H_o, is the statement about a parameter that is assumed to be correct unless there is convincing statistical evidence suggesting otherwise. It is the status quo condition. The alternative hypothesis,H_a, is the claim or belief about a parameter for which evidence is being collected. A researcher's claim or belief about the population parameter is represented by the alternative hypothesis.
- The null hypothesis contains an equality reference (=, ≤, or≥) . Although the null hypothesis for a one-sided test may include an inequality symbol, in AP Statistics it is tested at the boundary of equality. The alternative hypothesis with < or > is called one-sided, and the alternative hypothesis with ≠ is called two-sided.
- The null hypothesis for a one-sample z-test for a population proportion is as follows: H_o: p= p_0, where p_0 is the null hypothesized value for the population proportion. A one-sided alternative

hypothesis for a one-sample z-test for a population proportion is either H_a: $p < p_0$ or H_a: $p > p_0$. A two-sided alternative hypothesis is H_a: $p \neq p_0$.

- A one-sample z-test for a population proportion requires that three conditions be met: 3.5.C.1.i The randomization condition—the data should be collected using a random sample. 3.5.C.1.ii The 10% condition—when sampling without replacement, the population size must be at least 10 times larger than the sample size 3.5.C.1.iii The normality condition—the expected number of successes, np_0, and the expected number of failures, $n(1 - p_0)$, both have to be at least 10.
- Given the null hypothesis is true, there is a probability distribution of the test statistic called the null distribution. Using the null distribution, the p-value is the probability of obtaining a test statistic as extreme or more extreme (i.e., in the direction of the alternative hypothesis) than the test statistic that is observed given that the null hypothesis is true. That is, when x is the test statistic, the p-value is determined by finding the following:
  The probability at or above the observed value of the test statistic $P(z \geq x)$, if the alternative is > 3.6.A.1.ii The probability at or below the observed value of the test statistic $P(z \leq x)$, if the alternative is < 3.6.A.1.iii The probability less than or equal to the negative of the absolute value of the test statistic plus the probability greater than or equal to the absolute value of the test statistic, $P(z \leq -|x|) + P(z \leq |x|)$, if the alternative is ≠.
- If the distribution of the test statistic has been simulated, the p-value is the proportion of values in the null distribution that are as extreme or more extreme than the observed value of the test statistic. This is as follows: 3.6.A.2.i The proportion at or above the observed value of the test statistic, if the alternative is > 3.6.A.2.ii The proportion at or below the observed value of the test statistic, if the alternative is < 3.6.A.2.iii The proportion less than or equal to the negative of the absolute value of the test statistic plus the proportion greater than or equal to the absolute value of the test statistic, if the alternative is ≠.
- An interpretation of the p-value of a hypothesis test for a population proportion should include a statement that the p-value is computed by assuming the null hypothesis is true (i.e., by assuming the true population proportion is equal to the particular value stated in the null hypothesis in context).
- Small p-values indicate that the observed value of the test statistic would be unusual if the null hypothesis were true and therefore provide evidence for the alternative hypothesis. The lower the p-value, the more convincing the statistical evidence for the alternative hypothesis.
- p-values that are not small indicate that the observed value of the test statistic would not be unusual if the null hypothesis were true and therefore do not provide convincing statistical evidence for the alternative hypothesis, nor do they provide evidence that the null hypothesis is true.
- The test statistic for testing a population proportion is:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

  The z-statistic has a standard normal distribution when the null hypothesis is true.
- The distribution of the test statistic assuming the null hypothesis is true (null distribution) can be approximated by a probability model (e.g., a theoretical distribution such as the standard normal distribution).
- The p-value of a one-sample z-test for a population proportion is found from the standard normal distribution using a table or technology.
- The significance level of a hypothesis test, denoted by α, is the predetermined probability of rejecting the null hypothesis given that it is true. The significance level may be given or determined by the researcher. The relationship between a p-value and the significance level of a hypothesis test determines whether a result is statistically significant.

- A formal decision in a hypothesis test explicitly compares the p-value to the significance level, α. If the p-value ≤ α, then reject the null hypothesis, H_o: $p = p_0$. If the p-value ≥ α, then fail to reject the null hypothesis.
- Rejecting the null hypothesis means there is convincing statistical evidence to support the alternative hypothesis. Failing to reject the null hypothesis means there is not convincing statistical evidence to support the alternative hypothesis.
- A hypothesis test can lead to rejecting or not rejecting the null hypothesis but can never lead to concluding or proving that the null hypothesis is true. Lack of statistical evidence for the alternative hypothesis is not the same as evidence for the null hypothesis.
- The results of a hypothesis test for a population proportion can serve as the statistical reasoning to support the answer to an investigative question about the population that was sampled.
- A conclusion for the hypothesis test for a population proportion is stated in context consistent with, and in terms of, the alternative hypothesis using non-definitive language. The conclusion should contain a reference to the parameter and the population.
- A Type I error occurs when there is convincing statistical evidence that the alternative hypothesis is true (due to the small p-value), but it is not.
- A Type II error occurs when there is not convincing statistical evidence that the alternative hypothesis is true (due to the large p-value), but it is.
- The power of a hypothesis test is the probability that a hypothesis test will correctly reject the false null hypothesis.
- The probability of making a Type I error is defined as the significance level, α. For a given study and hypothesis test, the probability of making a Type I error is typically set to a small value (e.g., 0.01, 0.05, 0.10) prior to collecting the data.
- The probability of making a Type II error is 1 power.
- For a given study and hypothesis test, the probability of a Type II error should ideally be small, and thus, the power will be large (e.g., P (Type II error)= 0 2. and power = 0.80). The probability of a Type II error decreases and the power increases when any one of the following occurs, provided the others do not change: 3.8.C.1.i Sample size(s) increases. 3.8.C.1.ii Standard error decreases. 3.8.C.1.iii True parameter value is farther from the null hypothesis. 3.8.C.1.iv Significance level of a test increases.
- In some studies, making a Type I error may have more serious consequences than making a Type II error. In other studies, making a Type II error may have more serious consequences than making a Type I error. The consequences of each error should be considered prior to conducting the study.
- Because the significance level, α, is the probability of making a Type I error, the consequences of a Type I error influence decisions about a significance level.
- Because sample size influences the probability of making a Type II error, the consequences of a Type II error influence decisions about how large the sample size should be.
- For two independent populations, with population proportions p_1 and p_2, when the sampled values are independent, the sampling distribution for the difference in sample proportions, $\widehat{p1} - \widehat{p2}$, has a mean, $p_1 - p_2$, and standard deviation, $\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$.
- When sampling without replacement, two conditions must be met: 3.9.B.1.i The randomization condition—the data should be collected using two independent random samples. 3.9.B.1.ii The 10% condition—the size of each sample should be less than or equal to 10% of the respective population size.
- If the data come from an experiment, the data only need to meet the randomization condition. The treatments must be randomly assigned to the experimental units to meet the randomization condition.

- The sampling distribution for the difference between sample proportions, $p_1 - p_2$, will have an approximately normal distribution provided both sample sizes are large enough. To ensure that both samples are large enough, the data must meet the following conditions:$n_1p_1 \geq 10$, $n_1(1-p_1) \geq 10$, $n_2p_2 \geq 10$, and $n_2(1-p_2) \geq 10$.
- The mean, standard deviation, and probabilities for the sampling distribution for the difference between two sample proportions should be interpreted within the context of two specific populations.
- Based on the sample data, a confidence interval can be calculated to estimate the difference between two population proportions. The appropriate confidence interval procedure is a two-sample z-interval for a difference between population proportions.
- The parameters of a confidence interval for the difference between two population proportions should refer to the difference in the proportions, the response variable, and the populations in context.
- A two-sample z-interval for a difference between two population proportions requires that three conditions be met: 3.10.B.1.i The randomization condition—the data should be collected using two independent random samples or a randomized experiment.The 10% condition—when sampling without replacement, the size of each sample should be less than or equal to 10% of the respective population size (Note: This condition is unnecessary when the data are from a randomized experiment.) 3.10.B.1.iii The normality condition—the number of observed successes and number of observed failures, for both samples are all at least 10.
- The point estimate for the difference between two population proportions is $\widehat{p1} - \widehat{p2}$.
- For the difference between two population proportions, the interval estimate can be constructed as point estimate $\pm$ (margin of error). The interval estimate for the difference between two population proportions is :

$$(\hat{p}_1 - \hat{p}_2) \pm Z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

- The standard error (SE) for the difference between two population proportions is:

$$SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

- For the difference between two population proportions, the margin of error is the critical value z^* times the standard error (SE) of the difference between the two proportions.
- Because the confidence interval for the difference between two population proportions is calculated based on samples from two populations, the computed interval may or may not contain the value for the difference between those two population proportions.
- The interpretation of the confidence level is as follows: In repeated random sampling with the same sample sizes from the same populations, approximately C% of confidence intervals created will capture the difference between the two population proportions, where C represents the numerical value of the confidence level used.
- When interpreting a C% confidence interval for the difference between two population proportions, we say we are C% confident that the interval (a, b) contains the parameter for the difference between the populations, where a represents the lower limit and b represents the upper limit. An interpretation of a confidence interval for the difference between two population proportions includes a reference to the parameter with the details about the populations it represents in the context of the study.
- A confidence interval for the difference between two population proportions provides an interval of values that may provide convincing evidence to support a particular claim about the difference between the two population proportions. For example, if the interval contains 0, then there is insufficient evidence to conclude there is a difference between the two population proportions. If

the interval does not contain 0, there is sufficient evidence to conclude there is a difference between the two population proportions.

- The appropriate testing method for the difference between two population proportions is a two-sample z-test for the difference between two population proportions.
- The parameters for a hypothesis test for the difference between two population proportions should reference the population parameters, the response variables, and the populations in context.
- For a two-sample z-test for the difference between two population proportions, the null hypothesis indicates no difference.
- A two-sample z-test for a difference between two population proportions requires that three conditions be met: 3.12.C.1.i The randomization condition—the data should be collected using two independent random samples or a randomized experiment. 3.12.C.1.iiThe 10% condition—when sampling without replacement, the size of each sample should be less than or equal to 10% of the respective population size (Note: This condition is unnecessary when the data are from a randomized experiment.) 3.12.C.1.iii The normality condition - the number of successes and the number of failures must both be at least 10.
- The test statistic for the difference between two population proportions is: $z = \frac{(\widehat{p1} - \widehat{p2}) - 0}{\sqrt{\widehat{pc}(1 - \widehat{pc})}\sqrt{\frac{1}{n1} + \frac{1}{n2}}}$, where $\widehat{pc} = \frac{n1\widehat{p1} + n2\widehat{p2}}{n1 + n2}$ is the proportion of successes for the two groups combined. The z-statistic has a standard normal distribution when the null hypothesis is true.
- The p-value for a two-sample z-test for the difference between two population proportions can be found from the standard normal distribution using a table or technology.
- The p-value is the probability of obtaining a test statistic as extreme or more extreme than the test statistic that was observed (i.e., in the direction of the alternative hypothesis) given that the null hypothesis is true. An interpretation of the p-value of a hypothesis test for a difference between two population proportions should include a statement that the p-value is computed assuming the null hypothesis is true (i.e., by assuming that the true population proportions are equal to each other in context).
- A formal decision in a hypothesis test for the difference between two population proportions explicitly compares the p-value to the significance level, α. If the p-value $\leq$ α, then reject the null hypothesis, $H_0 : p_1 = p_2$. If the p-value > α, then fail to reject the null hypothesis.
- The results of a hypothesis test for the difference between two population proportions can serve as the statistical reasoning to support the answer to an investigative question about the two populations that were sampled.
- A conclusion for the hypothesis test for the difference between two population proportions is stated in context consistent with, and in terms of, the alternative hypothesis using nondefinitive language. The conclusion should contain a reference to the parameters and the populations.
- The chi-square statistic measures the distance between observed and expected counts relative to expected counts.
- Chi-square distributions have positive values and are skewed right. Within this family of density curves, the skew becomes less pronounced with increasing degrees of freedom.
- To determine whether the distributions of a categorical variable for two or more populations are different, the appropriate test is the chi-square test for homogeneity
- A chi-square test for homogeneity should reference the categorical variable and the populations in context.
- To determine whether row and column variables in a two-way table of categorical data might be associated in the single population from which the data were sampled, the appropriate test is the chi-square test for independence.
- A chi-square test for independence should reference the categorical variables and the population in context.

- The appropriate null hypothesis for a chi-square test for homogeneity is H_o: there is no difference in the distributions of the categorical variable across populations or treatments. The appropriate alternative hypothesis for a chi-square test for homogeneity is Ha: there is a difference in the distributions of the categorical variable across populations or treatments.
- The appropriate null hypothesis for a chi-square test for independence is H_o:: there is no association between two categorical variables in a given population or the two categorical variables in a given population are independent of each other. The appropriate alternative hypothesis for a chi-square test for independence is Ha: there is an association between two categorical variables in a given population or the two categorical variables in a given population are not independent of each other.
- A chi-square test for homogeneity or independence requires that three conditions must be met: 3.14.D.1.i The randomization condition—the test of independence states that the data should be collected using a random sample. The test for homogeneity states that the data should be collected using independent random samples or a randomized experiment. 3.14.D.1.ii The 10% condition—when sampling without replacement, check that n ≤10%N, where N is the size of the population and n is the sample size. (Note: This condition is unnecessary when the data are from a randomized experiment.) 3.14.D.1.iii The expected counts condition—all expected counts should be greater than 5.
- The expected counts (under the null hypothesis) in a particular cell of a two-way table of categorical data can be calculated using the formula : $expected\ count = \frac{(row\ total)(column\ total)}{table\ total}$
- The appropriate test statistic for a chi-square test for homogeneity or independence is the chi-square statistic $\chi^2 = \sum \frac{(Observed\ count - Expected\ count)^2}{Expected\ count}$ where the sum is taken over all cells of the two-way table. The chi-square statistics have a chi-square distribution with degrees of freedom equal to (number of rows − 1) (number of columns −1) when the null hypothesis is true.
- The p-value for a chi-square test for independence or homogeneity is found from a chi-square distribution using a table or technology.
- The p-value is the probability of obtaining a test statistic as extreme or more extreme than the test statistic that was observed (i.e., in the direction of the alternative hypothesis) given that the null hypothesis is true. An interpretation of the p-value for the chi-square test for homogeneity or independence should include a statement that the p-value is computed by assuming the null hypothesis is true in context.
- A formal decision in a hypothesis test explicitly compares the p-value to the significance level, α . If the p-value ≤ α, then reject the null hypothesis for the appropriate chi-square test. If the p-value >α , then fail to reject the null hypothesis.
- The results of a chi-square test for homogeneity or independence can serve as the statistical reasoning to support the answer to an investigative question about the population that was sampled (independence) or the populations that were sampled (homogeneity)
- A conclusion for a chi-square test for homogeneity or independence is stated in context consistent with, and in terms of, the alternative hypothesis using non-definitive language. The conclusion should contain a reference to the population(s).

Students will be able to…

- Calculate means, standard deviations, and parameters for probability distributions.
- Justify a claim based on statistical calculations and results.
- Interpret statistical calculations and results to assess meaning or a claim.
- Justify the use of a chosen statistical inference method by verifying conditions.
- Identify appropriate statistical inference methods.
- Calculate appropriate statistical inference method results.

- Identify types of errors and relationships among components in statistical inference methods.
- Interpret results of statistical inference methods.
- Justify a claim based on statistical inference method results.
- Identify the null and alternative hypotheses.
- Calculate and estimate expected counts, percentages, probabilities, and intervals.
- Describe distributions and compare relative positions of points within a distribution.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment
- FRQ (Free Response Questions)
- Daily observation of student work
- Student/group presentations

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Khan Academy assignments
- AP Classroom assignments
- **Error Analysis:** Give spairs a worksheet with 20 sets of hypotheses, each with a common student mistake. Be sure to include hypotheses for a test for a population proportion, a test for the difference between two proportions, a chi-square test for independence, and a chi-square test for homogeneity. Have students circle the incorrect part, write why the circled component is incorrect, and then write the correct hypotheses. Include errors such as using statistics instead of parameters and interchanging the = and > in the two hypotheses
- **Sentence Starters:** For a given question, provide students with a set of hypotheses, p-value, significance level, and context. Have them compare the p-value to the significance level to determine whether to reject the null hypothesis. Use a sentence starter with blanks to fill in, and have students write a sentence in context explaining whether they have enough evidence to reject H0 or if they will fail to reject H0. Make sure students avoid the common mistake of implying that evidence supports an accept H0 conclusion or a reject H0 conclusion.
- **The Scribe and the Calculator:** Have students work with a partner to construct and interpret a confidence interval for a population proportion. Only one partner is allowed to use the calculator, and only the other partner is allowed to write. When a calculation needs to be made, only the scribe can describe to the calculator operator what buttons to push; when writing needs to be done, only the calculator operator can describe to the scribe what needs to be written. Have students switch roles when constructing and interpreting a confidence interval for the difference of two population proportions.

RESOURCES

Teacher Resources:

- **Textbook:** Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.

- Khan Academy
- AP Classroom
- Math Medic

Equipment Needed:

- Internet
- Laptop/Chromebook
- Document Camera
- Newline Board
- Graphing Calculator

UNIT 4 OVERVIEW

Content Area: Mathematics

Unit Title: Inference for Quantitative Data: Means

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: In this unit, students analyze quantitative data to make inferences about population means. Students learn that t ∗ and t-tests are used for inference with means when the population standard deviation, σ , is unknown. Using s for σ in the formula for z gives a slightly different value, t. The distribution of t depends on sample size and has more area in the tails than a normal distribution. The boundaries for rejecting a null hypothesis using a t-distribution tend to be further from the mean than for a normal distribution. Students discover how and why conditions for inference with proportions and means are similar and different.

Approximate Length of Unit: 5 Wweks

LEARNING TARGETS

NJ Student Learning Standards:

S.IC.A.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.MD.A.1: Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

College Board Standards:

4.1.A Calculate the mean and standard deviation of a sampling distribution of a sample mean.

4.1.B Justify the appropriateness of conditions for the sampling distribution of a sample mean.

4.1.C Interpret the mean, standard deviation, and probabilities for the sampling distribution of a sample mean.

4.2.A Describe t-distributions.

4.2.B Identify an appropriate confidence interval procedure including the parameter for a population mean or population mean difference.

4.2.C Justify the appropriateness of constructing a confidence interval for a population mean or population mean difference by verifying conditions.

4.2.D Calculate an appropriate confidence interval for a population mean or population mean difference.

4.2.E Calculate the standard error and margin of error for a sample size for a one-sample t-interval.

4.3.A Interpret a confidence interval in context for a population mean or population mean difference.

4.3.B Justify a claim based on a confidence interval for a population mean or population mean difference.

4.3.C Identify the relationships among sample size, confidence interval width, confidence level, and margin of error for a population mean or population mean difference.

4.4.A Identify an appropriate testing method and parameter for a population mean or population mean difference with unknown σ .

4.4.B Identify the null and alternative hypotheses for a population mean or population mean difference with unknown σ.

4.4.C Justify the appropriateness of a hypothesis test for a population mean or population mean difference by verifying conditions.
4.5.A Calculate an appropriate test statistic and p-value for testing a hypothesis about a population mean or population mean difference.
4.5.B Interpret the p-value of a hypothesis test for a population mean or population mean difference.
4.5.C Justify a claim about the population based on the results of a hypothesis test for a population mean or population mean difference.
4.6.A Calculate the mean and standard deviation of a sampling distribution for the difference between two sample means.
4.6.B Justify the appropriateness of conditions for the sampling distribution of the difference between two sample means.
4.6.C Interpret the mean, standard deviation, and probabilities for a sampling distribution for the difference between sample means.
4.7.A Identify an appropriate confidence interval procedure including the parameter for the difference between two population means.
4.7.B Justify the appropriateness of constructing a confidence interval for the difference between two population means by verifying conditions.
4.7.C Calculate an appropriate confidence interval for the difference between two population means.
4.7.D Calculate the standard error and margin of error for estimating the difference between two population means.
4.8.A Interpret a confidence interval in context for the difference between two population means.
4.8.B Justify a claim based on a confidence interval for the difference between two population means.
4.9.A Identify an appropriate testing method for the difference between two population means including the parameters for the difference between the two population means.
4.9.B Identify the null and alternative hypotheses for the difference between two population means.
4.9.C Justify the appropriateness of a hypothesis test for the difference between two population means by verifying conditions.
4.10.A Calculate an appropriate test statistic and p-value for testing a hypothesis for the difference between two population means.
4.10.B Interpret the p-value of a hypothesis test for the difference between two population means.
4.10.C Justify a claim about the populations based on the results of a hypothesis test for the difference between two population means.

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.

L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.

SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

HS-ETS1-4 Both physical models and computers can be used in various ways to aid in the engineering design process. Computers are useful for a variety of purposes, such as running simulations to test different ways of solving a problem or to see which one is most efficient or economical; and in making a persuasive presentation to a client about how a given design will meet his or her needs.

Unit Understandings:

Students will understand that…

- Given that variation may be random or not, conclusions are uncertain..
- The t-distribution may be used to model variation.
- An interval of values should be used to estimate parameters, in order to account for uncertainty.
- Significance testing allows us to make decisions about hypotheses within a particular context.

Unit Essential Questions:

- How can we compare the average salaries of different professions across different regions of a country?
- How can we evaluate the effectiveness of a new medicine in lowering cholesterol for older adults?
- How can a pre-test and a post-test be used to evaluate student learning?

Knowledge and Skills:

Students will know...

- For a population with population mean μ and population standard deviation σ , when the sampled values are independent, the sampling distribution of the sample mean has mean $\mu_{\bar{x}} = \mu$ and a standard deviation of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.
- Sampling without replacement requires that two conditions must be met: 4.1.B.1.i The randomization condition—the data should be collected using a random sample. 4.1.B.1.ii The 10% condition—the population size must be at least 10 times larger than the sample size ($n \leq 10\%N$), where N is the size of the population and n is the sample size.
- For a quantitative variable, if the population distribution can be modeled by a normal distribution, the sampling distribution of the sample mean, $\bar{x}$, can be modeled with a normal distribution regardless of the sample size.
- For a quantitative variable, if the population distribution cannot be modeled by a normal distribution, the sampling distribution of the sample mean,$\bar{x}$, can be modeled approximately by a normal distribution, provided $n \geq 30$. If the population distribution is extremely skewed, a sample size much larger than 30 may be needed to ensure the sampling distribution is approximately normal.
- The mean, standard deviation, and probabilities for a sampling distribution of a sample mean should be interpreted within the context of a specific population.
- t-distributions, also called Student's t-distributions, form a family of symmetric, bell-shaped, standardized distributions with wider tails than that of the standard normal distribution. Specific t-distributions are identified using a parameter known as the number of degrees of freedom (df), which is based on the sample size(s). When the degrees of freedom are small, the t-distribution has a much narrower peak and fatter tails than a normal distribution. As the degrees of freedom increase, the t-distribution more closely resembles the standard normal distribution (mean $\mu = 0$ and standard deviation $\sigma = 1$).
- t-distributions are used for finding critical values and test statistics for inferences about a population mean, μ, when the population standard deviation, σ , is unknown and the sample standard deviation, s, must be used instead.
- The appropriate confidence interval procedure for estimating the population mean of a quantitative variable for one sample is a one-sample t-interval for a population mean. (The population standard deviation, σ , is not typically known for distributions for quantitative variables.)
- For a matched pairs design with two dependent samples, the appropriate analysis calculates differences between pairs of values to produce one sample of differences. The confidence interval procedure for the matched pairs design is a one-sample t-interval for a population mean difference.
- The parameter for a confidence interval for a population mean or population mean difference should reference the population mean or population mean difference and the response variable, in context. For the population mean difference, it is important to state the order of subtraction for the difference.
- A one-sample t-interval for a population mean or population mean difference requires that three conditions be met: 4.2.C.1.i The randomization condition—the data should be collected using a random sample or a randomized experiment. 4.2.C.1.ii The 10% condition—when sampling without replacement, the population size must be at least 10 times larger than the sample size. 4.2.C.1.iii The sample data condition—it is indicated the population distribution is approximately normal, or $n \geq 30$, or if $n < 30$, the sample data distribution should be free from strong skewness and outliers. For matched pairs, the number of differences should be greater than or equal to 30. If

the number of differences is less than 30, the sample of differences should be free from strong skewness and outliers.

- A point estimate for a population mean is the sample mean, $\bar{x}$, or $\bar{x}_d$ for the sample mean difference.
- To estimate the population mean for one sample or the population mean difference between values in matched pairs, when the population standard deviation is unknown, the confidence interval is $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$, where t^* is the critical value for the central C% of a t-distribution with degrees of freedom n −1.
- The standard error (SE) for a sample mean is given by $SE = \frac{s}{\sqrt{n}}$
- For a one-sample t-interval for a population mean, the margin of error is the critical value (t^*) times the standard error (SE), which equals $t^* (\frac{s}{\sqrt{n}})$.
- Because the confidence interval for a population mean or population mean difference is calculated based on a sample from a population, the computed interval may or may not contain the value of the population mean or population mean difference.
- The interpretation of the confidence level is as follows: In repeated random sampling with the same sample size from the same population, approximately C% of confidence intervals created will capture the population mean or population mean difference, where C represents the numerical value of the confidence level used.
- When interpreting a C% confidence interval for a population mean or population mean difference, we say we are C% confident the interval (a,b) contains the value of the population mean or population mean difference, where a represents the lower limit and b represents the upper limit. An interpretation of a confidence interval for a population mean or population mean difference includes a reference to the parameter.
- A confidence interval for a population mean or population mean difference provides an interval of values that may serve as convincing evidence to support a particular claim about the population mean or population mean difference.
- For a given sample, increasing the confidence level will result in the following: 4.3.C.1.i The critical value will increase. 4.3.C.1.ii The margin of error will increase. 4.3.C.1.iii The width of the confidence interval will increase.
- Increasing the sample size decreases the standard error. Thus, when all other things remain the same, the width of a confidence interval for a population mean or population mean difference tends to decrease as the sample size increases. For a confidence interval for a population mean or population mean difference with a given confidence level, the width of the interval is approximately proportional to $\frac{1}{\sqrt{n}}$.
- The appropriate test for a population mean with unknown population standard deviation σ is a one-sample t-test for a population mean.
- For a matched pairs design with two dependent samples, the appropriate analysis calculates differences between pairs of values to produce one sample of differences. The hypothesis testing procedure for the matched pairs design is a one-sample t-test for the population mean difference.
- The parameter for a hypothesis test for a population mean and population mean difference should reference the population parameter, the response variable, and the population in context.
- The null hypothesis for a one-sample t-test for a population mean is $H_o: \mu = u_o$ where u_o is the hypothesized value. Depending upon the situation, the alternative hypothesis is $H_a: \mu < u_o$, $H_a: \mu > u_o$ or $H_a: \mu \neq u_o$.
- A one-sample t-test for a population mean or a population mean difference requires that three conditions be met: 4.4.C.1.i The randomization condition—the data should be collected using a

random sample or a randomized experiment. 4.4.C.1.ii The 10% condition—when sampling without replacement, the population size must be at least 10 times larger than the sample size 4.4.C.1.iii The sample data condition—it is indicated the population distribution is approximately normal, or $n \geq 30$, or if $n < 30$, the sample data distribution should be free from strong skewness and outliers. For matched pairs, the number of differences should be greater than or equal to 30. If the number of differences is less than 30, the sample of differences should be free from strong skewness and outliers.

- The test statistic for a one-sample t-test for a population mean or population mean difference is $t = \frac{(\bar{x} - u_o)}{\frac{s}{\sqrt{n}}}$, where t has degrees of freedom n −1. The t-statistic has a t-distribution with degrees of freedom n −1 when the null hypothesis is true.
- The p-value for a one-sample t-test for a population mean or population mean difference is found using the appropriate t-distribution table or technology.
- The p-value is the probability of obtaining a test statistic as extreme or more extreme than the test statistic that was observed (i.e., in the direction of the alternative hypothesis) given that the null hypothesis is true. An interpretation of the p-value of a hypothesis test for a population mean or population mean difference should include a statement that the p-value is computed by assuming that the null hypothesis is true (i.e., by assuming that the population mean is equal to the particular value stated in the null hypothesis in context).
- A formal decision explicitly compares the p-value to the significance . If the p-value $\leq \alpha$, then reject the null hypothesis, $H_a: \mu = u_o$. If the p-value $>\alpha$, then fail to reject the null hypothesis.
- The results of a hypothesis test for a population mean or population mean difference can serve as the statistical reasoning to support the answer to an investigative question about the population that was sampled.
- A conclusion for the hypothesis test for a population mean or population mean difference is stated in context consistent with, and in terms of, the alternative hypothesis using nondefinitive language. The conclusion should contain a reference to the parameter and the population.
- Consider a simple random sample from population 1 of size n_1 , mean μ_1 , and standard deviation $\sigma_1$and a second simple random sample from population 2 of size n_2 , mean μ_2 , and standard deviation σ_2 . If the distributions of populations 1 and 2 are normal or if both n_1 and n_2 are greater than 30, then the sampling distribution of the difference of means, $\bar{x}_1 - \bar{x}_2$ is also normal. The mean for the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is $\mu_1 - \mu_2$. The standard deviation is $\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}$.
- Sampling without replacement requires that two conditions must be met: 4.6.B.1.i The randomization condition—the data should be collected using two independent random samples. 4.6.B.1.ii The 10% condition—the size of each sample should be less than or equal to 10% of the respective population size. 4.6.B.2 If the data come from an experiment, the data only need to meet the randomization condition. The treatments must be randomly assigned to experimental units to meet the randomization condition.
- The sampling distribution for the difference between sample means, $x_1 - x_2$ can be modeled with a normal distribution if the two population distributions can each be modeled by a normal distribution.
- The sampling distribution for the difference between sample means, $x_1 - x_2$ can be modeled approximately by a normal distribution if the two population distributions cannot be modeled by a normal distribution but $n_1 \geq 30$ and $n_2 \geq 30$.

- The mean, standard deviation, and probabilities for a sampling distribution for the difference between sample means should be interpreted within the context of specific populations.
- Based on the sample data, a confidence interval can be calculated to estimate the difference between two population means. The appropriate confidence interval procedure for two independent samples is a twosample t-interval for the difference between population means.
- The parameter for a confidence interval for a two-sample t-interval for the difference between population means should reference the difference in the means, the response variable, and the populations in context.
- A two-sample t-interval for a difference between population means requires that three conditions be met: 4.7.B.1.i The randomization condition—the data should be collected using two independent random samples or a randomized experiment. 4.7.B.1.ii The 10% condition—when sampling without replacement, the size of each sample should be less than or equal to 10% of the respective population size. (Note: This condition is unnecessary when the data are from a randomized experiment). 4.7.B.1.iii The sample data condition—both samples should have a sample size greater than or equal to 30 or it is indicated that both population distributions are approximately normal. If either sample size is less than 30, both sample data distributions should be free from strong skewness and outliers.
- A point estimate for the difference between two population means is the difference in sample means, $x_1 - x_2$.
- For the difference between population means when the population standard deviations are unknown, the confidence interval can be constructed as point estimate ±(margin of error). The confidence interval for the difference between population means is

  $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}$ where t^* are the critical values for the central C% of a t-distribution with appropriate degrees of freedom that can be found using technology. The degrees of f.

- The standard error for the difference between two sample means is $SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}$, where s_1 and s_2 are the sample standard deviations.
- For the difference between two sample means, the margin of error is the critical value ($t*$) times the standard error (SE) of the difference of two sample means, which equals

  $t^* \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}$.
- Because the confidence interval for the difference between two population means is calculated based on samples from two populations, the computed interval may or may not contain the value for the difference between the two population means.
- The interpretation of the confidence level is as follows: In repeated random sampling with the same sample size from the same populations, approximately C% of confidence intervals created will capture the difference between the two population means, where C represents the numerical value of the confidence level used.
- When interpreting a C% confidence interval for the difference between two population means, we say we are C% confident that the interval (a,b) contains the value of the difference in the population means, where a represents the lower limit and b represents the upper limit. An interpretation of a confidence interval for the difference between two population means includes a reference to the difference in the population means with the details about the populations it represents in the context of the study.
- A confidence interval for the difference between two population means provides an interval of values that may serve as convincing evidence to support a particular claim about the difference in

two population means. For example, if the interval contains 0, then there is insufficient evidence to conclude there is a difference between the two population means. If the interval does not contain 0, then there is sufficient evidence to conclude there is a difference between the two population means.

- The appropriate test for the difference between two population means is a twosample t-test for a difference between two population means.
- The parameters for a hypothesis test for the difference between two population means should reference the population parameters, the response variables, and the populations in context.
- The null hypothesis for a two-sample t-test for the difference between two population means, μ1 and μ2 , can be written as either μ_1 and μ_2 is H_o: $\mu_1 - \mu_2 = 0$ or H_o: $\mu_1 = \mu_2$. The alternative hypothesis is H_a: $\mu_1 < \mu_2$, H_a: $\mu_1 > \mu_2$ or H_a: $\mu_1 \neq \mu_2$.
- A two-sample t-test for a difference between population means requires that three conditions be met: 4.9.C.1.i The randomization condition—the data should be collected using two independent random samples or a randomized experiment. 4.9.C.1.ii The 10% condition—when sampling without replacement, the size of each sample should be less than or equal to 10% of the respective population. (Note: This condition is unnecessary when the data are from a randomized experiment.) 4.9.C.1.iii The sample data condition—both samples should have a sample size greater than or equal to 30 or it is indicated that both population distributions are approximately normal. If either sample size is less than 30, both sample data distributions should be free from strong skewness and outliers.
- The test statistic for a two-sample t-test for the difference between two population means is $t = \frac{(\overline{x1} - \overline{x2}) - 0}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$. The t-statistic has a t-distribution when the null hypothesis is true. The t-statistics with the degrees of freedom can be found using technology
- The p-value for a two-sample t-test for the difference between two population means can be found using the appropriate t-distribution table or from the appropriate t-distribution using technology.
- The p-value is the probability of obtaining a test statistic as extreme or more extreme than the test statistic that was observed (i.e., in the direction of the alternative hypothesis) given that the null hypothesis is true. An interpretation of the p-value of a hypothesis test for a two-sample test for the difference between two population means should include a statement that the p-value is computed by assuming that the null hypothesis is true (i.e., by assuming the population means are equal to each other in context).
- A formal decision explicitly compares the p-value to the significance α . If the p-value ≤ α, then reject the null hypothesis H_o: $\mu_1 - \mu_2 = 0$ or H_o: $\mu_1 = \mu_2$. If the p-value >α , then fail to reject the null hypothesis.
- A conclusion for the hypothesis test for the difference between two population means is stated in context consistent with, and in terms of, the alternative hypothesis using nondefinitive language. The conclusion should contain a reference to the parameters and the population.

Students will be able to...

- Interpret statistical calculations and results to assess meaning or a claim.
- Calculate means, standard deviations, and parameters for probability distributions.
- Justify the use of a chosen statistical inference method by verifying conditions.
- Interpret the mean, standard deviation, and probabilities for the sampling distribution of a sample mean.
- Identify appropriate statistical inference methods.
- Calculate appropriate statistical inference method results.
- Describe distributions and compare relative positions of points within a distribution.

- Justify the appropriateness of constructing a confidence interval for a population mean or population mean difference by verifying conditions.
- Identify types of errors and relationships among components in statistical inference methods.
- Interpret results of statistical inference methods.
- Justify a claim based on statistical inference method results.
- Identify the null and alternative hypotheses.
- Interpret statistical calculations and results to assess meaning or a claim.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment
- FRQ (Free Response Questions)
- Daily observation of student work
- Student/group presentations

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Khan Academy assignments
- AP Classroom assignments
- **Quick Write:** Provide students with a prompt such as "Describe how to construct a sampling distribution." Then allow 1 or 2 minutes for them to write a response (e.g., "obtain sample, collect responses, calculate statistic, plot statistic, repeat"). Allow more writing time for more difficult prompts.
- **Discussion Groups:** Place students in groups of three or four and ask each group to identify the conditions for performing a test about a population mean. For each condition, have them explain why the condition is required and what would go wrong with the test if the condition were violated. Have groups pair up and compare answers.
- **Team Challenge - Marking the Text:** Give students examples of statistical situations. Have them highlight or underline the key phrases to help them determine which statistical test to use for each situation. For example, does it require a significance test or a confidence interval? Does it involve means or proportions? Is there only one variable, or are there two or more variables?
- **Round Table:** Arrange students in groups of four and give each student an identical paper containing four different inference procedure problems from Unit 4. Have students complete the first one on their paper and then pass the paper clockwise to another member in their group. That student checks the first problem and then completes the second problem on the paper. Have students rotate again, continuing the process until each student has their original paper back.
- **Graphic Organizer:** Have students work in teams of two or three to develop a flowchart for determining which inference procedure from Units 3 and 4 to use in a given setting.

RESOURCES

Teacher Resources:

- **Textbook:** Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom
- Math Medic

Equipment Needed:

- Internet
- Laptop/Chromebook
- Document Camera
- Newline Board
- Graphing Calculator

UNIT 5 OVERVIEW

Content Area: Mathematics

Unit Title: Regression Analysis

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: In this unit, students explore relationships in two-variable quantitative data sets. Students describe form, direction, strength, and unusual features for an association between two quantitative variables. They assess correlation and, if appropriate, use a linear model to predict values of the response variable from values of the explanatory variable. Students interpret the least-squares regression line in context, analyze prediction errors (i.e., residuals), and explore departures from a linear pattern.

Approximate Length of Unit: 3 weeks

LEARNING TARGETS

NJ Student Learning Standards:

S.ID.B.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S.ID.B.6: Represent data on two quantitative variables on a scatter plot and describe how the variables are related.

College Board Standards:

5.1.A Construct scatterplots depicting the relationship between two quantitative variables.
5.1.B Describe the characteristics of a scatterplot.
5.1.C Justify a claim using scatterplots depicting the relationship between two quantitative variables.
5.2.A Interpret the correlation for a linear relationship.
5.3.A Calculate a predicted response value using a linear regression model.
5.4.A Calculate the differences between the observed and predicted values.
5.4.B Interpret the differences between the observed and predicted values.
5.4.C Describe the form of association of bivariate data using residual plots.
5.5.A Calculate the coefficients for the least-squares regression line model.
5.5.B Interpret coefficients for the least-squares regression line model.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:

ELA

RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.

L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.

L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.

SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

HS-ETS1-4 Both physical models and computers can be used in various ways to aid in the engineering design process. Computers are useful for a variety of purposes, such as running simulations to test different ways of solving a problem or to see which one is most efficient or economical; and in making a persuasive presentation to a client about how a given design will meet his or her needs.

Unit Understandings:

Students will understand that…

- Given that variation may be random or not, conclusions are uncertain.
- Graphical representations and statistics allow us to identify and represent key features of data.
- Regression models may allow us to predict responses to changes in an explanatory variable.
- Interpret coefficients for the least-squares regression line model.
- Calculate a predicted response using a least squares regression line for a transformed data set.

Unit Essential Questions:

- Does the fact that the number of shark attacks increases as ice cream sales increase necessarily mean that ice cream sales cause shark attacks?

- How can we determine the effectiveness of a linear model that uses the number of cricket chirps per minute to predict temperature?

Knowledge and Skills:

Students will know...

- A bivariate quantitative data set consists of observations of ordered pairs from two quantitative variables, collected from the same individuals in a sample or population, and can be used to construct a scatterplot.
- A scatterplot shows the relationship between two quantitative variables for each observation, one corresponding to the value on the x-axis and one corresponding to the value on the y-axis. The explanatory variable is placed on the x-axis and is the variable whose values are used to explain or predict the corresponding values for the response variable, which is placed on the y-axis.
- A description of the association shown in a scatterplot includes form, direction, strength, and unusual features.
- The form of the association shown in a scatterplot, if any, can be described as linear or non-linear.
- The direction of the association shown in a scatterplot, if any, can be described as positive or negative. A positive association means that as values of the explanatory variable increase, the values of the response variable tend to increase. A negative association means that as values of the explanatory variable increase, the values of the response variable tend to decrease.
- The strength of the association shown in a scatterplot is how closely the points follow the general pattern. Strength can be described as strong, moderate, or weak.
- Unusual features of a scatterplot include clusters of individual points or points that don't fit in the general pattern of association between the two variables.
- Scatterplots depicting the relationship between two numeric variables may reveal information that can be used to justify claims about the variable in context.
- The correlation coefficient, r, summarizes the strength and direction of the linear association between two quantitative variables. The correlation coefficient r is unit-free and always between −1 and 1, inclusive. A negative correlation coefficient value indicates a negative association, and a positive correlation coefficient value indicates a positive association.
- The strength of the linear association is determined by how close the correlation coefficient is to −1 or 1. A value of r = 0 indicates that there is no linear association. A value of r = -1 or r =1 indicates that there is a perfect linear association.
- A correlation coefficient close to −1 or 1 does not necessarily mean that a linear model is appropriate.
- A perceived or real relationship between two variables does not mean that changes in one variable cause changes in the other. That is, correlation does not necessarily imply causation.
- If the form of the relationship between x and y appears linear, we can approximate the relationship between x and y using a linear regression model, which is a linear equation that uses an explanatory variable, x, to predict the response variable, y.
- In a linear regression model, the predicted response value, denoted by $\hat{y}$, is calculated as $\hat{y} = a + bx$, where a is the y-intercept, b is the slope of the regression line, and x is the explanatory variable.
- Extrapolation is predicting a response value using a value for the explanatory variable that is beyond the interval of x-values used to determine the regression line. The predicted value is less reliable the further the estimate is extrapolated.
- Interpolation is predicting a response value using a value for the explanatory variable that is within the interval of x-values used to determine the regression line.
- A residual is the difference between the observed response value and the predicted response value for the given value of the explanatory variable: residual = $y - \hat{y}$ or

(residual = observed y - predicted y).

- If the residual is positive, the model underpredicts (underestimates) the value of the response variable. If the residual is negative, the model overpredicts (overestimates) the value of the response variable.
- A residual plot is a scatterplot of the residuals versus the predicted response values (or the explanatory variable values).
- Residual plots can be used to investigate the appropriateness of the linear regression model for the observed data.
- The linear regression model should only be fit to the data if the data exhibit a linear trend. Apparent randomness in a residual plot for a linear regression model is confirmation of a linear form in the association between the two variables and indicates that the simple linear regression model is an appropriate model for the data.
- Curvature in the residual plot for a linear regression model suggests that the linear model is not the most appropriate model for the data.
- The simple linear regression model is fit to the data by minimizing the sum of the squares of the residuals. Because of this, the resulting equation is often called the least-squares regression line (LSRL) and is calculated using technology. This regression line will pass through the point $(\bar{x}, \bar{y})$.
- The slope of the regression line, b, is calculated using technology.
- The y-intercept of the regression line, a, is calculated using technology.
- In simple linear regression, the correlation coefficient, r, is calculated using technology.
- In simple linear regression, the square of the correlation coefficient, r^2, is called the coefficient of determination. The value of r^2 is the proportion of variation in the response variable that is explained by the linear relationship with the explanatory variable.
- The coefficients of the least-squares regression line model (line of best fit) are the slope, b, and the y-intercept, a, because they are based on a sample of values.
- The slope of the least-squares regression line can be interpreted as the predicted increase or decrease in the response variable for a oneunit increase in the explanatory variable, and it should be interpreted in context.
- The y-intercept in the least-squares regression line is the predicted value of the response variable when the explanatory variable is equal to 0, and it should be interpreted in context. Sometimes, the y-intercept of the line does not have a reasonable interpretation in context because x = 0 might be beyond the interval of x-values used to determine the regression line (extrapolation). At other times, the y-intercept of the line does not have a logical interpretation in context because it might be a negative value for a response variable that has no negative values, such as height.

Students will be able to…

- Construct tabular and graphical representations of data and distributions.
- Describe and compare tabular and graphical representations of data, as well as summary statistics.
- Justify a claim based on statistical calculations and results.
- Interpret statistical calculations and results to assess meaning or a claim.
- Calculate summary statistics, relative positions of points within a distribution, and predicted responses.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment
- FRQ (Free Response Questions)
- Daily observation of student work
- Student/group presentations

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Khan Academy assignments
- AP Classroom assignments
- **Sketch and Switch:** Ask students to create a sketch of a scatterplot using a prompt such as a "weak negative, non-linear association." Then have them switch papers to review each other's work.
- **Reversing Interpretations:** Instead of asking students to interpret a residual, give them the residual and the equation of the least-squares regression line and ask them to make a prediction for a particular observation (e.g., "One wolf in the pack had a length of 1 4. m and a residual of −9 8. 7. What does that −9 8. 7 tell us about that particular wolf?")
- **Think-Aloud:** Have students toss a handful of coins and record how many land heads up. This is trial 1. Then have them remove the coins that landed heads up. For trial 2, have students toss the remaining coins (i.e., the coins left over after removing those that landed heads up) and record how many land heads up on the second toss. Ask students to think about the trend and make a prediction, discussing their ideas with a partner. Will it be linear? A scatterplot of many trials should show a nonlinear relationship.
- **Team Challenge:** Set up teams of three or four and task students with solving different multiple-part problems in Unit 5 with different datasets to answer investigative questions. Have teams explain their work to other teams.
- **Relief Pitcher:** Use this strategy to review multiple concepts at the end of a unit or to spiral review at the end of the course. For instance, when reviewing for the Unit 5 test, students can earn 5 points if they get a question correct when up at the "pitcher's mound" and 3 points if they do not know the answer but their team in the "bullpen" helps them answer it.

RESOURCES

Teacher Resources:

- **Textbook:** Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom
- Math Medic

Equipment Needed:

- Internet
- Laptop/Chromebook
- Document Camera
- Newline Board
- Graphing Calculator