

1.43 c

1.44 The individuals are the hotels. The variables in the data set are whether there is a pool (categorical), whether there is an exercise room (categorical), Internet cost (\$/day) (quantitative), number of restaurants (quantitative), distance to site (mi) (quantitative), room service availability (categorical), and room rate (\$/day) (quantitative).

43. The following partially completed two-way table shows the marginal distributions of gender and handedness for a sample of 100 high school students.

Dominant hand	Gender		
	Right	Left	Total
	x	10	90
Total	40	60	100

If there is no association between gender and handedness for the members of the sample, which of the following is the correct value of x ?

- (a) 20 (d) 45
(b) 30 (e) Impossible to determine without more information.
(c) 36

Recycle and Review

44. **Hotels (Introduction)** A high school lacrosse team is planning to go to Buffalo for a three-day tournament. The tournament's sponsor provides a list of available hotels, along with some information about each

hotel. The following table displays data about hotel options. Identify the individuals and variables in this data set. Classify each variable as categorical or quantitative.

Hotel	Pool	Exercise room?	Internet (\$/day)	Restaurants	Distance to site (mi)	Room service?	Room rate (\$/day)
Comfort Inn	Out	Y	0.00	1	8.2	Y	149
Fairfield Inn & Suites	In	Y	0.00	1	8.3	N	119
Baymont Inn & Suites	Out	Y	0.00	1	3.7	Y	60
Chase Suite Hotel	Out	N	15.00	0	1.5	N	139
Courtyard	In	Y	0.00	1	0.2	Dinner	114
Hilton	In	Y	10.00	2	0.1	Y	156
Marriott	In	Y	9.95	2	0.0	Y	145

PD Section 1.2 Overview

To watch the video overview of Section 1.2, click on the link in the TE-Book, look on the TRFD, or download from the Teacher's Resources on the book's digital platform.

Teaching Tip

Emphasize the switch from categorical (Section 1.1) to quantitative variables (Section 1.2). It is important that students begin to use this distinction to classify the different methods of data analysis.

SECTION 1.2 Displaying Quantitative Data with Graphs

LEARNING TARGETS By the end of the section, you should be able to:

- Make and interpret dotplots, stemplots, and histograms of quantitative data.
- Identify the shape of a distribution from a graph.
- Describe the overall pattern (shape, center, and variability) of a distribution and identify any major departures from the pattern (outliers).
- Compare distributions of quantitative data using dotplots, stemplots, and histograms.

To display the distribution of a categorical variable, use a bar graph or a pie chart. How can we picture the distribution of a quantitative variable? In this section, we present several types of graphs that can be used to display quantitative data.

Dotplots

One of the simplest graphs to construct and interpret is a dotplot.

DEFINITION Dotplot

A dotplot shows each data value as a dot above its location on a number line.

Preparing for Inference

Be aware of the power of the dotplot. Instructionally, the dotplot can be created very quickly to display class data. More importantly, we will use the dotplot later to display the distribution of a statistic calculated from many samples, which will lead us to an understanding of sampling distributions.

Note that goals scored is a discrete variable.

Here are data on the number of goals scored in 20 games played by the 2016 U.S. women's soccer team:

5 5 1 10 5 2 1 1 2 3 3 2 1 4 2 1 2 1 9 3

Figure 1.5 shows a dotplot of these data.

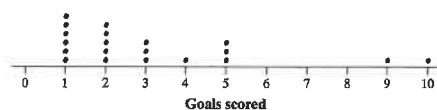


FIGURE 1.5 Dotplot of goals scored in 20 games by the 2016 U.S. women's soccer team.

It is fairly easy to make a dotplot by hand for small sets of quantitative data.

HOW TO MAKE A DOTPLOT

- **Draw and label the axis.** Draw a horizontal axis and put the name of the quantitative variable underneath. Be sure to include units of measurement for continuous variables.
- **Scale the axis.** Look at the smallest and largest values in the data set. Start the horizontal axis at a convenient number equal to or less than the smallest value and place tick marks at equal intervals until you equal or exceed the largest value.
- **Plot the values.** Mark a dot above the location on the horizontal axis corresponding to each data value. Try to make all the dots the same size and space them out equally as you stack them.

Remember what we said in Section 1.1: Making a graph is not an end in itself. When you look at a graph, always ask, "What do I see?" From Figure 1.5, we see that the 2016 U.S. women's soccer team scored 4 or more goals in $6/20 = 0.30$ or 30% of its games. That's quite an offense! Unfortunately, the team lost to Sweden on penalty kicks in the 2016 Summer Olympics.

EXAMPLE

Give it some gas! Making and interpreting dotplots

PROBLEM: The Environmental Protection Agency (EPA) is in charge of determining and reporting fuel economy ratings for cars. To estimate fuel economy, the EPA performs tests on several vehicles of the same make, model, and year. Here are data on the highway fuel economy ratings for a sample of 25 model year 2018 Toyota 4Runners tested by the EPA:

22.4 22.4 22.3 23.3 22.3 22.3 22.5 22.4 22.1 21.5 22.0 22.2 22.7
22.8 22.4 22.6 22.9 22.5 22.1 22.4 22.2 22.9 22.6 21.9 22.4



Section 1.2 Alternate Examples

You can find the Alternate Examples for this section in Microsoft Word format by clicking on the link in the TE-Book, opening the TRFD, or downloading from the Teacher's Resources on the book's digital platform.

Teaching Tip: Using Technology

All the graphs in this section can be created with technology. Go to the Student Site and choose "One Quantitative Variable" from the Extra Applets. Type the variable name and then the values for the variable. Then click "Begin Analysis." Graph options include dotplot, stemplot, histogram, and boxplot.

Teaching Tip

It is more important that students can interpret graphs and use them to answer questions than it is for them to create graphs. Don't spend too much class time constructing graphs by hand. Technology can usually create the graphs much more quickly.

Teaching Tip: Using Technology

Go to the Student Site and choose "One Quantitative Variable" from the Extra Applets. Type in the variable name and the data values. Click "Begin Analysis." The default graph type is dotplot.

ALTERNATE EXAMPLE

Skill 2.B

Fair soda cans?

Making and interpreting dotplots

PROBLEM:

Phoebe and Lisa bought a 12-pack of soda and measured the number of fluid ounces in each can. Here are the data:

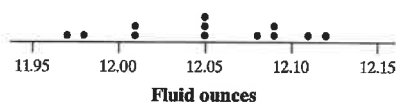
12.01	12.05	11.98	12.12
12.08	12.05	12.01	12.09
12.11	12.05	12.09	11.97

(a) Make a dotplot of these data.

(b) The company claims that its cans contain 12 ounces of soda. Do these data give Phoebe and Lisa reason to doubt that claim?

SOLUTION:

(a)



(b) No; of the 12 cans tested, 10 had at least 12 ounces of soda.

Preparing for Inference

Inferential thinking requires students to use data to evaluate the evidence against a claim. At the end of this Example, ask students, "Is Toyota's claim plausible (believable) or do we have convincing evidence against the claim?"

Teaching Tip

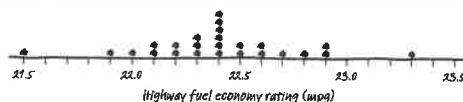
The word *skew* has a looser meaning in the English language than in statistics class. In statistics, it should only be used to describe the shape of a distribution. Students should not say, for example, "The outlier *skews* the data" or, in later chapters, that "the sampling method *skews* the data."

(a) Make a dotplot of these data.

(b) Toyota reports the highway gas mileage of its 2018 model year 4Runners as 22 mpg. Do these data give the EPA sufficient reason to investigate that claim?

SOLUTION:

(a)



(b) No. 23 of the 25 cars tested had an estimated highway fuel economy of 22 mpg or greater.

To make the dotplot:

- Draw and label the axis. Note the name and units for this continuous variable.
- Scale the axis. The smallest value is 21.5 and the largest value is 23.3. So we choose a scale from 21.5 to 23.5 with tick marks 0.1 units apart.
- Plot the values.

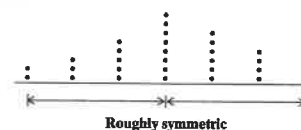
FOR PRACTICE, TRY EXERCISE 45

Describing Shape

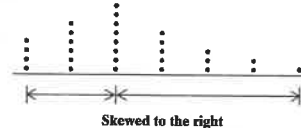
When you describe the shape of a dotplot or another graph of quantitative data, focus on the main features. Look for major *peaks*, not for minor ups and downs in the graph. Look for *clusters* of values and obvious *gaps*. Decide if the distribution is roughly symmetric or clearly skewed.

DEFINITION Symmetric and skewed distributions

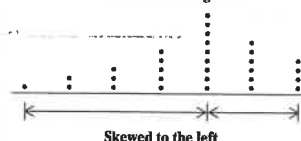
A distribution is roughly **symmetric** if the right side of the graph (containing the half of observations with the largest values) is approximately a mirror image of the left side.



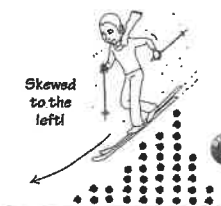
A distribution is **skewed to the right** if the right side of the graph is much longer than the left side.



A distribution is **skewed to the left** if the left side of the graph is much longer than the right side.



We could also describe a distribution with a long tail to the left as "skewed toward negative values" or "negatively skewed" and a distribution with a long right tail as "positively skewed."



For ease, we sometimes say "left-skewed" instead of "skewed to the left" and "right-skewed" instead of "skewed to the right." The direction of skewness is toward the long tail, not the direction where most observations are clustered. The drawing is a cute but corny way to help you keep this straight. To avoid danger, Mr. Starnes skis on the gentler slope—in the direction of the skewness.

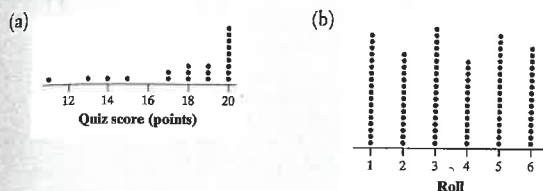
Teaching Tip

Distributions seldom have perfect shapes. Tell students to use *-ly* words to modify their description of the shape. For example, say "roughly symmetric," or "slightly skewed to the right," or "strongly skewed to the left."

EXAMPLE

Quiz scores and die rolls
Describing shape

PROBLEM: The dotplots display two different sets of quantitative data. Graph (a) shows the scores of 21 statistics students on a 20-point quiz. Graph (b) shows the results of 100 rolls of a 6-sided die. Describe the shape of each distribution.



SOLUTION:

- (a) The distribution of statistics quiz scores is skewed to the left, with a single peak at 20 (a perfect score). There are two small gaps at 12 and 16.
- (b) The distribution of die rolls is roughly symmetric. It has no clear peak.

We can describe the shape of the distribution in part (b) as *approximately uniform* because the frequencies are about the same for all possible rolls.

FOR PRACTICE, TRY EXERCISE 49

Some people refer to graphs with a single peak, like the dotplot of quiz scores in part (a) of the example, as *unimodal*. Figure 1.6 is a dotplot of the duration (in minutes) of 220 eruptions of the Old Faithful geyser. The graph has two distinct clusters and two peaks: one at about 2 minutes and one at about 4.5 minutes. We would describe this distribution's shape as roughly symmetric and *bimodal*. (Although we could continue the pattern with "trimodal" for three peaks and so on, it's more common to refer to distributions with more than two clear peaks as *multimodal*.) When you examine a graph of quantitative data, describe any pattern you see as clearly as you can.

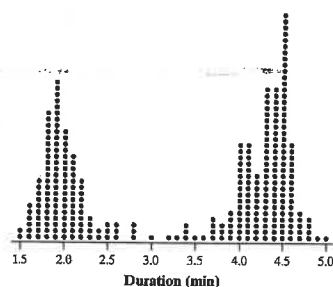


FIGURE 1.6 Dotplot displaying duration (in minutes) of 220 Old Faithful eruptions. This graph has two distinct clusters and two clear peaks (bimodal).

AP® EXAM TIP

Student solutions on the free response section of the AP® Statistics exam should always include the context of the problem. Exam questions are unlikely to say "in context" because that is always the expectation. Establish early in the course the importance of including context in solutions. In the example on this page, a student solution for part (a) that says "skewed to the left" is insufficient; however, "the distribution of statistics quiz scores is skewed to the left" includes context.

AP® EXAM TIP

When describing shape, the first priority is to use the terms *symmetric*, *skewed to the right*, or *skewed to the left*, if appropriate. Next identify peaks and gaps, if appropriate.

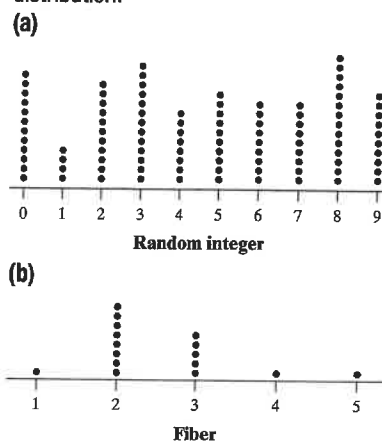
ALTERNATE EXAMPLE

Skill 2.A

Pick a random number for pizza?
Describing shape

PROBLEM:

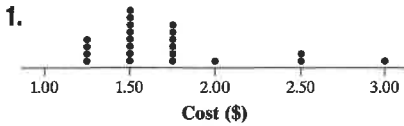
The dotplots below display two different sets of quantitative data. Graph (a) shows the results of 100 integers from a random digit generator. Graph (b) shows the number of grams of fiber in one serving of frozen cheese pizza for several different brands. Describe the shape of each distribution.



SOLUTION:

- (a) The distribution of random integers is roughly symmetric: It has no clear peak. We can describe the distribution as "approximately uniform."
- (b) The distribution of the number of grams of fiber in one serving of frozen cheese pizza is skewed to the right with a peak at 2 grams of fiber.

✓ Answers to CYU



2. The distribution of cost is skewed to the right, with a single peak at \$1.50. There are two small gaps at \$2.25 and \$2.75.

TRM Motivating SOCV

Consider using the “Motivating SOCV” activity, found by clicking on the link in the TE-Book, opening the TRFD, or downloading from the Teacher’s Resources on the book’s digital platform. It presents several different distributions and leads students to discover that the most important characteristics to identify are shape, outliers, center, and variability.

Variability is sometimes referred to as *spread*. We prefer variability because students sometimes think that spread refers only to the distance between the maximum and minimum value of a quantitative data set (the *range*). There are several ways to measure the variability (spread) of a distribution, including the range.

Some quantitative variables have distributions with predictable shapes. Many biological measurements on individuals from the same species and gender—lengths of bird bills, heights of young women—have roughly symmetric distributions. Salaries and home prices, on the other hand, usually have right-skewed distributions. There are many moderately priced houses, for example, but the few very expensive mansions give the distribution of house prices a strong right skew.



CHECK YOUR UNDERSTANDING

Knoebels Amusement Park in Elysburg, Pennsylvania, has earned acclaim for being an affordable, family-friendly entertainment venue. Knoebels does not charge for general admission or parking, but it does charge customers for each ride they take. How much do the rides cost at Knoebels? The table shows the cost for each ride in a sample of 22 rides in a recent year.

Name	Cost	Name	Cost
Merry Mixer	\$1.50	Looper	\$1.75
Italian Trapeze	\$1.50	Flying Turns	\$3.00
Satellite	\$1.50	Flyer	\$1.50
Galleon	\$1.50	The Haunted Mansion	\$1.75
Whipper	\$1.25	StratosFear	\$2.00
Skooters	\$1.75	Twister	\$2.50
Ribbit	\$1.25	Cosmotron	\$1.75
Roundup	\$1.50	Paratrooper	\$1.50
Paradrop	\$1.25	Downdraft	\$1.50
The Phoenix	\$2.50	Rockin' Tug	\$1.25
Gasoline Alley	\$1.75	Sklooosh!	\$1.75

1. Make a dotplot of the data.
2. Describe the shape of the distribution.

Describing Distributions

Here is a general strategy for describing a distribution of quantitative data.

HOW TO DESCRIBE THE DISTRIBUTION OF A QUANTITATIVE VARIABLE

In any graph, look for the *overall pattern* and for clear *departures* from that pattern.

- You can describe the overall pattern of a distribution by its **shape**, **center**, and **variability**.
- An important kind of departure is an **outlier**, an observation that falls outside the overall pattern.

AP® EXAM TIP

Always be sure to include context when you are asked to describe a distribution. This means using the variable name, not just the units the variable is measured in.

We will discuss more formal ways to measure center and variability and to identify outliers in Section 1.3. For now, just use the *median* (middle value in the ordered data set) when describing center and the *minimum* and *maximum* when describing variability.

Let's practice with the dotplot of goals scored in 20 games played by the 2016 U.S. women's soccer team.



Shape: The distribution of goals scored is skewed to the right, with a single peak at 1 goal. There is a gap between 5 and 9 goals.

Outliers: The games when the team scored 9 and 10 goals appear to be outliers.

Center: The median is 2 goals scored.

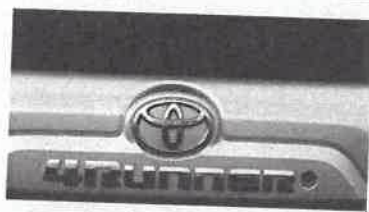
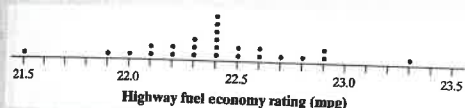
Variability: The data vary from 1 to 10 goals scored.

When describing a distribution of quantitative data, don't forget: Statistical Opinions Can Vary (Shape, Outliers, Center, Variability).

EXAMPLE

Give it some gas!
Describing a distribution

PROBLEM: Here is a dotplot of the highway fuel economy ratings for a sample of 25 model year 2018 Toyota 4Runners tested by the EPA. Describe the distribution.



SOLUTION:

Shape: The distribution of highway fuel economy ratings is roughly symmetric, with a single peak at 22.4 mpg. There are two clear gaps: between 21.5 and 21.9 mpg and between 22.9 and 23.3 mpg.

Outliers: The cars with 21.5 mpg and 23.3 mpg ratings are possible outliers.

Center: The median rating is 22.4 mpg.

Variability: The ratings vary from 21.5 to 23.3 mpg.

Be sure to include context by discussing the variable of interest, highway fuel economy ratings. And give the units of measurement: miles per gallon (mpg).

FOR PRACTICE, TRY EXERCISE 53

ALTERNATE EXAMPLE

How many calories in that slice of pizza? Describing a distribution

Skill 2.A

PROBLEM:

Here are the calories per serving for 16 brands of frozen cheese pizza, along with a dotplot of the data. Describe the distribution.



SOLUTION:

Shape: The distribution of number of calories per serving is roughly symmetric with peaks at 320 and 340 calories. There are three gaps: between 260 and 290 calories, between 290 and 310 calories, and between 360 and 380 calories.

Outliers: One pizza has an unusually small number of calories (260).

Center: The median number of calories is 330 calories.

Variability: The values vary from 260 to 380 calories.

AP® EXAM TIP

One of the free response questions on the AP® Statistics exam often is a scenario where students are asked to describe or compare two distributions of quantitative data. It is important that they address shape, outliers, center, and variability *in the context of the problem*. The student discussion of context should make it clear what variable is being measured. Once context is established (say, in describing the shape), it is no longer necessary to include context over and over again.

+ Ask the StatsMedic

Knix the Trix: Except for SOCV and DU

In this post, we discuss consequences for when students use acronyms to memorize procedures. DUFS will be introduced in Chapter 3.

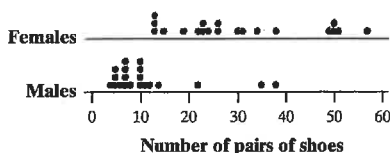
ALTERNATE EXAMPLE

Skill 2.D

How many shoes are too many shoes?
Comparing distributions

PROBLEM:

How many pairs of shoes does a typical teenager own? To find out, a group of statistics students surveyed separate random samples of 20 female students and 20 male students from their large high school. Here are dotplots of the number of pairs of shoes owned by the survey respondents. Compare the distributions of number of pairs of shoes for females and males.



SOLUTION:

Shape: The distribution of numbers of pairs of shoes for the female students is slightly skewed right, with a single peak at 13 pairs of shoes. The distribution of numbers of pairs of shoes for the male students is also skewed to the right, with peaks at 7 and 10 pairs of shoes.

Outliers: There don't appear to be any outliers in the female student distribution. The male distribution seems to have three outliers: the male students with 22, 35, and 38 pairs of shoes.

Center: The numbers of pairs of shoes for female students tend to be greater (median = 26 pairs) than for the male students (median = 9 pairs).

Variability: The numbers of pairs of shoes for female students vary more (from 13 to 57 pairs) than for the males (from 4 to 38 pairs).

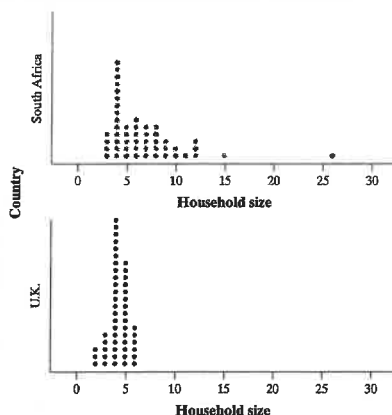
Comparing Distributions

Some of the most interesting statistics questions involve comparing two or more groups. Which of two popular diets leads to greater long-term weight loss? Who texts more—males or females? As the following example suggests, you should always discuss shape, outliers, center, and variability whenever you compare distributions of a quantitative variable.

EXAMPLE

Household size: U.K. versus South Africa
Comparing distributions

PROBLEM: How do the numbers of people living in households in the United Kingdom (U.K.) and South Africa compare? To help answer this question, we used Census At School's "Random Data Selector" to choose 50 students from each country. Here are dotplots of the household sizes reported by the survey respondents. Compare the distributions of household size for these two countries.



SOLUTION:

Shape: The distribution of household size for the U.K. sample is roughly symmetric, with a single peak at 4 people. The distribution of household size for the South Africa sample is skewed to the right, with a single peak at 4 people and a clear gap between 15 and 26.

Outliers: There don't appear to be any outliers in the U.K. distribution.

The South African distribution seems to have two outliers: the households with 15 and 26 people.

Center: Household sizes for the South African students tend to be larger (median = 6 people) than for the U.K. students (median = 4 people).

Variability: The household sizes for the South African students vary more (from 3 to 26 people) than for the U.K. students (from 2 to 6 people).

AP® EXAM TIP

When comparing distributions of quantitative data, it's not enough just to list values for the center and variability of each distribution. You have to explicitly *compare* these values, using words like "greater than," "less than," or "about the same as."

Don't forget to include context! It isn't enough to refer to the U.K. distribution or the South Africa distribution. You need to mention the variable of interest, household size.

FOR PRACTICE, TRY EXERCISE 55

Teaching Tip

It is incorrect to say that the "center of South Africa" is higher than the "center of the United Kingdom." After all, the middle of South Africa is much farther south than the middle of the U.K.! It is better to say that the "center of the distribution of household size in South Africa is much greater than the center of the distribution of household size in the U.K."



Frank van der Berg/Getty Images

A stemplot stem-and-

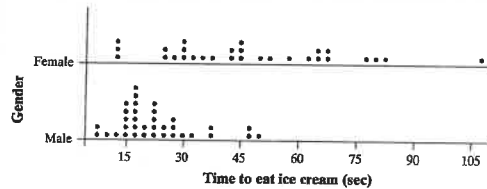
FIGURE 1.
resting pu
school stu

Notice that in the preceding example, we discussed the distributions of household size only for the two *samples* of 50 students. We might be interested in whether the sample data give us convincing evidence of a difference in the *population* distributions of household size for South Africa and the United Kingdom. We'll have to wait a few chapters to decide whether we can reach such a conclusion, but our ability to make such an inference later will be helped by the fact that the students in our samples were chosen at random.



CHECK YOUR UNDERSTANDING

For a statistics class project, Jonathan and Crystal hosted an ice-cream-eating contest. Each student in the contest was given a small cup of ice cream and instructed to eat it as fast as possible. Jonathan and Crystal then recorded each contestant's gender and time (in seconds), as shown in the dotplots. Compare the distributions of eating times for males and females.



Stemplots

Another simple type of graph for displaying quantitative data is a stemplot.

DEFINITION Stemplot

A **stemplot** shows each data value separated into two parts: a **stem**, which consists of all but the final digit, and a **leaf**, the final digit. The stems are ordered from lowest to highest and arranged in a vertical column. The leaves are arranged in increasing order out from the appropriate stems.

Here are data on the resting pulse rates (beats per minute) of 19 middle school students:

71 104 76 88 78 71 68 86 70 90 74 76 69 68 88 96 68 82 120

Figure 1.7 shows a stemplot of these data.

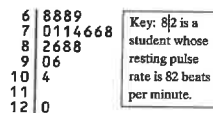


FIGURE 1.7 Stemplot of the resting pulse rates of 19 middle school students.

According to the American Heart Association, a resting pulse rate above 100 beats per minute is considered high for this age group. We can see that $2/19 = 0.105 = 10.5\%$

Preparing for Inference

In Chapter 11, we could use a two-sample *t*-test for a difference in means to determine if there is convincing evidence of a difference in mean household size between the U.K. and South Africa. The fact that we have two random samples will allow us to generalize our results to both populations.

✓ Answers to CYU

Shape: The distribution of time to eat ice cream (in seconds) for the female sample is skewed to the right, with no clear peak. The distribution of time to eat ice cream (in seconds) for the male sample is skewed to the right, with a single peak at about 17.5 seconds.

Outliers: The female distribution appears to have one outlier: the female who took approximately 105 seconds to eat the ice cream. The male distribution does not appear to contain any outliers.

Center: The time it took female students to eat ice cream was generally longer (median ≈ 45 seconds) than for male students (median ≈ 20 seconds).

Variability: The ice-cream-eating times for female students vary more (from about 13 seconds to about 107 seconds) than for the male students (from about 5 seconds to about 50 seconds).



COMMON STUDENT ERROR

The stem of 11 has no leaves. Students will often omit this stem. This is not appropriate, because doing so will hide the gap that exists in the distribution. Also, students will often forget the key or labels on a stemplot.

Teaching Tip: Using Technology

Go to the Student Site and choose "One Quantitative Variable" from the Extra Applets. Type in the variable name and the data values. Click "Begin Analysis." Change the graph option to stemplot.

ALTERNATE EXAMPLE

Skill 2.B

How many medals?**Making and interpreting stemplots****PROBLEM:**

The table displays the total number of medals 28 countries won during the 2016 Summer Olympic Games in Rio de Janeiro.

Country	Medals
United States	121
China	70
Great Britain	67
Russia	56
Germany	42
France	42
Japan	41
Australia	29
Italy	28
Canada	22
South Korea	21
Netherlands	19
Brazil	19
Spain	18
New Zealand	18
Azerbaijan	18
Kazakhstan	17
Hungary	15
Denmark	15
Kenya	13
Uzbekistan	13
Jamaica	11
Cuba	11
Sweden	11
Ukraine	11
Poland	11
Croatia	10
South Africa	10

(a) Make a stemplot of these data.

(b) Describe the shape of the distribution. Are there any obvious outliers?

SOLUTION:

(a)	1	0 0 1 1 1 1 1 3 3 5 5 7 8 8 9 9
	2	1 2 8 9
	3	
	4	1 2 2
	5	6
	6	7
	7	0
	8	
	9	
	10	
	11	
	12	1

Key: 4|1 is a country with 41 total medals

of these students have high resting pulse rates by this standard. Also, the distribution of pulse rates for these 19 students is skewed to the right (toward the larger values).

Stemplots give us a quick picture of a distribution that includes the individual observations in the graph. It is fairly easy to make a stemplot by hand for small sets of quantitative data.

HOW TO MAKE A STEMLOT

- **Make stems.** Separate each observation into a stem, consisting of all but the final digit, and a leaf, the final digit. Write the stems in a vertical column with the smallest at the top. Draw a vertical line at the right of this column. Do not skip any stems, even if there is no data value for a particular stem.
- **Add leaves.** Write each leaf in the row to the right of its stem.
- **Order leaves.** Arrange the leaves in increasing order out from the stem.
- **Add a key.** Provide a key that identifies the variable and explains what the stems and leaves represent.

EXAMPLE**Wear your helmets!****Making and interpreting stemplots**

PROBLEM: Many athletes (and their parents) worry about the risk of concussions when playing sports. A football coach plans to obtain specially made helmets for his players that are designed to reduce the chance of getting a concussion. Here are the measurements of head circumference (in inches) for the 30 players on the team:

23.0 22.2 21.7 22.0 22.3 22.6 22.7 21.5 22.7 25.6 20.8 23.0 24.2 23.5 20.8
24.0 22.7 22.6 23.9 22.5 23.1 21.9 21.0 22.4 23.5 22.5 23.9 23.4 21.6 23.3

(a) Make a stemplot of these data.

(b) Describe the shape of the distribution. Are there any obvious outliers?

SOLUTION:

(a)

20 | 38
21 | 05679
22 | 02345566777
23 | 001345533
24 | 02
25 | 6

Key: 23|5 is a player with a head circumference of 23.5 inches.

(b) The distribution of head circumferences for the 30 players on the team is roughly symmetric, with a single peak on the 22-inch stem. There are no obvious outliers.

To make the stemplot:

- **Make stems.** The smallest head circumference is 20.8 inches and the largest is 25.6 inches. We use the first two digits as the stem and the final digit as the leaf. So we need stems from 20 to 25.
- **Add leaves.**
- **Order leaves.**
- **Add a key.**

FOR PRACTICE, TRY EXERCISE 59

We can get a better picture of the head circumference data by *splitting stems*. In Figure 1.8(a), leaf values from 0 to 9 are placed on the same stem. Figure 1.8(b) shows another stemplot of the same data. This time, values with leaves from 0 to 4 are placed on one stem, while those with leaves from 5 to 9 are placed on another stem. Now we can see the shape of the distribution even more clearly—including the possible outlier at 25.6 inches.

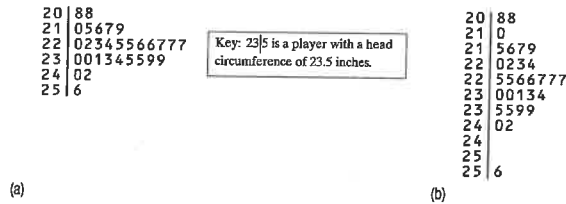


FIGURE 1.8 Two stemplots showing the head circumference data. The graph in (b) improves on the graph in (a) by splitting stems.

Here are a few tips to consider before making a stemplot:

- There is no magic number of stems to use. Too few or too many stems will make it difficult to see the distribution's shape. Five stems is a good minimum.
- If you split stems, be sure that each stem is assigned an equal number of possible leaf digits.
- When the data have too many digits, you can get more flexibility by rounding or truncating the data. See Exercises 61 and 62 for an illustration of rounding data before making a stemplot.

You can use a *back-to-back stemplot* with common stems to compare the distribution of a quantitative variable in two groups. The leaves are placed in order on each side of the common stem. For example, Figure 1.9 shows a back-to-back stemplot of the 19 middle school students' resting pulse rates and their pulse rates after 5 minutes of running.

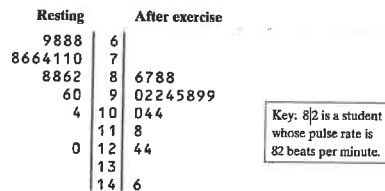


FIGURE 1.9 Back-to-back stemplot of 19 middle school students' resting pulse rates and their pulse rates after 5 minutes of running.



COMMON STUDENT ERROR

When identifying the shape of each distribution using a back-to-back stemplot, students will often give the wrong direction of the skew for the distribution displayed on the left side of the back-to-back stemplot. For this example, the resting pulse rates are mostly in the 70s, with values extending farther to larger values than to smaller values (skewed to the right).

✓ Answers to CYU

1. **Shape:** The distribution of resting pulse rates is skewed to the right, with a single peak on the 70s stem. The distribution of after-exercise pulse rates is also skewed to the right, with a single peak on the 90s stem. **Outliers:** The distribution of resting pulse rates appears to have one outlier: the student whose resting pulse rate was 120 bpm. The distribution of after-exercise pulse rates appears to have one outlier as well: the student whose after-exercise pulse rate was 146 bpm. **Center:** The students' resting pulse rates tended to be lower (median = 76 bpm) than their "after-exercise" pulse rates (median = 98 bpm). **Variability:** The "after-exercise" pulse rates vary more (from 86 bpm to 146 bpm) than the resting pulse rates (which vary from 68 bpm to 120 bpm).

2. b

3. e

4. c



CHECK YOUR UNDERSTANDING

1. Write a few sentences comparing the distributions of resting and after-exercise pulse rates in Figure 1.9.

Multiple Choice: Select the best answer for Questions 2–4.

Here is a stemplot of the percent of residents aged 65 and older in the 50 states and District of Columbia:

6	8
7	
8	8
9	79
10	08
11	15566
12	0122234444578889999
13	01233333444899
14	02666
15	23
16	8

Key: 8|8 represents a state in which 8.8% of residents are 65 and older.

- The low outlier is Alaska. What percent of Alaska residents are 65 or older?
(a) 0.68 (b) 6.8 (c) 8.8 (d) 16.8 (e) 68
- Ignoring the outlier, the shape of the distribution is
(a) skewed to the right.
(b) skewed to the left.
(c) skewed to the middle.
(d) double-peaked (bimodal).
(e) roughly symmetric.
- The center of the distribution is close to
(a) 11.6%. (b) 12.0%. (c) 12.8%. (d) 13.3%. (e) 6.8% to 16.8%

Histograms

You can use a dotplot or stemplot to display quantitative data. Both graphs show every individual data value. For large data sets, this can make it difficult to see the overall pattern in the graph. We often get a clearer picture of the distribution by grouping together nearby values. Doing so allows us to make a new type of graph called a histogram.

DEFINITION Histogram

A histogram shows each interval of values as a bar. The heights of the bars show the frequencies or relative frequencies of values in each interval.

Figure 1.10 shows a dotplot and a histogram of the durations (in minutes) of 220 eruptions of the Old Faithful geyser. Notice how the histogram groups together nearby values.

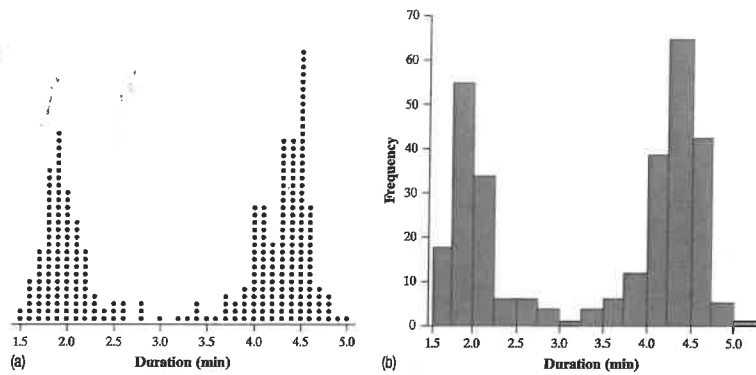


FIGURE 1.10 (a) Dotplot and (b) histogram of the duration (in minutes) of 220 eruptions of the Old Faithful geyser.

It is fairly easy to make a histogram by hand. Here's how you do it.

HOW TO MAKE A HISTOGRAM

- **Choose equal-width intervals** that span the data. Five intervals is a good minimum.
- **Make a table** that shows the frequency (count) or relative frequency (percent or proportion) of individuals in each interval. Put values that fall on an interval boundary in the interval containing larger values.
- **Draw and label the axes.** Draw horizontal and vertical axes. Put the name of the quantitative variable under the horizontal axis. To the left of the vertical axis, indicate whether the graph shows the frequency (count) or relative frequency (percent or proportion) of individuals in each interval.
- **Scale the axes.** Place equally spaced tick marks at the smallest value in each interval along the horizontal axis. On the vertical axis, start at 0 and place equally spaced tick marks until you exceed the largest frequency or relative frequency in any interval.
- **Draw bars above the intervals.** Make the bars equal in width and leave no gaps between them. Be sure that the height of each bar corresponds to the frequency or relative frequency of individuals in that interval. An interval with no data values will appear as a bar of height 0 on the graph.

It is possible to choose intervals of unequal widths when making a histogram. Such graphs are beyond the scope of this book.

Teaching Tip: Using Technology

Go to the Student Site and choose "One Quantitative Variable" from the Extra Applets. Type in the variable name and the data values. Click "Begin Analysis." Change the graph option to histogram.

ALTERNATE EXAMPLE Skill 2.B**How many medals?
Making and interpreting a histogram****PROBLEM:**

The table displays the total number of medals 28 countries won during the 2016 Summer Olympic Games in Rio de Janeiro.

Country	Medals
United States	121
China	70
Great Britain	67
Russia	56
Germany	42
France	42
Japan	41
Australia	29
Italy	28
Canada	22
South Korea	21
Netherlands	19
Brazil	19
Spain	18
New Zealand	18
Azerbaijan	18
Kazakhstan	17
Hungary	15
Denmark	15
Kenya	13
Uzbekistan	13
Jamaica	11
Cuba	11
Sweden	11
Ukraine	11
Poland	11
Croatia	10
South Africa	10

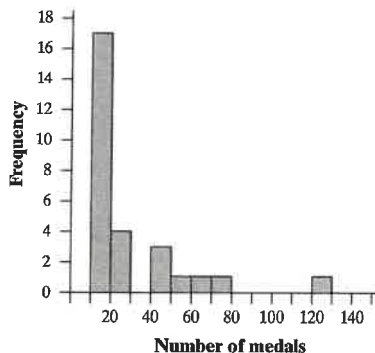
(a) Make a frequency histogram to display the data.

(b) What percent of values in the graph are less than 30? Interpret this result in context.

SOLUTION:

(a) The data vary from 10 to 121 medals. I will choose intervals of width 10, starting at 10 medals.

Interval	Frequency
10 to <20	17
20 to <30	4
30 to <40	0
40 to <50	3
50 to <60	1
60 to <70	1
70 to <80	1
80 to <90	0
90 to <100	0
100 to <110	0
110 to <120	0
120 to <130	1



(b) $21/28 = 75\%$. 75% of the countries represented in the table won a total of fewer than 30 medals at the 2016 Summer Olympics.

EXAMPLE**How much tax?
Making and interpreting histograms**

PROBLEM: Sales tax rates vary widely across the United States. Four states charge no state or local sales tax: Delaware, Montana, New Hampshire, and Oregon. The table shows data on the average total tax rate for each of the remaining 46 states and the District of Columbia.²³

State	Tax rate (%)	State	Tax rate (%)	State	Tax rate (%)
Alabama	9.0	Louisiana	9.0	Oklahoma	8.8
Alaska	1.8	Maine	5.5	Pennsylvania	6.3
Arizona	8.3	Maryland	6.0	Rhode Island	7.0
Arkansas	9.3	Massachusetts	6.3	South Carolina	7.2
California	8.5	Michigan	6.0	South Dakota	5.8
Colorado	7.5	Minnesota	7.3	Tennessee	9.5
Connecticut	6.4	Mississippi	7.1	Texas	8.2
Florida	6.7	Missouri	7.9	Utah	6.7
Georgia	7.0	Nebraska	6.9	Vermont	6.2
Hawaii	4.4	Nevada	8.0	Virginia	5.6
Idaho	6.0	New Jersey	7.0	Washington	8.9
Illinois	8.6	New Mexico	7.5	West Virginia	6.2
Indiana	7.0	New York	8.5	Wisconsin	5.4
Iowa	6.8	North Carolina	6.9	Wyoming	5.4
Kansas	8.6	North Dakota	6.8	District of Columbia	5.8
Kentucky	6.0	Ohio	7.1		

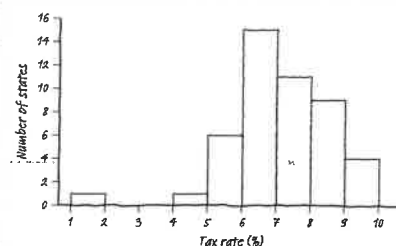
(a) Make a frequency histogram to display the data.

(b) What percent of values in the distribution are less than 6.0? Interpret this result in context.

SOLUTION:

(a)

Interval	Frequency
1.0 to <2.0	1
2.0 to <3.0	0
3.0 to <4.0	0
4.0 to <5.0	1
5.0 to <6.0	6
6.0 to <7.0	15
7.0 to <8.0	11
8.0 to <9.0	9
9.0 to <10.0	4



(b) $8/47 = 0.170 = 17.0\%$; 17% of the states (including the District of Columbia) have tax rates less than 6%.

To make the histogram:

- Choose equal-width intervals that span the data. The data vary from 1.8 percent to 9.5 percent. So we choose intervals of width 1.0, starting at 1.0%.
- Make a table. Record the number of states in each interval to make a frequency histogram.
- Draw and label the axes. Don't forget units (percent) for the variable (tax rate).
- Scale the axes.
- Draw bars.

FOR PRACTICE, TRY EXERCISE 67

Figure 1.11 shows two different histograms of the state sales tax data. Graph (a) uses the intervals of width 1% from the preceding example. The distribution has a single peak in the 6.0 to <7.0 interval. Graph (b) uses intervals half as wide: 1.0 to <1.5, 1.5 to <2.0, and so on. Now we see a distribution with more than one distinct peak. The choice of intervals in a histogram can affect the appearance of a distribution. Histograms with more intervals show more detail but may have a less clear overall pattern.

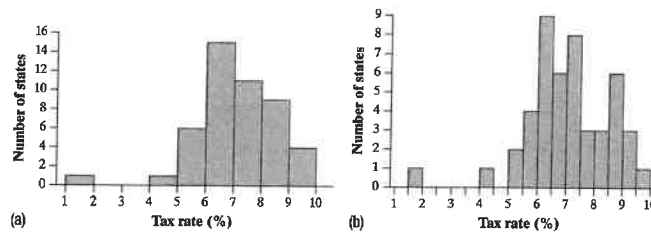


FIGURE 1.11 (a) Frequency histogram of the sales tax rate in the states that have local or state sales taxes and the District of Columbia with intervals of width 1.0%, from the preceding example. (b) Frequency histogram of the data with intervals of width 0.5%.

You can use a graphing calculator, statistical software, or an applet to make a histogram. The technology's default choice of intervals is a good starting point, but you should adjust the intervals to fit with common sense.

2. Technology Corner

MAKING HISTOGRAMS

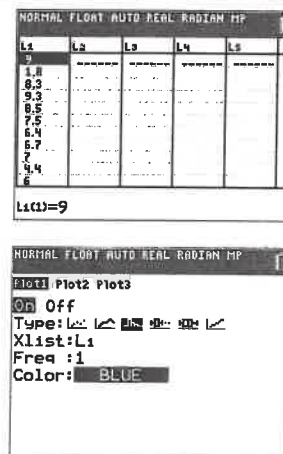
TI-Nspire and other technology instructions are on the book's website at highschool.bfwpub.com/updatedtps6e.

1. Enter the data from the sales tax example in your Statistics/List Editor.

- Press **STAT** and choose Edit...
- Type the values into list L1.

2. Set up a histogram in the Statistics Plots menu.

- Press **2nd** **Y=** (STAT PLOT).
- Press **ENTER** or **1** to go into Plot1.
- Adjust the settings as shown.



Teaching Tip: Using Technology

The *Histogram Bin-Width* applet allows students to explore how changing the classes in a histogram can greatly alter the apparent shape of a distribution. Go to www.rossmanchance.com/applets and choose "Histogram Bin-Width" and be sure to click on the JavaScript version.

Teaching Tip

Some people prefer "single-peaked" to "unimodal." In a histogram with one peak, we can't ascertain that there is one mode (because values are grouped together). "Unimodal" is too strong in many cases, but "single-peaked" is OK even if there is more than 1 mode (or no modes).

Teaching Tip: Using Technology

The screen shots we show for the TI-83/84 are from the TI-84 Plus CE. In many cases, the instructions are the same for older and newer operating systems.

Teaching Tip: Using Technology

Remind students that instructions for using the TI-Nspire are available on the Student Site.

Teaching Tip: Using Technology

Students may ask what the "Freq" setting is for. "Freq" stands for frequency, so when $\text{Freq} = 1$, each observation in L1 is counted once. The only time you might use something other than 1 is if you have data already in a frequency table. For example, suppose that the distribution of scores for your students on last year's AP[®] Statistics exam was

Score	Frequency
1	3
2	5
3	14
4	6
5	4

If you enter the values 1 to 5 in L1 and the frequencies in L2, you should use $\text{Freq} = \text{L2}$ when making a graph or calculating summary statistics.

3. Use ZoomStat to let the calculator choose intervals and make a histogram.

- Press **ZOOM** and choose ZoomStat.
- Press **TRACE** to examine the intervals.

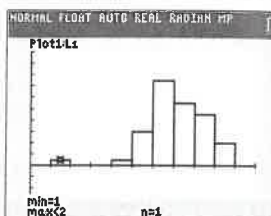
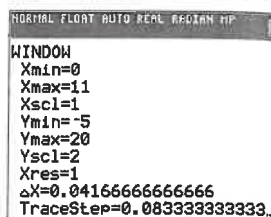
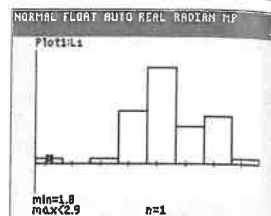
4. Adjust the intervals to match those in Figure 1.11(a), and then graph the histogram.

- Press **WINDOW** and enter the values shown for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl.
- Press **GRAPH**.
- Press **TRACE** to examine the intervals.

5. See if you can match the histogram in Figure 1.11(b).

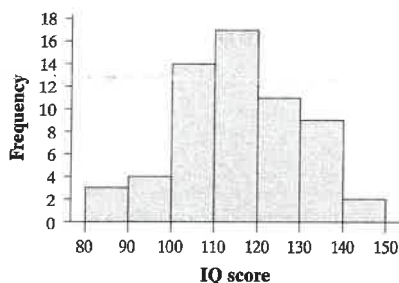
AP® EXAM TIP

If you're asked to make a graph on a free-response question, be sure to label and scale your axes. Unless your calculator shows labels and scaling, don't just transfer a calculator screen shot to your paper.



✓ Answers to CYU

1. One possible histogram is displayed below.

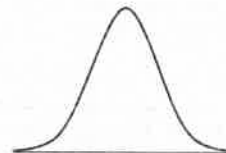


2. The distribution of IQ scores is roughly symmetric and bell-shaped with a single peak (unimodal). There are no obvious outliers. The typical IQ score appears to be between 110 and 120 (median = 114). The IQ scores vary from about 80 to 150.



CHECK YOUR UNDERSTANDING

Many people believe that the distribution of IQ scores follows a “bell curve,” like the one shown. But is this really how such scores are distributed? The IQ scores of 60 fifth-grade students chosen at random from one school are shown here.²⁴



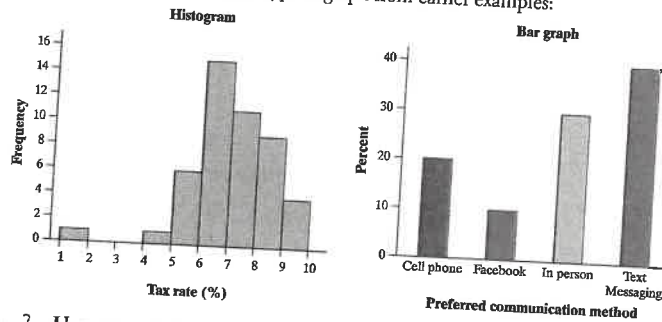
145	139	126	122	125	130	96	110	118	118
101	142	134	124	112	109	134	113	81	113
123	94	100	136	109	131	117	110	127	124
106	124	115	133	116	102	127	117	109	137
117	90	103	114	139	101	122	105	97	89
102	108	110	128	114	112	114	102	82	101

1. Construct a histogram that displays the distribution of IQ scores effectively.
2. Describe what you see. Is the distribution bell-shaped?

Using Histograms Wisely

We offer several cautions based on common mistakes students make when using histograms.

1. **Don't confuse histograms and bar graphs.** Although histograms resemble bar graphs, their details and uses are different. A histogram displays the distribution of a quantitative variable. Its horizontal axis identifies intervals of values that the variable takes. A bar graph displays the distribution of a categorical variable. Its horizontal axis identifies the categories. Be sure to draw bar graphs with blank space between the bars to separate the categories. Draw histograms with no space between bars for adjacent intervals. For comparison, here is one of each type of graph from earlier examples:



2. **Use percents or proportions instead of counts on the vertical axis when comparing distributions with different numbers of observations.** Mary was interested in comparing the reading levels of a biology journal and an airline magazine. She counted the number of letters in the first 400 words of an article in the journal and of the first 100 words of an article in the airline magazine. Mary then used statistical software to produce the histograms shown in Figure 1.12(a). This figure is misleading—it compares frequencies, but the two samples were of very different sizes (400 and 100). Using the same data, Mary's teacher produced the histograms in Figure 1.12(b). By using relative frequencies, this figure makes the comparison of word lengths in the two samples much easier.

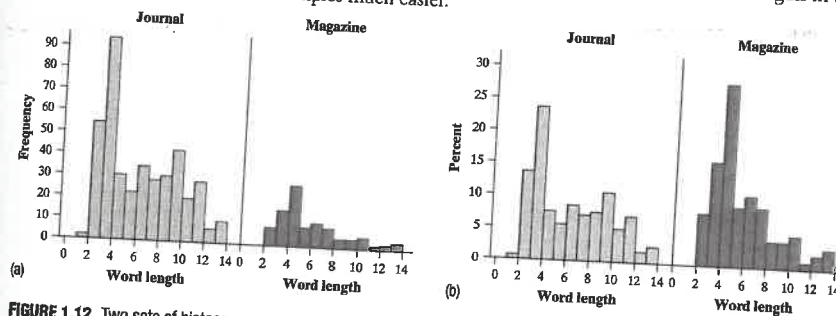


FIGURE 1.12 Two sets of histograms comparing word lengths in articles from a biology journal and from an airline magazine. In graph (a), the vertical scale uses frequencies. Graph (b) fixes the problem of different sample sizes by using percents (relative frequencies) on the vertical scale.

Teaching Tip: Using Technology

The bar graph on the left is not helpful because it doesn't allow us to identify the shape, center, or variability of the distribution of name length. To be a proper histogram, the variable should be on the horizontal axis and frequency (or relative frequency) should be on the vertical axis. Warn your students against using Excel for graphing quantitative variables unless they have a statistics add-on (which they probably don't). It is much better to use statistical software or applets designed for such a purpose.

✓ Answers to CYU

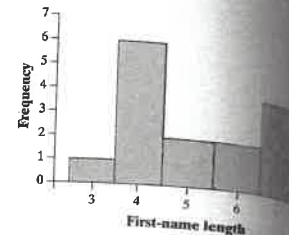
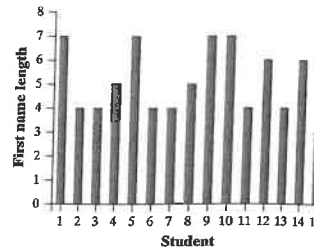
1. The distribution of word length for both the Journal and the Magazine have shapes that are skewed to the right and single-peaked. Neither distribution appears to have any outliers. The centers for both distributions are about the same at approximately 5–6 letters per word. Both distributions have similar variability, as the length of words in the Journal varies from 1 letter to 14 letters and the length of words in the Magazine varies from 2 letters to 14 letters.

2. This is a bar graph. It displays categorical data about first-year students' planned field of study.

3. No, because the variable is categorical and the categories could be listed in any order on the horizontal axis.



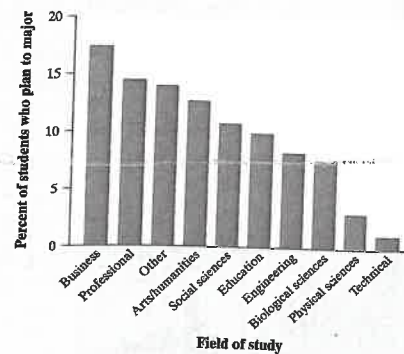
3. Just because a graph looks nice doesn't make it a meaningful data. The 15 students in a small statistics class recorded the number of letters in their first names. One student entered the data into an Excel spreadsheet and then used Excel's "chart maker" to produce the graph shown on the left. What kind of graph is this? It's a bar graph that compares the raw data. But first-name length is a quantitative variable, so a bar graph is not the appropriate way to display its distribution. The histogram on the right is a better choice because the graph makes it easier to identify the shape and variability of the distribution of name length.



CHECK YOUR UNDERSTANDING

1. Write a few sentences comparing the distributions of word length shown in Figure 1.12(b).

Questions 2 and 3 refer to the following setting. About 3 million first-year students enroll in colleges and universities each year. What do they plan to study? The graph displays data on the percent of first-year students who plan to major in several disciplines.²⁵



2. Is this a bar graph or a histogram? Explain.
3. Would it be correct to describe this distribution as right-skewed? Why or why not?

Section 1.2 Summary

- You can use a **dotplot**, **stemplot**, or **histogram** to show the distribution of a quantitative variable. A dotplot displays individual values on a number line. Stemplots separate each observation into a stem and a one-digit leaf. Histograms plot the frequencies (counts) or relative frequencies (proportions or percents) of values in equal-width intervals.
- Some distributions have simple shapes, such as **symmetric**, **skewed to the left**, or **skewed to the right**. The number of peaks is another aspect of overall shape. So are distinct clusters and gaps.
- A single-peaked graph is sometimes called **unimodal**, and a double-peaked graph is sometimes called **bimodal**.
- When examining any graph of quantitative data, look for an **overall pattern** and for clear **departures** from that pattern. Shape, center, and variability describe the overall pattern of the distribution of a quantitative variable. Outliers are observations that lie outside the overall pattern of a distribution.
- When comparing distributions of quantitative data, be sure to compare shape, center, variability, and possible outliers.
- Remember: histograms are for quantitative data; bar graphs are for categorical data. Be sure to use relative frequencies when comparing data sets of different sizes.

1.2 Technology Corner

TI-Nspire and other technology instructions are on the book's website at highschool.bfwpub.com/updatedtps6e.

2. Making histograms

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Section 1.2 Exercises

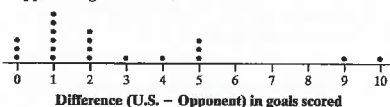
45. **Feeling sleepy?** Students in a high school statistics class responded to a survey designed by their teacher. One of the survey questions was "How much sleep did you get last night?" Here are the data (in hours):

9	6	8	7	8	8	6	6.5	7	7	9.0	4	3	4
5	6	11	6	3	7	6	10.0	7	8	4.5	9	7	7

- (a) Make a dotplot to display the data.
- (b) Experts recommend that high school students sleep at least 9 hours per night. What proportion of students in this class got the recommended amount of sleep?
46. **Easy reading?** Here are data on the lengths of the first 25 words on a randomly selected page from Toni Morrison's *Song of Solomon*:

2	3	4	10	2	11	2	8	4	3	7	2	7
5	3	6	4	4	2	5	8	2	3	4	4	

- (a) Make a dotplot of these data.
- (b) Long words can make a book hard to read. What percentage of words in the sample have 8 or more letters?
47. **U.S. women's soccer—2016** Earlier, we examined data on the number of goals scored by the 2016 U.S. women's soccer team in 20 games played. The following dotplot displays the goal differential for those same games, computed as U.S. goals scored minus opponent goals scored.



- (a) Explain what the dot above 3 represents.
- (b) What does the graph tell us about how well the team did in 2016? Be specific.

Section 1.2 Quiz

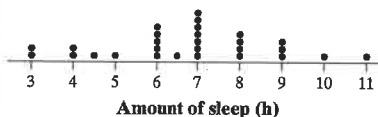
There is one quiz available for this section in the Teacher's Resource Materials. Click on the link in the TE-Book, open in the TRFD, or download from the Teacher's Resources on the book's digital platform. You can also create your own quiz using the ExamView® Assessment Suite that is part of the TPS6 program. Questions are coded by Learning Target to make it easy to build parallel quizzes.

Full Solutions to Section 1.2 Exercises

The full solutions can be found by clicking on the link in the TE-Book, opening the TRFD, or downloading from the Teacher's Resources on the book's digital platform.

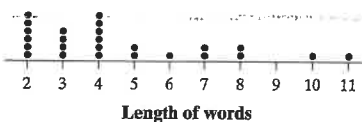
Answers to Section 1.2 Exercises

1.45 (a)



(b) 0.179

1.46 (a)



(b) 16%

1.47 (a) The dot above 3 indicates that there was one game in which the U.S. women's soccer team scored 3 more goals than their opponent. (b) All 20 of the values are zero or more, which indicates that the U.S. women's soccer team had a very good season. They won $17/20 = 85\%$ of their games, tied with the other team in $3/20 = 15\%$ of their games, and never lost.

1.48 (a) The dot above -3 represents a car that got 3 mpg less on the highway than it did in city driving. **(b)** From the dotplot, we see that the EPA mpg rating is greater on the highway than in the city for all but one of the cars. These other 23 cars all averaged at least 7 mpg more on the highway than in the city.

1.49 The shape of the distribution is left-skewed with a peak between 90 and 100 years. There is a small gap around 70 years.

1.50 The shape of the distribution is roughly symmetric, with a single peak at 7.

1.51 The shape of the distribution is roughly symmetric, with a single peak at 7.

1.52 The shape of the distribution is skewed to the right, with peaks at 2 and 4 letters.

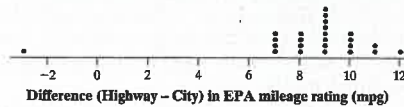
1.53 The distribution of difference (U.S. — Opponent) in goals scored is skewed to the right, with two potential outliers: when the U.S. team outscored their opponents by 9 and 10 goals. The median difference was 2 goals and the differences varied from 0 to 10 goals.

1.54 The distribution of difference (Highway — City) in EPA mileage rating is roughly symmetric, with a low outlier at -3 . The median difference is 9 mpg and the differences vary from -3 to 12 miles per gallon.

1.55 The distribution of total family income for Indiana is roughly symmetric, while the distribution of total family income for New Jersey is slightly skewed right. The value of \$125,000 may be an outlier in the Indiana distribution. There are no obvious outliers in the New Jersey distribution. The median for both distributions is about the same, approximately \$49,000. The distribution of total family income in Indiana is less variable than the New Jersey distribution. The incomes in Indiana vary from \$0 to about \$125,000. The incomes in New Jersey vary from \$0 to about \$170,000.

1.56 The distributions of nitrate concentration for Stony Brook and for Mill Brook are roughly symmetric. The values 18 mg/L and 20 mg/L in the Mill Brook distribution may be outliers. There do not appear to be any outliers in the Stony Brook distribution. The typical nitrate concentration in Stony Brook (median ≈ 5 mg/L) is less

48. Fuel efficiency The dotplot shows the difference (Highway — City) in EPA mileage ratings, in miles per gallon (mpg) for each of 24 model year 2018 cars.

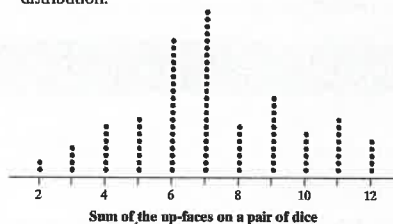


- (a) Explain what the dot above -3 represents.
(b) What does the graph tell us about fuel economy in the city versus on the highway for these car models? Be specific.

49. Getting older How old is the oldest person you know? Prudential Insurance Company asked 400 people to place a blue sticker on a huge wall next to the age of the oldest person they have ever known. An image of the graph is shown here. Describe the shape of the distribution.

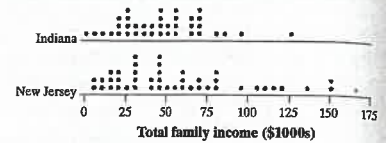


50. Pair-o-dice The dotplot shows the results of rolling a pair of fair, six-sided dice and finding the sum of the up-faces 100 times. Describe the shape of the distribution.

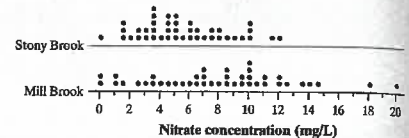


51. Feeling sleepy? Refer to Exercise 45. Describe the shape of the distribution.
52. Easy reading? Refer to Exercise 46. Describe the shape of the distribution.
53. U.S. women's soccer—2016 Refer to Exercise 47. Describe the distribution.
54. Fuel efficiency Refer to Exercise 48. Describe the distribution.
55. Making money The parallel dotplots show the total family income of randomly chosen individuals from Indiana (38 individuals) and New Jersey

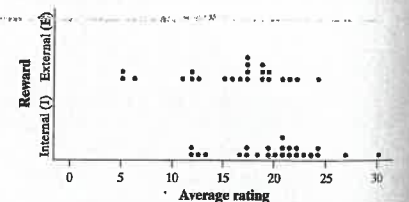
(44 individuals). Compare the distributions of total family incomes in these two samples.



56. Healthy streams Nitrates are organic compounds that are a main ingredient in fertilizers. When those fertilizers run off into streams, the nitrates can have a toxic effect on fish. An ecologist studying nitrate pollution in two streams measures nitrate concentrations at 42 places on Stony Brook and 42 places on Mill Brook. The parallel dotplots display the data. Compare the distributions of nitrate concentration in these two streams.



57. Enhancing creativity Do external rewards—things like money, praise, fame, and grades—promote creativity? Researcher Teresa Amabile recruited 47 experienced creative writers who were college students and divided them at random into two groups. The students in one group were given a list of statements about external reasons (E) for writing, such as public recognition, making money, or pleasing their parents. Students in the other group were given a list of statements about internal reasons (I) for writing, such as expressing yourself and enjoying wordplay. Both groups were then instructed to write a poem about laughter. Each student's poem was rated separately by 12 different poets using a creativity scale.²⁶ These ratings were averaged to obtain an overall creativity score for each poem. Parallel dotplots of the two groups' creativity scores are shown here.

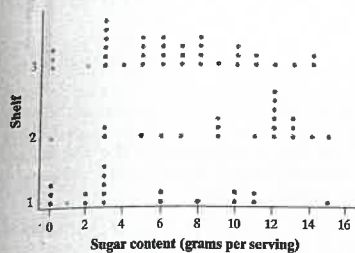


- (a) Is the variability in creativity scores similar or different for the two groups? Justify your answer.
(b) Do the data suggest that external rewards promote creativity? Justify your answer.

than the typical nitrate concentration in Mill Brook (median ≈ 8 mg/L). The distribution of nitrate concentration for Stony Brook is less variable than the Mill Brook distribution. The nitrate concentrations in the Stony Brook distribution vary from 0 to 12 mg/L. The nitrate concentrations in the Mill Brook distribution vary from 0 to 20 mg/L.

1.57 (a) Both distributions have about the same amount of variability. The “external reward” distribution varies from 5 to about 24. And the “internal reward” distribution varies from about 12 to 30. **(b)** The center of the internal reward distribution is greater than the center of the external distribution, indicating that external rewards do not promote creativity.

58. **Healthy cereal?** Researchers collected data on 76 brands of cereal at a local supermarket.²⁷ For each brand, the sugar content (grams per serving) and the shelf in the store on which the cereal was located (1 = bottom, 2 = middle, 3 = top) were recorded. A dot-plot of the data is shown here.



- (a) Is the variability in sugar content of the cereals on the three shelves similar or different? Justify your answer.
(b) Critics claim that supermarkets tend to put sugary cereals where kids can see them. Do the data from this study support this claim? Justify your answer. (Note that Shelf 2 is at about eye level for kids in most supermarkets.)

59. **Snickers® are fun!** Here are the weights (in grams) of 17 Snickers Fun Size bars from a single bag:

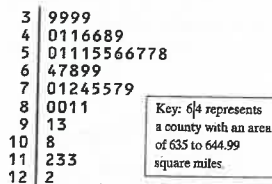
17.1 17.4 16.6 17.4 17.7 17.1 17.3 17.7 17.8
19.2 16.0 15.9 16.5 16.8 16.5 17.1 16.7

- (a) Make a stemplot of these data.
(b) What interesting feature does the graph reveal?
(c) The advertised weight of a Snickers Fun Size bar is 17 grams. What proportion of candy bars in this sample weigh less than advertised?
60. **Eat your beans!** Beans and other legumes are a great source of protein. The following data give the protein content of 30 different varieties of beans, in grams per 100 grams of cooked beans.²⁸

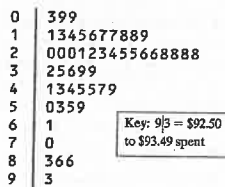
7.5 8.2 8.9 9.3 7.1 8.3 8.7 9.5 8.2 9.1
9.0 9.0 9.7 9.2 8.9 8.1 9.0 7.8 8.0 7.8
7.0 7.5 13.5 8.3 6.8 10.6 8.3 7.6 7.7 8.1

- (a) Make a stemplot of these data.
(b) What interesting feature does the graph reveal?
(c) What proportion of these bean varieties contain more than 9 grams of protein per 100 grams of cooked beans?

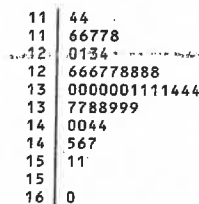
61. **South Carolina counties** Here is a stemplot of the areas of the 46 counties in South Carolina. Note that the data have been rounded to the nearest 10 square miles (mi^2).



- (a) What is the area of the largest South Carolina county?
(b) Describe the distribution of area for the 46 South Carolina counties.
62. **Shopping spree** The stemplot displays data on the amount spent by 50 shoppers at a grocery store. Note that the values have been rounded to the nearest dollar.



- (a) What was the smallest amount spent by any of the shoppers?
(b) Describe the distribution of amount spent by these 50 shoppers.
63. **Where do the young live?** Here is a stemplot of the percent of residents aged 25 to 34 in each of the 50 states:



- (a) Why did we split stems?
(b) Give an appropriate key for this stemplot.
(c) Describe the shape of the distribution. Are there any outliers?

others (13.5 grams per 100 grams of cooked beans). (c) 0.233

1.61 (a) The area of the largest South Carolina county is 1220 square miles (rounded to the nearest 10 mi^2). **(b)** The distribution of the area for the 46 South Carolina counties is right-skewed, with distinct peaks on the 500 mi^2 and 700 mi^2 stems. There are no clear outliers. A typical South Carolina county has an area of about 655 square miles. The area of the counties varies from about 390 square miles to 1220 square miles.

1.62 (a) The smallest amount spent by any of the shoppers is between \$2.50 and \$3.49. **(b)** The distribution of the amount of money spent by shoppers at this supermarket is skewed to the right, with distinct peaks on the \$20 and \$40 stems. There are some potential outliers on the high end (\$83, \$86, and \$93). A typical shopper spent about \$28. The amount spent by the 50 shoppers varies from about \$3 to about \$93.

1.63 (a) If we had not split the stems, most of the data would appear on just a few stems, making it hard to identify the shape of the distribution. **(b)** Key: 16 | 0 means that 16.0% of that state's residents are aged 25 to 34. **(c)** The distribution of percent of residents aged 25–34 is roughly symmetric, with a possible outlier at 16.0%.

1.58 (a) All three distributions of sugar content have about the same amount of variability, as they all vary from about 0 grams per serving to about 15 grams per serving. **(b)** Yes! The distribution of sugar content for shelf 2, which is right at eye level for most kids, has the largest center indicating that supermarkets tend to put sugary kids' cereals where the kids are most likely to see them.

1.59 (a) 15 | 9
16 | 0 5 5 6 7 8
17 | 1 1 1 3 4 4 7 7 8
18 |
19 | 2

Key: 15 | 9 = 15.9 grams

(b) The graph reveals that there was one Fun Size Snickers® bar that is "gigantic"! It weighs 19.2 grams. **(c)** 0.412

1.60 (a) 6 | 8
7 | 0 1 5 5 6 7 8 8
8 | 0 1 1 2 2 3 3 3 7 9 9
9 | 0 0 0 1 2 3 5 7
10 | 6
11 |
12 |
13 | 5

Key 6 | 8 = 6.8 grams per 100 grams of cooked beans

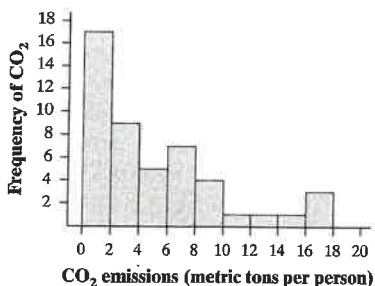
(b) The graph reveals that there was one variety of bean that has much more protein than the

1.64 (a) If we had not split the stems, most of the data would appear on just a few stems, making it hard to identify the shape of the distribution. **(b)** Key: 1 | 5 = a soft drink for which an 8-oz serving has 15 mg of caffeine. **(c)** This distribution is slightly skewed to the right, with two distinct peaks, one on the 25–29 stem and the other on the 35–39 stem. There do not appear to be any outliers.

1.65 The distribution of acorn volume for the Atlantic coast is skewed to the right. The distribution of acorn volume for California is roughly symmetric, with one high outlier of 17.1 cubic centimeters. The distribution of volume of acorn for the Atlantic coast has 3 potential outliers: 8.1, 9.1, and 10.5 cubic centimeters. The typical acorn volume for Atlantic coast oak tree species (median = 1.7 cubic centimeters) is less than the typical acorn volume for California oak tree species (median = 4.1 cubic centimeters). The Atlantic coast distribution (with acorn volumes from 0.3 to 10.5 cubic centimeters) varies less than the California distribution (with acorn volumes from 0.4 to 17.1 cubic centimeters).

1.66 The distributions of study time for men and for women are both roughly symmetric. Both distributions have one upper outlier: one woman who studied 360 minutes and one man who studied 300 minutes. The women (median = 175 minutes) tended to study more than the men (median = 120 minutes). The variability of the two study time distributions is about the same: the women's study times varied from 60 to 360 minutes and the men's study times varied from 0 to 300 minutes.

1.67 (a)



(b) The distribution of amount of carbon dioxide emissions per person in these 48 countries is right-skewed. Visually, none of the countries appear to be outliers.

- 64. Watch that caffeine!** The U.S. Food and Drug Administration (USFDA) limits the amount of caffeine in a 12-ounce can of carbonated beverage to 72 milligrams. That translates to a maximum of 48 milligrams of caffeine per 8-ounce serving. Data on the caffeine content of popular soft drinks (in milligrams per 8-ounce serving) are displayed in the stemplot.

1	556
2	033344
2	55667778888899
3	113
3	55567778
4	33
4	77

- (a) Why did we split stems?
 (b) Give an appropriate key for this graph.
 (c) Describe the shape of the distribution. Are there any outliers?

- 65. Acorns and oak trees** Of the many species of oak trees in the United States, 28 grow on the Atlantic Coast and 11 grow in California. The back-to-back stemplot displays data on the average volume of acorns (in cubic centimeters) for these 39 oak species.²⁹ Write a few sentences comparing the distributions of acorn size for the oak trees in these two regions.

Atlantic Coast		California
998643	0	4
88864211111	1	06
50	2	06
6640	3	
8	4	1
	5	59
8	6	0
7	7	1
1	8	
1	9	
5	10	
	11	
	12	
	13	
	14	
	15	
	16	
	17	1

Key: 2|6 = An oak species whose acorn volume is 2.6 cm³.

- 66. Who studies more?** Researchers asked the students in a large first-year college class how many minutes they studied on a typical weeknight. The back-to-back stemplot displays the responses from random samples of 30 women and 30 men from the class, rounded to the nearest 10 minutes. Write a few sentences comparing the male and female distributions of study time.

Women		Men
	0	03333
96	0	56668999
22222222	1	02222222
88888888887555	1	558
4440	2	00344
	2	
	3	0
	3	

Key: 2|3 = 225–234 minutes

- 67. Carbon dioxide emissions** Burning fuels in power plants and motor vehicles emits carbon dioxide (CO₂), which contributes to global warming. The table displays CO₂ emissions per person from countries with populations of at least 20 million.³⁰

- (a) Make a histogram of the data using intervals of width 2, starting at 0.
 (b) Describe the shape of the distribution. Which countries appear to be outliers?

Country	CO ₂	Country	CO ₂
Algeria	3.3	Mexico	3.8
Argentina	4.5	Morocco	1.6
Australia	16.9	Myanmar	0.2
Bangladesh	0.4	Nepal	0.1
Brazil	2.2	Nigeria	0.5
Canada	14.7	Pakistan	0.9
China	6.2	Peru	2.0
Colombia	1.6	Philippines	0.9
Congo	0.5	Poland	8.3
Egypt	2.6	Romania	3.9
Ethiopia	0.1	Russia	12.2
France	5.6	Saudi Arabia	17.0
Germany	9.1	South Africa	9.0
Ghana	0.4	Spain	5.8
India	1.7	Sudan	0.3
Indonesia	1.8	Tanzania	0.2
Iran	7.7	Thailand	4.4
Iraq	3.7	Turkey	4.1
Italy	6.7	Ukraine	6.6
Japan	9.2	United Kingdom	7.9
Kenya	0.3	United States	17.6
Korea, North	11.5	Uzbekistan	3.7
Korea, South	2.9	Venezuela	6.9
Malaysia	7.7	Vietnam	1.7

- 68. Traveling to work** How long do people travel each day to get to work? The following table gives the average travel times to work (in minutes) for workers in each state and the District of Columbia who are at least 16 years old and don't work at home.³¹

1.68 (a)



(b) The distribution of travel times is right-skewed. Visually, none of the states appear to be outliers.

1.69 The distribution of travel times is right-skewed. Visually, none of the states appear to be outliers.

AL	23.6	LA	25.1	OH	22.1
AK	17.7	ME	22.3	OK	20.0
AZ	25.0	MD	30.6	OR	21.8
AR	20.7	MA	26.6	PA	25.0
CA	26.8	MI	23.4	RI	22.3
CO	23.9	MN	22.0	SC	22.9
CT	24.1	MS	24.0	SD	15.9
DE	23.6	MO	22.9	TN	23.5
FL	25.9	MT	17.6	TX	24.6
GA	27.3	NE	17.7	UT	20.8
HI	25.5	NV	24.2	VT	21.2
ID	20.1	NH	24.6	VA	26.9
IL	27.9	NJ	29.1	WA	25.2
IN	22.3	NM	20.9	WV	25.6
IA	18.2	NY	30.9	WI	20.8
KS	18.5	NC	23.4	WY	17.9
KY	22.4	ND	15.5	DC	29.2

- (a) Make a histogram to display the travel time data using intervals of width 2 minutes, starting at 14 minutes.
- (b) Describe the shape of the distribution. What is the most common interval of travel times?
69. **DRP test scores** There are many ways to measure the reading ability of children. One frequently used test is the Degree of Reading Power (DRP). In a research study on third-grade students, the DRP was administered to 44 students.³² Their scores were as follows.

40	26	39	14	42	18	25	43	46	27	19
47	19	26	35	34	15	44	40	38	31	46
52	25	35	35	33	29	34	41	49	28	52
47	35	48	22	33	41	51	27	14	54	45

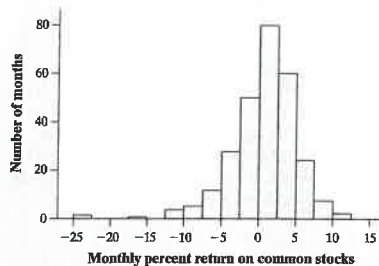
Make a histogram to display the data. Write a few sentences describing the distribution of DRP scores.

70. **Country music** The lengths, in minutes, of the 50 most popular mp3 downloads of songs by country artist Dierks Bentley are given here.

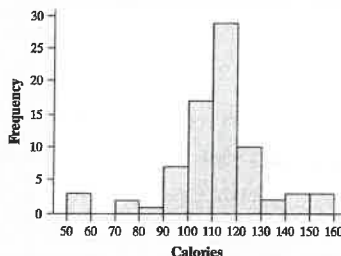
4.2	4.0	3.9	3.8	3.7	4.7
3.4	4.0	4.4	5.0	4.6	3.7
4.6	4.4	4.1	3.0	3.2	4.7
3.5	3.7	4.3	3.7	4.8	4.4
4.2	4.7	6.2	4.0	7.0	3.9
3.4	3.4	2.9	3.3	4.0	4.2
3.2	3.4	3.7	3.5	3.4	3.7
3.9	3.7	3.8	3.1	3.7	3.6
4.5	3.7				

Make a histogram to display the data. Write a few sentences describing the distribution of song lengths.

71. **Returns on common stocks** The return on a stock is the change in its market price plus any dividend payments made. Return is usually expressed as a percent of the beginning price. The figure shows a histogram of the distribution of monthly returns for the U.S. stock market over a 273-month period.³³

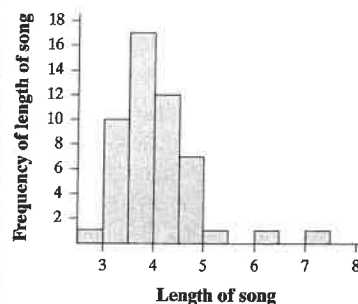


- (a) Describe the overall shape of the distribution of monthly returns.
- (b) What is the approximate center of this distribution?
- (c) Explain why you cannot find the exact value for the minimum return. Between what two values does it lie?
- (d) A return less than 0 means that stocks lost value in that month. About what percent of all months had returns less than 0?
72. **Healthy cereal?** Researchers collected data on calories per serving for 77 brands of breakfast cereal. The histogram displays the data.³⁴



- (a) Describe the overall shape of the distribution of calories.
- (b) What is the approximate center of this distribution?
- (c) Explain why you cannot find the exact value for the maximum number of calories per serving for

- 1.70 The data vary from 2.9 to 7.0 minutes. We chose intervals of width 0.5, beginning at 2.5.

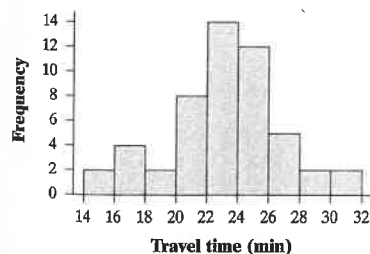


- (b) The distribution of song lengths of the most popular mp3 downloads from country artist Dierks Bentley is skewed to the right, with a single peak in the 3.5 to < 4.0 minute interval. There are gaps in the 5.5 to < 6.0 minute interval and the 6.5 to < 7.0 interval. The songs with lengths of 6.2 and 7 minutes are possible outliers. Song lengths vary from 2.9 to 7 minutes.

- 1.71 (a) The shape of the distribution is slightly skewed to the left, with a single peak. (b) The center is between 0% and 2.5% return on common stocks. (c) The exact value for the minimum return cannot be identified because we have a histogram of only the returns, not the actual data. The lowest return was in the interval -22.5% to -25%. (d) About 37% of these months (102 out of 273) had negative returns.

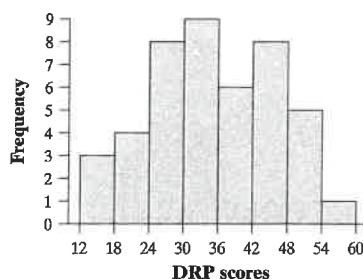
- 1.72 (a) The shape of the distribution is roughly symmetric, with a single peak. (b) The center of the distribution is between 110 and 120 calories. (c) The exact value for the maximum number of calories per serving cannot be identified because we have a histogram of only the calories, not the actual data. The maximum number of calories per serving is in the interval 150 to 160 calories.

1.68 (a)



- (b) The distribution of travel times is roughly symmetric. The most common interval of travel times is 22 to 24 minutes.

- 1.69 The data vary from 14 to 54. We chose intervals of width 6, beginning at 12.



- The distribution of DRP scores is roughly symmetric. There do not appear to be any outliers. The center of the DRP score distribution is between 30 and 36 (with median = 35). The DRP scores vary from 14 to 54.

1.73 It is difficult to effectively compare the salaries of the two teams with these two histograms because the scale on the horizontal axis is very different from one graph to the other. It also does not help that the scales on the y axis differ as well.

1.74 The distribution of salary for both teams is skewed to the right. It appears that the Yankees' distribution has an outlier somewhere between \$32 and \$36 million, while the Phillies' distribution does not appear to have any outliers. The salaries for the Yankees have a higher center of about \$16 million than the Phillies' center of about \$4 million. The distribution of salaries for the Yankees also has more variability as their salaries vary from \$0 to \$36 million and the Phillies' salaries vary from \$0 to \$16 million.

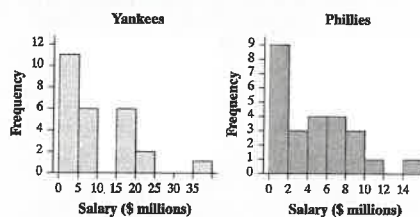
1.75 (a) No, it would not be appropriate to use frequency histograms instead of relative frequency histograms in this setting because there were many more graduates surveyed (314) than non-graduates (57). **(b)** The distribution of total personal income for each group is skewed to the right and single-peaked. There are some possible high outliers in the graduate distribution. There do not appear to be any outliers in the non-graduate distribution. The center of the personal income distribution is larger for graduates than non-graduates, indicating that graduates typically have higher incomes than non-graduates in this sample. The incomes for graduates vary a lot more (from \$0 to \$150,000) than non-graduates (from \$0 to \$60,000).

1.76 (a) Yes, it would be all right to use frequency histograms instead of relative frequency histograms in this setting because the sample sizes were similar. The study was based on 30 Bounty paper towels and 30 generic paper towels. **(b)** The distributions of the number of quarters the Bounty and generic paper towels could hold until breaking are roughly symmetric. Neither distribution appears to contain any outliers. The Bounty paper towels tended to hold more quarters before breaking, as their center was around 115 quarters, while the generic brand tended to only hold about 90 quarters before breaking. The variability of both brands was about the same as the number of quarters held by the Bounty paper towels varied from approximately 100 to 130 quarters and the number of quarters held by the generic paper towels varied from about 75 to 105.

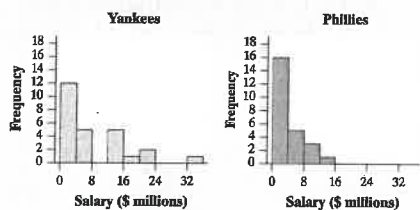
these cereal brands. Between what two values does it lie?

- (d) About what percent of the cereal brands have 130 or more calories per serving?

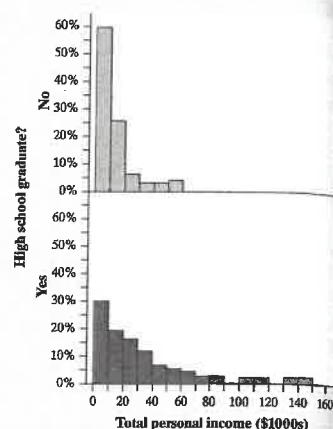
- 73. Paying for championships** Does paying high salaries lead to more victories in professional sports? The New York Yankees have long been known for having Major League Baseball's highest team payroll. And over the years, the team has won many championships. This strategy didn't pay off in 2008, when the Philadelphia Phillies won the World Series. Maybe the Yankees didn't spend enough money that year. The figure shows histograms of the salary distributions for the two teams during the 2008 season. Why can't you use these graphs to effectively compare the team payrolls?



- 74. Paying for championships** Refer to Exercise 73. Here is a better graph of the 2008 salary distributions for the Yankees and the Phillies. Write a few sentences comparing these two distributions.



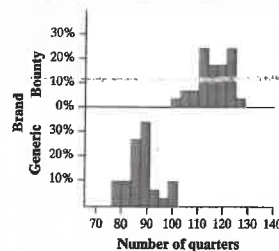
- 75. Value of a diploma** Do students who graduate from high school earn more money than students who do not? To find out, we took a random sample of 371 U.S. residents aged 18 and older. The educational level and total personal income of each person were recorded. The data for the 57 non-graduates (No) and the 314 graduates (Yes) are displayed in the relative frequency histograms.



- (a) Would it be appropriate to use frequency histograms instead of relative frequency histograms in this setting? Explain why or why not.

- (b) Compare the distributions of total personal income for the two groups.

- 76. Strong paper towels** In commercials for Bounty paper towels, the manufacturer claims that they are the "quick picker-upper," but are they also the stronger picker-upper? Two of Mr. Tabor's statistics students, Wesley and Mavrick, decided to find out. They selected a random sample of 30 Bounty paper towels and a random sample of 30 generic paper towels and measured their strength when wet. To do this, they uniformly soaked each paper towel with 4 ounces of water, held two opposite edges of the paper towel, and counted how many quarters each paper towel could hold until ripping, alternating brands. The data are displayed in the relative frequency histograms. Compare the distributions.



- (a) Would it be appropriate to use frequency histograms instead of relative frequency histograms in this setting? Explain why or why not.

- (b) Compare the distributions of number of quarters until breaking for the two paper towel brands.

1.77 A
month i
graph i

Percent
9
8
7
6
5
4
3
2
1
0
January
A

77. **Birth months** Imagine asking a random sample of 60 students from your school about their birth months. Draw a plausible (believable) graph of the distribution of birth months. Should you use a bar graph or a histogram to display the data?

78. **Die rolls** Imagine rolling a fair, six-sided die 60 times. Draw a plausible graph of the distribution of die rolls. Should you use a bar graph or a histogram to display the data?

79. **AP® exam scores** The table gives the distribution of grades earned by students taking the AP® Calculus AB and AP® Statistics exams in 2016.³⁵

	Grade					
	5	4	3	2	1	Total
Calculus AB	76,486	53,467	53,533	30,017	94,712	308,215
Statistics	29,627	44,884	51,367	32,120	48,565	206,563

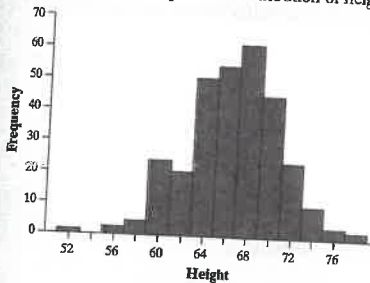
- (a) Make an appropriate graphical display to compare the grade distributions for AP® Calculus AB and AP® Statistics.
 (b) Write a few sentences comparing the two distributions of exam grades.

Multiple Choice: Select the best answer for Exercises 80–85.

80. Here are the amounts of money (cents) in coins carried by 10 students in a statistics class: 50, 35, 0, 46, 86, 0, 5, 47, 23, 65. To make a stemplot of these data, you would use stems

- (a) 0, 2, 3, 4, 6, 8. (d) 00, 10, 20, 30, 40, 50,
 (b) 0, 1, 2, 3, 4, 5, 6, 7, 8. 60, 70, 80, 90.
 (c) 0, 3, 5, 6, 7. (e) None of these.

81. The histogram shows the heights of 300 randomly selected high school students. Which of the following is the best description of the shape of the distribution of heights?



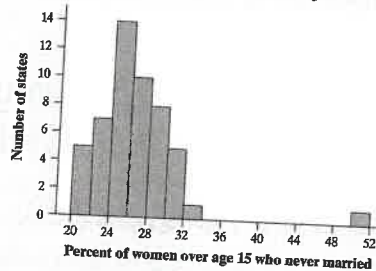
- (a) Roughly symmetric and single-peaked (unimodal)
 (b) Roughly symmetric and double-peaked (bimodal)
 (c) Roughly symmetric and multi-peaked (multimodal)
 (d) Skewed to the left
 (e) Skewed to the right

82. You look at real estate ads for houses in Naples, Florida. There are many houses ranging from \$200,000

to \$500,000 in price. The few houses on the water, however, are priced up to \$15 million. The distribution of house prices will be

- (a) skewed to the left. (d) single-peaked.
 (b) roughly symmetric. (e) approximately uniform.
 (c) skewed to the right.

83. The histogram shows the distribution of the percents of women aged 15 and over who have never married in each of the 50 states and the District of Columbia. Which of the following statements about the histogram is correct?



- (a) The center (median) of the distribution is about 36%.
 (b) There are more states with percentages above 32 than there are states with percentages less than 24.
 (c) It would be better if the values from 34 to 50 were deleted on the horizontal axis so there wouldn't be a large gap.
 (d) There was one state with a value of exactly 33%.
 (e) About half of the states had percentages between 24% and 28%.

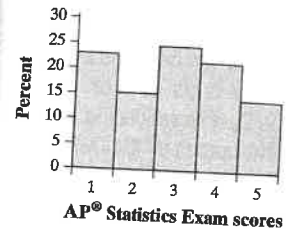
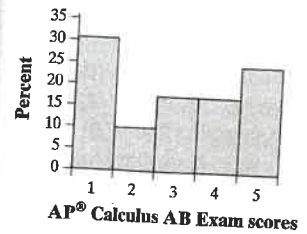
84. When comparing two distributions, it would be best to use relative frequency histograms rather than frequency histograms when

- (a) the distributions have different shapes.
 (b) the distributions have different amounts of variability.
 (c) the distributions have different centers.
 (d) the distributions have different numbers of observations.
 (e) at least one of the distributions has outliers.

85. Which of the following is the best reason for choosing a stemplot rather than a histogram to display the distribution of a quantitative variable?

- (a) Stemplots allow you to split stems; histograms don't.
 (b) Stemplots allow you to see the values of individual observations.
 (c) Stemplots are better for displaying very large sets of data.
 (d) Stemplots never require rounding of values.
 (e) Stemplots make it easier to determine the shape of a distribution.

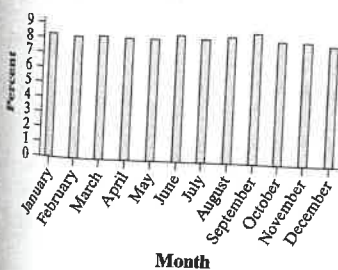
1.79 A score earned on the AP® Statistics exam is quantitative, so histograms are shown below using relative frequencies since there are many more students who take the AP® Calculus AB exam.



The shapes of the two distributions are very different. The distribution of scores on the AP® Calculus AB exam has a peak at 1 and another slightly lower peak at 5. The distribution of scores on the AP® Statistics exam, however, is more uniform, with scores of 1, 3, and 4 being the most frequent and scores of 5 being the least frequent. Neither distribution has any outliers. The center of both distributions is 3. Although scores on both exams vary from 1 to 5, there are more scores close to the center on the AP® Statistics exam and more scores at the extremes on the AP® Calculus exam.

1.80 b 1.82 c 1.84 d
 1.81 a 1.83 e 1.85 b

1.77 A bar graph should be used because birth month is a categorical variable. A possible bar graph is given below.



1.78 A histogram should be used because die roll is a quantitative variable. A possible histogram is given below.

