

Pre-Calculus: Trigonometry

Stage 1 Desired Results		
ESTABLISHED GOALS: Competencies: - Students will demonstrate perseverance and strategic thinking by identifying the underlying meaning of a problem, planning a logical solution pathway, working through challenges, and verifying the reasonableness of their answers. - Students will demonstrate the ability to decontextualize (represent symbols) and contextualize (understand the meaning of quantities) by converting real-world situations into mathematical symbols for manipulation, and interpreting symbolic results back into practical contexts. - Students will demonstrate logical thinking and communication skills by building justified mathematical arguments, comprehending the justification of others, and critiquing their reasoning. - Students will demonstrate application skills by using mathematical concepts and symbols to represent, analyze, and solve problems arising in everyday life, society, and the workplace. - Students will demonstrate accuracy and clarity using correct mathematical terminology, specifying units of measure, and calculating with the degree of precision appropriate for the problem context. - Students will demonstrate the ability to notice repetitive calculations or patterns, evaluate the reasonableness of results, and formulate general methods, formulas, or shortcuts while maintaining oversight of the process. - Students will demonstrate the ability to analyze and summarize text and integrate knowledge to make meaning of discipline-specific materials. - Students will demonstrate the ability to produce coherent and supported writing in order to communicate effectively for a range of discipline-specific tasks, purposes, and audiences. - Students will demonstrate the ability to speak purposefully and effectively by strategically making decisions about content, language use, and discourse style. Content Standards: - HSF.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. - HSF.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. - HSF.TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. - HSF.TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. - HSF.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. - HSF.TF.B.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	Transfer	
	Students will be able to independently use their learning to understand and make predictions involving periodic phenomena.	
	Meaning	
	ENDURING UNDERSTANDINGS <i>Students will understand that...</i> - a repeated pattern of outputs at regular intervals always results in a periodic relationship between quantities. - periodic functions can be used to analyze and predict cyclical events.	ESSENTIAL QUESTIONS How could we use trigonometry as a tool to better understand the world around us?
	Acquisition	
	Students will know... $180 \text{ degrees} = \pi \text{ radians}$ $s = r\theta$. $A = \frac{1}{2}r^2\theta$. $\omega = \theta/t$. $v = d/t$. $v = r\omega$. $\sin^2\theta + \cos^2\theta = 1$. $\sin(90 - \theta) = \cos\theta$. $\cos(90 - \theta) = \sin\theta$. $\sin(-t) = -\sin t$. $\tan(-t) = \tan t$. $\cos(-t) = \cos t$. $-1 \leq \sin x \leq 1$ for all x. $-1 \leq \cos x \leq 1$ for all x. $\sin(t + 2k\pi) = \sin t$ for any integer k. $\cos(t + 2k\pi) = \cos t$ for any integer k. $\tan(t + k\pi) = \tan t$ for any integer k. - given the point $P(x,y)$ on the unit circle, $\cos t = x$; $\sin t = y$; $\tan t = \frac{y}{x}$; $\sec t = \frac{1}{x}, x \neq 0$; $\csc t = \frac{1}{y}$; $\cot t = \frac{x}{y}$, $y \neq 0$. - the trig values for 30, 45, 60, and 90 degree	Students will be skilled at... - calculate the radian measure of an angle. - converting between radians and degrees. - calculating arc length. - calculating area of a sector. - calculating angular and linear speed. - proving $\sin^2\theta + \cos^2\theta = 1$. - calculating exact values of six trig functions given one point on the terminal side of theta in standard position. - calculating exact values of six trig functions given one angle and one side of a right triangle. - calculating exact and decimal values of remaining trig functions given one trig ratio in any quadrant. - evaluating trig functions using coterminal angles without a calculator. - simplifying trig expressions by using algebra skills. - simplifying trig expressions by using the trig identities. - factoring trig expressions. - using identities to calculate the value of trig functions. - proving identities. - using the value of a given trig function to find the remaining trig function values.

• HSF.TF.B.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. • HSF.TF.C.8 Prove the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ and use it to find $\sin \theta$, $\cos \theta$, or $\tan \theta$ given $\sin \theta$, $\cos \theta$, or $\tan \theta$ and the quadrant of the angle. • HSF.TF.C.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. • HSF.BF.B.4.D (+) Produce an invertible function from a non-invertible function by restricting the domain. • HSG.SRT.D.9 (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. • HSG.SRT.D.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems. • HSG.SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). • MP2 Reason abstractly and quantitatively. • MP3 Construct viable arguments and critique the reasoning of others. • MP4 Model with mathematics. • MP5 Use appropriate tools strategically. • MP6 Attend to precision. • MP7 Look for and make use of structure. • MP8 Look for and express regularity in repeated reasoning.	<i>angles.</i> • <i>the Pythagorean identities.</i> • <i>that sine is positive in quadrants I and II.</i> • <i>that cosine is positive in quadrants I and IV.</i> • <i>that tangent is positive in quadrants I and III.</i> • <i>that the period of sine and cosine is 2π.</i> • <i>that the period of tangent is π.</i> • <i>that the graph of $y = \sin x$, including the intercepts and turning points.</i> • <i>that the graph of $y = \cos x$, including the intercepts and turning points.</i> • <i>that the graph of $y = \tan x$, including intercepts and asymptotes.</i> • <i>that the graph of $y = \cot x$, including intercepts and asymptotes.</i> • <i>that the graph of $y = \csc x$, including intercepts, asymptotes, and turning points.</i> • <i>that the graph of $y = \sec x$, including intercepts, asymptotes, and turning points.</i> • <i>that the area of a triangle = $\frac{1}{2}bc \sin A$.</i> • <i>the addition and subtraction formulas for sine, cosine, and tangent</i> • <i>the double-angle formulas.</i> • <i>the half-angle formulas.</i> • <i>the product-to-sum identities.</i> • <i>the sum-to-product identities.</i> • <i>that we restrict the domain of the sine, cosine, and tangent functions so that they will be one-to-one and therefore have an inverse.</i> • <i>the domain, range, and graph for each of the inverse trig functions.</i> • <i>Laws of Sines and Cosines.</i> <u>vocabulary:</u> <i>radian, identity, secant, cosecant, cotangent, initial side, angle of rotation, terminal side, initial side, coterminal angle, standard position, reference angle, unit circle, even and odd functions, periodic function, periodicity, amplitude, inverse sine, inverse cosine, phase shift, asymptotes, arcsine, arccosine, arctangent</i>	• <i>evaluating trig functions using the opposite angle identities and periodicity.</i> • <i>sketching the graph of $y = A \sin b(x - h) + k$.</i> • <i>sketching the graph of $y = A \cos b(x - h) + k$.</i> • <i>sketching the graph of $y = A \tan b(x - h) + k$.</i> • <i>sketching the graph of $y = A \cot b(x - h) + k$.</i> • <i>sketching the graph of $y = A \csc b(x - h) + k$.</i> • <i>sketching the graph of $y = A \sec b(x - h) + k$.</i> • <i>proving the trigonometric formulas.</i> • <i>using the trigonometric formulas to evaluate trig functions.</i> • <i>using the trigonometric formulas to find exact expressions to represent the value of a trig function.</i> • <i>solving trig equations by applying knowledge of periods.</i> • <i>solving trig equations by factoring.</i> • <i>solving trig equations by applying inverse trig functions.</i> • <i>solving trig equations by using trig identities.</i> • <i>solving equations involving multiples of an angle.</i> • <i>finding exact and approximate values for inverse trig functions.</i> • <i>using graphing calculators to estimate the value of trig functions.</i> • <i>sketching the graph of transformations to the inverse trig functions.</i> • <i>using the trigonometric identities in formal and informal proofs.</i> • <i>using the laws of sine and cosine to solve problems.</i>
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Content Area Literacy Standards	21st Century Skills
• RST.11-12.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text. • RST.11-12.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to <i>grades 11-12 texts and topics</i>. • RST.11-12.7 Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem. • RST.11-12.8 Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information. • RST.11-12.9 Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	• <i>reason effectively</i> • <i>use systems thinking</i> • <i>make judgments and decisions</i> • <i>solve problems</i> • <i>assess and evaluate information</i> • <i>use and manage information</i>

Stage 2 - Evidence	
Evaluative Criteria	Assessment Evidence
	PERFORMANCE TASK(S):
	OTHER EVIDENCE:

Stage 3 – Learning Plan	
<i>Summary of Key Learning Events and Instruction</i>	
Language Arts Integration	Mathematics Integration
• 1.OA.1 Use	• 1.OA.1 Use
Technology Integration	District Materials
• 1.OA.1 Use	

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