E-Learning Day 1

Review: Set Theory

Learning Objectives

By the end of this module, Studnets will be able to:

1. Define and use basic terms in set theory.
2. Compare sets using equality, subset relations, equivalence, and power sets.
3. Determine the cardinality of sets and distinguish between finite and infinite sets.

DM.NR.1.1 – Basic Terms and Concepts in Set Theory

- **Set:** A well-defined collection of distinct objects, called *elements*.
  - Example: A={1,2,3,4}A = \{1, 2, 3, 4\}A={1,2,3,4}.

- **Element:** A member of a set.
  - Example: In A={1,2,3}A = \{1, 2, 3\}A={1,2,3}, 2∈A2 \in A2∈A.
  -
- **Well-defined set:** A set where it is clear whether an element belongs or not.
  - Example: “Set of all prime numbers less than 10” → {2,3,5,7}\{2, 3, 5, 7\}{2,3,5,7}.
  - Not well-defined: “Set of the most beautiful colors.”

- **Empty set (Null set):** A set with no elements.
  - Symbol: ∅\varnothing∅ or {}\{\}{}.

- **Cardinal number (Cardinality):** The number of elements in a set.
  - Example: If A={2,4,6}A = \{2, 4, 6\}A={2,4,6}, then n(A)=3n(A) = 3n(A)=3.

DM.NR.1.2 – Comparing Sets

- **Equality of sets:** Two sets are equal if they contain the exact same elements.
  - Example: A={1,2,3},B={3,1,2}A = \{1, 2, 3\}, B = \{3, 1, 2\}A={1,2,3},B={3,1,2}. Then A=BA = BA=B.

- **Subset:** A⊆BA \subseteq BA⊆B if every element of AAA is also in BBB.

- **Proper subset:** A⊂BA \subset BA⊂B if A⊆BA \subseteq BA⊆B but A≠BA \neq BA□=B.

- **Equivalence of sets:** Two sets are equivalent if they have the same cardinality (same number of elements), even if elements differ.
  - Example: {a,b,c}\{a, b, c\}{a,b,c} and {1,2,3}\{1, 2, 3\}{1,2,3} are equivalent.

- **Power set:** The set of all subsets of a given set.
  - If A={1,2}A = \{1, 2\}A={1,2}, then P(A)={∅,{1},{2},{1,2}}P(A) = \{\varnothing, \{1\}, \{2\}, \{1,2\}\}P(A)={∅,{1},{2},{1,2}}.
  - If AAA has nnn elements, then P(A)P(A)P(A) has 2n2^n2n elements.

DM.NR.1.3 – Cardinality of Sets

- **Finite set:** A set with a countable number of elements.
  - Example: A={1,2,3,4,5},n(A)=5A = \{1, 2, 3, 4, 5\}, n(A) = 5A={1,2,3,4,5},n(A)=5.

- **Infinite set:** A set with unlimited elements.
  - Example: N={1,2,3,4,…}N = \{1, 2, 3, 4, …\}N={1,2,3,4,…}.

- **Cardinality notation:** n(A)n(A)n(A) = number of elements in set AAA.

Set Theory Worksheet

Multiple Choice Questions

1. Which of the following is a well-defined set?

- A) Set of tall students in the class
- B) Set of even numbers less than 10
- C) Set of beautiful colors
- D) Set of popular movies

2. The symbol Ø represents:

- A) Universal set
- B) Null set
- C) Power set
- D) Subset

3. If A = {1,2,3}, then n(A) = ?

- A) 1
- B) 2
- C) 3
- D) 4

4. Which of the following is true about the empty set?

- A) It contains one element
- B) Its cardinality is zero
- C) It is not a set
- D) It is equal to the universal set

5. If A = {a, b, c}, then the number of elements in P(A) is:

- A) 3
- B) 6
- C) 8
- D) 9

6. If A = {1,2,3}, B = {3,2,1}, then:

- A) $A \neq B$
- B) $A = B$
- C) $A \subset B$ only
- D) None

7. Which of the following is an infinite set?

- A) {2,4,6,8}
- B) {x | x is a natural number}
- C) {1,2,3,4,5}

- D) {a,b,c}

8. If A = {1,2}, then P(A) = ?

- A) {1,2}
- B) {∅, {1}, {2}, {1,2}}
- C) {∅}
- D) {1}

9. Which of the following shows subset relation?

- A) {1,2} ⊆ {1,2,3}
- B) {1,2,3} ⊂ {1,2}
- C) {a,b} ⊂ {1,2}
- D) {x} ⊂ ∅

10. The set {x | x is a prime number less than 10} equals:

- A) {2,3,5,7}
- B) {1,2,3,5}
- C) {2,4,6,8}
- D) {3,5,7,9}

11. The cardinality of ∅ is:

- A) 0
- B) 1
- C) Undefined
- D) Infinite

12. Which notation means ‘is an element of’?

- A) $\in$
- B) $\subset$
- C) $\subseteq$
- D) $\emptyset$

13. If A = {1,2,3}, B = {1,2,3,4}, then:

- A) A = B
- B) $A \subset B$
- C) $B \subset A$
- D) $A \neq B$ but not a subset

14. Two sets are equivalent if:

- A) They are identical
- B) They have the same number of elements
- C) They are subsets
- D) They have different elements

15. Power set of Ø is:

- A) Ø
- B) {Ø}
- C) {Ø, {Ø}}
- D) None

16. Which of the following is finite?

- A) Set of integers
- B) Set of real numbers
- C) Set of weekdays
- D) Set of natural numbers

17. If A has 4 elements, the number of subsets is:

- A) 8
- B) 12
- C) 16
- D) 32

18. If A = {2,4,6}, B = {x | x is an even number less than 7}, then:

- A) A = B
- B) A ⊂ B

- C) $B \subset A$
- D) $A \neq B$

19. Which of the following is not a set?

- A) {1,2,3}
- B) Set of months of the year
- C) Set of popular songs
- D) {a,e,i,o,u}

20. If A = {a,b}, B = {1,2}, which statement is true?

- A) A = B
- B) $A \neq B$ but equivalent
- C) $A \subset B$
- D) None

21. If A = {1,2,3,4}, what is n(P(A))?

- A) 4
- B) 8
- C) 12
- D) 16

22. Universal set U = {1,2,3,4,5}, A = {2,4}. Then A ⊂ U?

- A) Yes
- B) No
- C) Maybe
- D) Cannot determine

23. The set of vowels in English alphabet is:

- A) {a,e,i,o,u}
- B) {a,b,c}
- C) {e,o,u}
- D) {a,e}

24. Which symbol shows ‘proper subset’?

- A) ⊂
- B) ⊆
- C) ∈
- D) =

25. The set of all subsets of {a,b,c} has:

- A) 6 elements
- B) 8 elements
- C) 9 elements

- D) 12 element

1. Identify the type of set: S={}

2. Compare: A={1,2,3},B={1,2,3,4}A = \{1, 2, 3\}, B = \{1, 2, 3, 4\}A={1,2,3},B={1,2,3,4}. Is A⊆B

3. Find the power set of {x,y}

4. State whether the set of natural numbers is finite or infinite.

E-learning **Day2**

DM.MGSR.1.4 — Converse, Inverse, and Contrapositive

Learning Objectives:

By the end of the lesson, Students will be able to :

• Translate between words and symbols using p, q, ¬ (not), → (implies).

• Given a conditional 'If p, then q', write its converse, inverse, and contrapositive in words and symbols.

• Explain the truth relationships: a statement is logically equivalent to its contrapositive; the converse is equivalent to the inverse.

Key Vocabulary & Symbols

• Proposition (statement): a sentence that is true or false, not both.

• p, q: variables that stand for propositions (e.g., p: “a number is even”).

• ¬p: negation of p (“not p”).

• p → q: conditional (“if p, then q”).

Definitions from a conditional 'If p, then q' (p → q)

• **Converse:** q → p — Swap hypothesis and conclusion.

• **Inverse:** ¬p → ¬q — Negate both parts.

• **Contrapositive:** ¬q → ¬p — Swap and negate both parts.

Truth Relationships

• A conditional (p → q) is logically equivalent to its contrapositive (¬q → ¬p).

• The converse (q → p) is logically equivalent to the inverse (¬p → ¬q).

• A statement and its converse are not necessarily equivalent.

Example 1

• **Context:**

Let p: “a figure is a square”,

q: “the figure has four equal sides”.

The conditional is: If a figure is a square, then it has four equal sides (p → q).

• **Converse:** If a figure has four equal sides, then it is a square (q → p).

(False in general—could be a rhombus that is not a square.)

• **Inverse:** If a figure is not a square, then it does not have four equal sides (¬p → ¬q). (False.)

• **Contrapositive:** If a figure does not have four equal sides, then it is not a square (¬q → ¬p). (True.)

• **Takeaway:** Original statement and contrapositive share truth value; converse and inverse share truth value.

Translation Hints

• “Only if” means q is necessary: p only if q ≡ p → q.

• “If and only if” (iff) means p ↔ q (both directions).

• Watch quantifiers: “All”, “Some”, “No” change truth, not just form.

Worksheet — Converse, Inverse, and Contrapositive (30 MCQs)

1. If p: 'It is raining', q: 'The ground is wet'. Which is the converse of p → q?

A. If the ground is wet, then it is raining.

B. If it is not raining, then the ground is not wet.

C. If the ground is not wet, then it is not raining.

D. If it is raining, then the ground is wet.

2. Which is the inverse of: If a figure is a square, then it has four equal sides?

A. If a figure is not a square, then it does not have four equal sides.

B. If a figure has four equal sides, then it is a square.

C. If a figure does not have four equal sides, then it is not a square.

D. If a figure is a square, then it has four equal sides.

3. The contrapositive of 'If $x > 4$, then $x > 2$' is:

A. If $x > 2$, then $x > 4$.

B. If $x \leq 2$, then $x \leq 4$.

C. If $x \leq 4$, then $x \leq 2$.

D. If $x \leq 2$, then $x \leq 4$.

4. Which pair is logically equivalent?

A. Conditional and Converse

B. Conditional and Inverse

C. Conditional and Contrapositive

D. Converse and Contrapositive

5. If p: 'a number is even', q: 'the number is divisible by 2'. Which is inverse?

A. If a number is not even, then it is not divisible by 2.

B. If a number is divisible by 2, then it is even.

C. If a number is not divisible by 2, then it is not even.

D. If a number is even, then it is divisible by 2.

6. Which is the Converse of: If a polygon is a triangle, then it has 3 sides.

A. If a polygon is not a triangle, then it does not have 3 sides.

B. If a polygon has 3 sides, then it is a triangle.

C. If a polygon is a triangle, then it has 3 sides.

D. If a polygon does not have 3 sides, then it is not a triangle.

7. Which is the Inverse of: If a number is divisible by 6, then it is divisible by 3.

A. If a number is divisible by 3, then it is divisible by 6.

B. If a number is not divisible by 3, then it is not divisible by 6.

C. If a number is not divisible by 6, then it is not divisible by 3.

D. If a number is divisible by 6, then it is divisible by 3.

8. Which is the Contrapositive of: If two lines are parallel, then they never meet.

A. If two lines are parallel, then they never meet.

B. If two lines meet, then they are not parallel.

C. If two lines never meet, then they are parallel.

D. If two lines are not parallel, then they meet.

9. Which is the Converse of: If a quadrilateral is a rectangle, then its diagonals are equal.

A. If a quadrilateral has equal diagonals, then it is a rectangle.

B. If a quadrilateral is a rectangle, then its diagonals are equal.

C. If a quadrilateral's diagonals are not equal, then it is not a rectangle.

D. If a quadrilateral is not a rectangle, then its diagonals are not equal.

10. Which is the Inverse of: If a person is a teenager, then they are at least 13 years old.

A. If a person is at least 13, then they are a teenager.

B. If a person is not at least 13, then they are not a teenager.

C. If a person is not a teenager, then they are not at least 13.

D. If a person is a teenager, then they are at least 13 years old.

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B. If a polygon has 3 sides, then it is a triangle.

C. If a polygon is not a triangle, then it does not have 3 sides.

D. If a polygon does not have 3 sides, then it is not a triangle.

12. Which is the Converse of: If a number is divisible by 6, then it is divisible by 3.

A. If a number is not divisible by 3, then it is not divisible by 6.

B. If a number is divisible by 6, then it is divisible by 3.

C. If a number is divisible by 3, then it is divisible by 6.

D. If a number is not divisible by 6, then it is not divisible by 3.

13. Which is the Inverse of: If two lines are parallel, then they never meet.

A. If two lines are not parallel, then they meet.

B. If two lines are parallel, then they never meet.

C. If two lines meet, then they are not parallel.

D. If two lines never meet, then they are parallel.

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C. If a quadrilateral is not a rectangle, then its diagonals are not equal.

D. If a quadrilateral has equal diagonals, then it is a rectangle.

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D. If a polygon is not a triangle, then it does not have 3 sides.

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B. If a number is divisible by 3, then it is divisible by 6.

C. If a number is not divisible by 3, then it is not divisible by 6.

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B. If a quadrilateral is a rectangle, then its diagonals are equal.

C. If a quadrilateral is not a rectangle, then its diagonals are not equal.

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A. If a person is not at least 13, then they are not a teenager.

B. If a person is not a teenager, then they are not at least 13.

C. If a person is at least 13, then they are a teenager.

D. If a person is a teenager, then they are at least 13 years old.

21. Which is the Converse of: If a polygon is a triangle, then it has 3 sides.

A. If a polygon is a triangle, then it has 3 sides.

B. If a polygon has 3 sides, then it is a triangle.

C. If a polygon does not have 3 sides, then it is not a triangle.

D. If a polygon is not a triangle, then it does not have 3 sides.

22. Which is the Inverse of: If a number is divisible by 6, then it is divisible by 3.

A. If a number is not divisible by 3, then it is not divisible by 6.

B. If a number is divisible by 6, then it is divisible by 3.

C. If a number is not divisible by 6, then it is not divisible by 3.

D. If a number is divisible by 3, then it is divisible by 6.

23. Which is the Contrapositive of: If two lines are parallel, then they never meet.

A. If two lines are not parallel, then they meet.

B. If two lines meet, then they are not parallel.

C. If two lines are parallel, then they never meet.

D. If two lines never meet, then they are parallel.

24. Which is the Converse of: If a quadrilateral is a rectangle, then its diagonals are equal.

A. If a quadrilateral has equal diagonals, then it is a rectangle.

B. If a quadrilateral is not a rectangle, then its diagonals are not equal.

C. If a quadrilateral is a rectangle, then its diagonals are equal.

D. If a quadrilateral's diagonals are not equal, then it is not a rectangle.

25. Which is the Inverse of: If a person is a teenager, then they are at least 13 years old.

A. If a person is a teenager, then they are at least 13 years old.

B. If a person is not at least 13, then they are not a teenager.

C. If a person is at least 13, then they are a teenager.

D. If a person is not a teenager, then they are not at least 13.

26. Which is the Contrapositive of: If a polygon is a triangle, then it has 3 sides.

A. If a polygon does not have 3 sides, then it is not a triangle.

B. If a polygon is a triangle, then it has 3 sides.

C. If a polygon has 3 sides, then it is a triangle.

D. If a polygon is not a triangle, then it does not have 3 sides.

27. Which is the Converse of: If a number is divisible by 6, then it is divisible by 3.

A. If a number is not divisible by 3, then it is not divisible by 6.

B. If a number is not divisible by 6, then it is not divisible by 3.

C. If a number is divisible by 3, then it is divisible by 6.

D. If a number is divisible by 6, then it is divisible by 3.

28. Which is the Inverse of: If two lines are parallel, then they never meet.

A. If two lines never meet, then they are parallel.

B. If two lines are not parallel, then they meet.

C. If two lines meet, then they are not parallel.

D. If two lines are parallel, then they never meet.

29. Which is the Contrapositive of: If a quadrilateral is a rectangle, then its diagonals are equal.

A. If a quadrilateral is a rectangle, then its diagonals are equal.

B. If a quadrilateral's diagonals are not equal, then it is not a rectangle.

C. If a quadrilateral is not a rectangle, then its diagonals are not equal.

D. If a quadrilateral has equal diagonals, then it is a rectangle.

30. Which is the Converse of: If a person is a teenager, then they are at least 13 years old.

A. If a person is a teenager, then they are at least 13 years old.

B. If a person is not a teenager, then they are not at least 13.

C. If a person is at least 13, then they are a teenager.

D. If a person is not at least 13, then they are not a teenager.

E-Learning Election Methods
Day3

Standard: DM.DPSR.1.1

Topic: Investigate and describe the results of various election methods.

Learning Objectives: By the end of the lesson Students will be

Investigate and describe the results of various election methods

Learn about different methods of voting preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners

Apply the concept in real life scenario to solve problems

Introduction

Elections are decision-making processes where voters select candidates or options. Different voting methods can produce different results, even with the same set of ballots. In this lesson, we will explore several election methods, including approval voting, preference voting, plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

1. Approval Voting

In approval voting, each voter can vote for (approve of) as many candidates as they wish. The candidate with the most approvals wins. This system reduces the impact of vote splitting and allows voters to support multiple acceptable candidates.

2. Preference Voting

In preference voting, voters rank candidates in order of preference (1st choice, 2nd choice, etc.). This ranking is used in different methods such as Borda count or Condorcet to determine a winner.

3. Plurality Voting

Plurality voting, also called 'first-past-the-post,' means the candidate with the most votes wins, even if they do not have a majority. It is the most common method but may not always reflect the will of the majority.

4. Majority Voting

Majority voting requires a candidate to receive more than 50% of the votes to win. If no candidate receives a majority, another method such as a run-off may be used.

5. Run-off Voting

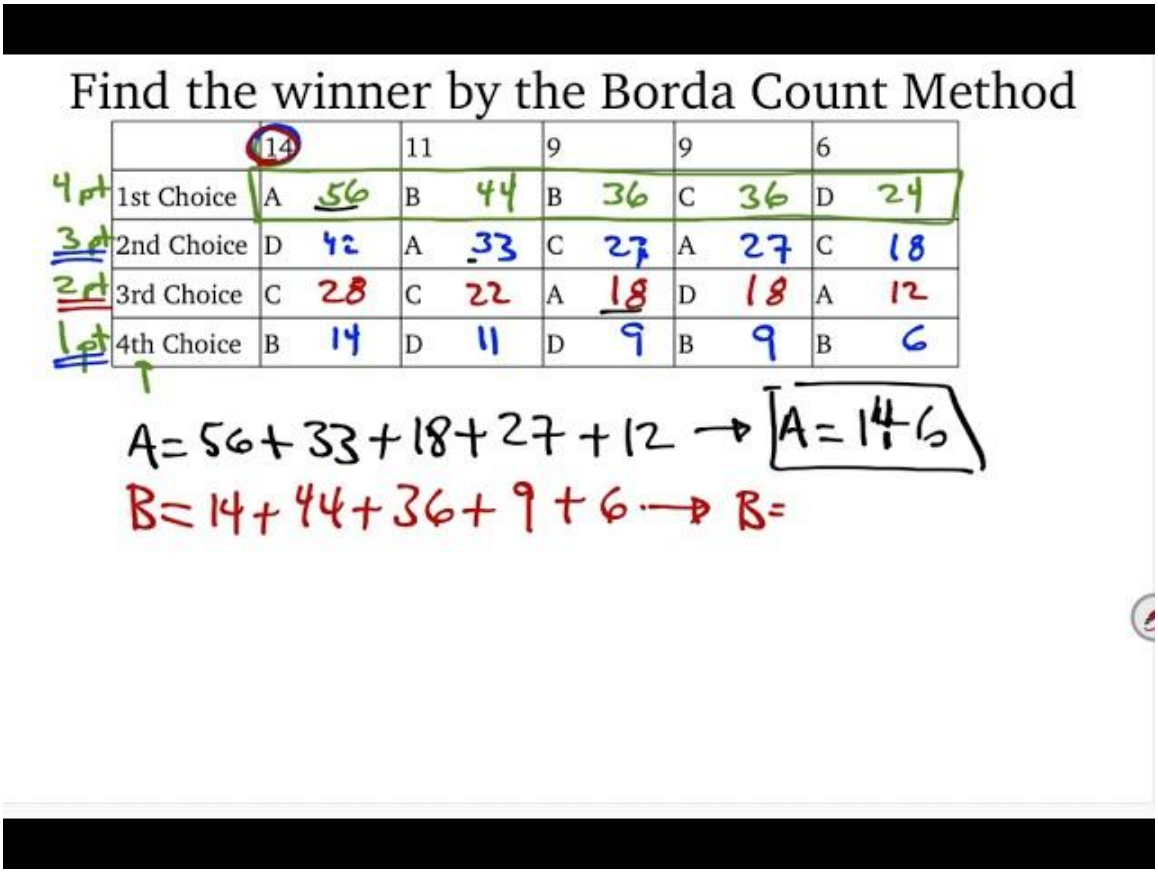
Run-off voting is used when no candidate has a majority. The two candidates with the most votes face off in a second round, ensuring the winner has majority support.

6. Sequential Run-off (Elimination Method)

In sequential run-off, also known as instant run-off or ranked-choice voting, voters rank candidates. If no candidate has a majority, the candidate with the fewest votes is eliminated, and votes are transferred based on next preferences. This continues until a candidate achieves a majority.

7. Borda Count

In the Borda count method, voters rank candidates, and points are assigned based on position (e.g., in a 4-candidate race, 1st choice = 4 points, 2nd = 3, etc.). The candidate with the highest total score wins.



8. Condorcet Winners

The Condorcet method compares each candidate head-to-head against every other candidate. If one candidate wins against all others in these one-on-one matchups, they are the Condorcet winner. However, cycles can occur, making it possible for no Condorcet winner to exist.

Conclusion

Different election methods can lead to different outcomes. While plurality is simple, it may not represent the majority. Methods like run-off, sequential run-off, Borda count, and Condorcet aim to ensure fairness but come with more complexity. Understanding these methods helps in analyzing real-world elections and voting systems.

E-Learning Worksheet: Election Methods

Standard: DM.DPSR.1.1

Topic: Investigate and describe the results of various election methods, including approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

Part A: Multiple Choice Questions (20)

1. Which election method selects the candidate with the most first-place votes?

A. Plurality

B. Majority

C. Approval

D. Borda count

2. In which election method must a candidate receive more than 50% of the votes to win?

A. Majority

B. Plurality

C. Approval

D. Condorcet

3. What is the key feature of approval voting?

A. Voters may select more than one candidate

B. Only top choice is counted

C. Candidates are ranked

D. Votes are redistributed

4. Which voting method eliminates the candidate with the fewest votes and redistributes votes until a winner is found?

A. Run-off

B. Sequential run-off

C. Borda count

D. Condorcet

5. Which method uses a system of points assigned based on ranking candidates?

A. Plurality

B. Majority

C. Borda count

D. Approval

6. What is the main principle of the Condorcet method?

A. Candidate with most points wins

B. Candidate preferred in all head-to-head wins

C. Candidate with majority wins

D. Random tie-breaking

7. In sequential run-off voting, what happens after the candidate with the least votes is eliminated?

A. Election ends immediately

B. Votes for eliminated candidate are reassigned

C. All votes are discarded

D. Winner is chosen randomly

8. Which method may elect a candidate who is most preferred in head-to-head matchups against all others?

A. Approval voting

B. Condorcet method

C. Borda count

D. Run-off

9. In plurality voting, is it possible for a candidate to win without a majority?

A. Yes

B. No

10. Which voting method best reflects overall voter preferences across rankings?

A. Borda count

B. Plurality

C. Run-off

D. Majority

11. What is one advantage of approval voting compared to plurality voting?

A. Reduces vote splitting

B. Guarantees majority winner

C. Requires rankings

D. Eliminates cycles

12. The Borda count method may allow which type of candidate to win?

A. Least popular

B. Compromise

C. Most extreme

D. Head-to-head winner

13. Which method can result in no clear winner due to cycles in preferences?

A. Condorcet

B. Borda count

C. Plurality

D. Run-off

14. In majority voting, how many votes are needed to win in a 101-voter election?

A. 50

B. 51

C. 100

D. 52

15. Which voting method can sometimes encourage strategic voting (ranking a weaker candidate higher)?

A. Plurality

B. Borda count

C. Approval

D. Majority

16. Condorcet winners are guaranteed to exist in every election. (True/False)

A. True

B. False

17. Which method can produce different results depending on how ties are broken?

A. Approval

B. Run-off

C. Sequential run-off

D. Borda count

18. Sequential run-off is also known as what voting system?

A. Instant Runoff Voting (IRV)

B. Approval Voting

C. Condorcet

D. Borda count

19. In approval voting, how many candidates can a voter approve?

A. Only one

B. Two

C. Any number

D. None

20. Which method is most likely to require multiple rounds of voting?

A. Majority

B. Plurality

C. Sequential run-off

D. Approval

Part B: Descriptive Questions (5)

1. Explain the differences between plurality voting and majority voting. Give an example for each.

2. Discuss the strengths and weaknesses of the Borda count method.

3.

Borda Count Method

Determine the winner using the Borda Count method.

Number of Voters	36	25	40	23	41	35
1st Choice	H	H	C	C	P	P
2nd Choice	C	P	H	P	H	C
3rd Choice	P	C	P	H	C	H

E-learning Day 4

DM.NR.3.1 — Calculating Permutations using Set Notation

Learning Objectives

By the end of this lesson, I will be able to:

1. Define permutations and distinguish them from combinations.
2. Use factorial notation ((n!)) to compute permutations.
3. Write permutation problems using set notation (e.g. (A = {1,2,3})).
4. Calculate permutations with and without repetition and for circular arrangements.
5. Apply permutation formulas to solve real-world problems.

Key Definitions

- **Permutation:** An ordered arrangement of elements selected from a set.

- **Factorial:** (n! = n(n-1)(n-2)), with (0! = 1).

- **Permutation of (n) objects taken (r) at a time (no repetition):** [P(n,r)=]

- **Permutation with repetition allowed:** (n^r) (choose one of (n) options for each of (r) positions)

- **Set notation:** We write a set using braces, e.g. (S={a,b,c,d}). When working permutations we often specify the universal set and the size (|S|=n).

- **Circular permutations (distinct rotations considered the same):** For arranging (n) distinct objects around a circle: ((n-1)!).

Writing Problems in Set Notation

1. Specify your set and its cardinality: e.g. (A={1,2,3,4,5}), so (|A|=5).

2. State what you are selecting (ordered) and whether repetition is allowed.

3. Use the appropriate permutation formula.

Tips & Common Mistakes

- Order matters for permutations — switching positions usually gives a different arrangement.
- When repetition is allowed, use (n^r). When it is not, use (P(n,r)=n!/(n-r)!).
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- Circular arrangements consider rotations identical unless seats are labelled.
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- Carefully read questions for words like “arrange”, “select”, “order”, “distinct”, and “repeat”.

Worksheet

Instructions: For each question, write the letter of the correct answer. Show your working if required.

1. Let (A={1,2,3,4}). How many permutations of all elements of (A) are there?

A. 4 B. 8 C. 24 D. 16

2. How many 3-letter 'words' can be formed from (B={a,b,c,d,e}) if repetition is not allowed?

A. 60 B. 125 C. 10 D. 20

3. How many 3-letter sequences from the digits ({0,1,2,3,4}) if repetition is allowed?

A. 60 B. 125 C. 20 D. 24

4. From set (S={1,2,3,4,5,6}), how many ordered selections of 4 elements (no repeats)?

A. 360 B. 720 C. 90 D. 96

5. How many distinct arrangements of the letters in the word "MATH"?

A. 24 B. 12 C. 4 D. 6

6. How many 4-digit PINs using digits (0)–(9) with no repeated digits?

A. 5040 B. 10,000 C. 504 D. 5040

7. A circular necklace is to be made with 5 distinct beads. How many distinct necklaces if rotations are considered identical (but reflections are different)?

A. 24 B. 5! C. 4! D. 120

8. From (T={A,B,C,D,E}), how many ways to choose an ordered pair (first, second) allowing repetition?

A. 25 B. 20 C. 10 D. 5

9. Number of ways to seat 7 people in a row (distinct seats)?

A. 6! B. 7! C. 7 D. 5040

10. Using set notation: if (X={1,2,3}), how many 2-length sequences from (X) allow repetition?

A. 6 B. 9 C. 3 D. 4

11. How many 5-digit numbers can be formed from digits ({1,2,3,4,5,6}) if digits cannot repeat and numbers cannot start with 0? (Assume 0 not in the set.)

A. 6P5 B. 6^5 C. 5^6 D. 120

12. From 8 different books, how many ways to arrange 3 of them on a shelf?

A. P(8,3) B. C(8,3) C. 8^3 D. 3^8

13. How many permutations of the letters in the word "BANANA"?

A. (6!) B. (6!/3!2!) C. (6!/3!) D. (6!/2!)

14. How many 2-letter codes from letters ({A,B,C,D}) are there if both letters must be different?

A. 16 B. 12 C. 8 D. 4

15. How many ways to choose president, vice-president, and secretary from a group of 10 students (distinct roles)?

A. P(10,3) B. C(10,3) C. 10^3 D. 720

E-Learning **Day5**

Standard: DM.MGSR.1.7
Topic: Graphs & Digraphs – Paths and Circuits

🎯 Learning Objectives

By the end of this lesson, students will be able to:

1. Define Euler paths, Euler circuits, Hamiltonian paths, and Hamiltonian circuits.
2. Differentiate between Eulerian and Hamiltonian problems.
3. Determine whether a graph or digraph contains Euler paths/circuits or Hamiltonian paths/circuits.
4. Apply Euler's Theorem to test for Euler paths and circuits.
5. Explore Hamiltonian paths/circuits through examples and reasoning.

📝 Key Concepts

1. Graph Basics

- **Graph (G):** A set of vertices (V) connected by edges (E).

- **Digraph:** Directed graph, where edges have a direction (arrows).

- **Path:** A sequence of edges that connects a series of distinct vertices.

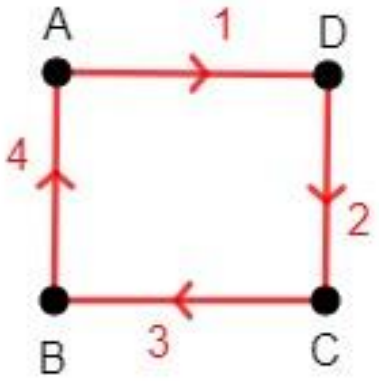
- **Circuit (or cycle):** A path that begins and ends at the same vertex.

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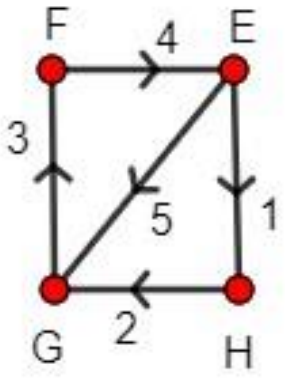
2. Euler Paths and Circuits

- **Euler Path:** A path that uses **every edge exactly once**.

- **Euler Circuit:** A circuit that uses **every edge exactly once** and returns to the starting vertex.



Euler circuit



Euler path

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Euler's Theorem:

- A connected graph has an **Euler Circuit** if **every vertex has an even degree**.

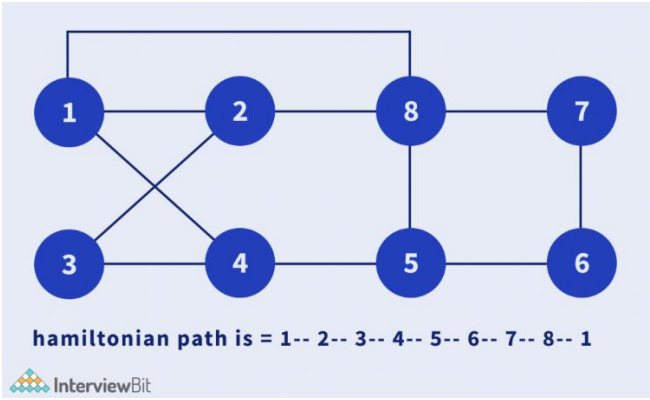
- A connected graph has an **Euler Path (but not a circuit)** if **exactly two vertices have odd degree**.

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3. Hamiltonian Paths and Circuits

- **Hamiltonian Path:** A path that visits **every vertex exactly once**.

- **Hamiltonian Circuit:** A circuit that visits **every vertex exactly once** and returns to the starting vertex.



🔎 Examples

📊 Summary Chart

Feature	Euler Path/Circuit	Hamiltonian Path/Circuit
Focus	**Edges**	**Vertices**
Uses every…	Edge exactly once	Vertex exactly once
Theorem Exists?	Yes (Euler's Theorem)	No simple test

Feature	Euler Path/Circuit	Hamiltonian Path/Circuit
Classic Problem	Seven Bridges of Königsberg	Traveling Salesperson Problem

Worksheet: Euler and Hamiltonian Paths and Circuits

Instructions: Choose the correct answer for each multiple-choice question.

1. A connected graph where all vertices have even degree must have:

a) Euler Path only

b) Euler Circuit

c) Hamiltonian Path

d) None

2. A connected graph with exactly two vertices of odd degree has:

a) Euler Path only

b) Euler Circuit

c) Hamiltonian Circuit

d) None

3. Euler paths are concerned with:

a) Vertices

b) Edges

c) Circuits only

d) Directed graphs

4. Hamiltonian paths are concerned with:

a) Vertices

b) Edges

c) Degrees

d) Odd numbers

5. Which property is necessary for an Euler Circuit?

a) All vertices even degree

b) Exactly one odd vertex

c) All vertices odd degree

d) None

6. The Seven Bridges of Königsberg problem is an example of:

a) Hamiltonian Path

b) Eulerian Path

c) Degree Sequence

d) Traveling Salesperson

7. The Traveling Salesperson Problem is related to:

a) Euler Circuits

b) Hamiltonian Circuits

c) Vertex Degree

d) Connectivity

8. A Hamiltonian circuit must visit:

a) Every edge once

b) Every vertex once and return to start

c) Every edge twice

d) Only odd-degree vertices

9. A graph with 0 or 2 odd-degree vertices always has:

a) Hamiltonian Circuit

b) Euler Path or Circuit

c) No paths

d) Directed edges only

10. In a digraph, if in-degree = out-degree at every vertex, then:

a) Euler Circuit exists

b) Only Hamiltonian Path exists

c) No Euler Path

d) None

11. Which focuses on edges rather than vertices?

a) Hamiltonian

b) Eulerian

c) Both

d) Neither

12. Which focuses on visiting every vertex once?

a) Hamiltonian

b) Eulerian

c) Degree sequence

d) None

13. A circuit that uses every edge exactly once is:

a) Hamiltonian Circuit

b) Euler Circuit

c) Path

d) None

14. A path that visits every vertex exactly once is:

a) Euler Path

b) Hamiltonian Path

c) Circuit

d) None

15. A Hamiltonian path may not necessarily be:

a) A circuit

b) Connected

c) Finite

d) Simple

16. Which theorem helps determine Euler paths/circuits?

a) Dirac's Theorem

b) Euler's Theorem

c) Pythagoras' Theorem

d) Fermat's Theorem

17. There is no simple test to check for:

a) Euler Circuit

b) Hamiltonian Path

c) Euler Path

d) Degrees

18. A Hamiltonian circuit must:

a) Use all edges

b) Visit all vertices once

c) Start and end at different points

d) Have only even degree vertices

19. Euler’s theorem applies to:

a) Degrees of vertices

b) Number of vertices

c) Edge colors

d) None

20. A graph with more than two odd-degree vertices:

a) Euler Circuit

b) Euler Path

c) Neither Euler path nor circuit

d) Hamiltonian Path

21. A Hamiltonian path may exist in:

a) Disconnected graphs

b) Connected graphs only

c) Directed graphs only

d) Trees only

22. Euler circuits and paths are based on:

a) Vertex degree

b) Edge weight

c) Path length

d) Number of vertices

23. Hamiltonian circuits are often related to:

a) Traveling Salesperson Problem

b) Bridge problems

c) Coloring problems

d) Bipartite graphs

24. In Euler's theorem, the degree of a vertex refers to:

a) Number of connected edges

b) Number of vertices

c) Distance measure

d) None

25. A Hamiltonian path visits:

a) Every edge at least once

b) Every vertex exactly once

c) Every vertex at least twice

d) Every edge exactly twice

26. Which of the following graphs always has an Euler Circuit?

a) A graph with all even-degree vertices

b) A graph with exactly two odd-degree vertices

c) A disconnected graph

d) A tree

27. A simple cycle through all vertices is:

a) Hamiltonian Circuit

b) Euler Circuit

c) Euler Path

d) None

28. A graph has an Euler Path but not an Euler Circuit if:

a) All vertices even degree

b) Exactly two vertices odd degree

c) All vertices odd degree

d) None

29. The Hamiltonian path problem is generally considered:

a) Easy to solve

b) NP-complete

c) Impossible

d) Eulerian

30. The difference between Euler and Hamiltonian problems is:

a) Euler uses edges, Hamiltonian uses vertices

b) Both use vertices

c) Euler uses vertices, Hamiltonian uses edges

d) Both use edges