

North Allegheny School District

AP Precalculus

Summer Tune Up Packet

College Board has created a set of 13 prerequisite topics that help students reinforce their readiness for the AP Precalculus course.

Complete the Skills Worksheets: These worksheets are designed to help reinforce your understanding of each prerequisite topic. Refer back to your notes as necessary to assist you in completing the worksheet accurately.

Check Your Solutions: Once you've completed each worksheet, carefully review your solutions. Check them against the answer key at the end of this document. Take note of any problems that you may have missed or answered incorrectly.

Redo Problems for Mastery: For any problems that were missed or answered incorrectly, take the time to redo them until mastery has been achieved. Refer back to relevant sections of your notes to identify and address any areas of confusion or misunderstanding.

By following these steps and actively engaging with the video content and accompanying worksheets, you'll be better prepared to excel in AP Precalculus. Consistent practice and review are key to building a strong foundation in Algebra II and succeeding in more advanced math courses.

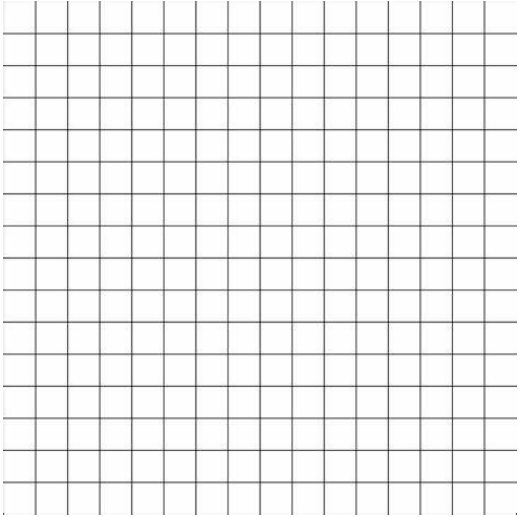
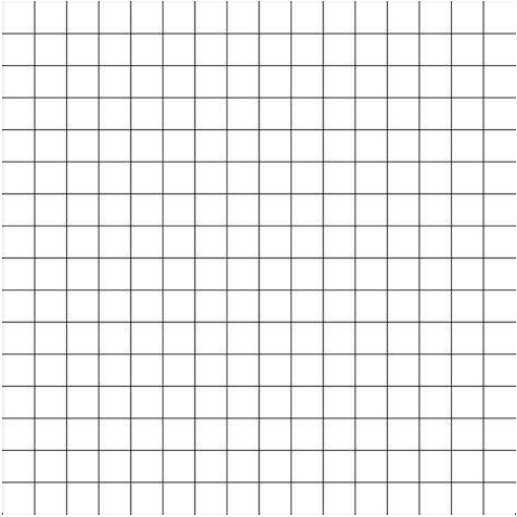


AP Precalculus
Prerequisites Review #1 – Algebraic Manipulation of Linear Functions

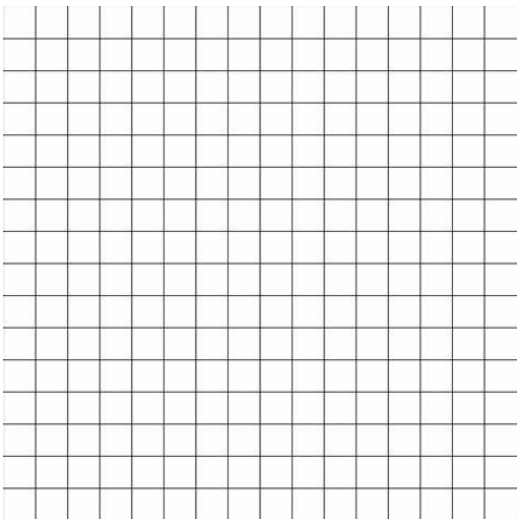
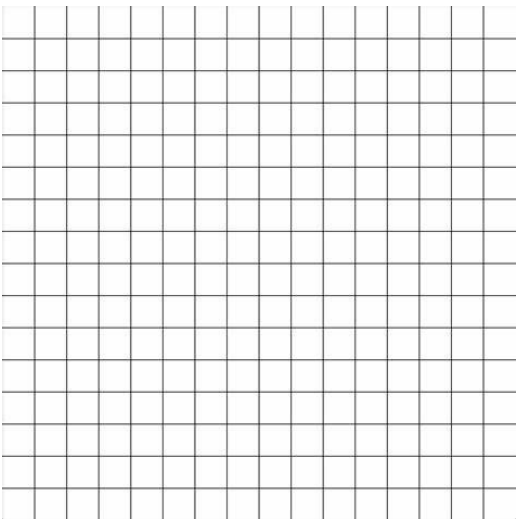
1. A line passes through (7,4) and (3,-4). Find an equation for the line in all three forms for linear equations.

Slope-intercept Form	Point-slope Form	Standard Form

2. Sketch the graph of each line.

a. $y = \frac{2}{3}x - 2$	b. $y = -3x + 1$
	

3. Sketch the graph of each line.

a. $2x - 5y = 10$	b. $-4x + 3y = -24$
	

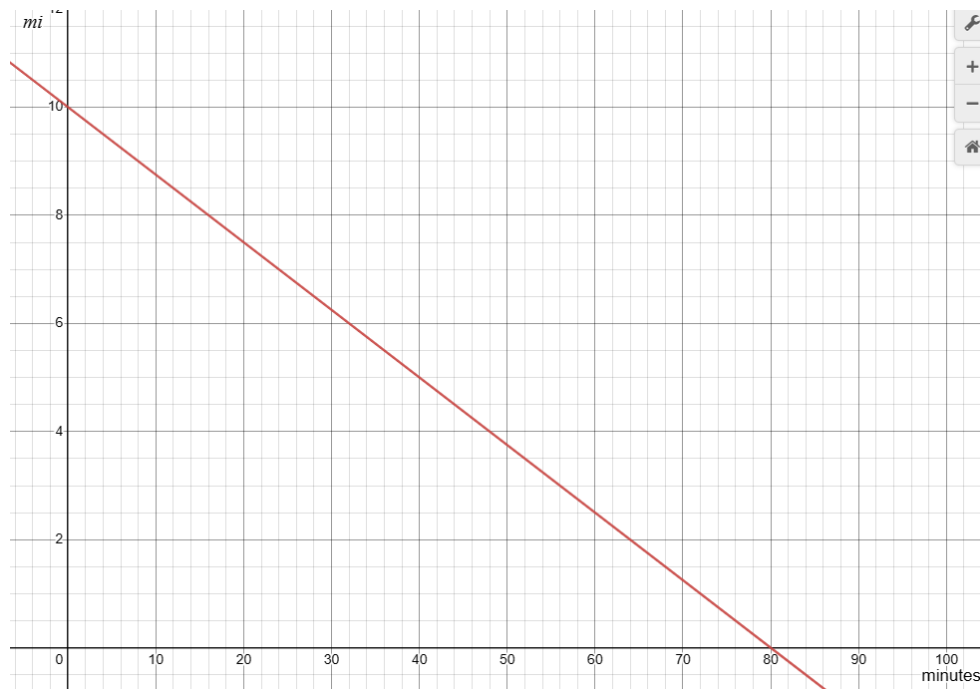
4. A recording studio charges a base fee for use of their facility plus a constant fee per hour of use. The table compares the number of hours the studio is used with the total cost, c , for use of the studio. Use the table to answer each of the questions below.

Hours of studio use (h)	2	4	6	8
Total cost to use the studio(C)	\$450	\$600	\$750	\$900

a. What is the fee charged per hour for use of the studio?	b. What is the base fee for rental of the studio?

c. Write a linear equation to model this situation.	d. Identify the domain and range for this function.

5. Jaden competes in a race, running at a constant pace from start to finish. The distance remaining in the race (in miles) as a function of time (in minutes) is shown in the graph. Use the graph to answer the following questions.



a. How long did it take Jaden to reach the finish line? Explain.	b. How long (distance) was the race? Explain your reasoning.

c. Write a linear equation to model this situation.	d. Identify the domain and range for this function.



AP Precalculus

Prerequisites Review #2 – Linear Functions: Solving Equations and Inequalities

1. Solve $4x - 9 < 7x + 15$

2. Solve $6(3x - 2) = -4(2x - 9)$

3. Solve $\frac{2}{3}x + 4 = \frac{4}{5}x - 3$



AP Precalculus

Prerequisites Review #3 – Polynomial Addition and Multiplication

1. Simplify the expression to a polynomial in standard form: $(4x^3 - 5x^2 - 3x + 7)(2x - 5)$.

2. Simplify the expression to a polynomial in standard form: $3(2x - 5)(x^2 - 4x + 2)$.

3. Simplify the expression to a polynomial in standard form: $(3x - 1)(-2x^2 + 4x - 7)$.

**AP Precalculus***Prerequisites Review #4 – Factoring Quadratic Trinomials*

Factor each quadratic trinomial.

1. $x^2 + 10x + 9$	2. $x^2 - 6x + 9$
3. $x^2 - 11x + 24$	4. $3x^2 - 5x - 12$
5. $4x^2 + 28x + 49$	6. $15x^2 - 11x - 12$

**AP Precalculus***Prerequisites Review #5 – Solving Quadratic Equations and Inequalities*

Solve each problem by factoring or using the quadratic formula. Round answers to the nearest hundredth as needed.

1. $x^2 + 4x + 3 = 0$

2. $x^2 - 5x = 6$

3. $3x^2 - 5x - 9 = x^2 + 3$

4. $4x^2 - 12x + 1 < 0$

5. $2x^2 + 8x = -7$

6. A ball is catapulted upward from the top of a building at a speed of 30 feet per second. The ball's height above the ground can be modeled as $H(t) = -16t^2 + 30t + 40$. How long does it take for the ball to reach a height of 50 feet?

**AP Precalculus***Prerequisites Review #6 – Quadratic Functions: Algebraic Manipulation*

1. A ball is launched straight up with a velocity of 40 feet per second. The ball's height above the ground can be modeled by $H(t) = -16t^2 + 40t + 5$. Use this information to answer the following questions.

a. How high is the ball when it is released? Explain your answer.	b. How long does it take the ball to reach its maximum height? Explain your answer.
c. What is the maximum height the ball reaches? Explain your answer.	d. How long is the ball in the air? Explain your answer.

2. A child uses 36 legos to build the rectangular frame for the base of her lego castle. Write a quadratic function to model this situation and determine the length of the side of the castle and the largest possible area covered by the castle's base.

3. Does the table of values below represent a quadratic equation? Justify your decision.

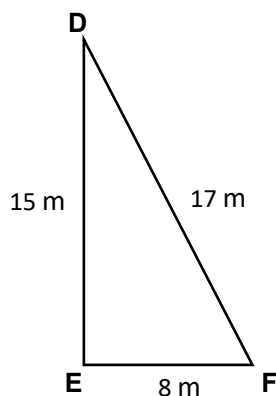
x	f(x)
-1	4
0	6
1	11
2	19
3	32



AP Precalculus

Prerequisites Review #7 – Solving Right Triangle Problems Using Trigonometry

1. Use the diagram to identify each ratio.



a. $\sin F^\circ =$	b. $\sin D^\circ =$
c. $\cos F^\circ =$	d. $\cos D^\circ =$
e. $\tan F^\circ =$	f. $\tan D^\circ =$

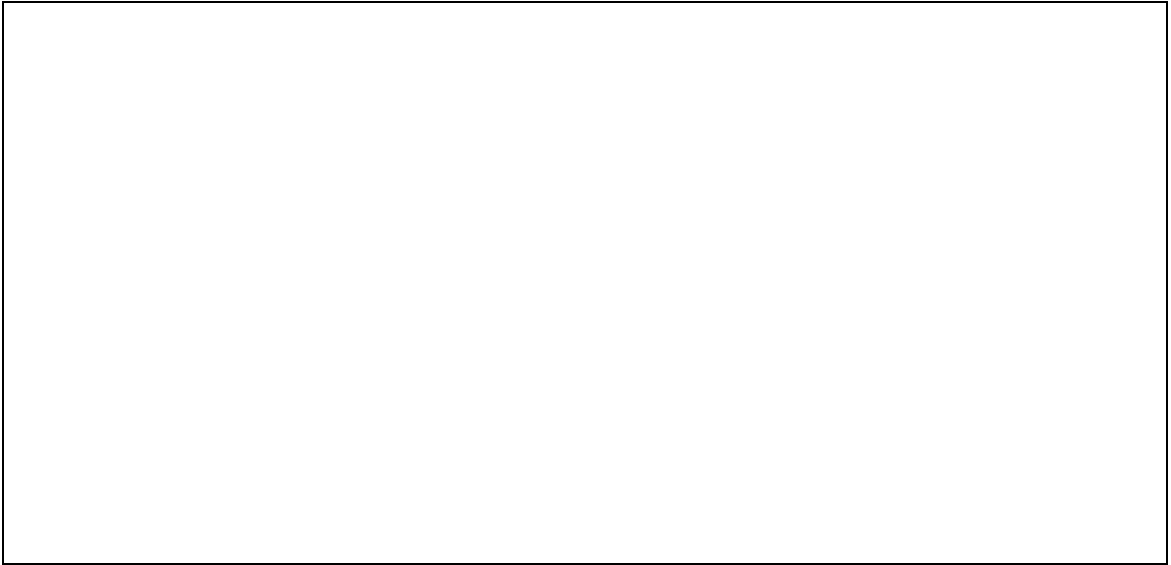
2. Using the diagram from #1 above, calculate the measure in degrees of $\angle F$.

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3. When a ladder leans against a wall, it reaches a height of 15 feet. The angle of incline is 60° . How far away from the wall is the base of the ladder?

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3. A kite is flying extended on 100 feet of string and is 30 feet high. What is the angle of elevation of the kite?





AP Precalculus

Prerequisites Review #8 – Solving Systems of Equations in 2 and 3 Variables

1. Solve $\begin{cases} x + 2y = 10 \\ y = 2x - 5 \end{cases}$

2. Solve $\begin{cases} 5x + 7y = 6 \\ 10x - 3y = 46 \end{cases}$

3. Solve $\begin{cases} 3x + y - 2z = -12 \\ 2x + 2y - 3z = -12 \\ 5x + 3y + 2z = 4 \end{cases}$

4. Solve $\begin{cases} y = x^2 + 4x - 2 \\ y = 3x + 5 \end{cases}$

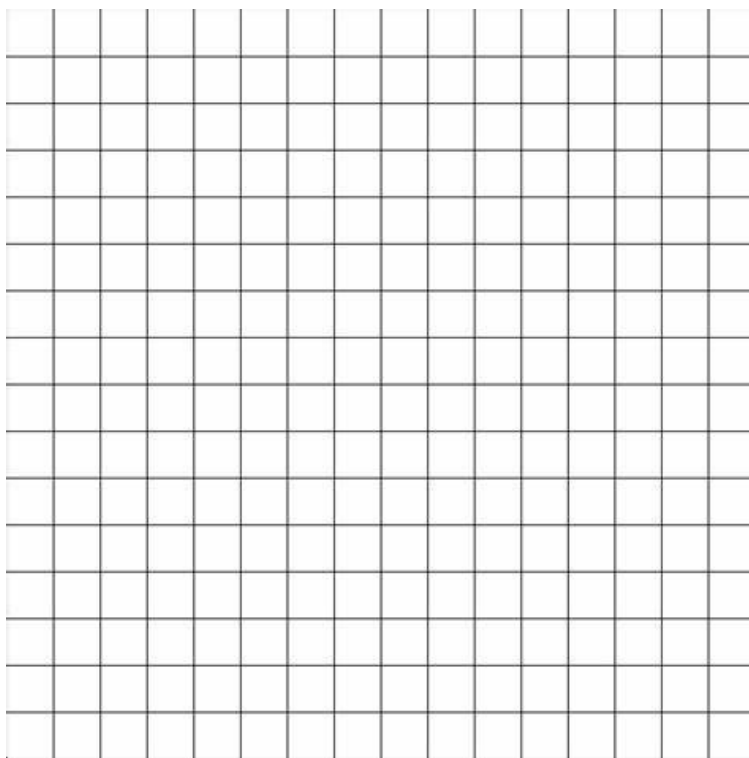


AP Precalculus
Prerequisites Review #9 – Piecewise Functions

1. An electrician charges \$250 for the first hour of work and \$75 for each additional hour.

a. Generate the piecewise function to define the cost of hiring this electrician.

b. Graph the piecewise function that would illustrate this situation.



2. Find each of the following values given that $f(x) = \begin{cases} x^3 - 4 & \text{when } x < -6 \\ 2x + 7 & \text{when } -6 \leq x < 1 \\ \frac{x}{x^2 + 2} & \text{when } x \geq 1 \end{cases}$

a. $f(-6)$	b. $f(1)$	c. $f(6)$	d. $f(0)$

3. Rewrite the function $g(x) = |3x| + 2$ as a piecewise function.



AP Precalculus
Prerequisites Review #10 – Exponential Functions

1. A certain bacteria population sample contains 500 bacteria and is known to grow by 20% every hour when left untreated.

a. Write an equation to model the untreated bacteria population (y) after x hours.

b. How many bacteria are in the sample after 5 hours? 7.5 hours?



AP Precalculus
Prerequisites Review #11 – Rules for Exponents

Simplify the following expressions. Write your answers with positive exponents only.

1. $(w^0x^5)^{-1}$

2. $c^{-3}(c^7)^4$

3. $(u^3v^5)^2(u^{-7}v^{-10})$

4. $\frac{x^3y^4}{w^7z^{-2}} * \frac{w^4y^{-3}}{x^5z^2}$

**AP Precalculus***Prerequisites Review #12 – Radicals (square roots and cube roots)*

1. Evaluate each of the following. Round to the nearest hundredth as needed.

a. $\sqrt{121}$	b. $\sqrt{175}$
c. $\sqrt[3]{125}$	d. $\sqrt[3]{8}$
e. $\sqrt[3]{36}$	

2. Solve for c .



3. Simplify each of the following expressions. Rationalize denominators as needed.

a. $\sqrt{50}$	b. $\frac{3\sqrt{6}}{4\sqrt{5}}$
c. $\sqrt{72a^5b^6}$	d. $3\sqrt{5} + 6\sqrt{20}$
e. $\frac{\sqrt{200x^{17}y^6}}{\sqrt{45x^{15}y^9}}$	



AP Precalculus
Prerequisites Review #13 – Complex Numbers

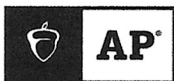
Simplify the following expressions and rationalize denominators as needed.

1. $(3 + 7i) + (4 - 9i)$

2. $(3 + 7i) - (4 - 9i)$

3. $(3 + 7i)(4 - 9i)$

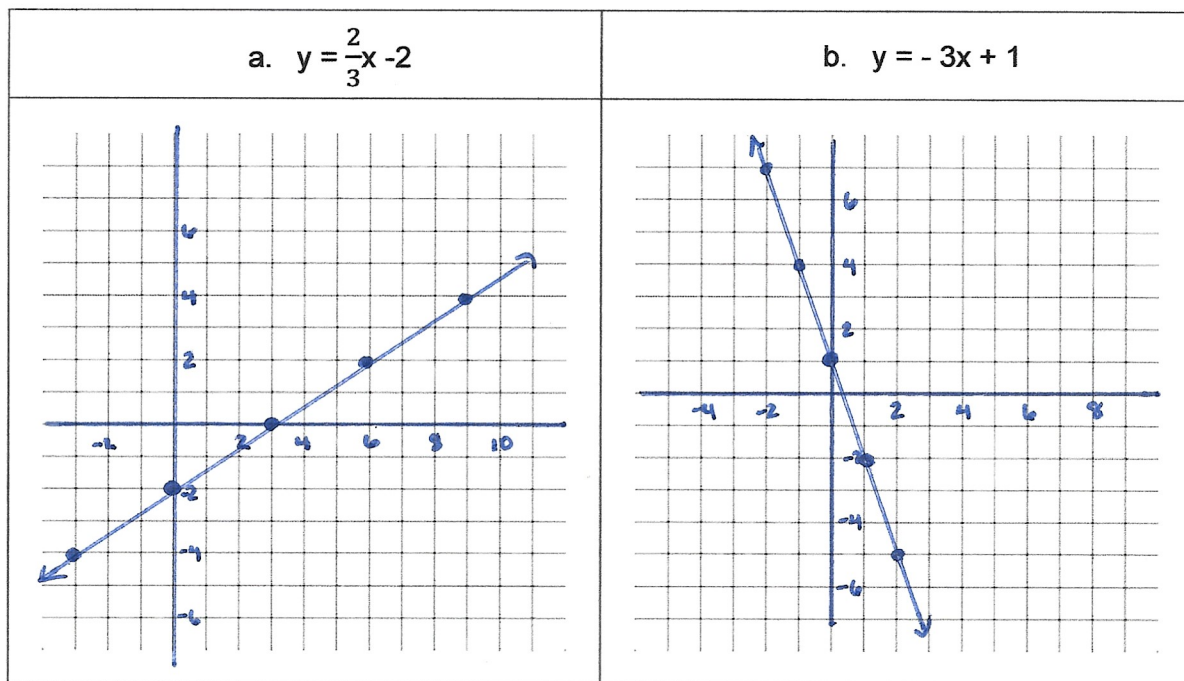
4. $\frac{10-2i}{3+4i}$

**AP Precalculus****Prerequisites Review #1 – Algebraic Manipulation of Linear Functions**

1. A line passes through (7,4) and (3,-4). Find an equation for the line in all three forms for linear equations.

② Slope-intercept Form	① Point-slope Form	③ Standard Form
$y = mx + b$ $y - 4 = 2(x - 7)$ $y - 4 = 2x - 14$ $y = 2x - 10$	$m = \frac{4 - (-4)}{7 - 3} = \frac{8}{4} = 2$ $y - y_1 = m(x - x_1)$ $y - 4 = 2(x - 7)$ OR $y + 4 = 2(x - 3)$	$Ax + By = C$ $y = 2x - 10$ $-2x + y = -10$

2. Sketch the graph of each line.

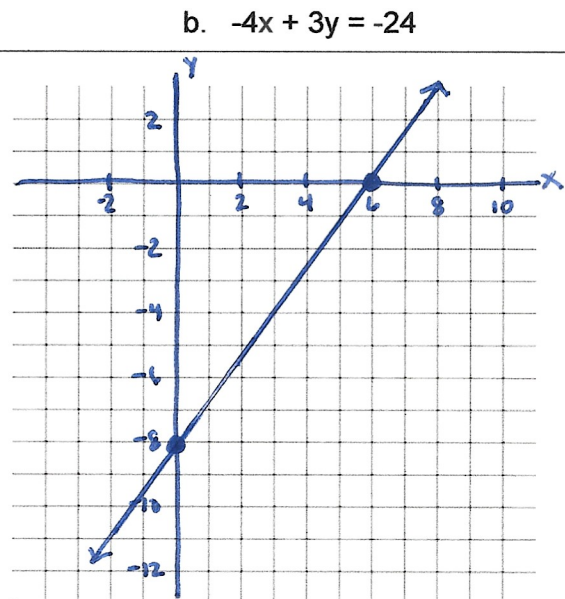
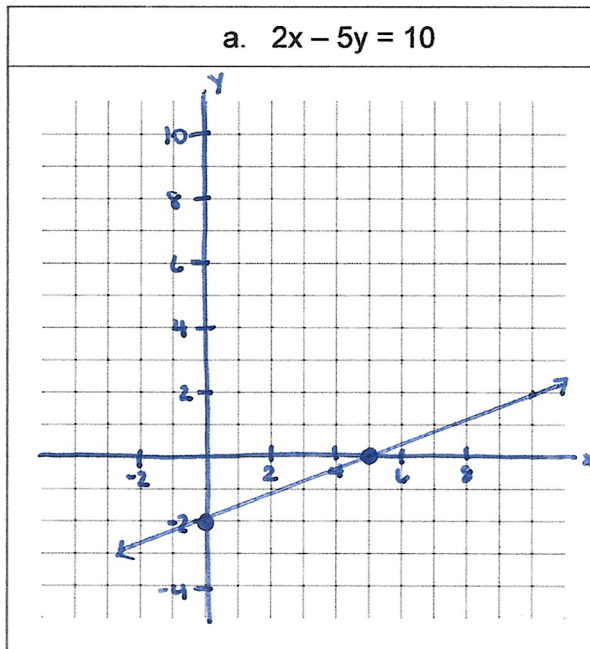


start at the y-intercept and use slope ($\frac{\text{rise}}{\text{run}}$) to find additional points

3. Sketch the graph of each line.

$$\begin{aligned} \text{x-int} \\ 2x &= 10 \\ x &= 5 \\ (5, 0) \end{aligned}$$

$$\begin{aligned} \text{y-int} \\ -5y &= 10 \\ y &= -2 \\ (0, -2) \end{aligned}$$



$$\begin{aligned} \text{x-int} \\ -4x &= -24 \\ x &= 6 \\ (6, 0) \end{aligned}$$

$$\begin{aligned} \text{y-int} \\ 3y &= -24 \\ y &= -8 \\ (0, -8) \end{aligned}$$

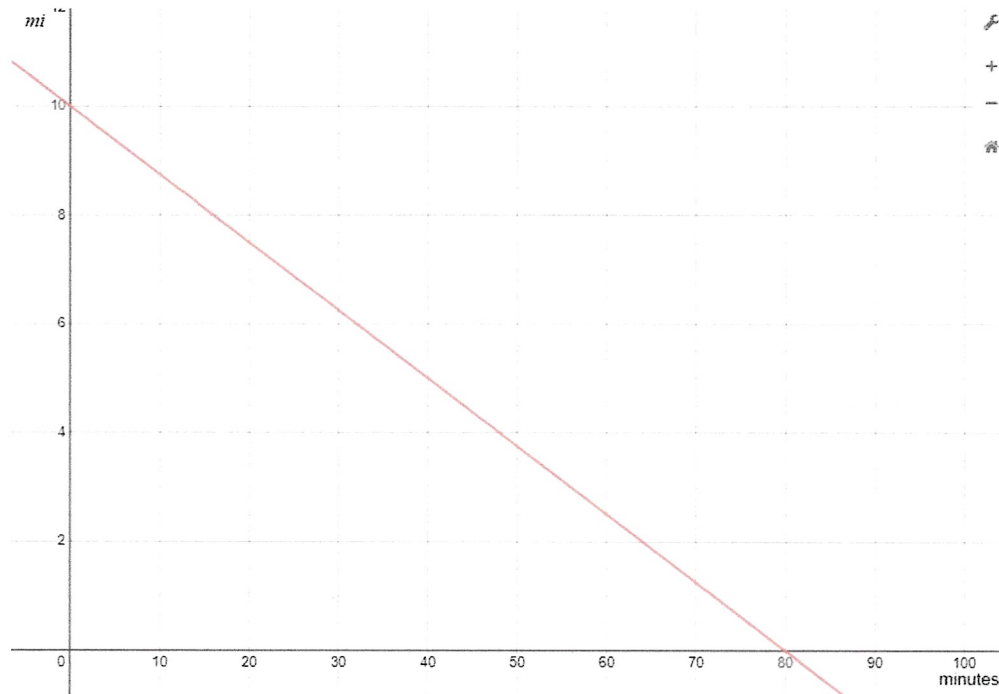
4. A recording studio charges a base fee for use of their facility plus a constant fee per hour of use. The table compares the number of hours the studio is used with the total cost, c , for use of the studio. Use the table to answer each of the questions below.

Hours of studio use (h)	2	4	6	8
Total cost to use the studio (C)	\$450	\$600	\$750	\$900

a. What is the fee charged per hour for use of the studio?	b. What is the base fee for rental of the studio?
$4 - 2 = 2 \text{ hours}$ $\$600 - \$450 = \$150$ $\frac{150}{2} = \$75 \text{ per hour}$	$\text{cost for 2 hours} = 450$ $\text{base fee} + 2(75) = 450$ $\text{base fee} + 150 = 450$ $\text{base fee} = \$300$

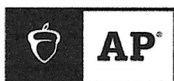
c. Write a linear equation to model this situation.	d. Identify the domain and range for this function.
$C = 75h + 300$	$\text{Domain: } h > 0 \text{ hours}$ $\text{Range: } C > \$300$

5. Jaden competes in a race, running at a constant pace from start to finish. The distance remaining in the race (in miles) as a function of time (in minutes) is shown in the graph. Use the graph to answer the following questions.



a. How long did it take Jaden to reach the finish line? Explain.	b. How long (distance) was the race? Explain your reasoning.
Jaden finished the race in 80 minutes (when there were zero miles remaining)	The race was 10 miles long, which is the distance remaining when no time has passed.

c. Write a linear equation to model this situation.	d. Identify the domain and range for this function.
$y = \text{miles remaining}$ $x = \text{minutes}$ $y = -\frac{1}{8}x + 10$	Domain: $0 \text{ minutes} \leq x \leq 80 \text{ minutes}$ Range: $0 \text{ miles} \leq y \leq 10 \text{ miles}$

**AP Precalculus***Prerequisites Review #2 – Linear Functions: Solving Equations and Inequalities*

1. Solve $4x - 9 < 7x + 15$

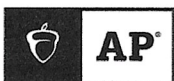
$$\begin{array}{r} 4x - 9 < 7x + 15 \\ -7x + 9 \quad -7x + 9 \\ \hline -3x < 24 \\ \frac{-3x}{-3} & \begin{array}{c} \textcircled{<} \\ \downarrow \end{array} & \frac{24}{-3} \\ x & & > -8 \end{array}$$

2. Solve $6(3x - 2) = -4(2x - 9)$

$$\begin{array}{r} 6(3x - 2) = -4(2x - 9) \\ 18x - 12 = -8x + 36 \\ +8x + 12 \quad +8x + 12 \\ \hline 26x = 48 \\ \frac{26x}{26} = \frac{48}{26} \\ x = \frac{24}{13} \end{array}$$

3. Solve $\frac{2}{3}x + 4 = \frac{4}{5}x - 3$

$$\begin{array}{r} 15 \left(\frac{2}{3}x + 4 \right) = \left(\frac{4}{5}x - 3 \right) 15 \\ 10x + 60 = 12x - 45 \\ -12x - 60 \quad -12x - 60 \\ \hline -2x = -105 \\ \frac{-2x}{-2} = \frac{-105}{-2} \\ x = 52.5 \end{array}$$

**AP Precalculus***Prerequisites Review #3 – Polynomial Addition and Multiplication*

1. Simplify the expression to a polynomial in standard form: $(4x^3 - 5x^2 - 3x + 7)(2x - 5)$.

Distribute
Multiply
Combine
like terms

$$\begin{aligned} &4x^3(2x) + 4x^3(-5) - 5x^2(2x) - 5x^2(-5) - 3x(2x) - 3x(-5) + 7(2x) + 7(-5) \\ &8x^4 - 20x^3 - 10x^3 + 25x^2 - 6x^2 + 15x + 14x - 35 \\ &8x^4 - 30x^3 + 19x^2 + 29x - 35 \end{aligned}$$

2. Simplify the expression to a polynomial in standard form: $3(2x - 5)(x^2 - 4x + 2)$.

Distribute
Multiply
Combine
like terms

Multiply the first two expressions

$$3(2x - 5) = 6x - 15$$

Multiply that answer by the third expression

$$(6x - 15)(x^2 - 4x + 2)$$

$$6x(x^2) + 6x(-4x) + 6x(2) - 15(x^2) - 15(-4x) - 15(2)$$

$$6x^3 - 24x^2 + 12x - 15x^2 + 60x - 30$$

$$6x^3 - 39x^2 + 72x - 30$$

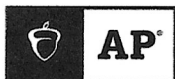
3. Simplify the expression to a polynomial in standard form: $(3x - 1)(-2x^2 + 4x - 7)$.

Distribute
Multiply
Combine
like terms

$$3x(-2x^2) + 3x(4x) + 3x(-7) - 1(-2x^2) - 1(4x) - 1(-7)$$

$$-6x^3 + 12x^2 - 21x + 2x^2 - 4x + 7$$

$$-6x^3 + 14x^2 - 25x + 7$$

**AP Precalculus****Prerequisites Review #4 – Factoring Quadratic Trinomials**

Factor each quadratic trinomial.

1. $x^2 + 10x + 9$ <u>1, 9</u> 3, 3 $(x+1)(x+9)$	2. $x^2 - 6x + 9$ -1, -9 <u>-3, -3</u> $(x-3)(x-3)$ or $(x-3)^2$
3. $x^2 - 11x + 24$ -1, -24 -2, -12 <u>-3, -8</u> -4, -6 $(x-3)(x-8)$	4. $3x^2 - 5x - 12$ $a \cdot c = -36$ 1, -36 -1, 36 2, -18 -2, 18 3, -12 -3, 12 <u>4, -9</u> -4, 9 -6, 6 $3x^2 + 4x - 9x - 12$ $x(3x+4) - 3(3x+4)$ $(3x+4)(x-3)$
5. $4x^2 + 28x + 49$ 1, 49 2, 98 $a \cdot c = 196$ 4, 49 8, 26 <u>14, 14</u> $4x^2 + 14x + 14x + 49$ $2x(2x+7) + 7(2x+7)$ $(2x+7)(2x+7)$	6. $15x^2 - 11x - 12$ -1, 180 1, -180 $a \cdot c = -180$ -2, 90 2, -90 -3, 60 3, -60 -4, 45 4, -45 -5, 36 5, -36 -6, 30 6, -30 -8, 25 8, -25 -9, 20 <u>9, -20</u> -10, 18 10, -18 -12, 15 12, -15 $15x^2 + 9x - 20x - 12$ $3x(5x+3) - 4(5x+3)$ $(5x+3)(3x-4)$

**AP Precalculus****Prerequisites Review #5 – Solving Quadratic Equations and Inequalities**

Solve each problem by factoring or using the quadratic formula. Round answers to the nearest hundredth as needed.

1. $x^2 + 4x + 3 = 0$

$a \cdot c = 3$ 1, 3

$(x+1)(x+3) = 0$

$x+1=0$ $x+3=0$

$x=-1$ or $x=-3$

2. $x^2 - 5x = 6$

$x^2 - 5x - 6 = 0$

$a \cdot c = -6$ $\boxed{1, -6}$ -1, 6
2, -3 -2, 3

$(x+1)(x-6) = 0$

$x+1=0$ $x-6=0$

$x=-1$ or $x=6$

3. $3x^2 - 5x - 9 = x^2 + 3$

$2x^2 - 5x - 12 = 0$

$a \cdot c = -24$ 1, -24 -1, 24

2, -12 -2, 12

$\boxed{3, -8}$ -3, 8

4, -6 -4, 6

$2x^2 + \underline{3x} - \underline{8x} - 12 = 0$

$x(2x+3) - 4(2x+3) = 0$

$(2x+3)(x-4) = 0$

$2x+3=0$ $x-4=0$

$x = -\frac{3}{2}$ or $x=4$

4. $4x^2 - 12x + 1 < 0$

$a=4$ $b=-12$ $c=1$

After using the quadratic formula to find these values, I tested a number in between them to see if it made the original inequality true and it did.

$$\frac{12 \pm \sqrt{(-12)^2 - 4(4)(1)}}{2(4)}$$

$$\frac{12 \pm \sqrt{144 - 16}}{8}$$

$$\frac{12 \pm \sqrt{128}}{8}$$

$$\frac{12 + \sqrt{128}}{8} \quad \text{or} \quad \frac{12 - \sqrt{128}}{8}$$

$$\frac{23.31}{8} \quad \text{or} \quad \frac{.69}{8}$$

$$2.91 \quad .09$$

$.09 < x < 2.91$

x is a value between the two solutions.

5. $2x^2 + 8x = -7$

$$2x^2 + 8x + 7 = 0$$

$$a = 2 \quad b = 8 \quad c = 7$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(2)(7)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{64 - 56}}{4}$$

$$x = \frac{-8 \pm \sqrt{8}}{4}$$

$$x = \frac{-8 + \sqrt{8}}{4} \quad \text{or} \quad \frac{-8 - \sqrt{8}}{4}$$

$$x = \frac{-5.17}{4} \quad \text{or} \quad \frac{-10.83}{4}$$

$$x = -1.29 \quad \text{or} \quad -2.71$$

6. A ball is catapulted upward from the top of a building at a speed of 30 feet per second. The ball's height above the ground can be modeled as $H(t) = -16t^2 + 30t + 40$. How long does it take for the ball to reach a height of 50 feet?

$$50 = -16t^2 + 30t + 40$$

$$0 = -16t^2 + 30t - 10$$

$$a = -16 \quad b = 30 \quad c = -10$$

$$t = \frac{-30 \pm \sqrt{(30)^2 - 4(-16)(-10)}}{2(-16)}$$

$$t = \frac{-30 \pm \sqrt{900 - 640}}{-32}$$

$$t = \frac{-30 \pm \sqrt{260}}{-32}$$

$$t = \frac{-30 + \sqrt{260}}{-32} \quad \text{or} \quad \frac{-30 - \sqrt{260}}{-32}$$

$$t = \frac{-13.87}{-32} \quad \text{or} \quad \frac{-46.12}{-32}$$

$$t = 0.43 \text{ sec} \quad \text{or} \quad 1.44 \text{ sec.}$$

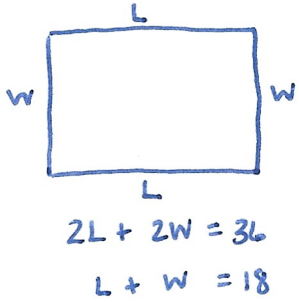
The ball reaches 50 ft on its way up at 0.43 seconds and on its way down at 1.44 seconds.

**AP Precalculus****Prerequisites Review #6 – Quadratic Functions: Algebraic Manipulation**

1. A ball is launched straight up with a velocity of 40 feet per second. The ball's height above the ground can be modeled by $H(t) = -16t^2 + 40t + 5$. Use this information to answer the following questions.

a. How high is the ball when it is released? Explain your answer. At the time the ball is released, no time has passed. $-16(0)^2 + 40(0) + 5 = 5$ The ball is 5 feet high when it is released.	b. How long does it take the ball to reach its maximum height? Explain your answer. Maximum indicates vertex How long indicates seconds Seconds is the x-coordinate $\frac{-b}{2a} = \frac{-40}{2(-16)} = \frac{-40}{-32} = 1.25 \text{ sec.}$
c. What is the maximum height the ball reaches? Explain your answer. Maximum indicates vertex Height is the y-coordinate Substitute the x-coordinate to find the height. $\begin{aligned} -16(1.25)^2 + 40(1.25) + 5 &= \\ -25 + 50 + 5 &= \\ 30 \text{ feet} \end{aligned}$	d. How long is the ball in the air? Explain your answer. When the ball is no longer in the air, it is on the ground which means the height is zero. Substitute 0 for height and solve. $\begin{aligned} 0 &= -16t^2 + 40t + 5 \\ a &= -16, b = 40, c = 5 \\ t &= \frac{-40 \pm \sqrt{(40)^2 - 4(-16)(5)}}{2(-16)} \\ t &= \frac{-40 \pm \sqrt{1920}}{-32} \\ t &= \frac{3.92}{-32} \text{ or } \frac{-83.82}{-32} = 2.62 \end{aligned}$ Ignore this answer because time cannot be negative. <u>$t = 2.62 \text{ seconds}$</u>

2. A child uses 36 legos to build the rectangular frame for the base of her lego castle. Write a quadratic function to model this situation and determine the length of the side of the castle and the largest possible area covered by the castle's base.



$2L + 2W = 36$
 $L + W = 18$

if $L = x$, then $W = 18 - x$.

Quadratic model for area is

$$A = x(18 - x)$$

x-intercepts are at $x = 0$ and $x = 18$
 Vertex must be at $x = 9$, so this is the maximized area.

$L = 9$
 $W = 18 - 9 = 9$
 $\text{Area} = 9 \cdot 9 = 81$

3. Does the table of values below represent a quadratic equation? Justify your decision.

x	f(x)
-1	4
0	6
1	11
2	19
3	32

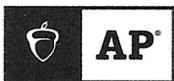
$$\frac{6-4}{0-(-1)} = \frac{2}{1} = 2$$

$$\frac{11-6}{1-0} = \frac{5}{1} = 5$$

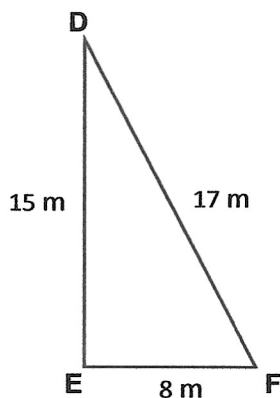
$$\frac{19-11}{2-1} = \frac{8}{1} = 8$$

$$\frac{32-19}{3-2} = \frac{13}{1} = 13$$

When calculating the rate of change, the relationship initially appears linear because the differences vary by 3. However the difference in the last two rates of change is 5 so the rate of change is not linear and this is not a quadratic model.

**AP Precalculus****Prerequisites Review #7 – Solving Right Triangle Problems Using Trigonometry**

1. Use the diagram to identify each ratio.



a. $\sin F^\circ = \frac{15}{17}$

b. $\sin D^\circ = \frac{8}{17}$

c. $\cos F^\circ = \frac{8}{17}$

d. $\cos D^\circ = \frac{15}{17}$

e. $\tan F^\circ = \frac{15}{8}$

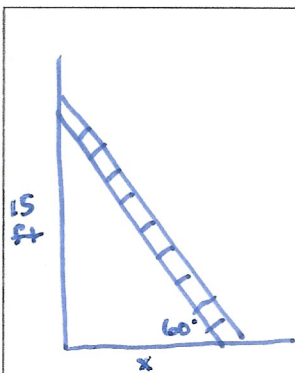
f. $\tan D^\circ = \frac{8}{15}$

2. Using the diagram from #1 above, calculate the measure in degrees of $\angle F$.

$$\sin F^\circ = \frac{15}{17}$$

$$\sin^{-1}\left(\frac{15}{17}\right) = F^\circ = 62^\circ$$

3. When a ladder leans against a wall, it reaches a height of 15 feet. The angle of incline is 60° . How far away from the wall is the base of the ladder?



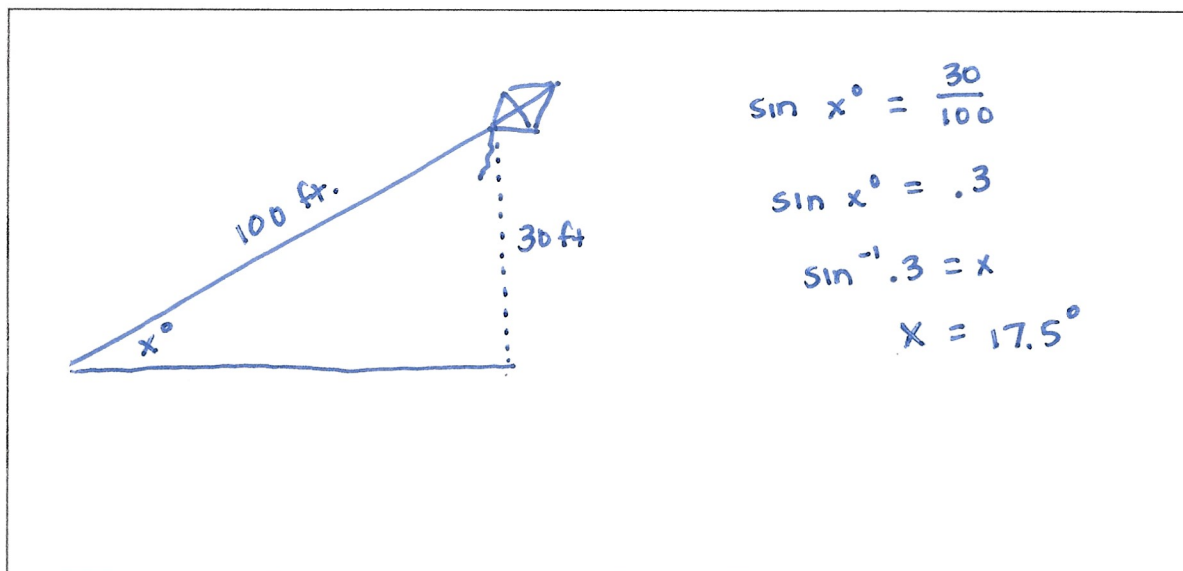
$$\tan 60^\circ = \frac{15}{x}$$

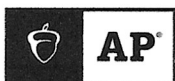
$$1.73 = \frac{15}{x}$$

$$1.73x = 15$$

$$x = 8.66 \text{ ft.}$$

3. A kite is flying extended on 100 feet of string and is 30 feet high. What is the angle of elevation of the kite?



**AP Precalculus****Prerequisites Review #8 – Solving Systems of Equations in 2 and 3 Variables**

1. Solve $\begin{cases} x + 2y = 10 \\ y = 2x - 5 \end{cases}$

Solve
Using
Substitution

$$x + 2(2x - 5) = 10$$

$$x + 4x - 10 = 10$$

$$5x - 10 = 10$$

$$5x = 20$$

$$x = 4$$

$$y = 2(4) - 5$$

$$y = 8 - 5$$

$$y = 3$$

solution: $(4, 3)$

2. Solve $\begin{cases} 5x + 7y = 6 \\ 10x - 3y = 46 \end{cases}$

Solve
using
elimination

$$-2(5x + 7y = 6) \rightarrow -10x - 14y = -12$$

$$10x - 3y = 46 \rightarrow 10x - 3y = 46$$

$$-17y = 34$$

$$y = -2$$

$$5x + 7(-2) = 6$$

$$5x - 14 = 6$$

$$5x = 20$$

$$x = 4$$

solution: $(4, -2)$

3.

$$\text{Solve } \begin{cases} 3x + y - 2z = -12 & \textcircled{1} \\ 2x + 2y - 3z = -12 & \textcircled{2} \\ 5x + 3y + 2z = 4 & \textcircled{3} \end{cases}$$

Pair ① and ③ to eliminate z .

$$3x + y - 2z = -12$$

$$5x + 3y + 2z = 4$$

$$8x + 4y = -8$$

Pair ① and ② to eliminate z .

$$3(3x + y - 2z = -12)$$

$$-2(2x + 2y - 3z = -12)$$

$$9x + 3y - 6z = -36$$

$$-4x - 4y + 6z = 24$$

$$5x - y = -12$$

Pair the two smaller equations to eliminate y , solve for x .

$$8x + 4y = -8 \rightarrow 8x + 4y = -8$$

$$4(5x - y = -12) \rightarrow 20x - 4y = -48$$

$$28x = -56$$

$$x = -2$$

Substitute solution for x into one of the smaller equations, solve for y .

$$8(-2) + 4y = -8$$

$$-16 + 4y = -8$$

$$4y = 8$$

$$y = 2$$

$$5(-2) - y = -12$$

$$-10 - y = -12$$

$$-y = -2$$

$$y = 2$$

Substitute x and y into one of the original equations, solve for z .

$$3x + y - 2z = -12$$

$$3(-2) + 2 - 2z = -12$$

$$-6 + 2 - 2z = -12$$

$$-4 - 2z = -12$$

$$-2z = -8$$

$$z = 4$$

$$2x + 2y - 3z = -12$$

$$2(-2) + 2(2) - 3z = -12$$

$$-4 + 4 - 3z = -12$$

$$-3z = -12$$

$$z = 4$$

$$5x + 3y + 2z = 4$$

$$5(-2) + 3(2) + 2z = 4$$

$$-10 + 6 + 2z = 4$$

$$-4 + 2z = 4$$

$$2z = 8$$

$$z = 4$$

Write answer as an ordered triple $(x, y, z) = (-2, 2, 4)$

4. Solve $\begin{cases} y = x^2 + 4x - 2 \\ y = 3x + 5 \end{cases}$

Use substitution to write an equation with one variable.

$$3x + 5 = x^2 + 4x - 2$$

$$0 = x^2 + x - 7 \quad (\text{standard form})$$

$$a = 1 \quad b = 1 \quad c = -7$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1+28}}{2}$$

$$x = \frac{-1 \pm \sqrt{29}}{2}$$

$$x = \frac{4.39}{2} \quad \text{or} \quad \frac{-6.39}{2}$$

$$x = 2.195 \quad \text{or} \quad -3.195$$

Substitute x in an original equation to find y .

$$y = 3(2.195) + 5 = 6.585 + 5 = 11.585$$

$$y = 3(-3.195) + 5 = -9.585 + 5 = -4.585$$

Write solutions as ordered pairs to show points of intersection.

$$(2.195, 11.585) \quad \text{and} \quad (-3.195, -4.585)$$



AP Precalculus
Prerequisites Review #9 – Piecewise Functions

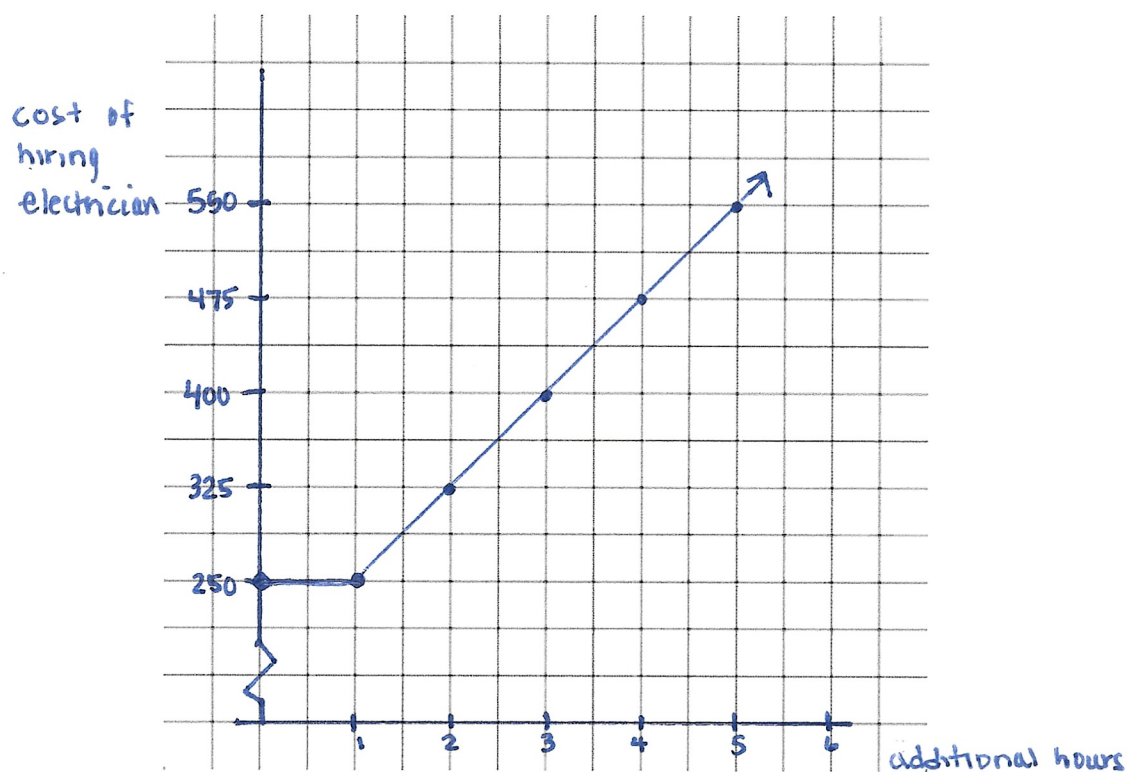
1. An electrician charges \$250 for the first hour of work and \$75 for each additional hour.

- a. Generate the piecewise function to define the cost of hiring this electrician.

h = additional hours
 C = cost of hiring
the electrician

$$\begin{cases} C = 250 & 0 \leq h \leq 1 \\ C = 250 + 75h & h > 1 \end{cases}$$

- b. Graph the piecewise function that would illustrate this situation.



2. Find each of the following values given that $f(x) = \begin{cases} x^3 - 4 & \text{when } x < -6 \\ 2x + 7 & \text{when } -6 \leq x < 1 \\ \frac{x}{x^2 + 2} & \text{when } x \geq 1 \end{cases}$

a. $f(-6)$ $2(-6) + 7 =$ $-12 + 7 =$ -5 $f(-6) = -5$	b. $f(1)$ $\frac{1}{1^2 + 2} =$ $\frac{1}{1 + 2} =$ $\frac{1}{3}$ $f(1) = \frac{1}{3}$	c. $f(6)$ $\frac{6}{6^2 + 2} =$ $\frac{6}{36 + 2} =$ $\frac{6}{38} =$ $\frac{3}{19}$ $f(6) = \frac{3}{19}$	d. $f(0)$ $2(0) + 7 =$ $0 + 7 =$ 7 $f(0) = 7$
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3. Rewrite the function $g(x) = |3x| + 2$ as a piecewise function.

$$g(x) = \begin{cases} 3x + 2 & \text{when } x \geq 0 \\ -3x + 2 & \text{when } x < 0 \end{cases}$$



AP Precalculus
Prerequisites Review #10 – Exponential Functions

1. A certain bacteria population sample contains 500 bacteria and is known to grow by 20% every hour when left untreated.

- a. Write an equation to model the untreated bacteria population (y) after x hours.

$$y = 500(1 + .20)^x$$

$$y = 500(1.2)^x$$

- b. How many bacteria are in the sample after 5 hours? 7.5 hours?

$$500(1.2)^5$$

$$500(2.48832)$$

1,244 bacteria
after 5 hours

$$500(1.2)^{7.5}$$

$$500(3.92517)$$

1962 bacteria
after 7.5 hours

**AP Precalculus****Prerequisites Review #11 – Rules for Exponents**

Simplify the following expressions. Write your answers with positive exponents only.

1. $(w^0x^5)^{-1}$

$$w^{0 \cdot -1} x^{5 \cdot -1} =$$

$$w^0 x^{-5} =$$

$$\frac{w^0}{x^5} =$$

$$\boxed{\frac{1}{x^5}}$$

2. $c^{-3}(c^7)^4$

$$c^{-3} \cdot c^{7 \cdot 4} =$$

$$c^{-3} \cdot c^{28} =$$

$$c^{-3+28} =$$

$$\boxed{c^{25}}$$

3. $(u^3v^5)^2(u^{-7}v^{-10})$

$$(u^{3 \cdot 2} v^{5 \cdot 2})(u^{-7} v^{-10}) =$$

$$(u^6 v^{10})(u^{-7} v^{-10}) =$$

$$u^{6+(-7)} v^{10+(-10)} =$$

$$u^{-1} v^0 =$$

$$\frac{v^0}{u^{-1}} =$$

$$\boxed{\frac{1}{u}}$$

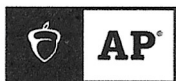
4. $\frac{x^3y^4}{w^7z^{-2}} * \frac{w^4y^{-3}}{x^5z^2}$

$$\frac{x^{3-5} y^{4+(-3)} w^{4-7}}{z^{-2+2}} =$$

$$\frac{x^{-2} y w^{-3}}{z^0} =$$

$$\frac{y}{w^3 x^2 z^0} =$$

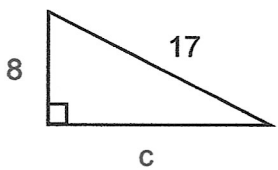
$$\boxed{\frac{y}{w^3 x^2}}$$

**AP Precalculus***Prerequisites Review #12 – Radicals (square roots and cube roots)*

1. Evaluate each of the following. Round to the nearest hundredth as needed.

a. $\sqrt{121}$ 11	b. $\sqrt{175}$ 13.23
c. $\sqrt[3]{125}$ 5	d. $\sqrt[3]{8}$ 2
e. $\sqrt[3]{36}$ 3.30	

2. Solve for c.



Pythagorean Theorem $a^2 + b^2 = c^2$
a and b are legs
c is the hypotenuse

$$8^2 + c^2 = 17^2$$
$$64 + c^2 = 289$$
$$c^2 = 225$$
$$c = \sqrt{225}$$
$$c = 15$$

3. Simplify each of the following expressions. Rationalize denominators as needed.

a. $\sqrt{50}$ $\sqrt{5 \cdot 5 \cdot 2}$ $5\sqrt{2}$	b. $\frac{3\sqrt{6}}{4\sqrt{5}}$ $\frac{3\sqrt{6}}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{30}}{4 \cdot 5} =$ $\frac{3\sqrt{30}}{20}$
c. $\sqrt{72a^5b^6}$ $\sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot a^2 \cdot a^2 \cdot a \cdot b^2 \cdot b^2 \cdot b^2}$ $2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b \sqrt{2a}$ $6a^2b^3\sqrt{2a}$	d. $3\sqrt{5} + 6\sqrt{20}$ $6\sqrt{20} = 6\sqrt{2 \cdot 2 \cdot 5} =$ $6 \cdot 2\sqrt{5} =$ $12\sqrt{5}$ $3\sqrt{5} + 12\sqrt{5} =$ $15\sqrt{5}$
e. $\frac{\sqrt{200x^{17}y^6}}{\sqrt{45x^{15}y^9}}$ Reduce the fraction then simplify the root $\sqrt{\frac{40x^2}{9y^3}} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 5 \cdot x^2}}{\sqrt{3 \cdot 3 \cdot y^2 \cdot y}} = \frac{2x\sqrt{10}}{3y\sqrt{y}}$ Rationalize the denominator. $\frac{2x\sqrt{10}}{3y\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}} = \frac{2x\sqrt{10y}}{3y^2}$	



AP Precalculus
Prerequisites Review #13 – Complex Numbers

Simplify the following expressions and rationalize denominators as needed.

1. $(3 + 7i) + (4 - 9i)$

$$3 + 7i + 4 - 9i$$

$$\boxed{7 - 2i}$$

2. $(3 + 7i) - (4 - 9i)$

$$3 + 7i - 4 + 9i$$

$$\boxed{-1 + 16i}$$

3. $(3 + 7i)(4 - 9i)$

$$3(4) + 3(-9i) + 7i(4) + 7i(-9i)$$

$$12 - 27i + 28i - 63i^2$$

$$12 + i - 63(-1)$$

$$12 + i + 63$$

$$\boxed{75 + i}$$

4. $\frac{10 - 2i}{3 + 4i}$

$$\frac{(10 - 2i)(3 - 4i)}{(3 + 4i)(3 - 4i)} =$$

$$\frac{30 - 40i - 6i + 8i^2}{9 - 12i + 12i - 16i^2} =$$

$$\frac{30 - 46i + 8(-1)}{9 - 16(-1)} =$$

$$\boxed{\frac{22 - 46i}{25}}$$