

Pre-Calculus: Polynomial & Rational Functions

Stage 1 Desired Results		
ESTABLISHED GOALS: <u>Competencies:</u> - Students will demonstrate perseverance and strategic thinking by identifying the underlying meaning of a problem, planning a logical solution pathway, working through challenges, and verifying the reasonableness of their answers. - Students will demonstrate resourcefulness by evaluating and selecting the most effective tools to solve mathematical problems and deepen their understanding of concepts. - Students will demonstrate accuracy and clarity using correct mathematical terminology, specifying units of measure, and calculating with the degree of precision appropriate for the problem context. - Students will demonstrate structural awareness by discerning patterns and functional relationships within complex mathematical concepts to simplify and solve problems. - Students will demonstrate the ability to notice repetitive calculations or patterns, evaluate the reasonableness of results, and formulate general methods, formulas, or shortcuts while maintaining oversight of the process. - Students will demonstrate the ability to analyze and summarize text and integrate knowledge to make meaning of discipline-specific materials. - Students will demonstrate the ability to produce coherent and supported writing in order to communicate effectively for a range of discipline-specific tasks, purposes, and audiences. - Students will demonstrate the ability to speak purposefully and effectively by strategically making decisions about content, language use, and discourse style. <u>Content Standards:</u> - HSN.CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. - HSN.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. - HSN.CN.A.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. - HSN.CN.B.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. - HSN.CN.B.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. - HSN.CN.B.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	Transfer	
	<i>Students will be able to independently use their learning to represent relationships between quantities, manipulate and analyze the representation, and interpret its meaning.</i>	
	Meaning	
	ENDURING UNDERSTANDINGS <i>Students will understand that...</i> - equivalent expressions can be created by using multiple strategies and the guiding rules of mathematics. - the graph and equation of a function communicate information that can be used to answer questions about real world problems. - it is possible to get closer and closer to something but to never be able to touch it.	ESSENTIAL QUESTIONS How can the multiple representations of a function be used to best analyze the relationships between the two quantities it models?
	Acquisition	
	<i>Students will know...</i> - the average rate of change of a function f on the interval $[a,b]$ is the slope of the line joining two points $(a, f(a))$ and $(b, f(b))$. - that a graph of a function can be sketched by identifying the parent function and then analyzing and interpreting the changes made to $f(x)$, for example $-f(x)$ or $f(x+c)$. - that given $f(x)=x^n$, when n is even, the graph of the function is symmetric with respect to the y-axis, and when n is odd, it is symmetric with respect to the origin. - that, if the multiplicity of a polynomial's zero is odd, the graph crosses the x-axis at that value. If it is even, then it touches, but does not cross, the x-axis at that value. - that the graph of a polynomial function of degree 2 or greater is a continuous smooth	<i>Students will be skilled at...</i> - calculating the instantaneous rate of change using the average rate of change. - computing, interpreting, and comparing average rate of change for a function. - simplifying difference quotients of the form $\frac{f(x)-f(a)}{x-a}$. - determining the domain and range of a function from both an equation and from a graph. - graphing the parent function and transformations for linear, quadratic, square root, and absolute value functions. - graphing transformations by viewing them as changes made to the parent function. - setting up equations that define functions for problem-solving, including maximum and minimum value problems.

• HSN.CN.C.8 (+) Extend polynomial identities to the complex numbers. <i>For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.</i> • HSN.CN.C.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. • HSA.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. • HSA.APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. • HSA.APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. • HSA.APR.C.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. • HSA.APR.C.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.1 • HSA.APR.D.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system • HSA.APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. • HSA.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. • HSF.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. • HSF.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* • HSF.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. • HSF.IF.C.7.C Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. • HSF.IF.C.7.D (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing	<i>curve.</i> • <i>leading Coefficient Test for polynomial functions.</i> • <i>that, for a polynomial function of degree, n has at most, n real zeros.</i> • <i>that the graph of a polynomial function of degree n has at most $n-1$ turning points.</i> • <i>the relationships between roots and coefficients for polynomials equations of any degree.</i> • <i>that complex zeros occur in conjugate pairs.</i> • <i>Descartes's Rule of Sign.</i> • <i>that, if the degree of the numerator is exactly one more than the degree of the denominator, then the graph of the function has a slant asymptote.</i> • <i>that, within an interval of the number line created by the x-intercepts, the sign of the value of the function stays the same.</i> • <i>the following algorithms and theorems:</i> - o <i>Division Algorithm</i> - o <i>Remainder Theorem</i> - o <i>Intermediate Value Theorem</i> - o <i>Factor Theorem</i> - o <i>Fundamental Theorem of Algebra</i> - o <i>Linear Factorization Theorem</i> - o <i>Rational Roots Theorem</i> - o <i>Upper and Lower Bound Theorem for</i> - o <i>Upper and Lower Bound Theorem for Real Roots.</i> <u><i>vocabulary:</i></u> <i>rational function, key numbers, vertical/horizontal/slant asymptote, complex conjugate, double root, extraneous roots, persistence of sign, , synthetic division, even and odd functions</i>	• <i>sketching polynomial functions, using the leading coefficient test, the real zeros, and test intervals.</i> • <i>using the Intermediate Value Theorem to approximate a real zero.</i> • <i>sketching rational functions.</i> • <i>using long and synthetic division to find quotients and remainders and to show that a value of x is a solution to the equation.</i> • <i>factoring polynomials.</i> • <i>finding solutions to polynomial equations using the Factor Theorem.</i> • <i>simplifying rational expressions by factoring and by using long/synthetic division.</i> • <i>adding, subtracting, and multiplying complex numbers.</i> • <i>determining the quotient of complex numbers in standard form.</i> • <i>writing complex numbers in standard form</i> • <i>expressing polynomial equations in the form $a_n(x-r_1)(x-r_2)...(x-r_n)$</i> • <i>determining a polynomial equation with prescribed roots and their multiplicities.</i> • <i>finding the rational roots of a polynomial equation.</i> • <i>solving an equation using the rational roots theorem and the upper and lower bound theorem.</i> • <i>solving an equation using the conjugate roots theorem.</i> • <i>determining remaining roots when provided one root.</i> • <i>using Descartes's Rule to verify the number of positive/negative roots.</i> • <i>determining the vertical and horizontal asymptotes of a rational function.</i> • <i>sketching the graph of rational functions.</i> • <i>finding the equation of a slant asymptote.</i> • <i>solving polynomial inequalities using factoring, key numbers, and the persistence of sign.</i>
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end behavior. • MP1 Make sense of problems and persevere in solving them. • MP2 Reason abstractly and quantitatively. • MP4 Model with mathematics. • MP5 Use appropriate tools strategically. • MP7 Look for and make use of structure. • MP8 Look for and express regularity in repeated reasoning.		
Content Area Literacy Standards		21st Century Skills
• RST.11-12.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text. • RST.11-12.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to <i>grades 11-12 texts and topics</i>. • RST.11-12.5 Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas. • RST.11-12.8 Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.		• <i>reason effectively</i> • <i>use systems thinking</i> • <i>solve problems</i> • <i>apply technology effectively</i>

Stage 3 – Learning Plan	
Summary of Key Learning Events and Instruction	
Language Arts Integration	Mathematics Integration
1.OA.1 Use	1.OA.1 Use

<i>Technology Integration</i>	<i>District Materials</i>
• 1.OA.1 Use	