ENGLISH

Bluebonnet Learning
Secondary Mathematics
EDITION 1

Algebra II

Family Guides

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Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

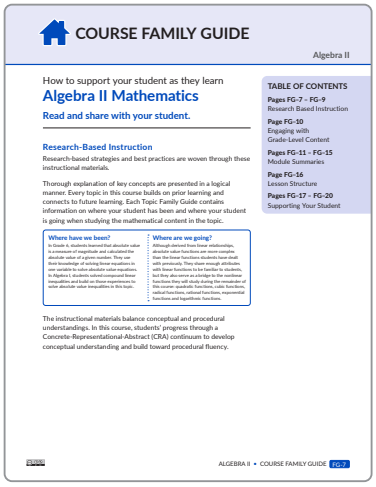
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Dear Family,

We recognize that learning outside of the classroom is crucial to your student's success at school. This letter serves as an introduction to the resources designed to assist you as you talk to your student about what they are learning. Resources available to you include:

- Course Family Guide
- Topic Family Guides
- Topic Summaries
- Math Glossary

Course Family Guide


COURSE FAMILY GUIDE

Algebra II

How to support your student as they learn
Algebra II Mathematics
Read and share with your student.

TABLE OF CONTENTS
Pages FG-7 – FG-9 Research Based Instruction
Page FG-10 Engaging with Grade-Level Content
Pages FG-11 – FG-15 Module Summaries
Page FG-16 Lesson Structure
Pages FG-17 – FG-20 Supporting Your Student

Research-Based Instruction
Research-based strategies and best practices are woven through these instructional materials.

Thorough explanation of key concepts are presented in a logical manner. Every topic in this course builds on prior learning and connects to future learning. Each Topic Family Guide contains information on where your student has been and where your student is going when studying the mathematical content in the topic.

Where have we been?
Where are we going?

The instructional materials balance conceptual and procedural understandings. In this course, students' progress through a Concrete-Representational-Abstract (CRA) continuum to develop conceptual understanding and build toward procedural fluency.

ALGEBRA II • COURSE FAMILY GUIDE FG-7

Following this letter, there is the Course Family Guide that will walk you through the research-based instructional approach, how the course is structured, how the mathematics applies to the real-world, using Talking Points from the Topic Family Guide, and using the TEKS mathematical process standards to initiate discussions.

Research and classroom experience guided course development, with the foundation being a scientific understanding of how people learn and a real-world understanding of how to apply that science to mathematics instructional materials. The instructional design elements presented in the Course Family Guide incorporate research-based strategies to develop conceptual understanding and creative problem solvers.

The Course Family Guide provides an overview of the structure of the course. The course consists of both a Learning Together component and a Learning Individually component. The teacher facilitates a collaborative learning experience during the Learning Together Days and uses data to target specific skills on the Learning Individually Days.

Next, the Course Family Guide includes Module Overviews of each module in the course, which include a detailed summary of what your student will be learning in each topic within the module. Below the topic summaries are facts and information that connect the concepts to the real world. Read and discuss the information below the topic summaries with your student and continue to come back to these pages as your student moves from one topic to the next within each module.

The Course Family Guide also highlights the lesson structure. Each lesson is structured the same way and includes four parts: Objectives & Essential Question, Getting Started, Activities, and the Talk the Talk.

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Topic Family Guide

Each course is organized into modules. Each module consists of topics with corresponding Topic Family Guides. These guides all have the same structure. This consistency will allow you and your student to understand how to reference the content of each topic.

The Topic Family Guide begins with an overview of the content in the topic. This introduction includes a brief explanation of what your student will learn in the topic, the prior knowledge they will use to help them understand this topic, and a connection to future learning.

The next section of the Topic Family Guide is the Talking Points section. The Talking Points section provides skills you can discuss with your students and a sample question based on the math in the topic that you can talk through with them.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Absolute value functions have many real-world applications including financial planning, measuring the magnitudes of earthquakes, and measuring heart rate variability.

HERE IS A SAMPLE QUESTION

What are the values of n and p so that $-n|2p - 6| > 0$?

For the product to be greater than 0, the factors must be either both greater than 0 or both less than 0.

Since one of the factors is an absolute value, the factors cannot be less than 0, so they are both greater than 0.

This means that n must be less than 0, and p cannot be equal to 3.

The solution is all values such that $n < 0$ and $p \neq 3$.

Next, the Topic Family Guide lists all the key terms of the topic and details some of the math strategies students will learn in the topic. Finally, each Topic Family Guide contains a Math in the Real-World section. Understanding how the math content is used in the real-world boosts student engagement, bridges theory and practice, and helps students to think about connections to other courses and careers they may be interested in.

Math in the Real World

Absolute value functions connect to many real-world situations in health sciences. For example, consider measuring how far a person's heart rate or temperature is from the normal range. Whether it's above or below that range, what matters is the size of the difference, not the direction. That is precisely what absolute value shows us—it focuses on the distance from zero, whether positive or negative.

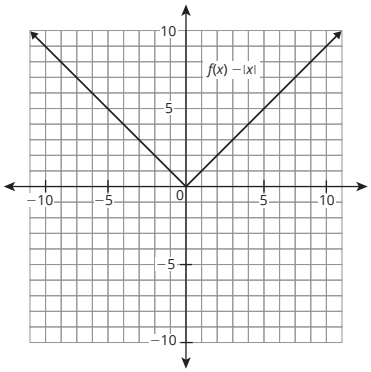
Take blood pressure as another example. Doctors track how much a patient's blood pressure numbers go up or down compared to a healthy level. Absolute value helps show how significant the change is, whether it's an increase or a decrease. Absolute value equations solve real problems in healthcare, like monitoring patients to see the effects of treatments. It is incredible to think about how these absolute value equations can help save lives!

Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or Worked Examples. The Topic Summary provides an opportunity for you and your student to discuss the key concepts from each lesson, review the examples, and do the math together.

LESSON 1 **Defining Absolute Value Functions**

The graph of the parent **absolute value function**, $f(x) = |x|$, is in a *V* shape. It is the combination of the graphs of $f(x) = x$ and $f(x) = -x$.



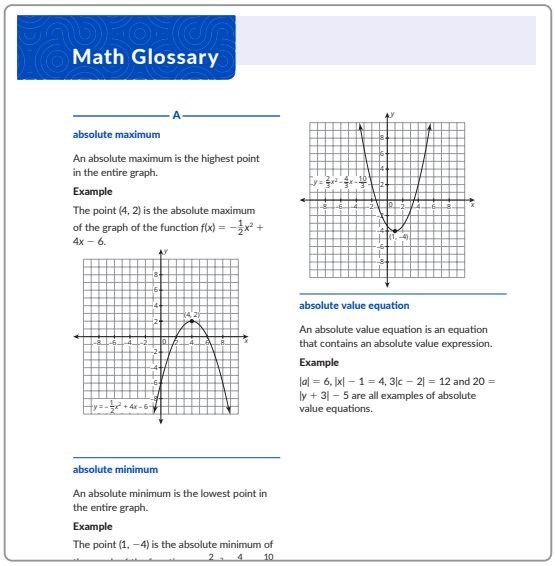
You can model many real-world situations with an absolute value function. For example, the equation $y = |x - 97.5|$ represents the average human internal body temperature. A deviation, *y*, of around 2.9°F from the average indicates a serious medical issue. Hypothermia can start at around 2.9°F less than the average, and a fever can start around 2.9°F more than the average.

Evidence of the TEKS mathematical process standards are present in the Topic Summaries. Each lesson within the topic highlights one or more of the TEKS mathematical process standards. These processes will help your student develop effective communication and collaboration skills that are essential for becoming a successful learner. Discuss with your student the “I can” statements associated with each of the TEKS mathematical process standards to help them develop their mathematical learning and understanding. The "I can" statements for each of the TEKS mathematical process standards are included in the Course Family Guide. With your help, your student can develop the habits of a productive mathematical thinker.

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Math Glossary

The Math Glossary for each course is a tool for your student to utilize and reference during their learning. Along with the definition of a term, the glossary provides examples to help further their understanding.



We all have the same goal for your student: to become a successful problem solver and use mathematics efficiently and effectively in daily life. Encourage them to use the mathematics they already know when seeing new concepts and communicate their thinking while providing a critical ear to the thinking of others.

Thank you for supporting your student.

How to support your student as they learn

Algebra II Mathematics

Read and share with your student.

TABLE OF CONTENTS

Pages FG–7 – FG–9
Research Based Instruction

Page FG–10
Engaging with Grade-Level Content

Pages FG–11 – FG–15
Module Summaries

Page FG–16
Lesson Structure

Pages FG–17 – FG–21
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Research-Based Instruction

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Where have we been?

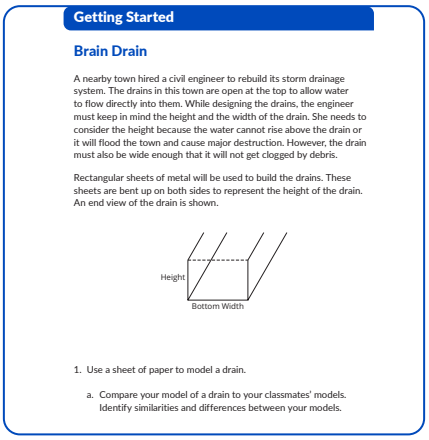
In Grade 6, students learned that absolute value is a measure of magnitude and calculated the absolute value of a given number. They use their knowledge of solving linear equations in one variable to solve absolute value equations. In Algebra I, students solved compound linear inequalities and build on those experiences to solve absolute value inequalities in this topic.

Where are we going?

Although derived from linear relationships, absolute value functions are more complex than the linear functions students have dealt with previously. They share enough attributes with linear functions to be familiar to students, but they also serve as a bridge to the nonlinear functions they will study during the remainder of this course: quadratic functions, cubic functions, radical functions, rational functions, exponential functions and logarithmic functions.

The instructional materials balance conceptual and procedural understandings. In this course, students' progress through a Concrete-Representational-Abstract (CRA) continuum to develop conceptual understanding and build toward procedural fluency.

Concrete	Representational	Abstract
Students explore a real-world scenario to develop their intuitive understanding of quadratic functions. They use a sheet of paper to model a drain.	Students use a table and a graph to represent the dimensions and area of the drain.	Students write and solve equations related to the problem situation. They also interpret the equation in terms of the problem situation.



Getting Started

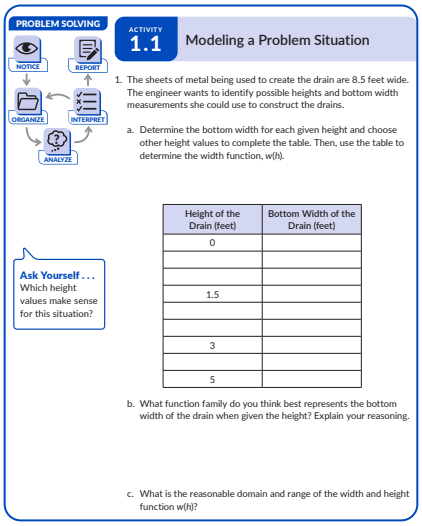
Brain Drain

A nearby town hired a civil engineer to rebuild its storm drainage system. The drains in this town are open at the top to allow water to flow directly into them. While designing the drains, the engineer must keep in mind the height and the width of the drain. She needs to consider the height because the water cannot rise above the drain or it will flood the town and cause major destruction. However, the drain must also be wide enough that it will not get clogged by debris.

Rectangular sheets of metal will be used to build the drains. These sheets are bent up on both sides to represent the height of the drain. An end view of the drain is shown.

1. Use a sheet of paper to model a drain.

   a. Compare your model of a drain to your classmates' models. Identify similarities and differences between your models.



PROBLEM SOLVING

NOTICE REPORT ORGANIZE INTERPRET ANALYZE

ACTIVITY 1.1 Modeling a Problem Situation

1. The sheets of metal being used to create the drain are 8.5 feet wide. The engineer wants to identify possible heights and bottom width measurements she could use to construct the drains.

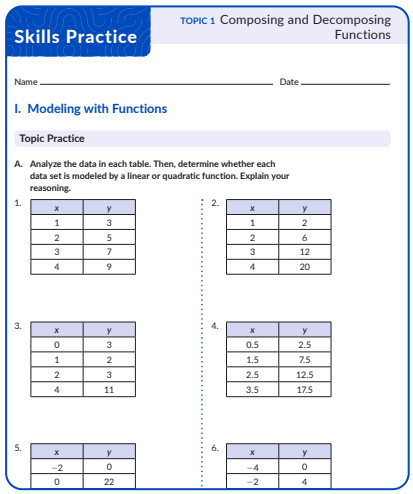
   a. Determine the bottom width for each given height and choose other height values to complete the table. Then, use the table to determine the width function, $w(h)$.

Ask Yourself . . . Which height values make sense for this situation?

Height of the Drain (feet)	Bottom Width of the Drain (feet)
0	
1.5	
3	
5	

   b. What function family do you think best represents the bottom width of the drain when given the height? Explain your reasoning.

   c. What is the reasonable domain and range of the width and height function $w(h)$?



Skills Practice TOPIC 1 Composing and Decomposing Functions

Name _____ Date _____

I. Modeling with Functions

Topic Practice

A. Analyze the data in each table. Then, determine whether each data set is modeled by a linear or quadratic function. Explain your reasoning.

1.

x	y
1	3
2	5
3	7
4	9

2.

x	y
1	2
2	6
3	12
4	20

3.

x	y
0	3
1	2
2	3
4	11

4.

x	y
0.5	2.5
1.5	7.5
2.5	12.5
3.5	17.5

5.

x	y
−2	0
0	22

6.

x	y
−4	0
−2	4



Support is provided to students as they persevere in problem solving. These instructional materials include a problem-solving model, which includes questions your student can ask when productively engaging in real-world and mathematical problems. Prompts will encourage your student to use the problem-solving model throughout the course.

These instructional materials include features that support learners. Worked Examples throughout the product provide explicit instruction and provide a model your student can continually reference.

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connection between steps.

WORKED EXAMPLE

Consider this absolute value equation.

$$|a| = 6$$

There are two points that are 6 units away from zero on the number line: one to the right of zero, and one to the left of zero.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$
$$a = 6 \quad \text{or} \quad a = -6$$

Now consider the case where $a = x - 1$.

$$|x - 1| = 6$$

If you know that $|a| = 6$ can be written as two separate equations, you can rewrite any absolute value equation.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$
$$+(x - 1) = 6 \quad \text{or} \quad -(x - 1) = 6$$

Thumbs Up, Thumbs Down, and Who's Correct questions address your student's common misconceptions and provide opportunities for peer work analysis.

Thumbs Up Thumbs Down

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connection between steps.

Ask Yourself

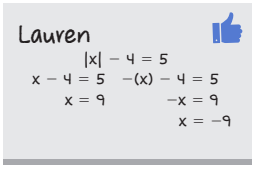
- Why is this method correct?
- Have I used this method before?

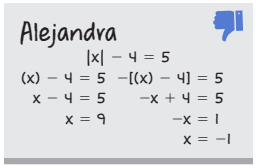
When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself

- Where is the error?
- Why is it an error?
- How can I correct it?





Who's Correct

When you see a Who's Correct icon:

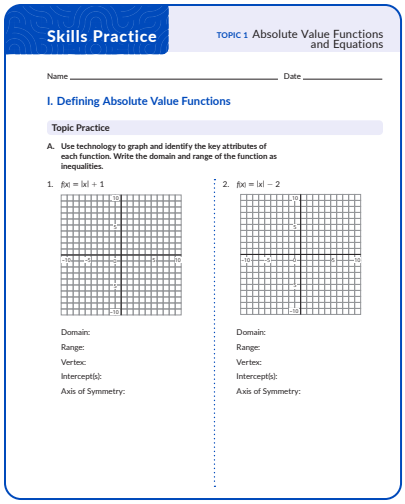
- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or incorrect.

Ask Yourself

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

1. Xavier and Lucia are working on a homework assignment for which they must identify all functions that are one-to-one functions. Xavier says that all linear functions are one-to-one functions, so they don't even need to look at the linear functions. Lucia disagrees and says that not all linear functions are one-to-one functions. Who is correct? Explain how you determined which student is correct.

Targeted Skills Practice provides support to your student as they work to gain proficiency of the course material. Spaced Practice provides a spaced retrieval of key concepts to your student. Extension opportunities provide challenges to accelerate your student's learning.


Skills Practice

TOPIC 1 Absolute Value Functions and Equations

Name _____ Date _____

I. Defining Absolute Value Functions

Topic Practice

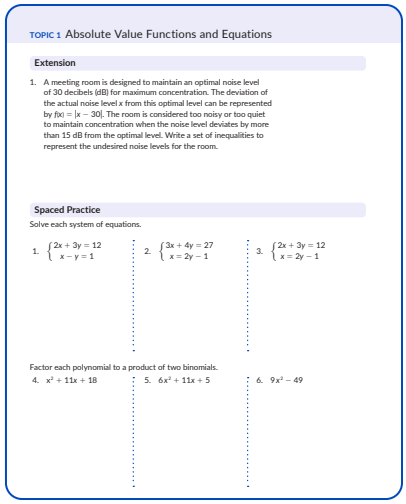
A. Use technology to graph and identify the key attributes of each function. Write the domain and range of the function as inequalities.

1. $f(x) = |x| + 1$

2. $f(x) = |x| - 2$

Domain:
Range:
Vertex:
Intercept(s):
Axis of Symmetry:

Domain:
Range:
Vertex:
Intercept(s):
Axis of Symmetry:


TOPIC 1 Absolute Value Functions and Equations

Extension

1. A meeting room is designed to maintain an optimal noise level of 30 decibels (dB) for maximum concentration. The deviation of the actual noise level x from this optimal level can be represented by $f(x) = |x - 30|$. The room is considered too noisy or too quiet to maintain concentration when the noise level deviates by more than 15 dB from the optimal level. Write a set of inequalities to represent the undesired noise levels for the room.

Spaced Practice

Solve each system of equations.

1. $\begin{cases} 2x + 3y = 12 \\ x - y = 1 \end{cases}$
2. $\begin{cases} 3x + 4y = 27 \\ x = 2y - 1 \end{cases}$
3. $\begin{cases} 2x + 3y = 12 \\ x = 2y - 1 \end{cases}$

Factor each polynomial to a product of two binomials.

4. $x^2 + 11x + 18$
5. $6x^2 + 11x + 5$
6. $9x^2 - 49$

KEY TERMS

- absolute value function [función de valor absoluto]
- reflection [reflexión]
- line of reflection [línea de reflexión]
- argument of a function [argumento de una función]
- absolute value equation [ecuación de valor absoluto]
- absolute value inequality [desigualdad de valor absoluto]
- equivalent compound inequality [desigualdad compuesta equivalente]

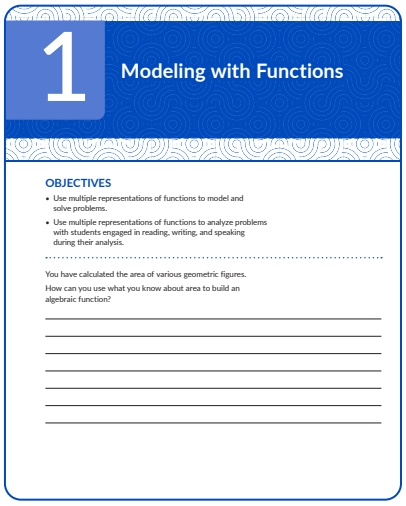
Each lesson features one or more ELPS (English Language Proficiency Standards) and provides the teacher with implementation strategies incorporating best practices for supporting language acquisition. In addition, students are provided with cognates for Key Terms in the Topic Summaries and Topic Family Guides.

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Engaging with Grade Level Content

Your student will engage with grade-level content in multiple ways with the support of the teacher.

Learning Together	Learning Individually
The teacher facilitates active learning of lessons so that students feel confident in sharing ideas, listening to each other, and learning together. Students become creators of their mathematical knowledge.	Skills Practice provides students the opportunity to engage in additional skill building that aligns to each Learning Together lesson. The Learning Individually Days target discrete skills that may require additional practice to achieve proficiency.


1

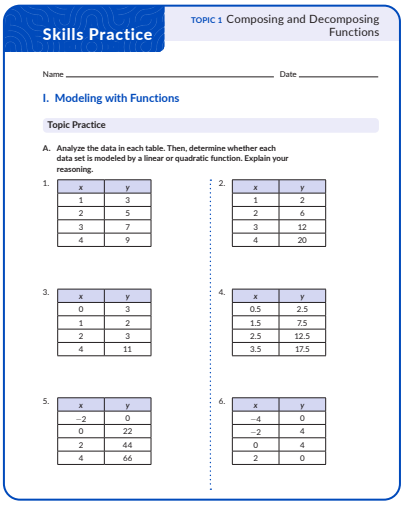
Modeling with Functions

OBJECTIVES

- Use multiple representations of functions to model and solve problems.
- Use multiple representations of functions to analyze problems with students engaged in reading, writing, and speaking during their analysis.

You have calculated the area of various geometric figures.

How can you use what you know about area to build an algebraic function?


Skills Practice

TOPIC 1 Composing and Decomposing Functions

Name _____ Date _____

I. Modeling with Functions

Topic Practice

A. Analyze the data in each table. Then, determine whether each data set is modeled by a linear or quadratic function. Explain your reasoning.

1.

x	y
1	3
2	5
3	7
4	9

2.

x	y
1	2
2	6
3	12
4	20

3.

x	y
0	3
1	2
2	3
4	11

4.

x	y
0.5	2.5
1.5	7.5
2.5	12.5
3.5	17.5

5.

x	y
−2	0
0	22
2	44
4	66

6.

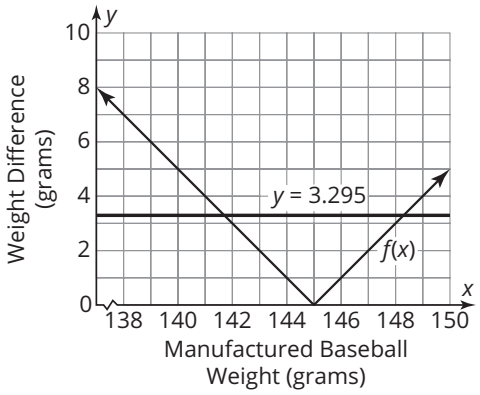
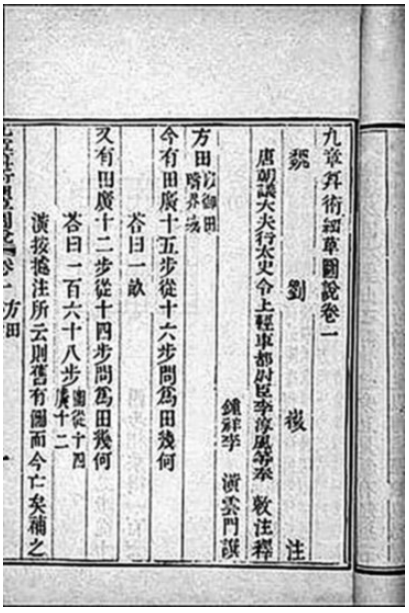
x	y
−4	0
−2	4
0	4
2	0

At the end of each topic, your student will take an assessment aligned to the standards covered in the topic. This assessment consists of multiple-choice, multiselect, and open-ended questions designed for your student to demonstrate learning. Each assessment also includes a scoring guide for teachers to ensure consistent scoring. The scoring guide includes ways to support or challenge your student based on their responses to the questions on the assessment. The purpose of the assessment is for the teacher and student to reflect on the learning. Teachers will use your student's assessment results to target individual skills your student needs for proficiency or to accelerate and challenge your student.

Response to Student Performance		
TEKS	**Question(s)**	**Recommendations**
2A.2A	1, 7	**To support students:** • Review graphing absolute value functions and identifying key attributes. • Use Skills Practice Set I.A, II.B, and II.C for additional practice.
2A.6C	6, 8	**To support students:** • Review transformations of absolute value functions. • Use Skills Practice Set II.A, II.B, II.C, and II.D for additional practice. **To challenge students:** • Extend student knowledge with Skill Practice Extension Set II.
2A.6D	2, 5	**To support students:** • Review formulating absolute value equations. • Use Skills Practice Set III.B and III.C for additional practice.

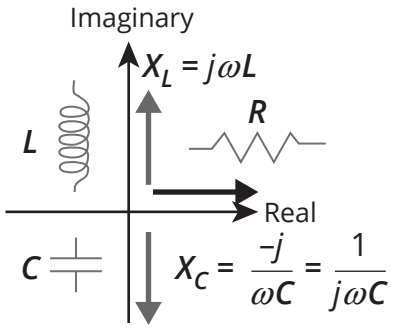
MODULE 1 Extending Linear Relationships

In this module, your student will use their understanding of linear functions to explore absolute value functions, systems of three linear equations, systems of inequalities, and inverse functions. There are two topics in this module: *Absolute value Functions and Equations* and *Applications of Linear Relationships*.

TOPIC 1 Absolute Value Functions and Equations	TOPIC 2 Applications of Linear Relationships
Your student will use what they know about solving equations and inequalities to solve absolute value equations and inequalities.	Your student will solve systems of three linear equations in three variables using substitution, Gaussian elimination, and technology with Matrices.
Did you know? Absolute value functions are useful for modeling real-world scenarios like measuring distance or the acceptable weight of a baseball that fits within specifications.  	Did you know that? The first signs of **matrix** usage was in 10th–2nd century BC in Chinese math books. 

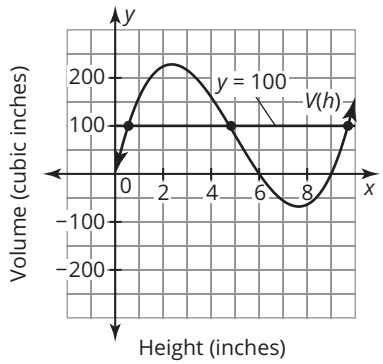
MODULE 2 Exploring Quadratic Functions

In this module, your student will expand on their understanding of quadratic functions. There are two topics in this module: *Quadratic Functions and Equations* and *Applications of Quadratics*.

TOPIC 1 Quadratic Functions and Equations	TOPIC 2 Applications of Quadratics
Your student will explore complex numbers and use complex numbers to solve quadratic equations.	Your student will model real-world scenarios using quadratic functions, inequalities, systems with a linear and quadratic function, and quadratic regression functions. They will explore inverses of functions.
Did you know that? **Imaginary numbers** are often used in electricity, specifically alternating current (AC) electronics. AC electricity changes between positive and negative. Using imaginary and real numbers helps those working with AC electricity do calculations and avoid electrocution.  	What in the world? A simple form of **inverse functions** that we use everyday but maybe don't think about is the temperature outside! Normally we use Fahrenheit but Celsius is actually just the inverse of it! **Fahrenheit To Celsius** $C = \frac{5}{9}(F - 32)$ **Celsius To Fahrenheit** $F = \frac{5}{9}C + 32$

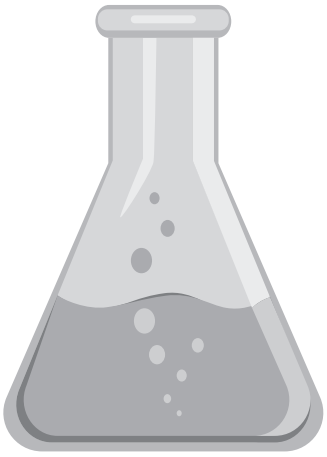
MODULE 3 Analyzing Structure

In this module, your student will explore cubic functions and their key attributes. There are three topics in this module: *Composing and Decomposing Functions*, *Attributes of Cubic Functions*, and *Relating Factors and Zeros*.

TOPIC 1 Composing and Decomposing Functions	TOPIC 2 Attributes of Cubic Functions	TOPIC 3 Relating Factors and Zeros
Your student will be introduced to building cubic functions on the coordinate plane by operating on or transforming linear and quadratic functions.	Your student will learn about the key attributes of polynomial functions, transformations applied to them, and how to build polynomial functions using these attributes and transformations.	Your student will expand their ability to operate with polynomials. They will add, subtract, multiply, factor, and divide polynomials.
Did you know? You can use a cubic function to model many real world situations. For example, consider a situation in which someone wants a planter box with a volume of 100 cubic inches. You can determine the possible heights for the planter from the graph of $V(h)$.  	Did you know? Just as we look for specific patterns, characteristics, and attributes in functions, scientists find certain characteristics within strands of DNA that allow them to predict what offspring may look like. 	Did you know? You can divide polynomials using the same long division algorithm you use to divide numbers! $$\begin{array}{r} x^2 + 0x + 3 \\ x + 1 \overline{)\, x^3 + x^2 + 3x + 3} \\ -(x^3 + x^2) \\ \hline 0x^2 + 3x \\ -(0x^2 + 0x) \\ \hline 3x + 3 \\ -(3x + 3) \\ \hline 0 \end{array}$$

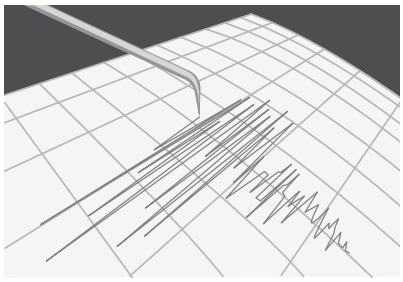
MODULE 4 Extending Beyond Polynomials

In this module, your student will explore rational functions, perform operations with rational expressions, and solve rational equations. There are three topics in this module: *Rational Functions*, *Radical Functions*, and *Radical Equations*.

TOPIC 1 Rational Functions	TOPIC 2 Radical Functions	TOPIC 3 Radical Equations
Your student will create rational functions, identify their key attributes, apply transformations, and explore how these functions are used in various types of problem situations.	Your student will expand what they know about inverses from Module 1 as they explore radical functions.	Your student will operate with radical expressions and solve radical equations.
Mixing it up! Chemistry is an exact science that doesn't always have the perfect ingredients. Diluting and concentrating ingredients is important to get correct and safe reactions. Rational equations are an important part of the pre-work! 	Did you know? Predicting tsunamis is not an exact science, but scientists use a variety of radical functions to quickly estimate the arrival time of any potential dangers so people can head for safety. 	Did you know? Radial equations are solved using the same reverse operations and properties of equality that you have used to solve other equations. However, radical equations can have extraneous solutions, so always check your answers! Solve $3\sqrt{x} + 7 = 25$. $3\sqrt{x} + 7 = 25$ $3\sqrt{x} = 18$ $\sqrt{x} = 6$ $(\sqrt{x})^2 = (6)^2$ $x = 36$ Check: $3\sqrt{36} + 7 \stackrel{?}{=} 25$ $3(6) + 7 \stackrel{?}{=} 25$ $25 = 25$✔

MODULE 5 Exploring Exponentials and Logarithmics

In this module, your student will deepen their understanding of exponential functions and learn that the inverse of an exponential function is a logarithmic function. There are two topics in this module: *Exponential and Logarithmic Functions* and *Exponential and Logarithmic Equations*.

TOPIC 1 Exponential and Logarithmic Functions	TOPIC 2 Exponential and Logarithmic Equations
Your student will expand on what they already know about exponential functions and be introduced to their inverses, logarithmic functions.	Your student will solve exponential and logarithmic equations.
Did you know? The Richter Scale is used to calculate the energy released by earthquakes. The Richter scale is logarithmic, meaning that whole-number jumps in the rating indicate a tenfold increase in the wave amplitude of the earthquake. For example, the wave amplitude in a Level 5 earthquake is ten times greater than the amplitude of a Level 4 earthquake. 	Cooling it Down Newton's Law of Cooling states that the rate at which an object's temperature changes is proportional to the difference between its temperature and the surrounding environment. This principle is used to determine when an object will reach a specific temperature. This method can be applied to cooling coffee, predicting when it will reach a drinkable temperature. 

Lesson Structure

Each lesson in the course is laid out in the same way to develop deep understanding. Read through the parts of the lesson to learn more about your student's learning in their math classroom.

Objectives & Essential Question

Each lesson begins with objectives, listed to help students understand the objectives. Also included is an essential statement connecting students' learning with a question to ponder. The question is asked again at the end of each lesson to see how much your student understands.

Getting Started

The Getting Started engages your student in the learning. In the Getting Started, your student uses what they already know about the world, what they've already learned, and their intuition to get them thinking mathematically and prepare them for what's to come in the lesson.

Activities

In the Activities, students develop their mathematical knowledge and build a deep understanding of the math. These activities provide your student with the opportunity to communicate and work with others in their math classroom.

When your student is working through these activities, keep in mind:

- It's not just about answer-getting. Doing the math and talking about it is important.
- Making mistakes is an important part of learning, so take risks.
- There is often more than one way to solve a problem.

Talk the Talk

The Talk the Talk gives your student an opportunity to reflect on the main ideas of the lesson and demonstrate their learning.

Lesson Assignment

The lesson assignment provides your students with practice to develop fluency and build proficiency. The lesson assignment also includes a section to help prepare students for the next lesson.

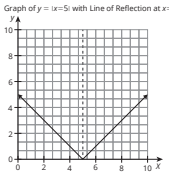
Key Concepts of the Lesson

At the end of each topic, the Topic Summary provides a summary of each lesson in the topic. Encourage your student to use these as a tool to review and retrieve the key concepts of a lesson.

Supporting Your Student

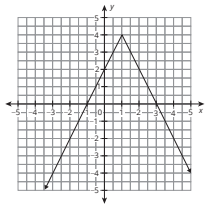
Where are we now?

A **reflection** of a graph is the mirror image of the graph about a **line of reflection**.



An **absolute value function** is a function of the form $f(x) = |x|$.

For example, the coordinate plane shows the graph of the absolute value function $f(x) = -2|x - 1| + 4$. The graph increases to a vertex and then decreases and is symmetric across a vertical line through the vertex.



In **Lesson 1: Defining Absolute Value Functions**, students explore the parent absolute value function. Given a real-world context, students formulate an absolute value equation and solve the equation graphically.

The Topic Family Guide

The Topic Family Guide provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It provides an example of a math model or strategy your student is learning in the topic, real-world connections to the mathematics, talking points to discuss and/or questions to ask your student, and the key terms your student will learn. You and your student can use the Math Glossary to look up the definitions to the key terms. Encourage your student to reference the key terms in the Topic Family Guide and/or Math Glossary when completing math tasks.

Learning outside of the classroom is crucial for your student's success. While we don't expect you to be a math teacher, the Topic Family Guide can assist you as you talk to your student about the mathematical content of the course. The hope is that both you and your student will read and benefit from the guides.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Absolute value functions have many real-world applications including financial planning, measuring the magnitudes of earthquakes, and measuring heart rate variability.

HERE IS A SAMPLE QUESTION

What are the values of n and p so that $-n|2p - 6| > 0$?

For the product to be greater than 0, the factors must be either both greater than 0 or both less than 0.

Since one of the factors is an absolute value, the factors cannot be less than 0, so they are both greater than 0.

This means that n must be less than 0, and p cannot be equal to 3.

The solution is all values such that $n < 0$ and $p \neq 3$.

Math in the Real-World

Math is used in everyday activities like budgeting, cooking, and sports, making it a valuable life skill. It also plays a crucial role in financial literacy, logical reasoning, and decision-making. By connecting math to real-life situations, students gain confidence, see its relevance, and develop critical thinking skills that will benefit them in both academics and daily life. The Math in the Real-World section in the Topic Family Guide provides examples of where the math concepts of the topic are used beyond the classroom.

Examples of these real-world connections include:

Math in the Real World: Absolute value functions connect to many real-world situations in health sciences. For example, consider measuring how far a person's heart rate or temperature is from the normal range. Whether it's above or below that range, what matters is the size of the difference, not the

Math in the Real World

Absolute value functions connect to many real-world situations in health sciences. For example, consider measuring how far a person's heart rate or temperature is from the normal range. Whether it's above or below that range, what matters is the size of the difference, not the direction. That is precisely what absolute value shows us—it focuses on the distance from zero, whether positive or negative.

Take blood pressure as another example. Doctors track how much a patient's blood pressure numbers go up or down compared to a healthy level. Absolute value helps show how significant the change is, whether it's an increase or a decrease. Absolute value equations solve real problems in healthcare, like monitoring patients to see the effects of treatments. It is incredible to think about how these absolute value equations can help save lives!

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direction. That is precisely what absolute value shows us—it focuses on the distance from zero, whether positive or negative.

Take blood pressure as another example. Doctors track how much a patient's blood pressure numbers go up or down compared to a healthy level. Absolute value helps show how significant the change is, whether it's an increase or a decrease. Absolute value equations solve real problems in healthcare, like monitoring patients to see the effects of treatments. It is incredible to think about how these absolute value equations can help save lives!

Math in the Real-World: Your body is an amazing collection of different systems. Your cardiovascular system pumps blood throughout your body, your skeletal system provides shape and support, and your nervous system controls communication between your senses and your brain. Your skin, including your hair and fingernails, is a system all by itself—the integumentary system—and it protects all of your body's other systems. You also have a digestive system, endocrine system, excretory system, immune system, muscular system, reproductive system, and respiratory system.

Your body solves problems every day to stay balanced. Scientists use systems of equations to model how the human body works. For example, how oxygen levels in your blood affect your heart rate. So, your body is basically a walking math problem!

The TEKS Mathematical Process Standards

Each module will focus on TEKS mathematical process standards that will help your student become a mathematical thinker. The TEKS mathematical process standards are listed below. Discuss with your student the "I can" statements below the standards to help them develop their mathematical learning and understanding. With your help, your student can become a productive mathematical thinker.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Supporting Your Student

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it when necessary.
- ask useful questions to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions when trying to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Create and use representations to organize, record, and communicate mathematical ideas.

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Supporting Your Student

Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

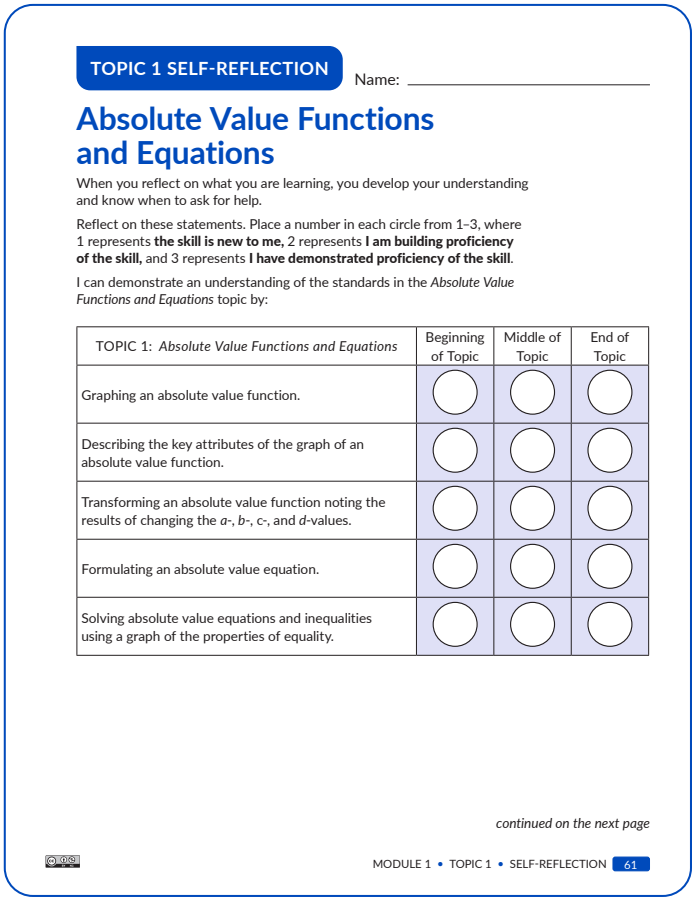
I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

Supporting Your Student

Reflecting on Learning and Progress

You can support your student by encouraging your student to reflect on the learning process. The instructional resources include a student Topic Self-Reflection for each topic. Encourage your student to accurately and frequently reflect on learning and progress throughout each topic. Talk about the specific concepts in the Topic Self-Reflection with your student and celebrate the progress from the beginning to the end of the topic. Remind your student to refer to the Topic Self-Reflection on Learning Individual Days after targeting specific skills and concepts. You can have your student explain concepts from the self-reflection using the topic summaries or lesson assignments to demonstrate understanding. In addition, encourage your student to reflect after taking an assessment. An Assessment Reflection is available to your student to assist with this process. Encourage your student to consider what went well and how to prepare for the next assessment. Ask your student how you can support them when preparing for the next assessment.



TOPIC 1 SELF-REFLECTION

Name: _____

Absolute Value Functions and Equations

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill,** and 3 represents **I have demonstrated proficiency of the skill.**

I can demonstrate an understanding of the standards in the *Absolute Value Functions and Equations* topic by:

TOPIC 1: *Absolute Value Functions and Equations*	Beginning of Topic	Middle of Topic	End of Topic
Graphing an absolute value function.	○	○	○
Describing the key attributes of the graph of an absolute value function.	○	○	○
Transforming an absolute value function noting the results of changing the *a*-, *b*-, *c*-, and *d*-values.	○	○	○
Formulating an absolute value equation.	○	○	○
Solving absolute value equations and inequalities using a graph of the properties of equality.	○	○	○

continued on the next page

MODULE 1 • TOPIC 1 • SELF-REFLECTION 61

Thanks!

Enjoy the fun mathematical adventure that is ahead for you and your student! Remember the supports available to you, and thank you for supporting your student's learning.

MODULE
1

Extending Linear Relationships

TOPIC 1 Absolute Value Functions and Equations 3

TOPIC 2 Applications of Linear Relationships 7

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Family Guide

MODULE 1 Extending Linear Relationships — Algebra II



TOPIC 1 Absolute Value Functions and Equations

Students begin this topic by reviewing what they know about absolute value from Grade 6. Students graph the function $f(x) = x$, and then graph $h(x) = |x|$ and $j(x) = |-x|$, discussing how each graph changed. Next, given a real-world situation, they formulate an absolute value function, and solve an absolute value equation graphically. Students explore transformations of the absolute value parent function before moving on to solve absolute value equations and inequalities based on real-world situations.

Where have we been?

In Grade 6, students learned that absolute value is a measure of magnitude and calculated the absolute value of a given number. They use their knowledge of solving linear equations in one variable to solve absolute value equations. In Algebra I, students solved compound linear inequalities and build on those experiences to solve absolute value inequalities in this topic.

Where are we going?

Although derived from linear relationships, absolute value functions are more complex than the linear functions students have dealt with previously. They share enough attributes with linear functions to be familiar to students, but they also serve as a bridge to the nonlinear functions they will study during the remainder of this course: quadratic functions, cubic functions, radical functions, rational functions, exponential functions and logarithmic functions.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Absolute value functions have many real-world applications including financial planning, measuring the magnitudes of earthquakes, and measuring heart rate variability.

HERE IS A SAMPLE QUESTION

What are the values of n and p so that $-n|2p - 6| > 0$?

For the product to be greater than 0, the factors must be either both greater than 0 or both less than 0.

Since one of the factors is an absolute value, the factors cannot be less than 0, so they are both greater than 0.

This means that n must be less than 0, and p cannot be equal to 3.

The solution is all values such that $n < 0$ and $p \neq 3$.

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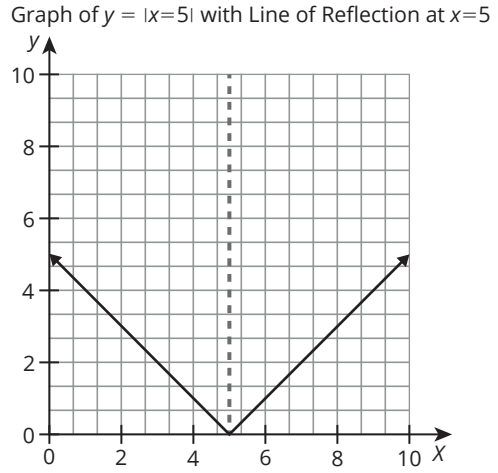
KEY TERMS

- absolute value function [función de valor absoluto]
- reflection [reflexión]
- line of reflection [línea de reflexión]
- argument of a function [argumento de una función]
- absolute value equation [ecuación de valor absoluto]
- absolute value inequality [desigualdad de valor absoluto]
- equivalent compound inequality [desigualdad compuesta equivalente]

Refer to the Math Glossary for definitions of the Key Terms.

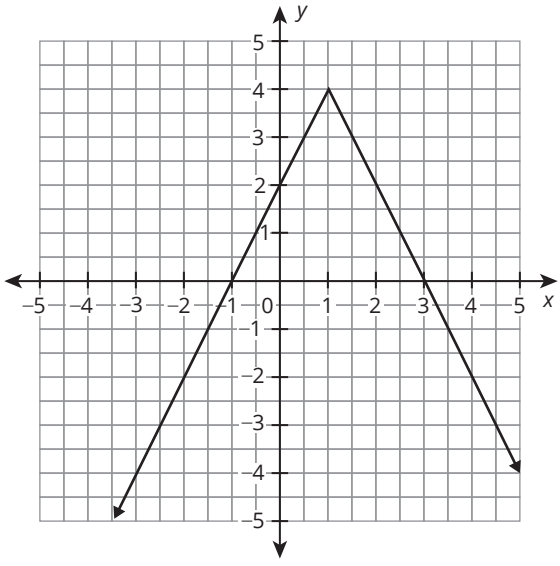
Where are we now?

A **reflection** of a graph is the mirror image of the graph about a **line of reflection**.



An **absolute value function** is a function of the form $f(x) = |x|$.

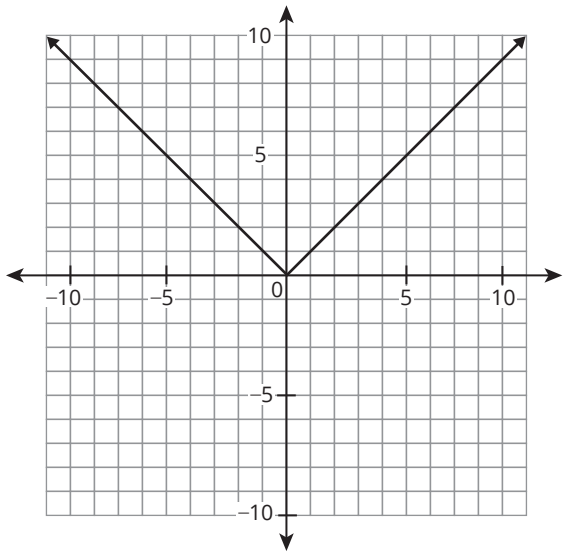
For example, the coordinate plane shows the graph of the absolute value function $f(x) = -2|x - 1| + 4$. The graph increases to a vertex and then decreases and is symmetric across a vertical line through the vertex.



In **Lesson 1: Defining Absolute Value Functions**, students explore the parent absolute value function. Given a real-world context, students formulate an absolute value equation and solve the equation graphically.

Parent Absolute Value Function

The parent absolute value function is $f(x) = |x|$.



The function has the following key attributes:

Domain: $-\infty < x < \infty$

Range: $y \geq 0$

Minimum: (0, 0)

Axis of symmetry: y-axis

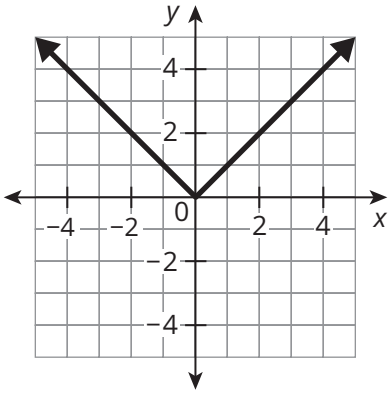
x-intercept: (0, 0)

y-intercept: (0, 0)

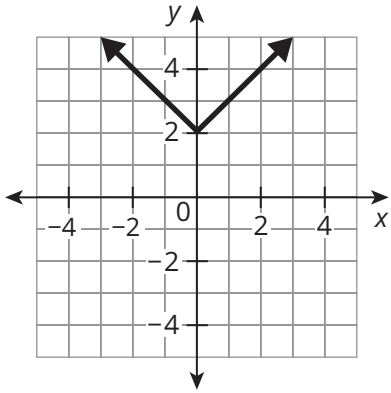
Transformations of Absolute Value Functions

In **Lesson 2: Transformations of Absolute Value functions**, students transform the function $f(x) = |x|$ using transformation form $g(x) = af(b(x - c)) + d$.

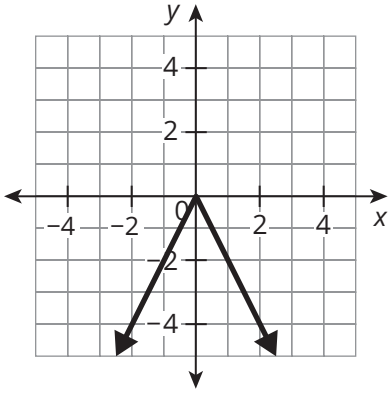
$f(x) = |x|$



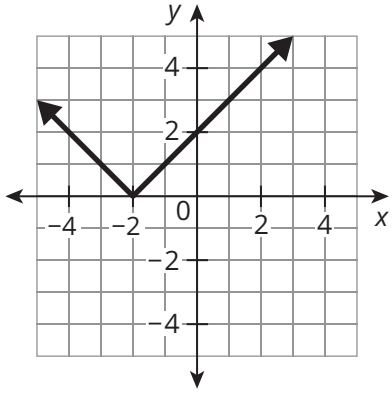
$y = |x| + 2$



$f(x) = -2|x|$



$f(x) = |x - (-2)|$



Math in the Real World

Absolute value functions connect to many real-world situations in health sciences. For example, consider measuring how far a person's heart rate or temperature is from the normal range. Whether it's above or below that range, what matters is the size of the difference, not the direction. That is precisely what absolute value shows us—it focuses on the distance from zero, whether positive or negative.

Take blood pressure as another example. Doctors track how much a patient's blood pressure numbers go up or down compared to a healthy level. Absolute value helps show how significant the change is, whether it's an increase or a decrease. Absolute value equations solve real problems in healthcare, like monitoring patients to see the effects of treatments. It is incredible to think about how these absolute value equations can help save lives!

In **Lesson 3: Absolute Value Equations and Inequalities,** student extend their understanding of solving equations to include those that include absolute values.

Absolute Value Equations and Inequalities

For example, solve the equation $|5x - 4| = 21$.

$$|5x - 4| = 21$$

$+(5x - 4) = 21$	$-(5x - 4) = 21$
$5x - 4 = 21$	$5x - 4 = -21$
$5x - 4 + 4 = 21 + 4$	$5x - 4 + 4 = -21 + 4$
$5x = 25$	$5x = -17$
$\frac{5x}{5} = \frac{25}{5}$	$\frac{5x}{5} = \frac{(-17)}{5}$
$x = 5$	$x = -3\frac{2}{5}$

Absolute value inequalities can take four different forms with the absolute value expression compared to a value, c. To solve an absolute value inequality, you must first write it as an **equivalent compound inequality**. "Less than" inequalities will be conjunctions and "greater than" inequalities will be disjunctions.

Absolute Value Inequality	Equivalent Compound Inequality
$\|ax + b\| < c$	$-c < ax + b < c$
$\|ax + b\| \leq c$	$-c \leq ax + b \leq c$
$\|ax + b\| > c$	$ax + b < -c$ or $ax + b > c$
$\|ax + b\| \geq c$	$ax + b \leq -c$ or $ax + b \geq c$

TOPIC 2 Absolute Value Functions and Equations



Students review what they know about systems of linear equations from Grade 8 and Algebra I. They apply this knowledge to solve systems of three linear equations in three variables. Students also use systems of linear inequalities and linear programming to model optimal solutions to real-world situations. They will learn matrix operations to better grasp the computations that a calculator uses to determine a solution to a system of equations. Finally, students explore inverses of linear functions.

Where have we been?

Students enter this topic with a wide range of experiences with linear functions. They have solved systems of equations since Grade 8 and Algebra I. They have investigated properties of real numbers, including the multiplicative identity and multiplicative inverse. In this topic, students will extend these properties to a new object—a matrix.

Where are we going?

In this course, students will solve increasingly complex equations, including quadratic, rational, radical, exponential, and logarithmic equations. They will also continue to solve more complicated systems of equations. In Module 2, students extend their understanding of solving systems of equations to solve systems consisting of a linear equation and a quadratic equation.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Solving systems of equations in three variables plays a crucial role in tasks like budgeting, where individuals balance income, expenses, and savings simultaneously. Students who choose to pursue engineering, computer science, economics, and mathematics fields will use systems of equations to analyze and solve complex problems.

HERE IS A SAMPLE QUESTION

Given the system of equations: $2x + y + z = 11$, $2x - y + 3z = 13$, and $3x + 2y - z = 8$
Determine the value of $x + y + z$.

Use a matrix equation to solve the system.

$$\begin{bmatrix} 2 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 13 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$x + y + z = 2 + 3 + 4 = 9$

The solution is 9.

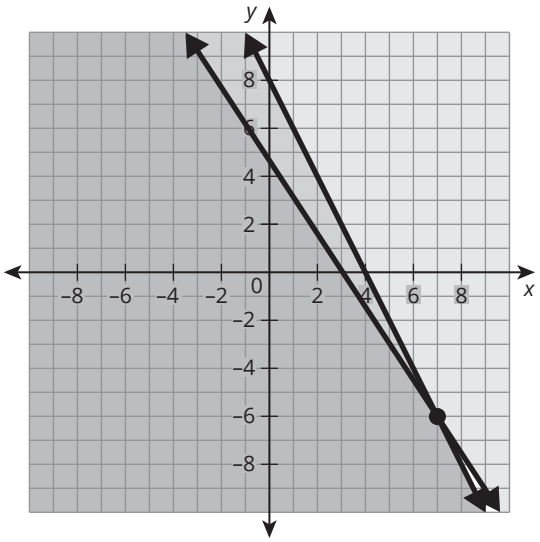
KEY TERMS

- Gaussian elimination [eliminación gaussian]
- solution set of a system of linear inequalities [solución de un sistema de desigualdades lineales]
- linear programming [programación lineal]
- matrix [matriz]
- dimensions [dimensiones]
- square matrix
- matrix element [elemento de una matriz]
- matrix multiplication [multiplicación de matrices]
- identity matrix [matriz de identidad]
- multiplicative inverse of a matrix [inversa multiplicativa de una matriz]
- matrix equation [ecuación matricial]
- coefficient matrix [matriz de coeficientes]
- variable matrix [matriz de variables]
- constant matrix [matriz constante]
- inverse of a function [inversa de una función]
- one-to-one function

Refer to the Math Glossary for definitions of the Key Terms.

Where are we now?

The **solution set of a system of inequalities** is the intersection of the solutions to each inequality. Every point that both solutions share satisfies all inequalities in the system.



A **matrix (plural matrices)** is an array of numbers composed of rows and columns. A matrix is usually designated by a capital letter.

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 2 & 1 & 0 & 1 & 1 \end{bmatrix}$$

In **Lesson 1: Solving Systems of Equations**, students review solving linear equations in two variables. Then, they use substitution and *Gaussian elimination* to solve systems of three linear equations in three variables.

The goal of **Gaussian elimination** is to use linear combinations to isolate one variable for each equation. When using this method, you can:

- swap the positions of two equations.
- multiply an equation by a nonzero constant.
- add one equation to the multiple of another.

For example, you can solve the system using Gaussian elimination.

$$\begin{cases} x + 5y - 6z = 24 \\ -x - 4y + 5z = -21 \\ 5x - 4y + 2z = 21 \end{cases}$$

Add the first and second equation and replace the second equation.

$$\begin{array}{r} x + 5y - 6z = 24 \\ -x - 4y + 5z = -21 \\ \hline y - z = 3 \end{array} \longrightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ 5x - 4y + 2z = 21 \end{cases}$$

Multiply the first equation by -5 and add it to the third equation. Replace the third equation.

$$\begin{array}{r} -5x - 25y + 30z = -120 \\ 5x - 4y + 2z = 21 \\ \hline -29y + 32z = -99 \end{array} \longrightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ -29y + 32z = -99 \end{cases}$$

Multiply the second equation by -9 and add it to the third equation. Replace the third equation.

$$\begin{array}{r} 29y - 29z = 87 \\ -29y + 32z = -99 \\ \hline 3z = -12 \end{array} \longrightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ 3z = -12 \end{cases}$$

Multiply the third equation by $\frac{1}{3}$ and add it to the second equation. Replace the second equation.

$$\begin{array}{r} y - z = 3 \\ z = -4 \\ \hline y = -1 \end{array} \longrightarrow \begin{cases} x + 5y - 6z = 24 \\ y = -3 \\ 3z = -12 \end{cases}$$

Multiply the second equation by -5 and add it to the first equation. Replace the first equation.

$$\begin{array}{r} x + 5y - 6z = 24 \\ -5y = 5 \\ \hline x - 6z = 29 \end{array} \longrightarrow \begin{cases} x - 6z = 29 \\ y = -1 \\ 3z = -12 \end{cases}$$

Multiply the third equation by 2 and add it to the first equation. Replace the first equation.

$$\begin{array}{r} x - 6z = 29 \\ 6z = -24 \\ \hline x = 5 \end{array} \longrightarrow \begin{cases} x = 5 \\ y = -1 \\ 3z = -12 \end{cases}$$

Multiply the third equation by $\frac{1}{3}$. Replace the third equation.

$$\begin{array}{r} \frac{1}{3}(3z = -12) \\ z = -4 \end{array} \longrightarrow \begin{cases} x = 5 \\ y = -1 \\ z = -4 \end{cases}$$

The solution to the system is $(x, y, z) = (5, -1, -4)$.

Math in the Real World

Your body is an amazing collection of different systems. Your cardiovascular system pumps blood throughout your body, your skeletal system provides shape and support, and your nervous system controls communication between your senses and your brain. Your skin, including your hair and fingernails, is a system all by itself—the integumentary system—and it protects all of your body's other systems. You also have a digestive system, endocrine system, excretory system, immune system, muscular system, reproductive system, and respiratory system.

Your body solves problems every day to stay balanced. Scientists use systems of equations to model how the human body works. For example, how oxygen levels in your blood affect your heart rate. So, your body is basically a walking math problem!

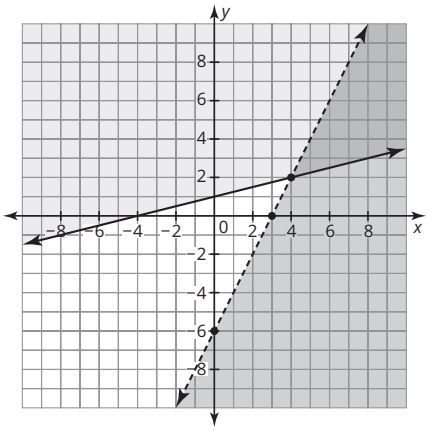
In **Lesson 2: Optimization**, students solve systems of two or more linear equalities.

Systems of Linear Inequalities

The **solution set of a system of inequalities** is the intersection of the solutions to each inequality. Every point in the intersection satisfies all inequalities in the system.

For example, consider the system $\begin{cases} y < 2x - 6 \\ y \geq \frac{1}{4}x + 1 \end{cases}$.

Graph the system of linear inequalities.



The solution to the system of linear inequalities is the overlapping region of the solutions to each inequality in the system. For example, the points (6, 4) and (8, 8) are both solutions to the system of linear inequalities.

Students then use technology with matrices to solve systems of linear equations in three variables In **Lesson 3: Solving Systems with Matrices**. The last lesson in the topic, **Lesson 4: Inverses of Linear Functions** introduces students to the *inverse of a function*.

Inverse of a Function

The **inverse of a function** is an equation that results from exchanging the independent and dependent variables.

For example, determine the inverse of the linear function $f(x) = 2x - 1$.

To determine the inverse, first rewrite the equation using y instead of $f(x)$.

$y = 2x - 1$

Then swap the x and y variables and solve for y.

$x = 2y - 1$

$\frac{x + 1}{2} = y$

$x + 1 = 2y$

The inverse of $y = 2x - 1$ is $y = \frac{x + 1}{2} = y$

MODULE
2

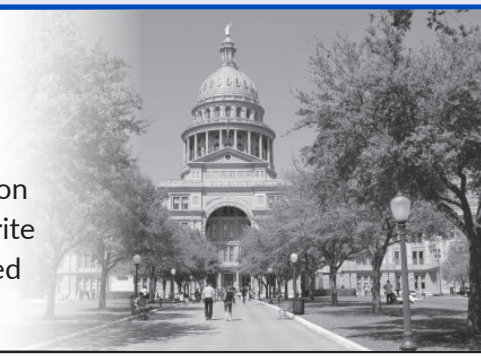
Exploring Quadratic Functions

TOPIC 1 Quadratic Functions and Equations 13

TOPIC 2 Applications of Quadratics 19

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TOPIC 1 Quadratic Functions and Equations



This topic provides students with an opportunity to review and expand upon what they already know about quadratic functions from Algebra I. They write and solve quadratic equations in various forms. Students are also introduced to the complex number system.

Where have we been?

In Algebra I, students gained extensive experience with multiple representations of linear, exponential, and quadratic functions. This topic serves as an opportunity for students to recall what they already know and prepare for how they will use this knowledge throughout the rest of the course. They are familiar with solving quadratic equations and have considered the case in which the graph of a quadratic equation does not cross the *x*-axis, yielding no real roots. When learning to solve quadratic equations with imaginary roots, students build upon these experiences and expand their knowledge of the real number system to the complex number system.

Where are we going?

Throughout the rest of this course, students will explore more complex functions: polynomials, rational functions, radical functions, and logarithmic functions. *Quadratic Functions and Equations* is a critical topic for preparing students for this work, which includes identifying key attributes, connecting representations, writing equations, using equations to solve for unknown values, analyzing graphical representations, and making connections to the real world.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Functions are an important topic to know for modeling various phenomena such as the trajectory of objects, optimization problems in business and economics, and natural patterns like the shape of parabolic bridges or satellite dishes.

HERE IS A SAMPLE QUESTION

What are the coordinates of the vertex of the parabola whose equation is $y = 2x^2 + 4x - 5$?

If you represent the quadratic equation as $ax^2 + bx + c$, then the *x*-coordinate of the vertex is $\frac{-b}{2a} = \frac{-4}{2(2)} = -1$. The *y*-coordinate of the vertex is then $2(-1)^2 + 4(-1) - 5 = -7$.

So, the vertex is at $(-1, -7)$.

KEY TERMS

- standard form of a quadratic equation [forma estandarizada/general de una ecuación cuadrática]
- factored form of a quadratic equation [forma factorizada de una ecuación cuadrática]
- vertex form of a quadratic equation [forma de vértice de una ecuación cuadrática]
- concavity of a parabola [concavidad de una parábola]
- quadratic formula [fórmula cuadrática]
- the number *i* [el número *i*]
- imaginary zeros [ceros imaginarios]
- imaginary roots [raíces imaginarias]
- complex numbers [números complejos]
- real part of a complex number [parte real de un número complejo]
- imaginary part of a complex number [parte imaginaria de un número complejo]
- imaginary numbers [números imaginarios]
- pure imaginary number [número imaginario puro]
- Fundamental Theorem of Algebra [Teorema fundamental del álgebra]

Refer to the Math Glossary for definitions of the Key Terms.

Where are we now?

The **quadratic formula** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can be used to calculate the solutions to any quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c represent real numbers and $a \neq 0$.

The number *i* is a number such that $i^2 = -1$.

The **Fundamental Theorem of Algebra** states that any polynomial equation of degree n must have exactly n complex roots or solutions; also, every polynomial function of degree n must have exactly n complex zeros. However, any root or zero may be a multiple root or zero.

The polynomial equation $x^5 + x^2 - 6 = 0$ has 5 complex roots because the polynomial $x^5 + x^2 - 6$ has a degree of 5.

In **Lesson 1: Forms of Quadratic Functions**, students review standard form, factored form, and vertex form of a quadratic function. They rewrite quadratic functions in different forms and identify key attributes of quadratic functions.

Forms of Quadratics

standard form of a quadratic function	**factored form of a quadratic function**	**vertex form of a quadratic function**
A quadratic function written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$, is in standard form, or general form. The function $f(x) = x^2 - 10x + 1$ is written in standard form.	A quadratic function written in factored form is in the form $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$. The function $h(x) = x^2 - 8x + 12$ written in factored form is $h(x) = (x - 6)(x - 2)$.	A quadratic function written in vertex form is in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$. The quadratic equation $y = 2(x - 5)^2 + 10$ is written in vertex form. The vertex of the graph is the point (5, 10).

In **Lesson 2: Writing Quadratic Functions**, students write equations to represent a quadratic function that goes through specific points.

Writing a Unique Quadratic Function

You can write a unique quadratic function given a vertex and a point on the parabola.

For example, consider a parabola with vertex (5, 2) that passes through the point (4, 9).

Substitute the given values into the vertex form of the function and solve for a.

$$f(x) = a(x - h)^2 + k$$
$$9 = a(4 - 5)^2 + 2$$
$$9 = a(-1)^2 + 2$$
$$9 = 1a + 2$$
$$7 = a$$

Finally, substitute the a-value into the function.

$$f(x) = 7(x - 5)^2 + 2$$

You can also write a unique quadratic function given the roots and a point on the parabola.

For example, consider a parabola with roots at (−2, 0) and (4, 0) that passes through the point (1, 6).

Substitute the given values into the vertex form of the function and solve for a.

$$f(x) = a(x - r_1)(x - r_2)$$
$$6 = a(1 + 2)(1 - 4)$$
$$6 = a(3)(-3)$$
$$6 = -9a$$
$$-\frac{2}{3} = a$$

Finally, substitute the a-value into the function.

$$f(x) = -\frac{2}{3}(x + 2)(x - 4)$$

You can determine a unique quadratic function algebraically given three reference points. Substitute the x- and y-values of each point into the standard form, $y = ax^2 + bx + c$ to create a system of three equations. Then, use elimination and substitution to solve for a, b, and c.

In **Lesson 3: Solving Quadratic Equations**, students solve quadratic functions with real solutions using factoring, completing the square, and the quadratic formula.

Solving Quadratic Equations

Students will use three major ways to solve quadratic equations depending on the situation or the format of a question.

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Solving by Factoring

$$x^2 - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$

$$\begin{array}{rcl} (x-3)=0 & \text{or} & (x-1)=0 \\ x-3+3=0+3 & \text{or} & x-1+1=0+1 \\ x=3 & \text{or} & x=1 \end{array}$$

Solving by Completing the Square

$$x^2 - 4x + 2 = 0$$
$$x^2 - 4x + 2 - 2 = 0 - 2$$
$$x^2 - 4x = -2$$
$$x^2 - 4x + ? = -2 + ?$$
$$x^2 - 4x + 4 = -2 + 4$$
$$x^2 - 4x + 4 = 2$$
$$(x - 2)^2 = 2$$

Once the function is in this vertex form students can solve by using square roots.

$$\sqrt{(x-2)^2} = \pm\sqrt{2}$$
$$x - 2 = \pm\sqrt{2}$$
$$x - 2 = \sqrt{2} \text{ and } x - 2 = -\sqrt{2}$$
$$x = 2 + \sqrt{2} \text{ and } x = 2 - \sqrt{2}$$
$$x \approx 3.41 \text{ and } x \approx 0.59$$

Solving by the Quadratic Formula

The **quadratic formula**, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, can be used to calculate the solutions to any quadratic equation of the form $ax^2 + bx + c = 0$, where a, b and c represent real numbers and $a \neq 0$.

For example, given the function $f(x) = 2x^2 - 4x - 3$ you can identify the values of a, b and c.

$$a = 2; b = -4; c = -3$$

Then you use the quadratic formula to solve.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$$
$$x = \frac{4 \pm \sqrt{16 + 24}}{4}$$
$$x = \frac{4 \pm \sqrt{40}}{4}$$
$$x \approx \frac{4 + 6.325}{4} \approx 2.581 \quad \text{or} \quad x \approx \frac{4 - 6.325}{4} \approx -0.581$$

The roots are $x \approx 2.581$ and $x \approx -0.581$.

In **Lesson 4: Imaginary and Complex Numbers**, students learn that in order to calculate the square root of any real number, there must be some way to calculate the square root of a negative number. That is, there must be a number such that when it is squared, it is equal to a negative number. For this reason, mathematicians defined what is called **the number *i***. The number i is a number such that $i^2 = -1$.

Rewriting Expression with Complex Numbers

For example, you can rewrite the expression $\sqrt{-25}$ using i.

Factor out -1. $\sqrt{-25} = \sqrt{(-1)(25)}$

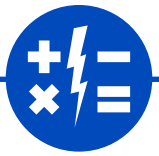
Rewrite the radical expression. $= \sqrt{-1} \cdot \sqrt{25}$

Apply the square root on $\sqrt{25}$. $= 5\sqrt{-1}$

Rewrite $\sqrt{-1}$ as i. $= 5i$

So, $\sqrt{-25}$ can be rewritten as $5i$.

Functions and equations that have imaginary solutions have **imaginary roots** or **imaginary zeros**, which are the solutions.



Math in the Real World

Have you ever watched a basketball shot arc through the air? That perfect swoosh follows a parabola — a path shaped by a quadratic function. Engineers rely on quadratics to design things like satellite dishes and suspension bridges because their curves are super strong and efficient. Even in business, companies use them to figure out how to maximize profits or minimize costs — because, let's face it, everyone loves more money! Quadratics even sneak into video games when calculating jumps or movement in physics engines. So, the next time you see something flying, bending, or optimizing, just remember: it's math working its magic behind the scenes!

Family Guide

MODULE 2 Exploring Quadratic Functions **Algebra II**

TOPIC 2 Applications of Quadratics



Applications of Quadratics provides students with an opportunity to extend what they have learned about quadratics in Algebra I and this course. They start by modeling and solving quadratic inequalities. From there, students model and solve systems of quadratic and linear functions. Students are presented with a real-world situation and use familiar strategies to fit a quadratic regression function.

Where have we been?

In Algebra I, students have experience with multiple representations of linear, exponential, and quadratic functions. This topic serves as an opportunity for students to extend what they already know about quadratic functions and prepare for how they will use this knowledge throughout the rest of the course.

Where are we going?

This topic is critical for preparing students to explore more complex functions, which includes identifying key attributes, connecting representations, analyzing graphical representations, and making connections to the real world.

TALKING POINTS

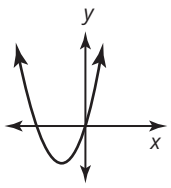
DISCUSS WITH YOUR STUDENT

Quadratic functions are an important topic to know for modeling real-world phenomena such as projectile motion and design of suspension bridges.

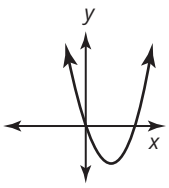
HERE IS A SAMPLE QUESTION

If $p = 0$ and $q < 0$, then which shows the graph of $f(x) = (x - p)(x - q)$?

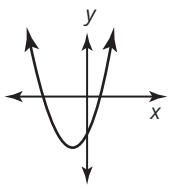
A.



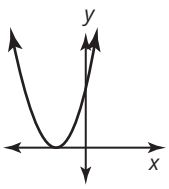
B.



C.



D.



The function crosses the x-axis at p and q. Since $p = 0$ and q is negative, the function crosses the x-axis at $x = 0$ and at some negative x-value. Choice A, then, is the correct graph.

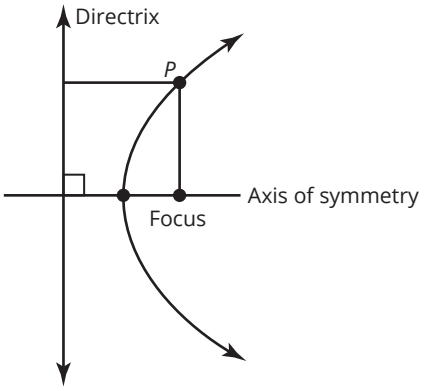
KEY TERMS

- interval notation [notación de intervalo]
- regression function [función de regresión]
- coefficient of determination [coeficiente de determinación]
- parabola [parábola]
- focus [foco]
- directrix [directriz]
- vertex of a parabola [vértice de una parábola]
- geometric form of a parabola [forma geométrica de una parábola]

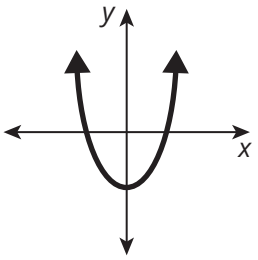
Refer to the Math Glossary for definitions of the Key Terms.

Where are we now?

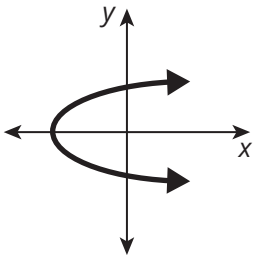
A **parabola** is a set of points in a plane equidistant from a *focus* and a *directrix*. The **focus** is a point that lies inside the parabola on the axis of symmetry. The **directrix** is a line perpendicular to the axis of symmetry that lies outside the parabola and does not intersect the parabola.



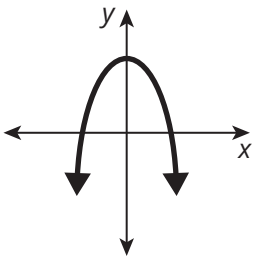
The concavity of a parabola describes the orientation of the curvature of the parabola.



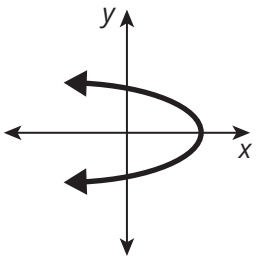
concave up



concave right



concave down



concave left

In **Lesson 1: Solving Quadratic Inequalities**, students graph and solve quadratic inequalities.

Solving Quadratic Inequalities

Consider the inequality $x^2 - 4x + 3 < 0$.

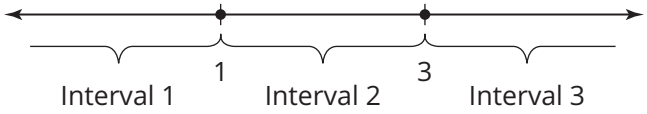
Write the corresponding quadratic equation. $x^2 - 4x + 3 = 0$

Calculate the roots of the quadratic equation using an appropriate method. $(x - 3)(x - 1) = 0$

$(x - 3) = 0$ or $(x - 1) = 0$

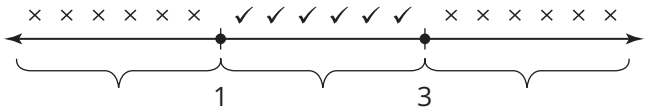
$x = 3$ or $x = 1$

Plot the roots to divide the number line into three regions.



Choose a value from each interval to test in the original inequality.

Identify the solution set as the interval(s) in which your test value satisfies the inequality.



Interval 1	Interval 2	Interval 3
Try $x = 0$	Try $x = 2$	Try $x = 4$
$0^2 - 4(0) + 3 < 0$	$2^2 - 4(2) + 3 < 0$	$4^2 - 4(4) + 3 < 0$
$3 < 0$ ×	$4 - 8 + 3 < 0$	$16 - 16 + 3 < 0$
	$-1 < 0$ ✓	$3 < 0$ ×

Interval 2 satisfies the original inequality, so the solution set includes all numbers between 1 and 3. You can write the solution set using **interval notation** or inequalities. Solution: $x \in (1, 3)$, or $1 < x < 3$.

In **Lesson 2: Systems of Linear and Quadratic Equations**, students represent real-world situations with a system of equations composed of a linear and quadratic equation. They solve the system graphically and algebraically.

Solving a System of Linear and Quadratic Equations

The graph of a linear equation and the graph of a quadratic equation can intersect at two points, at one point, or not at all.

The method to determine the solution or solutions of a system involving a quadratic equation and a linear equation is similar to solving a system of two linear equations. First, substitute one equation into the other. Then, solve the resulting equation for x and calculate the corresponding values for y. These values represent the point(s) of intersection. Finally, graph each equation of the system to verify the points of intersection.

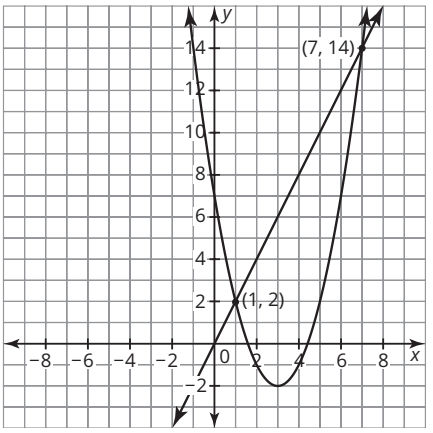
CC BY NC

$$\begin{cases} y = x^2 - 6x + 7 \\ y = 2x \end{cases}$$

$$2x = x^2 - 6x + 7$$
$$0 = x^2 - 8x + 7$$
$$0 = (x - 7)(x - 1)$$

$x - 7 = 0$	$x - 1 = 0$
$x = 7$	$x = 1$
$y = 2(7)$	$y = 2(1)$
$y = 14$	$y = 2$

The system has two solutions: (7, 14) and (1, 2).



In **Lesson 3: Modeling with Linear and Quadratic Functions**, students determine whether data sets are best represented by a linear or quadratic function. They write linear and quadratic regression functions and use the regression functions to make predictions about about a problem situation..

You can use first and second differences to determine whether a table of values represents a linear or quadratic function. Linear functions have constant first differences and second differences of 0. Quadratic functions have changing first differences and constant second differences.

Linear function: $f(x) = -x + 2$

x	$f(x)$	First Differences	Second Differences
0	2		
		−1	
1	1		0
		−1	
2	0		0
		−1	
3	−1		0
		−1	
4	−2		

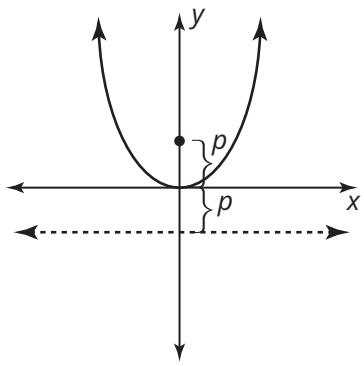
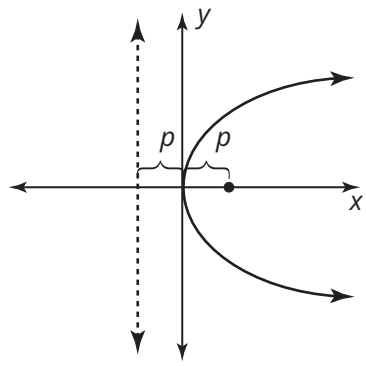
Quadratic function: $f(x) = 2x^2 - 3x$

x	$f(x)$	First Differences	Second Differences
0	0		
		−1	
1	−1		4
		3	
2	2		4
		7	
3	9		4
		11	
4	20		

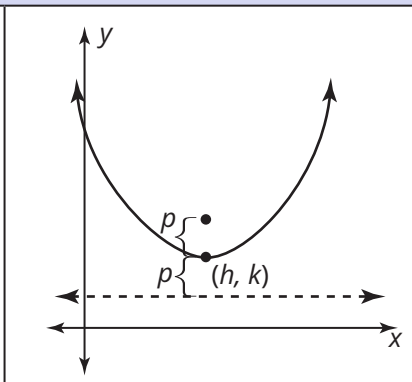
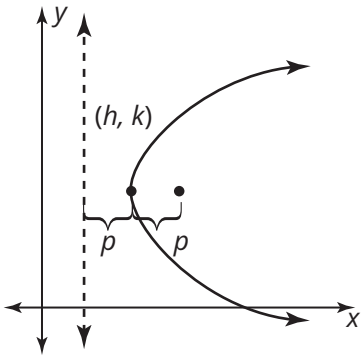
In **Lesson 4: Equation of a Parabola**, students explore the geometric form of a parabola.

Geometric Form of a Parabola

The **geometric form of a parabola with vertex** at the origin is an equation of the form $x^2 = 4py$ or $y^2 = 4px$, where p represents the distance from the vertex to the focus.

Parabola (Vertex at the Origin)		
Graph		
Equation of Parabola	$x^2 = 4py$	$y^2 = 4px$
Orientation of Parabola	vertical	horizontal
Axis of Symmetry	y-axis	x-axis
Coordinates of Vertex	(0, 0)	(0, 0)
Coordinates of Focus	$(0, p)$	$(p, 0)$
Equation of Directrix	$y = p$	$x = p$
Concavity	up or down	right or left

The geometric forms of parabolas with a vertex at (h, k) are $(x - h)^2 = 4p(y - k)$ and $(y - k)^2 = 4p(x - h)$.

Parabola (Vertex at (h, k))		
Graph		
Equation of Parabola	$(x - h)^2 = 4p(y - k)$	$(y - k)^2 = 4p(x - h)$
Orientation of Parabola	vertical	horizontal
Axis of Symmetry	$x = h$	$y = k$
Coordinates of Vertex	(h, k)	(h, k)
Coordinates of Focus	$(h, k + p)$	$(h + p, k)$
Equation of Directrix	$y = k - p$	$x = h - p$
Concavity	up or down	right or left

Math in the Real World

Your body is an incredible network of systems that work together to maintain health and support life, and understanding these systems is critical for careers in health and medical science. For example, in the cardiovascular system, healthcare professionals use systems of equations to calculate cardiac output, blood flow rates, and blood pressure to diagnose heart conditions. In respiratory therapy, systems of equations help analyze oxygen exchange and lung capacity to treat conditions like asthma or pneumonia. The endocrine system relies on equations to monitor hormone levels, such as insulin and glucose balance, which are vital for managing diabetes. In medical imaging, such as X-rays and MRIs, systems of equations are used to create accurate images of the skeletal and muscular systems, helping healthcare workers detect fractures or tissue damage. These examples show how understanding systems—interconnected parts working toward a specific function—plays a critical role in health science careers, from diagnosis to treatment. Why do you think these systems are called "systems," and how might their interconnectedness help you solve real-world health challenges?

MODULE
3

Analyzing Structure

TOPIC 1 Composing and Decomposing Functions 27

TOPIC 2 Attributes of Cubic Functions 33

TOPIC 3 Relating Factors and Zeros 37

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TOPIC 1 Composing and Decomposing Functions



This topic introduces students to the concept of building new functions on the coordinate plane by operating on or translating functions. Familiar with translating and dilating functions by a constant, students explore the higher-degree functions that result when translating and dilating functions by non-constant amounts. After building functions from linear and quadratic factors, students decompose functions.

Where have we been?

Starting in Grade 8, students studied the key attributes of linear functions, including the *x*- and *y*-intercept and slope. In Algebra I, students examined the graphs of linear and quadratic functions and learned that the *x*-intercepts of a graph are the zeros of the function. They have used the linear factors of a quadratic function to demonstrate the zero product property and have used this property to solve quadratic equations.

Where are we going?

In the upcoming topics in this course, students will continue to analyze the structure of polynomial functions. Understanding zeros graphically lays the foundation for understanding and solving for zeros algebraically, and the connection to graphical representations of functions helps students to factor polynomials using long division and synthetic division.

TALKING POINTS

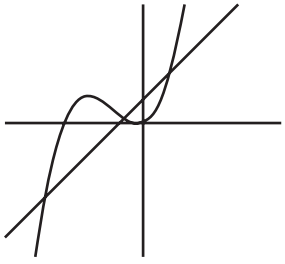
DISCUSS WITH YOUR STUDENT

Cubic functions are an important topic to know for modeling complex relationships such as vehicle dynamics, optimization, and growth in areas such as physics, engineering, economics, and computer graphics.

HERE IS A SAMPLE QUESTION

The graph of a cubic function and the graph of $y = x + 1$ in the *xy*-plane can intersect in at most how many points?

You can visualize the linear function and a cubic function as shown:



At most, there are 3 intersection points.

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KEY TERMS

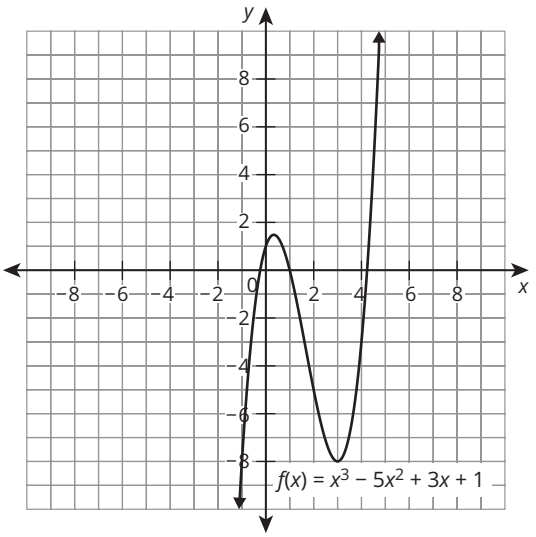
- cubic function [función cúbica]
- relative maximum [máximo relativo]
- relative minimum [mínimo relativo]
- multiplicity [multiplicidad]

Refer to the Math Glossary for definitions of the Key Terms.

Where are we now?

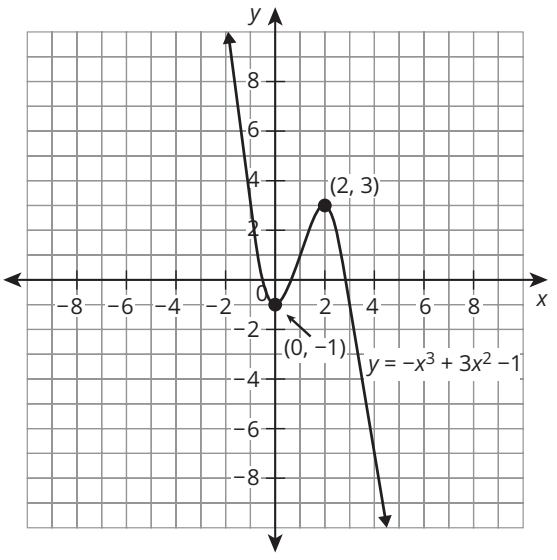
A **cubic function** is a function that can be written in the standard form $f(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$.

The function $f(x) = x^3 - 5x^2 + 3x + 1$ is a cubic function.



A **relative maximum** is the highest point in a particular section of a graph.

A **relative minimum** is the lowest point in a particular section of a graph.



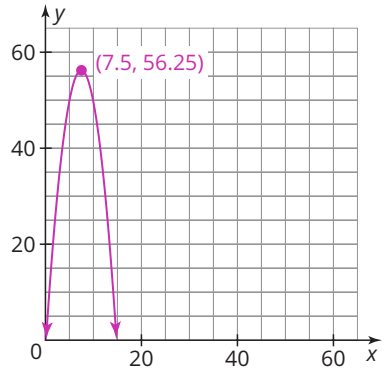
The graph shown has a relative maximum at (2, 3).

The graph shown has a relative minimum at (0, −1).

In this topic, students are introduced to cubic functions and discover how they are created from functions they already know. In **Lesson 1: Modeling with Functions**, students explore how a quadratic function can model area. They relate the factors of the quadratic function to the key attributes of its graph.

Modeling Area with a Quadratic Function

Consider a situation in which Jasmine wants to build a rectangular pen for her rabbit from a length of foldable mesh wire. She has a total length of 30 feet of mesh wire and wants to determine the maximum area for the pen.



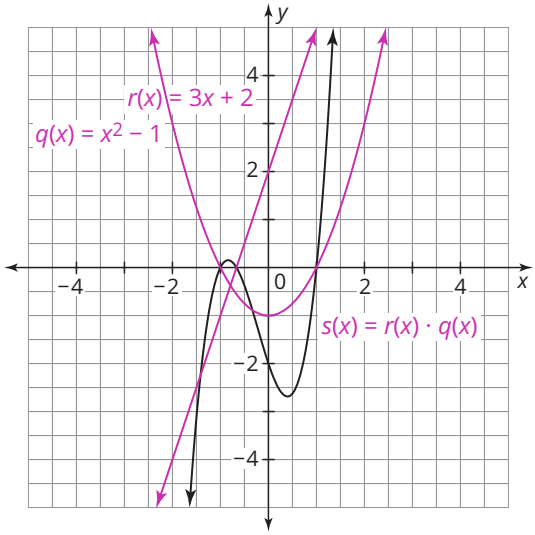
The function $A(w) = w(15 - w)$, or $A(w) = -w^2 + 15w$, models the area of the pen where w represents the width of the pen.

The graph of the function has a vertex at (7.5, 56.25). This means that the maximum area the pen can be is 56.25 square feet when the width of the pen is 7.5 feet.

In **Lesson 2: Transforming Function Shapes,** students consider the effect of transforming functions by non-constant factors.

Dilating a Quadratic Function by a Linear Function

When a quadratic function is dilated vertically by multiplying the function by a linear function that is not a constant function, the resulting function is a third-degree function.



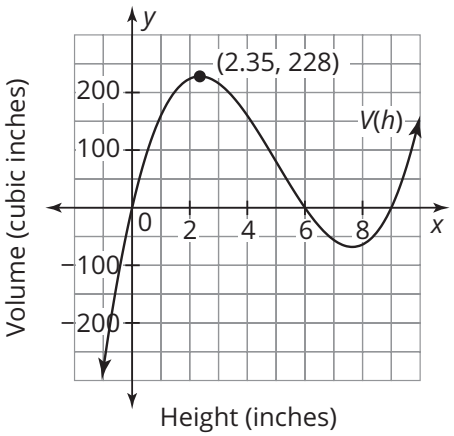
For example, consider the graphs of $q(x) = x^2 - 1$, $r(x) = 3x + 2$, and $s(x) = r(x) \cdot q(x)$. In this case, the a-value in $s(x)$ representing the vertical dilation of $q(x)$ is $r(x)$, or $(3x + 2)$.

Since $s(x)$ is a vertical dilation of $q(x)$ by a linear function, $r(x)$, the resulting function is a third-degree function. Written as an equation, $s(x) = (3x + 2)(x^2 - 1)$, or $s(x) = 3x^3 + 2x^2 - 3x - 2$.

The function $s(x)$ shares zeros with both $q(x)$ and $r(x)$.

In **Lesson 3: Exploring Cubic Functions,** students are formally introduced to cubic functions by investigating the volume of a planter.

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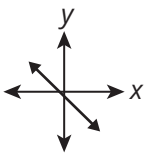


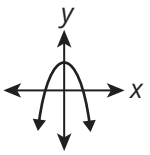
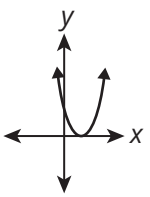
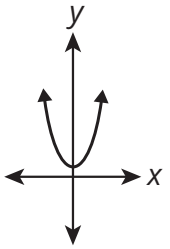
Modeling with a Cubic Function

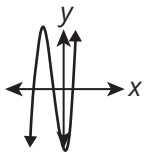
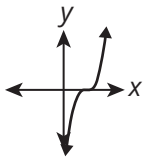
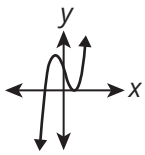
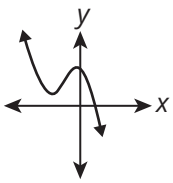
The function $V(h) = h(12 - 2h)(18 - 2h)$ models the volume of a planter box with height, h. The graph of the function is shown. The function has a relative maximum at (2.35, 228) and a relative minimum at (7.65, −68.16). The relative maximum can be interpreted in the context of the situation. A height of 2.35 inches will produce a prism with a maximum volume of 228 cubic inches.

In **Lesson 4: Decomposing Cubic Functions,** students investigate the multiplicity of the zeros of a polynomial function. They use these zeros, with multiplicity, to show the decompositions of quadratic and cubic functions into their linear and quadratic factors

Multiplicity is how many times a particular number is a zero for a given function. You can use the multiplicity of a zero to help determine the shape of a graph.

Linear Function	
Zeros	**Graph**
1 real	

Quadratic Function	
Zeros	**Graph**
2 real distinct	
1 real (multiplicity 2)	
2 imaginary	

Cubic Function	
Zeros	Graph
3 real distinct	
1 real (multiplicity 3)	
1 real (multiplicity 1) 1 real (multiplicity 2)	
1 real 2 imaginary	

Math in the Real World

Costs in a business don't always increase in a simple, straight line. Sometimes they grow quickly, slow down, and then speed up again as production scales. This is where the cubic cost function comes in—it's a model that uses a cubic equation in the form $C(x) = ax^3 + bx^2 + cx + d$ to show the relationship between the number of items produced (output) and the total cost of production. Each part of the equation tells a story:

- The d represents fixed costs, like rent or equipment, which you pay no matter how much (or little) you produce.
- The cx term shows variable costs, like the price of ingredients or wages, which grow steadily as you produce more.
- The bx^2 captures how economies of scale can make production more efficient for a while, reducing the overall growth rate of costs.
- The ax^3 shows how inefficiencies, like overcrowded factories or overworked equipment, can cause costs to increase sharply when production gets too high.

This function is super useful because it doesn't just show costs; it helps businesses make decisions. For example, they can use it to find the "sweet spot" where production is efficient without being overly expensive.

TOPIC 2 Attributes of Cubic Functions

This topic opens with a study of power functions of the form $P(x) = ax^n$. Students are then introduced to cubic function transformations. They build cubic functions both graphically and algebraically and then close the topic by analyzing cubic functions and applying a cubic function to a real-world situation. Throughout, students use their prior knowledge of function attributes and transformations to generalize to cubic functions.



Where have we been?

Students have extensive experience analyzing nonlinear functions, such as quadratic functions and absolute value functions, identifying their key attributes, and graphing their transformations. They have built functions from other functions. Students have manipulated and solved linear, quadratic, and absolute value equations.

Where are we going?

Students will work with polynomial expressions and equations closely in the next topic. Polynomial functions are used throughout the sciences and in engineering to model complex real-world situations and data sets.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Polynomial functions play a crucial role in fields like computer graphics, engineering, and bridge design, forming the foundation for advanced technologies and modern structures. They are also used to model growth rates and predict monthly revenue by business owners.

QUESTIONS TO ASK

The function $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, and d are constants, is graphed on the coordinate plane. Which statements about the graph of $f(x)$ must always be true?

A. The graph of $f(x)$ has exactly one relative maximum and one relative minimum.

B. The graph of $f(x)$ crosses the x-axis at least once.

C. The graph of $f(x)$ has at most three x-intercepts.

D. The graph of $f(x)$ is symmetric about the y-axis.

The correct answer is C. The other statements can be true but are not true for all values of a, b, c, and d.

KEY TERMS

- power function
- symmetric about a line [simétrico respecto a una línea/recta]
- symmetric about a point [simétrico respecto a un punto]
- polynomial function [función polinomial/ polinómica]
- absolute maximum [máximo absoluto]
- absolute minimum [mínimo absoluto]
- extrema [extremos]

Refer to the Math Glossary for definitions of the Key Terms.

Where are we now?

In this material, a **power function** is a function of the form $P(x) = ax^n$ where n is a non-negative integer.

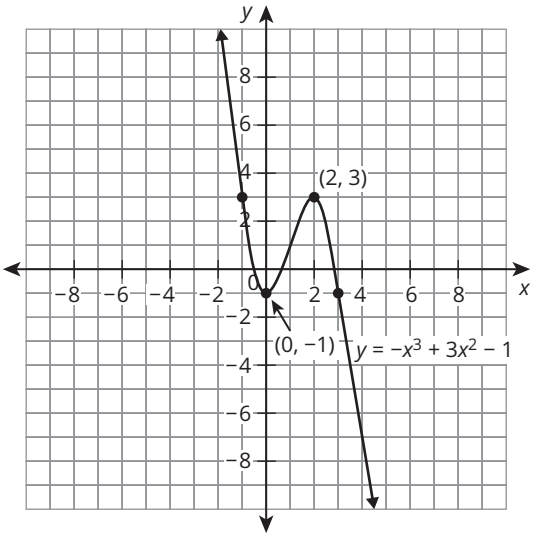
The functions $f(x) = x$, $f(x) = x^2$, and $f(x) = x^3$ are power functions.

A **polynomial function** is a function that can be written in the form $p(x) = \square x^n + \square x^{n-1} + \cdots + \square x^2 + \square x + \square$, where the coefficients, represented by each $\square$, are complex numbers and the exponents are nonnegative integers.

The function $f(x) = 5x^3 + 3x^2 + x + 1$ is a polynomial function.

Extrema are the set of all relative maximums, relative minimums, absolute maximums, and absolute minimums for a graph.

The graph shown has 2 extrema, a relative maximum at (2, 3) and a relative minimum at (0, −1).

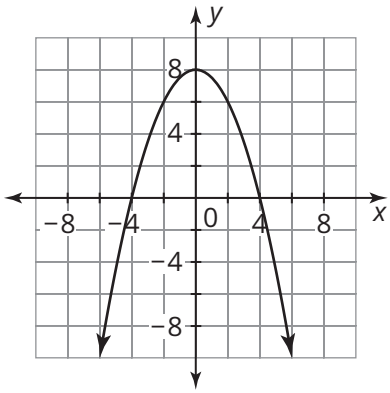


In **Lesson 1: Symmetric functions,** students explore power functions and their symmetry.

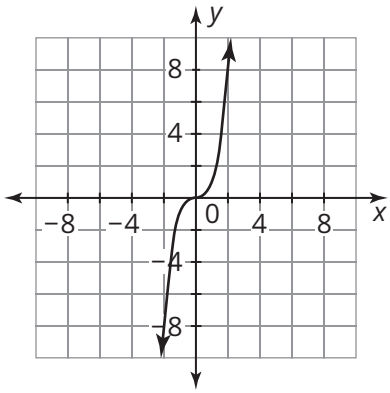
Identifying Symmetry

If a graph is symmetric about the y-axis, then $f(x) = f(-x)$. If a function is symmetric about the origin, then $f(x) = -f(-x)$.

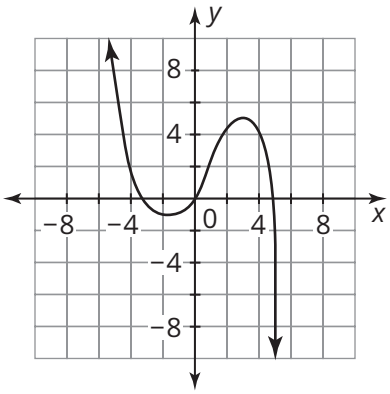
For example, consider the functions shown.



The function is symmetric about the y-axis.



The function symmetric about the origin.



The function is not symmetric.

In **Lesson 2: Transformations of Cubic Functions,** students explore the transformations of polynomial functions before specifically focusing on transforming cubic functions.

Transformations of Polynomials

Students have transformed linear, quadratic, and absolute value functions using the transformation function form $g(x) = af(b(x - c)) + d$. The same concept applies to cubic functions.

Function Form	Equation Information	Description of Transformation of Graph
$y = af(x)$	$\lvert a\rvert > 1$	vertical stretch of the graph by a factor of a units
	$0 < \lvert a\rvert < 1$	vertical compression of the graph by a factor of a units
	$a < 0$	reflection across the x-axis
$y = f(bx)$	$\lvert b\rvert > 1$	compressed horizontally by a factor of $\frac{1}{\lvert b\rvert}$
	$0 < \lvert b\rvert < 1$	stretched horizontally by a factor of $\frac{1}{\lvert b\rvert}$
	$b < 0$	reflection across the y-axis
$y = f(x - c)$	$c > 0$	horizontal shift right c units
	$c < 0$	horizontal shift left c units
$y = f(x) + d$	$d > 0$	vertical shift up d units
	$d < 0$	vertical shift down d units

Math in the Real World

Children typically resemble their parents due to the inheritance of genes passed from parents to offspring. These genes, carried on specific strands of DNA, are a biological code that, together with environmental conditions, determine the physical and behavioral traits of organisms. Similarly in mathematics, polynomials have "inherited" attributes that help us predict their behavior. Just as scientists can make predictions about the traits of offspring based on genetic information and the environmental conditions of the offspring, we can predict the graph and key attributes of a polynomial by understanding its components, such as its degree, coefficients, and leading term.

This comparison between genetics and polynomials is a great way to understand how patterns and structures influence outcomes, whether in science or math. Just like genes influence the traits of living things, the "attributes" of a polynomial shape its graph and behavior. It also shows how math is connected to the real world, especially to what you're learning in your health and science classes. Whether you're studying how traits are passed down in biology or analyzing data trends in health science, recognizing these patterns helps you see how math and science work together.

In **Lesson 3: Key Attributes of Cubic Functions,** students explore how they can use the key attributes of a function, such as the number of extrema, the *a*-value, and intercepts to sketch a graph of a function.

Key Attributes of Cubic Functions

Key attributes are features that make up different functions that students need to be able to identify from a graph or a function. Knowing what they are and how to identify them will help students be successful in later modules with new function types.

Key Attributes to Know	
• Domain • Range • Intercepts • Multiplicity	• Maximums/minimums/extrema • Intervals of increase and decrease • Leading coefficient (positive or negative) • Symmetrical behavior (if any)

TOPIC 3 Relating Factors and Zeros



In this topic, students perform operations with polynomial expressions. The factor theorem is introduced and used to determine whether a linear expression is a factor of a polynomial function. Students then perform polynomial long division, and the remainder theorem is stated and used to answer questions involving polynomial division with remainders. Methods of factoring polynomials are introduced, including using the rational roots theorem to identify possible roots.

Where have we been?

Students have factored mathematical expressions since elementary school. In Algebra I, they extended this skill set to factoring degree-2 polynomial equations in order to isolate key attributes of functions represented by those equations. Students can sketch the graph of a polynomial and identify its key attributes. Given a polynomial function, students can identify the number of zeros; given its graph, they can identify whether the roots are real or imaginary and whether they are distinct or have varying degrees of multiplicity.

Where are we going?

By relating the factors and zeros of polynomials, students are generalizing and extending their processes for solving quadratic equations and inequalities. The process of factoring will be necessary as students rewrite rational expressions and identify asymptotes. These skills are also applied in calculus to solve differential equations.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Factoring is an important topic to know for solving real-world problems, such as optimizing business processes and simplifying complex calculations in engineering.

HERE IS A SAMPLE QUESTION

For what value of the constant p does $\frac{6x^4 + 2x^2 - 8x - p}{x + 2}$ have no remainder?

When you use long division to divide the polynomials, you see that the quotient is $6x^3 - 12x^2 + 26x - 60$ with a remainder of $-p + 120$.

Thus, the quotient will have no remainder only if p has a value of 120.

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KEY TERMS

- factor theorem [teorema del factor]
- polynomial long division [división (larga) de polinomios]
- remainder theorem
- synthetic division [división sintética]
- rational root theorem

Refer to the Math Glossary for definitions of the Key Terms.

Where are we now?

The **factor theorem** states that a polynomial is divisible by $(x - r)$ if the value of the polynomial at r is zero.

The polynomial $x^3 - 2x^2 + 2x - 1$ is divisible by $x - 1$ because $(1)^3 - 2(1)^2 + 2(1) - 1 = 0$.

The **remainder theorem** states that the remainder when dividing a polynomial by $(x - r)$ is the value of the polynomial at r.

The value of the polynomial $x^2 + 5x + 2$ at 1 is $(1)^2 + 5(1) + 2 = 8$. So, the remainder when $x^2 + 5x + 2$ is divided by $x - 1$ is 8.

$$
\begin{array}{r}
x + 6 \\
x - 1 \overline{)\,x^2 + 5x + 2} \\
\underline{x^2 - x} \\
6x + 2 \\
\underline{6x - 6} \\
8
\end{array}
$$

In **Lesson 1: Adding, Subtracting, and Multiplying Polynomials,** students perform operations with higher-order polynomials.

Using a Multiplication Table to Multiply Polynomials

You can use a multiplication table to multiply higher-order polynomials.

Determine the product $(4x^3 - 2x^2 + 3x - 1)(2x^4 - x^2 + 4x)$.

Step 1: Use a multiplication table to determine the product of the factors.

·	$4x^3$	$-2x^2$	$3x$	-1
$2x^4$	$8x^7$	$-4x^6$	$6x^5$	$-2x^4$
$-x^2$	$-4x^5$	$2x^4$	$-3x^3$	x^2
$4x$	$16x^4$	$-8x^3$	$12x^2$	$-4x$

Step 2: Combine like terms.

$(4x^3 - 2x^2 + 3x - 1)(2x^4 - x^2 + 4x)$

$= 8x^7 - 4x^6 + 2x^5 + 16x^4 - 11x^3 + 13x^2 - 4x$

In **Lesson 2: Factoring Polynomials to Identify Zeros,** students extend what they have learned about factoring quadratic expressions to polynomial expressions.

Factoring by Grouping

In polynomials with 4 terms, you may notice that although not all terms share a common factor, pairs of terms might share a common factor. In this situation, you can *factor by grouping*.

For example, consider the polynomial $x^3 + 3x^2 - x - 3$.

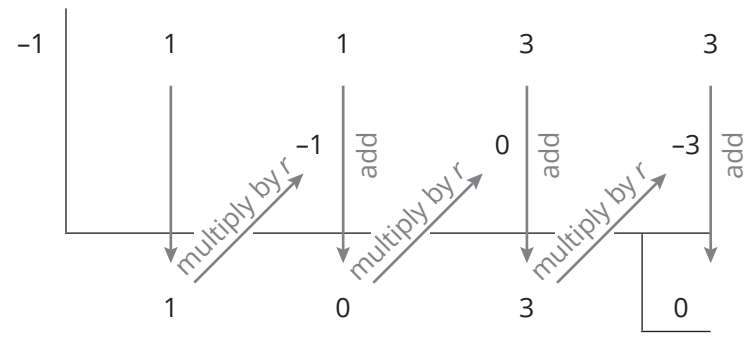
$(x^3 + 3x^2) + (-x - 3)$	Divide the polynomial into two groups.
$x^2(x + 3) - 1(x + 3)$	Find the common factor in each group.
$(x^2 - 1)(x + 3)$	Regroup the factors.
$(x - 1)(x + 1)(x + 3)$	Factor the first group using the difference of perfect squares.

The factored form of $x^3 + 3x^2 - x - 3$ is $(x - 1)(x + 1)(x + 3)$.

In **Lesson 3: Polynomial Division,** students expand their understanding of polynomial division to include higher order polynomials.

Polynomial Division

Two methods for dividing polynomials are long division and **synthetic division**. Long division is very similar to the method students used in previous grades to divide larger numbers. Synthetic division only works for linear divisors of the form $(x - r)$.

<table>
<tr><th>Long Division</th><th>Synthetic Division</th></tr>
<tr><td>$x^2 + 0x + 3$
$x + 1 \overline{)\, x^3 + x^2 + 3x + 3}$
$-(x^3 + x^2)$
$0x^2 + 3x$
$-(0x^2 + 0x)$
$3x + 3$
$-(3x + 3)$
0</td><td>$r = -1$

</td></tr>
<tr><td colspan="2">$(x^3 + x^2 + 3x + 3) \div (x + 1) = (x^2 + 3)$</td></tr>
</table>

Math in the Real World

In economics, businesses use polynomial equations to analyze supply and demand. By adding, subtracting, or multiplying polynomials, they can predict how changes in price or production will affect profits and consumer behavior. Cost analysis is another area where polynomials shine—businesses use them to calculate total costs, optimize pricing strategies, and determine the most efficient way to produce goods. Math is one thing that helps us make smarter decisions in the real world!

But it's not just about economics. Polynomials can help you understand patterns in technology, science, and design. For instance, they're used in computer graphics to create realistic animations in your favorite movies and games. They also help design highways, predict the motion of objects, optimize materials for buildings, and calculate trajectories in sports. Mastering polynomial operations isn't just about solving math problems—it's about decoding the world and unlocking the power to shape it.

In **Lesson 4: Rational Root Theorem,** students continue to add to their strategies for factoring polynomials. They use the rational roots theorem to identify all the possible roots of a polynomial.

Using the Rational Root Theorem

The rational root theorem states that a rational root of a polynomial $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x^1 + a_0 x^0$ with integer coefficients will be of the form $\frac{p}{q}$ where p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.

Given the polynomial: $3x^2 + 2x - 6$

$p = \pm 1, \pm 2, \pm 3, \pm 6$

$q = \pm 1, \pm 3$

The possible rational roots are $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}$, and $\pm\frac{2}{3}$.

MODULE
4

Extending Beyond Polynomials

TOPIC 1 Rational Functions 43

TOPIC 2 Radical Functions 49

TOPIC 3 Radical Equations 55

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Family Guide

MODULE 4 Extending Beyond Polynomials Algebra II

TOPIC 1 Rational Functions

To start this topic, students explore and compare the graphs, tables, and values of a linear function, $f(x) = x$, and its reciprocal function, $g(x) = \frac{1}{x}$. Rational functions are defined. Students then explore transformations of rational functions. Operations with rational expressions are introduced followed by problem solving, during which students create proportions, write rational expressions, describe the behavior of the ratios in the proportions, identify the domain and range, and calculate average costs.



Where have we been?

Students have been working with rational numbers since elementary school. They have extensive knowledge of function behaviors and key attributes to apply to the analysis of rational functions.

Where are we going?

As students experience in the final lesson of this topic, rational functions are applied in many real-world situations involving mixtures, distances, and costs. In precalculus, students will study more complex rational functions as they further explore discontinuities and rational functions composed of higher order polynomials in both the numerator and denominator.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Functions are used heavily in medical and econometric modeling applications for analysis and prediction. Rational functions also have applications in image resolution and acoustics.

HERE'S A SAMPLE QUESTION

Add the expression $\frac{x}{2x+4} + \frac{x}{x+2}$.

Determine a least common denominator:

$2x + 4 = 2(x + 2)$, so the LCD is $2(x + 2)$.

Multiply the second fraction by $\frac{2}{2}$, which is the same as multiplying it by 1, and rewrite the sum as:

$$\frac{x}{2(x+2)} + \frac{2x}{2(x+2)} = \frac{3x}{2(x+2)}$$

The given expression is equal to $\frac{3x}{2x+4}$.

KEY TERMS

- rational function [función racional]
- vertical asymptote [asíntota vertical]
- set notation
- horizontal asymptote [asíntota horizontal]
- inverse variation function [función de variación inversa]
- rational equation [ecuación racional]

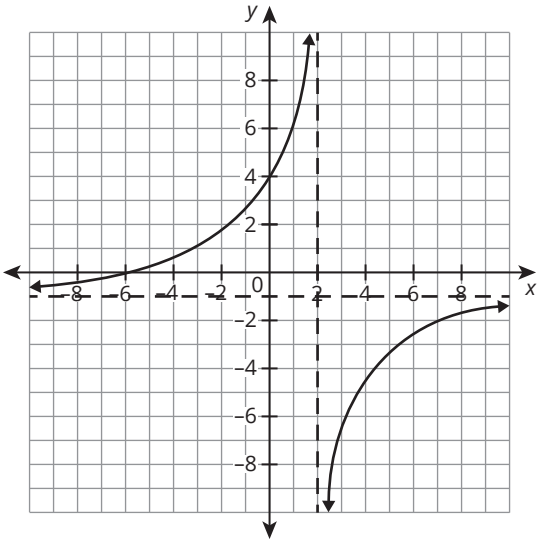
Refer to the Math Glossary for definitions of the Key Terms.

Where are we now?

A **vertical asymptote** is a vertical line that a function gets closer and closer to, but never intersects. The asymptote does not represent points on the graph of the function. It represents the value that the graph approaches.

A **horizontal asymptote** is a horizontal line that a function gets closer and closer to. In most cases, a graph never intersects a horizontal asymptote, but in some cases it does.

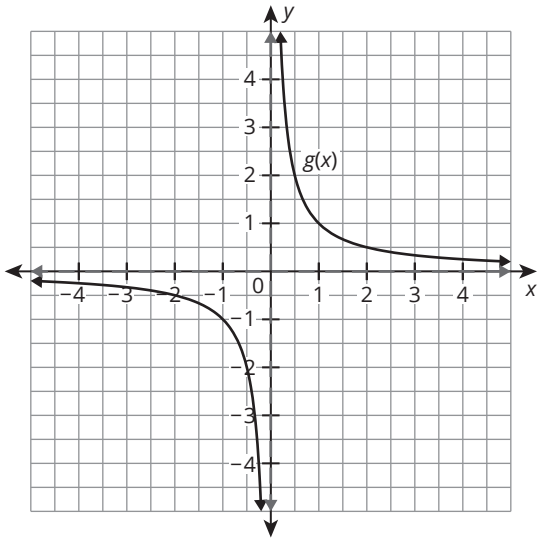
The graph has two asymptotes: a vertical asymptote $x = 2$ and a horizontal asymptote $y = -1$.



In **Lesson 1: Introduction to Rational Functions**, students explore the parent rational function. They also solve real-world problems that are represented by rational functions.

Rational Functions

A rational function is any function that can be written as the ratio of two polynomial functions. A rational function can be written in the form $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomial functions and $Q(x) \neq 0$. These are similar to exponential functions in that they can have horizontal asymptotes, which affect the function's domain. However, they can also have vertical asymptotes, which affect the function's range.



In **Lesson 2: Transformations of Rational Functions**, students apply what they know about transforming functions to the family of rational functions.

For a rational function, the transformation function form can be written as:

$$r(x) = a\left(\frac{1}{b(x-c)}\right) + d$$

For example, $g(x) = \frac{1}{x} - 1 + 2$ is created by transforming the graph of $f(x) = \frac{1}{x}$ right one unit and up two units.

In **Lesson 3: Exploring Asymptotes**, students further explore the effect transformations have on the asymptotes of rational functions. They also rewrite rational functions to determine the location of vertical asymptotes.

Identifying Vertical Asymptotes

Vertical asymptotes occur at x values that make the denominator of the rational function equal to zero. Sometimes algebra is needed to help identify the vertical asymptote(s).

$f(x) = \frac{1}{x^2 + 2x - 8}$

$$x^2 + 2x - 8 = 0$$

$$(x + 4)(x - 2) = 0$$

$$x = -4, x = 2$$

Vertical asymptotes exist at $x = -4$ and $x = 2$

In **Lesson 4, Operations with Rational Expressions**, students add, subtract, multiply, and divide rational functions. They use this knowledge in **Lesson 5: Solving Problem with Rational Equations** to solve rational equations.

Sidonie

$\frac{6}{x2 - 4x} + \frac{4}{x} = \frac{2}{x - 4}$

$\frac{6}{x(x - 4)} + \frac{4}{x} = \frac{2}{x - 4}$

$x \neq 0, 4$

$(x(x - 4)) \cdot \left[\frac{6}{x^2 - 4x} + \frac{4}{x} = \frac{2}{x - 4}\right]$

$6 + 4(x - 4) = 2x$

$6 + 4x - 16 = 2x$

$4x - 10 = 2x$

$2x = 10$

$x = 5$

Rational Equations

A rational equation is an equation that contains one or more rational expressions. Students will use different methods to solve equations depending on the operations used in the problem. Generally students will need previous skills learned, such as factoring polynomials, finding common denominators, and using proportional reasoning, to solve the equations they encounter.

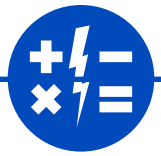
In **Lesson 6: Solving Work, Mixture, Distance, and Cost Problems**, students solve real-world problems involving rational equations.

Rational equations are used in many types of problems related to work, mixture, cost, and distance. Students will build tables to assist in building the equations that represent different situations. Below is an example of a mixture problem set up with a table.

Michael is taking a college chemistry course, and some of his time is spent in the chemistry lab.

He is conducting an experiment for which he needs a 2% salt solution. However, all he can find in the lab is 120 milliliters (mL) of 10% salt solution. Calculate the amount of water Michael would need to add to the 120 mL of 10% salt solution to make a 2% salt solution.

	Liquid	% Salt	Amount of Salt
Water	x	0	0
10% Solution	120	10	$0.10(120) = 12$
2% Solution	$120 + x$	2	$0.02(120 + x)$



Math in the Real World

Imagine you have a bag of chips, and you're sharing it with your friends. The more friends you invite, the less chips each person gets—that's a classic example of a rational function. You see this kind of relationship everywhere! When you walk faster, it takes less time to get somewhere. When more people work together on a project, it gets done faster.

When designing circuits, engineers need to control the relationship between voltage, current, and resistance. The function that models how current flows through a circuit often takes the form of a rational function. This allows engineers to predict how different components will interact, ensuring that devices like your phone or computer operate smoothly without overheating or short-circuiting. Whether you're splitting up snacks, dividing up chores, or powering up your devices, rational functions help us model many real-world scenarios.

Family Guide

MODULE 4 Extending Beyond Polynomials **Algebra II**

TOPIC 2 Radical Functions

This topic presents opportunities for students to explore radical functions. Using patty paper, students switch the *x*- and *y*-axes of power functions and generate their inverses, recognizing that they are transposing the axes when they are inverting a function. Students then transform square root and cubic functions. Finally, students use composition of functions to determine whether two functions are inverses.



Where have we been?

Students have experience with reflecting functions across the line $y = x$ to determine an inverse graphically and transposing x and y in an equation to determine an inverse of a linear equation algebraically. Students have explored power functions and have have extensive experience with transformation function form and discovering how changing the different values affect the graph of the function.

Where are we going?

Students will use their understanding of function inverses throughout the next module. They will explore the inverses of exponential functions, which introduces them to logarithmic functions.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Functions are important in physics to model phenomena involving rates of change, such as the relationship between velocity and stopping distance or the intensity of a wave as it diminishes over distance.

HERE'S A SAMPLE QUESTION

Let $g(x) = x^2 - 5$. If $f(g(x)) = \sqrt{x^2 + 4}$, **what is $f(x)$?**

The entire expression $x^2 - 5$ is the input of the function f, and the output is $\sqrt{x^2 + 4}$, so what does f do to the input to produce the output?

If you add 9 to $x^2 - 5$, and then take the square root of that, you get the correct output, so the function f adds 9 and takes the square root of its input.

Thus, $f(x) = \sqrt{x + 9}$.

KEY TERMS

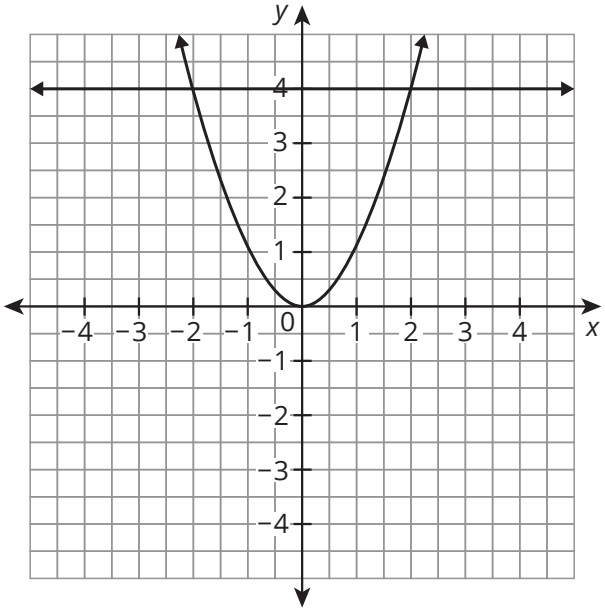
- invertible function [función invertible]
- Horizontal Line Test
- square root function
- restrict the domain [restringir el dominio]
- cube root function
- radical function [función radical]
- composition of functions [composición de funciones]
- identity function

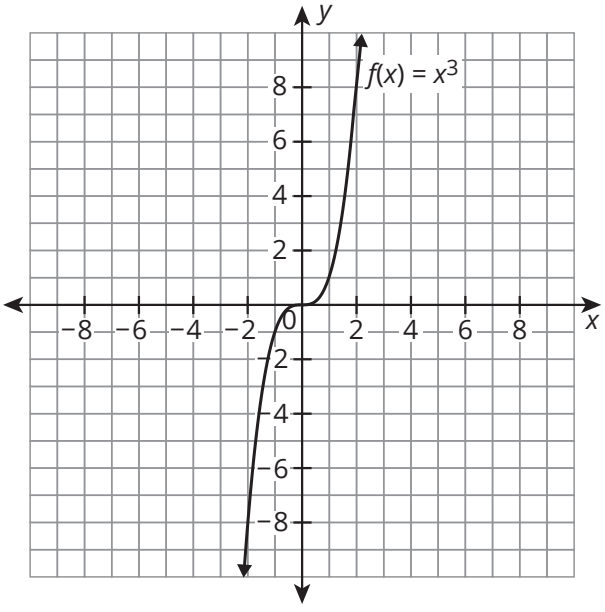
Refer to the Math Glossary for definitions of the Key Terms.

Where are we now?

The **Horizontal Line Test** is a test to determine if a function is one-to-one. To use the test, imagine drawing every possible horizontal line on the coordinate plane. If no horizontal line intersects the graph of a function at more than one point, then the function is one-to-one.

The function $y = x^2$ does not pass the Horizontal Line Test because a horizontal line can be drawn that intersects the graph at more than one point. So, the function is not one-to-one.





An **invertible function** is a function whose inverse exists. It is one-to-one and passes the Horizontal Line Test, so its inverse will also be a function.

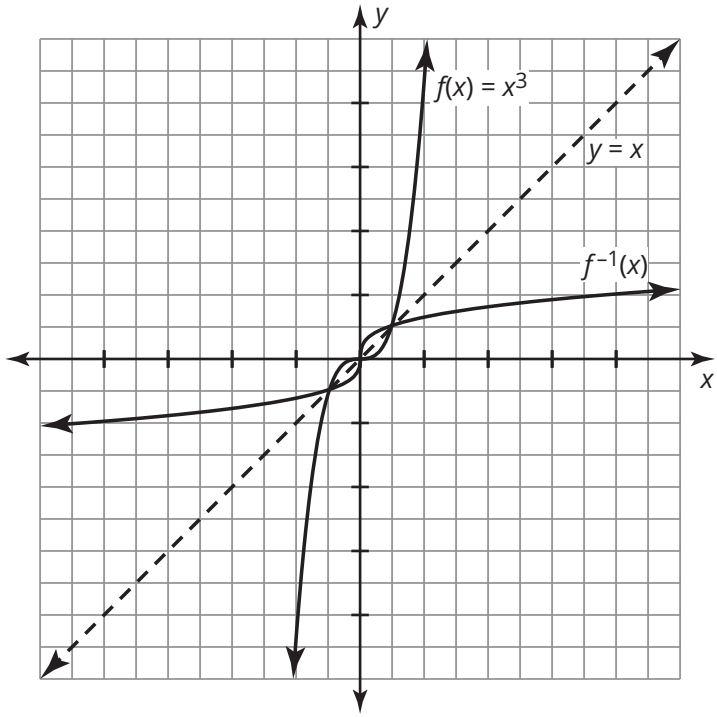
The graph of $f(x) = x^3$ is an invertible function because it is one-to-one and passes the Horizontal Line Test. Therefore, its inverse will also be a function.

In **Lesson 1: Inverses of Power Functions**, students apply what they know about inverses from Module 1 to determine the inverses of power functions.

Invertible Functions

A function f is the set of all ordered pairs (x, y), or $(x, f(x))$, where for every value of x there is one and only one value of y, or $f(x)$. Recall the inverse of a function is the set of all ordered pairs (y, x), or $(f(x), x)$.

For example, the graph shows $f(x) = x^3$ and its inverse, $f^{-1}(x)$.



If the inverse of a function f is also a function, then f is an **invertible function**, and its inverse is written as $f^{-1}(x)$.

In **Lesson 2: Radical Functions**, students continue to investigate power functions and their inverses. They are introduced to the square root and cube root functions.

To determine the equation of the inverse of a power function, transpose x and y in the equation and solve for y. For example, consider the function $y = x^3$. Determine the inverse of the function.

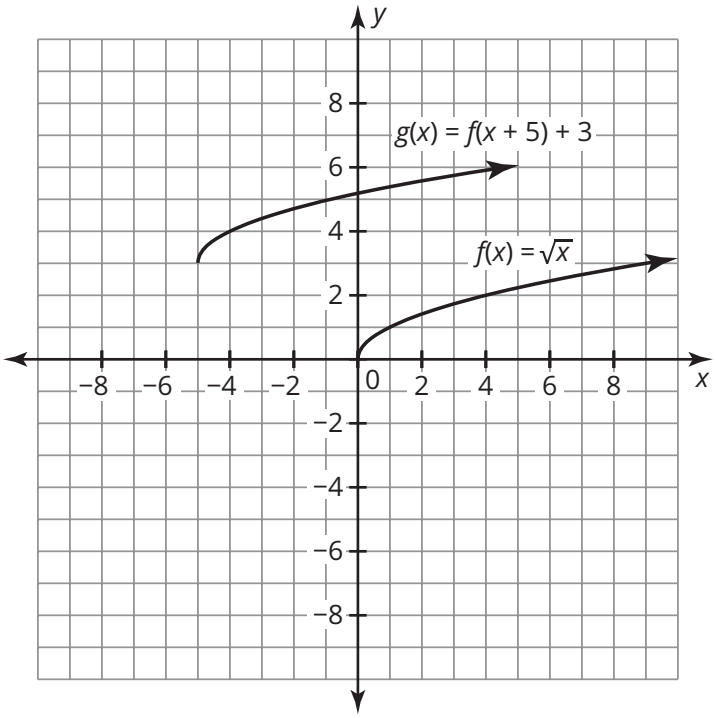
$$y = x^3$$
$$x = y^3$$
$$\sqrt[3]{x} = y$$

The equation for the inverse of the function $y = x^3$ is $y = \sqrt[3]{x}$. This function is called the **cube root function**.

The function $f(x) = x^2$ fails the Horizontal Line Test. Therefore, it is not an invertible function. But when you **restrict the domain** of the power function $f(x) = x^2$ to values greater than or equal to 0, the inverse of the function is called the **square root function** and is written as:

$$f^{-1}(x) = \sqrt{x}\text{ , for } x \geq 0$$

In **Lesson 3: Transformations of Radical Functions**, students transform cube root and square root functions.



For the square root function and cube root function respectively, the transformation function form can be written as $s(x) = a\sqrt{b(x - c)} + d$ or $c(x) = a\sqrt[3]{b(x - c)} + d$. Transformations that take place inside the radical shift the function left or right. Transformations that take place outside the radical shift the function up or down.

For example, consider the graph of $f(x) = \sqrt{x}$ and its transformation represented by the equation $g(x) = f(x + 5) + 3$.

The domain of $f(x)$ is $[0, \infty)$.
The range of $f(x)$ is $[0, \infty)$.

The range of $g(x)$ is $[3, \infty)$.
The domain of $g(x)$ is $[-5, \infty)$.

In **Lesson 4: Compositions of Functions**, students compose functions to determine whether they are inverses.

Two functions f and g are inverses of each other if the composite functions $(f \cdot g)(x)$ and $(g \cdot f)(x)$ both yield the **identity function**. That is, $(f \cdot g)(x) = x$ and $(g \cdot f)(x) = x$.

Use function composition to determine whether the functions $q(x) = \sqrt{x - 1}$ and $h(x) = x^2 + 1, x \geq 0$, are inverses of each other.

$(q \cdot h)(x) = q(h(x))$
$= q(x^2 + 1)$
$= \sqrt{(x^2 + 1) - 1}$
$= \sqrt{x^2}$
$= x$

$(h \cdot q)(x) = h(q(x))$
$= h(\sqrt{x - 1})$
$= (\sqrt{x - 1})^2 + 1$
$= (x - 1) + 1$
$= x$

The functions $q(x) = \sqrt{x - 1}$ and $h(x) = x^2 + 1, x \geq 0$, are inverses of each other, because $(q \cdot h)(x) = x$ and $(h \cdot q)(x) = x$.

Math in the Real World

Radical functions are important when it comes to sound, light, and waves. Ever notice how the music blasting from a speaker sounds super loud up close but fades as you walk away? That's because the intensity of sound decreases with the square root of the distance you are from the source. The same applies to light—think of how a flashlight beam dims the farther it travels. Understanding these relationships helps scientists design better audio systems, brighter flashlights, and even high-tech lasers.

Radical functions also play an important role in the medical field, especially in diagnostic imaging and treatments. For example, when doctors use X-rays or radiation therapy, the intensity of radiation decreases with the square root of the distance from the source. By understanding this relationship, medical professionals can safely target and limit exposure to healthy tissues. Additionally, kidney function is often analyzed using equations involving square roots to measure how well the body is filtering waste. Without radical functions, medical advances like these wouldn't be nearly as effective in helping doctors diagnose and treat patients!

Family Guide

MODULE 4 Extending Beyond Polynomials **Algebra II**

TOPIC 3 Radical Equations

In this topic, students explore radical expressions and equations now that they have an understanding of radical functions. They add, subtract, multiply, and divide radical expressions. Then, students solve equations involving radicals and rational exponents. Finally, students write square root equations given a table of values.



Where have we been?

Since Grade 6, students have been solving equations using the properties of equality. They extend their skills in this topic by solving radical equations. Students learned in Algebra I to write fractional exponents as radicals and have extracted perfect squares from under radicands.

Where are we going?

Students will apply their knowledge of solving radical equations in additional math and science classes such as calculus, physics, and chemistry. Learning about radical equations equips students with the problem-solving skills needed to model many real-world applications.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Radical equations are an important topic to know for solving real-world problems involving measurements, such as calculating distances, speeds, or areas where square roots naturally arise, like in engineering, physics, and construction projects.

HERE'S A SAMPLE QUESTION

Solve the equation: $\sqrt{x + 4} = 2$.

Square both sides of the equation: $(\sqrt{x + 4})^2 = 2^2$

This simplifies to $x + 4 = 4$ or $x = 0$.

Check for extraneous solutions: $\sqrt{0 + 4} = 2$ ✓

The solution is $x = 0$.

KEY TERM

- extraneous solution [solución extraña]

Refer to the Math Glossary for the definition of the Key Term.

Where are we now?

Raising both sides of an equation to a power may introduce an *extraneous solution*, so it is important to check your answers when solving radical equations.

Extraneous solutions are solutions that result from the process of solving an equation, but which are not valid solutions to the equation.

In **Lesson 1: Rewriting Radical Expressions**, students add, subtract, multiply, divide, and simplify radical expressions.

Extracting Roots

You can extract roots to rewrite radicals, using radicals or powers.

For example, you can extract the roots from $\sqrt[3]{8x^6}$ to rewrite the expression using radicals and using powers.

Using Radicals

$$\sqrt[3]{8x^6} = \sqrt[3]{2^3 \cdot x^6}$$
$$= \sqrt[3]{2^3 \cdot (x^2)^3}$$
$$= \sqrt[3]{2^3} \cdot \sqrt[3]{(x^2)^3}$$
$$= 2x^2$$

Using Powers

$$\sqrt[3]{8x^6} = (8x^6)^{\frac{1}{3}}$$
$$= (2^3 \cdot x^6)^{\frac{1}{3}}$$
$$= 2^{\frac{3}{3}} \cdot x^{\frac{6}{3}}$$
$$= 2^1 \cdot x^2$$
$$= 2x^2$$

In **Lesson 2: Solving Radical Equations**, students solve square root and cube root equations in and out of context.

Using Radial Equations to Solve Real-World Problems

Big Ben is the nickname of a well-known clock tower in London, England, that stands 316 feet tall. The clock is driven by a 660-pound pendulum in the tower that continually swings back and forth. The relationship between the length of the pendulum, L, in feet and the time it takes for a pendulum to swing back and forth one time, or its period T, is modeled by the equation $T = 2\pi\sqrt{\frac{L}{32}}$. If the pendulum's period is 4 seconds, solve for L.

$$4 = 2\pi\sqrt{\frac{L}{32}}$$

$$\frac{4}{2\pi} = \sqrt{\frac{L}{32}}$$

$$\left(\frac{2}{\pi}\right)^2 = \left(\sqrt{\frac{L}{32}}\right)^2$$

$$\frac{4}{\pi^2} = \frac{L}{32}$$

$$4 \cdot 32 = \pi^2 \cdot L$$

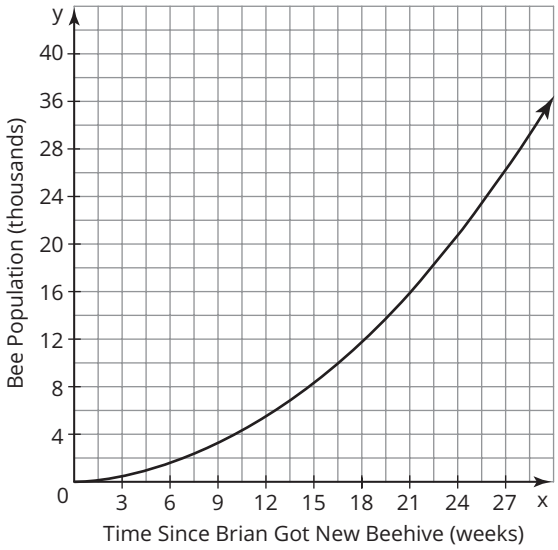
$$L = \frac{128}{\pi^2} \approx 12.97$$

The length of the pendulum is approximately 12.97 feet.

In **Lesson 3: Modeling with Radical Functions**, students use what they have learned about inverse functions to write square root functions when given a table of values.

Writing an Inverse Function

Jacob has a new beehive. He records the number of bees in the hive after x weeks and uses graphing technology to write an equation to represent the number of bees in the hive. The regression function $b(x) = 36x^2$ represents the data shown on the graph.



Write the inverse function $b^{-1}(x)$ to create a function that will determine the number of weeks given the bee population.

$$y = 36x^2 \rightarrow x = 36y^2$$

$$\frac{x}{36} = y^2$$

$$\sqrt{\frac{x}{36}} = \sqrt{y^2}$$

$$\pm \frac{\sqrt{x}}{6} = y$$

$$b^{-1}(x) = \frac{\sqrt{x}}{6}$$

Math in the Real World

Imagine you're designing the next big skateboard park, complete with ramps, half-pipes, and drop-ins. How do you calculate the perfect curve to launch a skater into the air and have them land smoothly without wiping out? Radical equations help you do just that by modeling the relationship between height, distance, and speed. Radical equations also come into play in physics to measure how long it takes objects to fall or travel a certain distance, helping everything from skydivers to drones perform as expected. They even show up in vehicle testing, where engineers use them to calculate braking distances and optimize speed for cars.

But it doesn't stop there! Radical equations are vital in science and technology, too. Chemists use them to understand reaction rates, like how fast a medicine dissolves in your body. Ecologists rely on them to model wildlife populations in changing ecosystems, predicting how a species might grow or decline. Even your favorite devices depend on these equations—signal processing for Wi-Fi, cell service, and sound systems often involves square roots and radicals to keep everything running smoothly. So whether you're skateboarding, exploring nature, or chatting with a friend on your phone, radical equations are quietly working behind the scenes!

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MODULE
5

Exploring Exponentials and Logarithmics

TOPIC 1 Exponential and Logarithmic Functions 63

TOPIC 2 Polynomial Operations 69

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TOPIC 1 Exponential and Logarithmic Functions



Students begin this topic by creating exponential graphs using their prior knowledge of geometric sequences. They write exponential growth and decay functions given specified characteristics and transform exponential functions. Logarithmic functions are introduced as the inverse of exponential functions. Students explore the key attributes of logarithmic functions and transform logarithmic functions.

Where have we been?

In Algebra I, students analyzed exponential functions, including their key attributes. They have also investigated inverse functions in earlier modules. Although they may not be familiar with the constant *e*, students have experience with irrational numbers, including some square roots and π.

Where are we going?

Students will use the intuitions they gain by studying the graphs of logarithmic functions in the next topic to analyze logarithmic equations and apply logarithmic functions to situations. As will be shown in the situations in these topics, logarithmic functions have a number of applications in astronomy, medicine, mechanics, physics, and seismology.

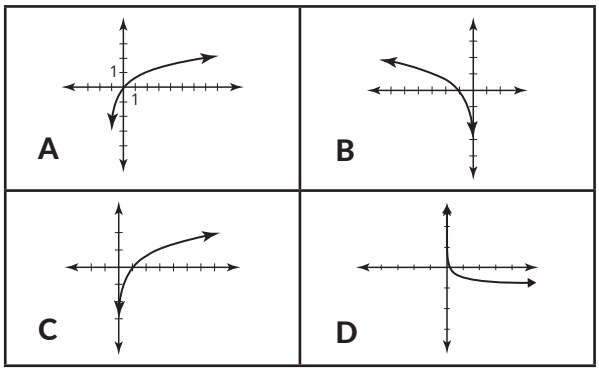
TALKING POINTS

DISCUSS WITH YOUR STUDENT

Exponential Functions are an important topic to know for measuring population growth, calculating compound interest, and measuring radioactive decay.

HERE IS A SAMPLE QUESTION

Identify the graph of $f(x) = -\ln(x)$.



The function $f(x) = -\ln(x)$ is a reflection of the function $f(x) = \ln(x)$ across the *x*-axis. Choice D is correct.

KEY TERMS

- half-life
- natural base e [base natural e]
- logarithm [logaritmo]
- logarithmic function [función logarítmica]
- common logarithm [logaritmo común/ decimal]
- natural logarithm [logaritmo natural]

Refer to the Math Glossary for definitions of the Key Terms.

Where are we now?

The natural base e is an irrational number equal to approximately 2.71828.
$e^2 \approx 2.7183^2 \approx 7.3892$

A common logarithm is a logarithm with a base of 10. Common logarithms are usually written without a base.

The expressions log (2x) and log x are examples of common logarithms.

In **Lesson 1: Comparing Linear and Exponential Functions**, students use exponential functions to model real-world situations.

Half-Life and Continuous Compounding

Calculating the half-life of a substance is a classic example of exponential decay. Half-life is used in carbon dating, radioactive decay, and to measure the effectiveness of medical drugs over time. Similar to exponential decay, there can also be growth, and how the growth happens depends on the situation.

The table shows the amount of caffeine in a person's system over time, starting with 80 milligrams of caffeine. The caffeine has a half-life of 5 hours.

Time Elapsed (hours)	0	5	10	15	20	25
Caffeine in System (mg)	80	40	20	10	5	2.5
Number of Half-Life Cycles	0	1	2	3	4	5

An equation to represent the half-life of the caffeine would be, $g(n) = 80\left(\frac{1}{2}\right)^{\left(\frac{n}{5}\right)}$, where n is the number of hours.

In **Lesson 2: Properties of Exponential Graphs**, students continue to explore the parent exponential function and its key attributes.

The Parent Exponential Function

Parent exponential functions, whether increasing or decreasing, have the same domain, range, asymptote, and intercepts.

	Parent Exponential Functions	
	Exponential Growth $f(x) = b^x, b > 1$	**Exponential Decay** $f(x) = b^x, 0 < b < 1$
Domain in Interval Notation	$(-\infty, \infty)$	$(-\infty, \infty)$
Range in Interval Notation	$(0, \infty)$	$(0, \infty)$
Asymptote	$y = 0$	$y = 0$
Intercept	(0, 1)	(0, 1)
Intervals of Increase or Decrease	Increasing over $(-\infty, \infty)$	Decreasing over $(-\infty, \infty)$

In **Lesson 3: Transformations of Exponential Functions**, students graph transformations of exponential functions.

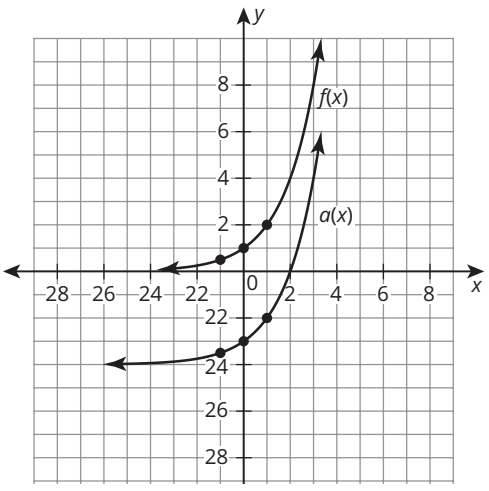
Transforming Exponential Functions

You can use reference points to graph transformations of exponential functions.

$f(x) = 2^x$

$a(x) = f(x) - 4$

Reference Points on $f(x)$	Corresponding Points on $a(x)$
$\left(-1, \frac{1}{2}\right)$	$\left(-1, -\frac{7}{2}\right)$
(0, 1)	(0, −3)
(1, 2)	(1, −2)



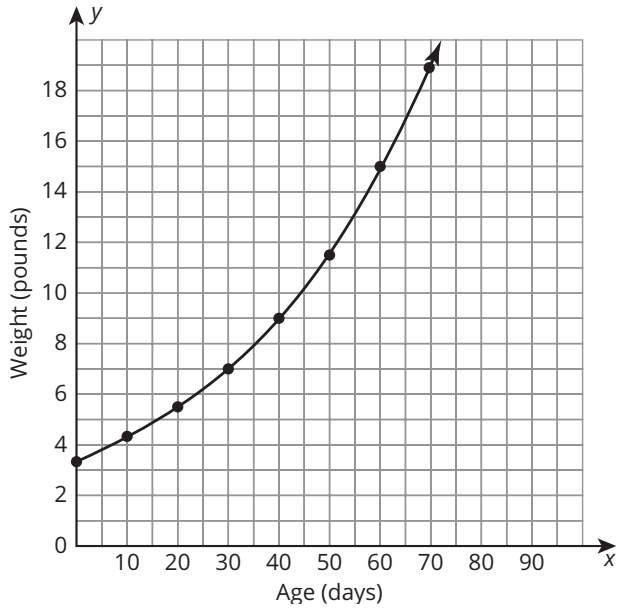
In **Lesson 4: Modeling with Exponential Functions**, students use graphing technology to determine an exponential regression functions a given data set.

Exponential Regression Function

The table shows the weight of a golden retriever puppy as recorded during its growth. A scatterplot and regression function of the data are shown.

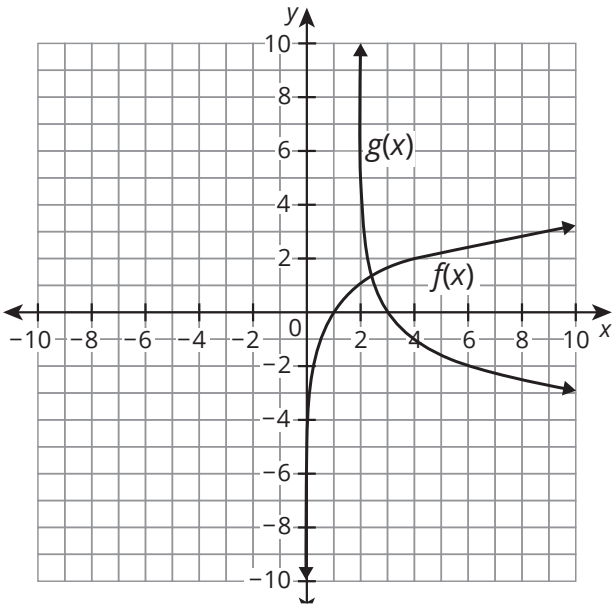
The exponential regression function for this scenario is $f(x) = 3.31(1.025)^x$.

Age (days)	Weight (pounds)
0	3.25
10	4.25
20	5.5
30	7
40	9
50	11.5
60	15
70	19



In **Lesson 5: Logarithmic Functions**, students are introduced to logarithmic functions as the inverse of an exponential function.

Logarithms and Exponents



In **Lesson 6: Transformations of Logarithmic Functions**, students apply what they know about transformations to logarithmic functions.

Transforming a Logarithmic Function

To create $g(x)$, the graph of $f(x)$ is horizontally translated right 2 units and reflected across the x-axis.

$g(x) = -f(x - 2)$

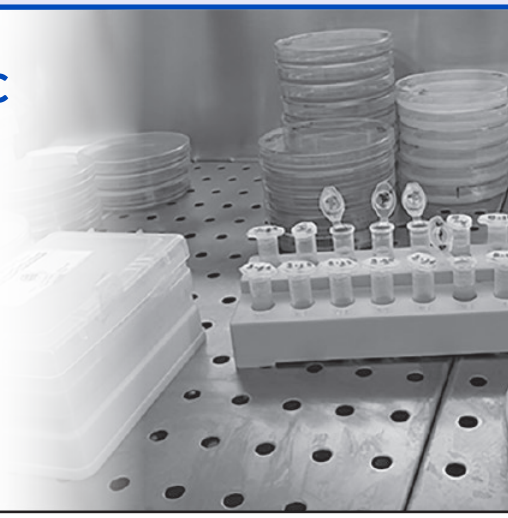
Math in the Real World

Imagine you're at a concert, and the speakers are blasting music at different volumes. The way we measure sound, called decibels, actually follows a logarithmic scale! That means a small increase in decibels represents a huge increase in sound intensity. So when a rock concert is at 120 decibels, it's not just a little louder than a conversation at 60 decibels—it's actually a million times more intense in terms of sound energy! Logarithms help scientists and engineers understand things that grow or shrink rapidly, like earthquake magnitudes, pH levels in chemistry, and even how information spreads on social media.

On the flip side, exponential functions describe things that grow super fast—like how bacteria multiply or how your social media followers can explode if your video goes viral. Imagine you start with just one follower, but every day, each person convinces two new friends to follow you. On day one, you have 1 follower, on day two you have 3, then 9, then 27, and soon you're at thousands! This kind of explosive growth happens in population growth, finance (like compound interest making your savings grow faster over time), and even in the spread of diseases. While logarithms help us manage massive numbers, exponential functions show us how quickly things can get out of hand!

Family Guide

TOPIC 2 Exponential and Logarithmic Equations



In this topic, students first convert between exponential and logarithmic forms of an equation. Students solve exponential equations using common bases or by taking the logarithm of both sides of the equation. They then solve logarithmic equations when the base, argument, or exponent is unknown. In the final lesson, students determine whether a data set is best represented by a linear, quadratic, or exponential function. They write regression functions for data sets and use the regression function to make predictions and critical decisions.

Where have we been?

In this topic, students first convert between exponential and logarithmic forms of an equation. Students solve exponential equations using common bases or by taking the logarithm of both sides of the equation. They then solve logarithmic equations when the base, argument, or exponent is unknown. In the final lesson, students determine whether a data set is best represented by a linear, quadratic, or exponential function. They write regression functions for data sets and use the regression function to make predictions and critical decisions.

Where are we going?

Logarithms are convenient ways to express large numbers and are used extensively in the sciences. Students are most familiar with the Richter Scale, which uses a logarithmic scale to classify the intensity of an earthquake. Logarithms are used to model real-world phenomena that decay logarithmically, including scenarios in science and physics. Exponential functions are used to model situations in banking and art.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Logarithms are an important topic to know for understanding exponential growth, solving complex equations, and applying mathematical concepts in fields like science, engineering, and computer science.

HERE IS A SAMPLE QUESTION

Solve $5^{x+3} = 15{,}625$.

Rewrite 15,625 as 5^6.

$5^{x+3} = 5^6$

Since the bases are the same, the exponents must be equal.

So, $x + 3 = 6$, and $x = 3$.

KEY TERM

- logarithmic expression [expresión logarítmica]

Refer to the Math Glossary for definitions of the Key Terms.

Where are we now?

A logarithmic equation is an equation that contains a logarithm. The equation $\log2(x) = 4$ is a logarithmic equation.

In **Lesson 1: Logarithmic Expressions**, students further their understanding of logarithmic expressions. They rewrite logarithmic equations as exponential equations and estimate the value of logarithms.

Estimating Logarithms

To estimate $\log_3 33$ to the tenths place, identify the closest logarithm whose argument is less than 33 and the closest logarithm whose argument is greater than 33 on the number line that represents base 3.

The closest logarithm whose argument is less than 33:	The logarithm you are estimating:	The closest logarithm whose argument is greater than 33:
$\log_3 27$	$\log_3 33$	$\log_3 81$

You know that $\log_3 27 = 3$ because $3^3 = 27$ and $\log_3 81 = 4$ because $3^4 = 81$. This means the estimate of $\log_3 33$ is between 3 and 4.

$$\log_3 27 < \log_3 33 < \log_3 81$$
$$3 < x < 4$$

Next, estimate the decimal digit. Because 33 is closer to 27 than it is to 81, the value of $\log_3 33$ is closer to 3 than it is to 4.



In this case, 3.2 is a good estimate for $\log_3 33$.

In **Lesson 2: Solving Exponential Equations**, students solve exponential equations in real-world and mathematical equations.

Using Common Bases to Solve an Exponential Equation

For example solve the equation $5^{x-3} = 625$.

$5^{x-3} = 625$

$5 \cdot 5 \cdot 5 \cdot 5 = 5^4 = 625$

$5^{x-3} = 5^4$

So, $x - 3 = 4$

Therefore, $x = 7$.

In **Lesson 3: Solving Logarithmic Equations**, students solve logarithmic equations for an unknown base, argument, or exponent.

Solving Logarithmic Equations

You can use the relationship between the base, argument, and exponent to solve for any unknown in a logarithmic equation. To solve for any unknown in a simple logarithmic equation, begin by converting it to an exponential equation.

Argument Is Unknown	Exponent Is Unknown	Base Is Unknown
$\log_4 y = 3$	$\log_4 64 = x$	$\log_b 64 = 3$
$4^3 = y$	$4^x = 64$	$b^3 = 64$
$64 = y$	$4^x = 4^3$	$b^3 = 4^3$
	$x = 3$	$b = 4$

In **Lesson 4: Applications of Exponential and Logarithmic Equations**, students solve exponential and logarithmic equations to solve real-world problems.

Using a Logarithmic Equation to Solve a Real-World Problem

The pH scale is a scale for measuring the acidity or alkalinity of a substance, which is determined by the concentration of hydrogen ions. The formula for pH is $pH = -\log H^+$ where H^+ is the concentration of hydrogen ions in moles per cubic liter.

Vinegar has a pH of 2.2. Determine the concentration of hydrogen ions in vinegar.

$$2.2 = -\log H^+$$

$$-2.2 = \log H^+$$

$$10^{-2.2} = H^+$$

$$H^+ \approx 0.006$$

The concentration of hydrogen ions in vinegar is 0.006 moles per cubic liter.

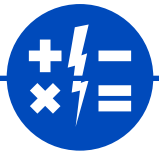
In **Lesson 5 Modeling with Linear, Quadratic, and Exponential Functions**, students determine whether a data set is best represented by a linear, quadratic, or exponential function. They use technology to write regression functions and use those functions to make predictions.

Choosing a Function

A cup of tea is poured into a teacup. The table shows the temperature of the tea, $t(x)$, in °F, x minutes after the tea is poured.

Time (Minutes)	0	5	7	9	11	15	18	22
Temperature (°F)	179.9	164.3	156.1	148.3	141.2	135.1	128.7	125.1

The data decreases over the given domain, but not at a constant rate, so an exponential regression best models the data. The function that models the data best is $t(x) = 176.1738 \cdot 0.9831^x$.



Math in the Real World

Imagine you just grabbed a steaming cup of hot chocolate on a chilly winter day. At first, it's way too hot to drink, so you wait. But how long should you wait before it cools to the perfect sipping temperature? That's where logarithmic and exponential equations come in! Newton's Law of Cooling explains that the hotter something is compared to its surroundings, the faster it cools down. But as it gets closer to room temperature, the cooling slows down. If you wanted to predict exactly when your hot chocolate will reach a cozy 130°F, you'd need logarithms to work backward from the cooling process.

Exponential and logarithmic equations aren't just for hot chocolate, though—they pop up all over the place! Scientists use them to predict how quickly viruses spread, how long radioactive materials take to decay, and even how your phone battery drains throughout the day. Exponential equations describe fast changes, like population growth or the way sound waves travel, while logarithms help us figure out unknowns, like how long something will take to reach a certain level. Earthquake magnitudes and the way music volume is measured also rely on logarithms! So next time you wait for your leftovers to cool down or check how much battery life you have left, remember—there's some serious math working behind the scenes!

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