PreCalculus CP Cover Page

Content Area: **Mathematics**
Course(s):
Time Period:
Length: **Full Year**
Status: **Published**

Course Overview: Objectives, benchmarking and screening plan

PreCalculus is a rigorous mathematics course designed to prepare students for Calculus and other college level mathematics courses. This course strengthens students' understanding of functions and emphasizes multiple representations - graphical, algebraic, and numerical - to analyze and interpret mathematic relationships. Throughout the course, students use graphing technology to explore mathematical concepts and verify solutions. They will develop critical thinking and problem solving skills through modeling, reasoning, and application based tasks. The course emphasizes communication of mathematical ideas, justification of solutions, and the ability to interpret and connect multiple representations of functions. By the end of the course, students are prepared to engage with mathematical concepts in Calculus and other STEM fields with strong foundations in functions, trigonometry, and analytical reasoning.

Course Name, Length, Date of Revision and Curriculum Writer

PreCalculus CP

Full Year Course

6/5/26

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Table of Contents

Unit 1: Function Analysis and Transformations

Content Area: **Mathematics**
Course(s):
Time Period: **1st Marking Period**
Length: **8 weeks**
Status: **Published**

Summary of the Unit

Unit 1 serves as a bridge between intermediate algebra and the foundational concepts of Calculus. Students will first review and master diverse algebraic methods for solving linear, quadratic, rational, and radical equations. The curriculum then shifts to an advanced, multi-faceted exploration of functions. Students will analyze functions algebraically and graphically, focusing on domain, range, increasing and decreasing intervals, extrema, and average rates of change. Students learn to manipulate expressions and utilize transformations to graph functions both manually and with technological assistance. Beyond abstract calculation, the unit emphasizes real-world modeling to show how these mathematical rules apply to practical scenarios in business, science, and industry. By analyzing relative extrema and optimization, students develop the critical thinking skills necessary to interpret changing data trends and make informed, data-driven decisions. The unit concludes with composite functions and the relationship between functions and their inverses both algebraically and graphically.

Enduring Understandings

- Identify the best method to solve an equation.
- Use function notation and evaluate functions abstractly and graphically.
- Analyze functions by determining intervals on which they are increasing, decreasing, or constant and determining their relative minimum and relative maximum values.
- Identify functions, and find their domain and range.
- Find the zeros of a function.
- Identify even and odd functions.
- Use functions to model and solve real-life problems.
- Rewrite polynomials in equivalent forms
- Analyze polynomial functions by finding all zeros using the Fundamental Theorem of Algebra, factoring, and graphing them, both manually and using technology.
- Use polynomials to model and solve real-life problems including ones with minimum and maximum values.
- Find the average rate of change of a function on an interval by calculating the slope of the secant line.
- Recognize graphs of common functions, and use vertical, and horizontal shifts, reflections, and non-rigid transformations to graph them

- Evaluate composite functions and decompose composite relationships.
- Find and verify inverse functions algebraically and graphically.
- Use the horizontal line test to determine one -to - one functions.

Essential Questions

- How do you determine the most efficient factoring strategy (e.g., GCF, grouping, difference of squares, or cubes) to factor a polynomial completely?
- Why must certain values be excluded from the domain of a rational expression, and how does factoring help simplify operations like multiplication and addition?
- When solving quadratic, radical, or rational equations, how do you evaluate which algebraic method is the most efficient and accurate?
- What does it mean for a relation to be a function, and how can you determine if a relation is a function?
- How are functions and their graphs related?
- What do domain and range tell us about a function in real world and mathematical contexts?
- How can we describe how a function behaves over intervals (increasing, decreasing, constant)?
- How can technology be used to investigate the properties of families of functions and their graphs?
- What does the degree of a polynomial tell you about its related polynomial function?
- For a polynomial function, how are factors, zeros, and x-intercepts related?
- How do the characteristics of graphs relate to their corresponding equations?
- How can algebra help us get information about a graph from an equation?
- What does the degree of a polynomial tell you about its related polynomial function?
- How do polynomial functions model real-world problems and their solutions?
- What does the average rate of change tell us about a function over a specific interval [a, b], and how does it connect to the slope of a line?
- How do transformations change the graph and equation of a function?
- How do algebraic and graphical methods (like the horizontal line test) help determine if a function is invertible, and what is the relationship between their input-output pairs?
- What does it mean for two functions to be inverses, and how can we verify that relationship?
- How do composite functions help us model multi-step processes in real life?

Unit Summative Assessment and Alternate Assessment Options

Suggested Formative/Summative Classroom Assessments

- Complete classwork Learning vertically at whiteboard
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Quizzes

- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Teacher-Student and Student-Student Conferencing
- WebAssign Problem Sets WebAssign Instructor Help
- Error analysis tasks
- Task based station assessments
- Homework
- Students will take formal assessments, such as tests and quizzes, to assess knowledge of concepts learned throughout the unit.
- Unit 1 summative assessment taken at the end of the unit
- Students will also demonstrate mastery through various assessment criteria included in the unit such as do nows, exit slips, graded classwork activities and assignments, and/or projects.

Resources

- Classpad.net (Casio): https://classpad.net/us/
- Desmos: https://www.desmos.com/
- Geogebra: https://www.geogebra.org/?lang=en
- Math Open Reference: https://www.mathopenref.com/
- TI Education (Texas Instruments): https://education.ti.com/en
- WolframAlpha: https://www.wolframalpha.com/
- Wolfram MathWorld: https://mathworld.wolfram.com/
- Khan Academy: https://www.khanacademy.org/math/precalculus
- Digital Mathematics Word Wall: http://www.mathwords.com/index_adv_alg_precal.htm
- Extra Notes for Pre-Calculus Content: https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates
- Review Documents for Pre-Calculus: https://sites.google.com/site/dgrahamcalculus/trigpre-

calculus/trig-pre-calculus-worksheets

- Pre-Calculus IXL Topics and Resources: https://www.ixl.com/math/precalculus
- Classroom Challenges to Support Teachers in Formative Assessments: http://map.mathshell.org/materials/lessons.php?gradeid=24
- Applications of Function Models: https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content
- Statistics Education Web (STEW). http://www.amstat.org/education/STEW/
- The Data and Story Library (DASL). http://lib.stat.cmu.edu/DASL/
- WebAssign Instructor Resources
- WebAssign Student Resources
- PreCalculus by Robert Blitzer 5th edition

Standards for Course Content Area and Cross Content Standards Addressed

MATH.9-12.A.APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MATH.9-12.F.BF.A.1.b	Combine standard function types using arithmetic operations.
MATH.9-12.A.APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MATH.9-12.F.BF.A.1.c	Compose functions.
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
MATH.9-12.A.APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MATH.9-12.A.APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MATH.9-12.F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write

	an expression for the inverse.
MATH.9-12.F.BF.B.4.b	Verify by composition that one function is the inverse of another.
MATH.9-12.F.BF.B.4.c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MATH.9-12.A.CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
MATH.9-12.F.BF.B.4.d	Produce an invertible function from a non-invertible function by restricting the domain.
MATH.9-12.A.CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
MATH.9-12.A.CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MATH.9-12.F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.A.REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MATH.9-12.A.REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MATH.9-12.A.REI.B	Solve equations and inequalities in one variable
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.A.REI.B.4	Solve quadratic equations in one variable. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
MATH.9-12.F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MATH.9-12.F.IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MATH.9-12.F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MATH.9-12.A.SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MATH.9-12.A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it.
MATH.9-12.F.LE.A.1.b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MATH.9-12.A.SSE.B	Write expressions in equivalent forms to solve problems

Unit Plan

Topic/Selection Timeframe	General Objectives	Instructional Activities	Formative and Summative Assessments
Factoring Polynomials (2 days)	SWBAT determine the best method to factor polynomials	Students will add, subtract, and multiply polynomials. Students will apply multiple factoring strategies such as the greatest common factor, factoring by grouping, product sum, difference of perfect squares, sum and difference of cubes to factor polynomials completely.	Circulate and monitor progress as students are working on classwork examples. Have students complete problems vertically at the whiteboard. Peer/self assessments Through questioning students will be able to analyze a polynomial and identify the most efficient factoring technique to factor completely. Closure
Rational Expressions (3 days)	SWBAT simplify, multiply, divide, add and subtract rational expressions. SWBAT simplify complex fractions.	Students will specify numbers that must be excluded from the domain of a rational expression. Students will simplify, multiply, and divide rational expressions by factoring each term and canceling out like factors. Students will add and subtract rational expressions by using the least common denominator.	Circulate and monitor progress as students are working on classwork examples. Have students complete problems vertically at the whiteboard. Peer/self assessments

			Through questioning students will be able to simplify, multiply, and divide rational expressions by factoring and simplifying. Through questioning students will be able to add and subtract rational expressions by successfully identify the least common denominator. Closure Prerequisite Skills Test # 1
Equations (6 days)	SWBAT use the best method to a solve an equation.	Students will solve linear equations in one variable and solve a formula for a variable. Students will solve quadratic equations by factoring, square roots, completing the square, and the quadratic formula. Students will determine of these methods which is the most efficient. Students will solve radical equations by isolating the radical and then raising each side to the power suggested by the index. Students will solve rational equations with variables in the denominator by using the least common	Circulate and monitor progress as students are working on classwork examples. Have students complete problems vertically at the whiteboard. Peer/self assessments Through questioning students will be able to analyze an equation and identify the best method to solve it.

		denominator.	Closure Prerequisite skills test # 2
1.1 - Change in Tandem (3 days)	SWBAT determine whether a relation is a function. SWBAT evaluate a function. SWBAT determine the domain and range of a function from a graph. SWBAT identify intercepts of a function both graphically and algebraically.	Students will use the definition of a function to determine whether a graph or an equation represents a function. Students will evaluate functions by substituting the input value into each variable. Students will recall the domain is the x value and the range is the y value. Students will analyze functions graphically and write the domain and range. Students will analyze graphs to identify the x and y intercepts. Students will find x intercept(s) algebraically by substituting $y = 0$ and the y - intercept algebraically by substituting $x = 0$.	Circulate and monitor progress as students are working on classwork examples. Have students complete problems vertically at the whiteboard. Peer/self assessments Through questioning students will be able to analyze and identify functions from graphical, numerical, analytical, and verbal representations. Closure
1.2 - Polynomial Functions (5 days)	SWBAT identify intervals on which a function increases, decreases, or is constant. SWBAT use the graph of a function to determine	Students will determine intervals of increase, decrease, or constant by visual inspection. Students will determine relative maxima/minima	Circulate and monitor progress as students are working on classwork examples.

	relative maxima and/or minima. SWBAT identify even and odd functions. SWBAT describe the end behavior of a polynomial related to even/odd functions.	by visual inspection. Students will determine whether a function is even, odd, or neither by using symmetry or algebraic tests.	Have students complete problems vertically at the whiteboard. Peer/self assessments Through questioning students will be able to analyze a graph and identify relative extrema , intervals of increase/decrease, and symmetry. Closure Quiz Chapter 1.1 - 1.2
1.3 - Average Rate of Change (4 days)	SWBAT calculate the slope of a line. SWBAT to write the equation of a line in point-slope and slope - intercept form. SWBAT find a function's average rate of change.	Given two points, students will use the slope formula to calculate the slope of a line. Given a point and slope or two points on a line, students will write linear equations in both point slope and slope intercept forms. Students will find a functions average rate of change on the closed interval [a, b] by applying the formula (f(b) - f(a)) / (b - a).	Circulate and monitor progress as students are working on classwork examples. Have students complete problems vertically at the whiteboard. Peer/self assessments Through questioning students will be able to calculate the average rate of change on the interval [a, b]

			Closure 1.3 quiz
1.4 - Recognize graphs of common functions, and use vertical, and horizontal shifts, reflections, and non rigid transformations to graph them (9 days)	SWBAT analyze the effect of the coefficients on the graph of a function. SWBAT use and apply vertical and horizontal shifts, reflections, and horizontal and vertical dilations. SWBAT graph functions involving a sequence of transformations.	Students will analyze the effect of each variable af(b(x-h))+k has on the graph of a function. Students will translate parent functions vertically and horizontally. Students will reflect parent functions over the x and y axis. Students will dilate functions both horizontally and vertically by multiplying either x or y values respectively. Students will identify transformations by analyzing a function graph. Students will graph functions involving a sequence of transformations.	Circulate and monitor progress as students are working on classwork examples. Have students complete problems vertically at the whiteboard. Peer/self assessments Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology Closure Topic 1.4 Quiz
1.5 - Composite Functions (3 days)	SWBAT evaluate the composition of two or more functions for given values. SWBAT construct a	Students will perform function operations. Students will find a composition of one	Circulate and monitor progress as students are working on classwork examples. Have students complete

	representation of the composition of two or more functions. SWBAT rewrite a given function as a composition of two or more functions.	function with another. Students will rewrite a given function as a composite of two or more functions. Students will recognize that the composition of functions is not commutative.	problems vertically at the whiteboard. Peer/self assessments Through questioning students will be able to construct new functions using transformations and compositions that may be useful in modeling contexts, criteria , or data with or without technology Closure
1.6 - Inverse Functions (4 days)	SWBAT determine the input - output pairs of the inverse of a function. SWBAT determine the inverse of a function on an invertible domain.	Students will verify that two functions are inverses both algebraically and graphically. Students will use the horizontal line test to determine if the functions are one to one. Students will find inverse functions both informally and algebraically. Students will use function notation to describe real world scenarios.	Circulate and monitor progress as students are working on classwork examples. Have students complete problems vertically at the whiteboard. Peer/self assessments Through questioning students will be able to find the inverse of a function both algebraically and graphically and determine if a function is invertible.

		Students will analyze and choose the appropriate method to find an inverse of a relation or function.	Closure 1.5 - 1.6 quiz

Suggested Modifications for Students with Disabilities, 504 eligible, Multilingual Learners, At Risk Students and Gifted Students

Special Education Students:

- Visual Aids and Manipulatives: Provide visual aids such as graphic organizers, anchor charts, and manipulatives like algebra tiles or graphing boards to help students visualize concepts and relationships.
- Simplified Instructions: Break down complex concepts into smaller, more manageable steps with clear and concise instructions.
- Extra Practice and Scaffolding: Offer additional practice problems with varying difficulty levels, along with scaffolding (hints, prompts, worked examples) to support students as they progress.
- Alternative Assessments: Provide alternative assessment options, such as oral presentations, projects, or portfolios, to demonstrate understanding.
- Individualized Instruction: Tailor instruction to meet individual needs, considering learning styles, strengths, and weaknesses.

Multilingual Learners:

- Visual Support: Incorporate visuals, such as diagrams, graphs, and real-world examples, to enhance understanding of vocabulary and concepts.
- Simplified Language: Use clear and concise language, avoiding jargon and idioms. Provide a glossary of key terms with definitions and translations.
- Bilingual Resources: Offer bilingual dictionaries, textbooks, or online resources to support students' understanding.
- Collaborative Learning: Encourage peer-to-peer interaction and group work to provide opportunities for language practice and collaborative problem-solving.
- Differentiated Instruction: This method tailors instruction to meet individual language proficiency levels, providing additional support for students who need it.

Gifted Students:

- Extension Activities: Provide opportunities for enrichment and extension beyond the standard curriculum, such as exploring advanced topics, independent research projects, or problem-solving challenges.
- Inquiry-Based Learning: Encourage students to ask questions, explore concepts independently, and develop their solutions.
- Higher-Order Thinking Skills: Challenge students with complex problems that require critical thinking, analysis, and synthesis.
- Acceleration: Consider accelerating gifted students to higher-level math courses if they demonstrate readiness.

- Mentorship: Pair gifted students with mentors who can guide and support their academic pursuits.

Computer Sci Design Thinking

CS.9-12.8.1.12.AP.1	Design algorithms to solve computational problems using a combination of original and existing algorithms.
CS.9-12.8.1.12.AP.2	Create generalized computational solutions using collections instead of repeatedly using simple variables.
CS.9-12.8.1.12.AP.5	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.
CS.9-12.8.1.12.DA.1	Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
CS.9-12.8.1.12.DA.6	Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
CS.9-12.8.1.12.IC.1	Evaluate the ways computing impacts personal, ethical, social, economic, and cultural practices.
CS.9-12.8.2.12.ED.5	Evaluate the effectiveness of a product or system based on factors that are related to its requirements, specifications, and constraints (e.g., safety, reliability, economic considerations, quality control, environmental concerns, manufacturability, maintenance and repair, ergonomics).
CS.9-12.8.2.12.ED.6	Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

Career Readiness, Life Literacies and Key Skills Practice

WRK.9.2.12.CAP.2	Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.
TECH.9.4.2.CT.1	Gather information about an issue, such as climate change, and collaboratively brainstorm ways to solve the problem (e.g., K-2-ETS1-1, 6.3.2.GeoGI.2).
TECH.9.4.2.TL.1	Identify the basic features of a digital tool and explain the purpose of the tool (e.g., 8.2.2.ED.1).
TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.IML.1	Compare search browsers and recognize features that allow for filtering of information.

Unit 2 : Rational Functions, Exponential, and Logarithmic Functions

Content Area: **Mathematics**
Course(s):
Time Period: **2nd Marking Period**
Length: **8 weeks**
Status: **Published**

Summary of the Unit

In Unit 2, students will expand their toolkit of functions beyond polynomials to explore the behavior, graphs, properties, and applications of rational, exponential, and logarithmic functions. Students will analyze and graph rational functions, identifying key features such as intercepts, asymptotes, discontinuities, domain, range, and end behavior. They will use multiple representations of rational functions to describe and interpret function behavior. Students will then explore exponential functions as models of growth and decay. They graph and transform exponential functions, examine the effects of parameters on function behavior, and interpret key characteristics. Building on their understanding of inverse relationships, students will investigate logarithmic functions as inverses of exponential functions. Throughout the unit, students will solve exponential and logarithmic equations using algebraic techniques, inverse operations, and technology when appropriate. They will develop mathematical models to represent and analyze real-world data, selecting and justifying appropriate function types based on context. Emphasis is placed on interpreting solutions. Exponential and logarithmic function models are widespread in the natural and social sciences. Exponential functions are key to modeling population growth, radioactive decay, interest rates, and the amount of medication in a patient. Logarithmic functions are useful in modeling sound intensity and frequency, the magnitude of earthquakes, the pH scale in chemistry, and the working memory in humans. The study of these two function types touches careers in business, medicine, chemistry, physics, education, and geography.

Enduring Understandings

- Analyze rational functions by finding all zeros, and asymptotes (horizontal and vertical) and graphing them, both manually and using technology.
- Rewrite rational expressions in equivalent form.
- Identify key characteristics of exponential functions.
- Use properties of exponents to simplify and evaluate expressions.
- Apply properties of logarithms to evaluate expressions and graph functions.
- Understand the characteristics and effects of constants in the exponential model y = ab^x.
- Apply exponential models about data sets in contextual scenarios and construct a model.
- Identify key characteristics of logarithmic functions
- Rewrite logarithmic expressions in equivalent forms.
- Solve exponential and logarithmic equations.
- Construct the inverse of exponential functions.
- Recognize, write, and find nth terms of arithmetic and geometric sequences.
- Use arithmetic and geometric sequences to model real world problems.
- Describe similarities and differences between linear and exponential functions.

- Construct functions that are comparable to arithmetic and geometric sequences.
- Solve real world applications using arithmetic and geometric sequences.

Essential Questions

- How do the key features of a function help us predict and interpret its graph?
- How are algebraic and graphical representations of functions connected?
- What are the key characteristics of an exponential function?
- How do the key characteristics of an exponential function graph—such as domain, range, continuity, and horizontal asymptotes—change depending on whether the base represents growth or decay?
- How can algebraic properties of exponents be applied to transform parent exponential functions, and how do these transformations alter their graphic behavior?
- How do I use exponential functions to model exponential behavior?
- What is the mathematical relationship between an exponential expression and a logarithm, and what does a single unit change on a logarithmic scale represent?
- How does reflecting an exponential graph over the identity line y = x visually and algebraically prove the inverse relationship between exponential and logarithmic functions?
- Why is the domain of a general-form logarithmic function restricted to numbers greater than zero, and how does this constraint dictate its vertical asymptotic behavior?

- What is the relationship between an exponential expression and a logarithm?

- Why is the number e important?
- How do the properties of logarithms simplify complex expressions?
- Why do logarithmic properties mirror exponent rules?
- How can logarithmic properties help solve equations?
- How do we use common differences and common ratios to distinguish between arithmetic and geometric patterns, and how do they help us model discrete real-world scenarios?
- How can I make a single model that merges the interest I earn from my bank with the taxes that are due so I can know how much I will have in the end?
- If different functions can be used to model data, how do we pick which one is best?

Unit Summative Assessment and Alternate Assessment Options

Suggested Formative/Summative Classroom Assessments

- Complete classwork Learning vertically at whiteboard

- Short/Extended Constructed Response Items

- Multiple-Choice Items (where multiple answer choices may be correct)
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Teacher-Student and Student-Student Conferencing
- WebAssign Problem Sets WebAssign Instructor Help
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- Homework
- Students will take formal assessments, such as tests and quizzes, to assess knowledge of concepts learned throughout the unit.
- Unit 2 summative assessment taken at the end of the unit
- Students will also demonstrate mastery through various assessment criteria included in the unit such as do nows, exit slips, graded classwork activities and assignments, and/or projects.

Resources

- Classpad.net (Casio): https://classpad.net/us/
- Desmos: https://www.desmos.com/
- Geogebra: https://www.geogebra.org/?lang=en
- Math Open Reference: https://www.mathopenref.com/
- TI Education (Texas Instruments): https://education.ti.com/en
- WolframAlpha: https://www.wolframalpha.com/
- Wolfram MathWorld: https://mathworld.wolfram.com/
- Khan Academy: https://www.khanacademy.org/math/precalculus
- Digital Mathematics Word Wall: http://www.mathwords.com/index_adv_alg_precal.htm

- Extra Notes for Pre-Calculus Content: https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates
- Review Documents for Pre-Calculus: https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets
- Pre-Calculus IXL Topics and Resources: https://www.ixl.com/math/precalculus
- Classroom Challenges to Support Teachers in Formative Assessments: http://map.mathshell.org/materials/lessons.php?gradeid=24
- Solving Exponential Equations: https://flexbooks.ck12.org/cbook/ck-12-precalculus-concepts-2.0/section/3.6/primary/lesson/exponential-equations-pcalc/
- Statistics Education Web (STEW). http://www.amstat.org/education/STEW/
- The Data and Story Library (DASL). http://lib.stat.cmu.edu/DASL/
- WebAssign Instructor Resources
- WebAssign Student Resources
- PreCalculus by Robert Blitzer 5th edition

Topic/Selection Timeframe	General Objectives	Instructional Activities	Benchmarks/Assessments
2.1 - Equivalent Representations of Rational Expressions, Vertical Asymptotes, Holes, and Zeros. (6 days)	SWBAT express rational functions in analytically equivalent forms that are useful in a given mathematical or applied context. SWBAT describe the characteristics of a rational function with varying levels of precision, depending on the function representation.	Students are presented with a nonconstant polynomial or rational function in analytical representations, and they then translate into a variety of representations: constructing a graph, writing the product of linear factors (x- a) when possible, and describing characteristics such as real zeros, x - intercepts, asymptotes, and holes. Students will check their graphs using technology.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology. Closure Quiz 2.1

2.2 - Exponential Functions (2 days)	SWBAT to identify key characteristics of exponential functions.	Students will investigate the basic behaviors of exponential functions and their graphs. Students will analyze exponential graphs for key characteristics, such as domain, range, continuity, symmetry, asymptotes, and intervals of increase and decrease.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to identify key characteristics and graph exponential functions. Closure
2.3 - Exponential Function Context and Data Modeling (4 days)	SWBAT construct a model for scenarios involving proportional output values related to real life scenarios. SWBAT apply exponential models about data sets in contextual scenarios.	Students will identify if a situation or equation represents exponential growth or decay in real world situations. Students will analyze and interpret exponential models in context, including: population growth, compound interest, depreciation, disease spread, and radioactive decay. Students will solve exponential equations	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to construct a model for scenarios involving exponential functions, apply exponential models about data sets ,and construct exponential

		graphically and algebraically and make predictions and draw conclusions using exponential models.	function models with and without technology. Closure Quiz 2.2 - 2.3
2.4 - Logarithmic Expressions (2 days)	SWBAT identify and apply the relationship between logarithmic and exponential functions. SWBAT to evaluate logarithmic expressions.	Students will convert logarithmic equations to exponential form and exponential functions to logarithmic form. Students will evaluate logarithmic expressions using properties of exponents and inverse relationships. Students will understand that each unit of the logarithmic scale is a multiplicative change of base 10.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to express exponential and logarithmic functions, equations, or expressions in analytically equivalent forms. Closure

2.5 -Logarithmic Functions (2 days)	SWBAT identify and graph key characteristics of logarithmic functions.	Students will understand the domain of logarithmic functions is any real number greater than zero. Students will understand that just like exponential functions, logarithmic functions are always increasing or decreasing. Students will understand how to apply transformations to logarithmic graphs. Students will graph logarithmic functions and identify key characteristics including domain, range, intercepts, asymptotes, and transformations.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to identify key characteristics and graph logarithmic functions. Closure
2.6 - Logarithmic Function Manipulation (4 days)	SWBAT rewrite logarithmic expressions in equivalent forms.	Students will apply the properties of logarithms to expand and condense logarithmic expressions. Students will use properties of exponents and logarithms to simplify and evaluate expressions Students will identify equivalent logarithmic	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to express functions, equations, or expressions in

		expressions. Students will recognize common errors when expanding and condensing logarithms.	analytically equivalent forms. Closure Quiz 2.4 - 2.6
2.7 - Exponential and Logarithmic Equations (7 days)	SWBAT solve exponential and logarithmic equations.	Students will solve exponential equations using inverse operations and logarithms. Students will solve logarithmic equations using exponential forms and algebraic methods. Students will apply properties of logarithms to simplify and solve equations. When solving logarithmic equations students will analyze for extraneous solutions. Students will compare multiple methods for solving exponential and logarithmic equations. Students will use	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to solve equations represented analytically with and without technology. Closure Quiz 2.7

		technology to analyze and verify solutions.	
2.8- Logarithmic Function Context and Data Modeling (4 days)	SWBAT construct a logarithmic function model.	Students will interpret logarithmic relationships in real world contexts. Students will use logarithms to model proportional growth. Students will use transformations based on context or data set to model logarithmic behavior. Students will use the natural log function to model phenomena, Students will use logarithmic models to predict values for dependent variables with or without technology.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to construct new functions using transformations, compositions, inverses, that may be useful in modeling contexts, criteria or data with and without technology. Closure Quiz 2.8
2.9 - Changes in Arithmetic and Geometric Sequences. (5 days)	SWBAT recognize, write, and find nth terms of arithmetic and geometric sequences. SWBAT use arithmetic and geometric sequences to model real world	Students will identify the common difference and first term of arithmetic sequences written in a variety of ways and use that to write a rule. Students will identify the common ratio and	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the whiteboard. Peer/self assessments

	problems.	the first term of geometric sequences written in a variety of ways and use that information to write a rule. Sequences will model arithmetic and geometric sequences from contextual and mathematical scenarios.	Through questioning students will be able to identify similarities and differences between arithmetic sequences and linear functions and geometric sequences and exponential functions. Students will be able to construct a model with and without technology. Quiz 2.9

Standards for Course Content Area and Cross Content Standards Addressed

MATH.K-12.4	Model with mathematics
MATH.9-12.A.APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x)$ + $r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning
MATH.9-12.A.SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties

	of the quantity represented by the expression.
MATH.9-12.A.SSE.B.3.b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MATH.9-12.A.SSE.B.3.c	Use the properties of exponents to transform expressions for exponential functions.
MATH.9-12.A.SSE.B.4	Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Suggested Modifications for Students with Disabilities, 504 eligible, Multilingual Learners, At Risk Students and Gifted Students

- Special Education Students:
    - Visual Aids and Manipulatives: Provide visual aids such as graphic organizers, anchor charts, and manipulatives like algebra tiles or graphing boards to help students visualize concepts and relationships.
    - Simplified Instructions: Break down complex concepts into smaller, more manageable steps with clear and concise instructions.
    - Extra Practice and Scaffolding: Offer additional practice problems with varying difficulty levels, along with scaffolding (hints, prompts, worked examples) to support students as they progress.
    - Alternative Assessments: Provide alternative assessment options, such as oral presentations, projects, or portfolios, to demonstrate understanding.
    - Individualized Instruction: Tailor instruction to meet individual needs, considering learning styles, strengths, and weaknesses.

Multilingual Learners:

- Visual Support: Incorporate visuals, such as diagrams, graphs, and real-world examples, to enhance understanding of vocabulary and concepts.
- Simplified Language: Use clear and concise language, avoiding jargon and idioms. Provide a glossary of key terms with definitions and translations.
- Bilingual Resources: Offer bilingual dictionaries, textbooks, or online resources to support students' understanding.
- Collaborative Learning: Encourage peer-to-peer interaction and group work to provide opportunities for language practice and collaborative problem-solving.
- Differentiated Instruction: This method tailors instruction to meet individual language proficiency levels, providing additional support for students who need it.

Gifted Students:

- Extension Activities: Provide opportunities for enrichment and extension beyond the standard curriculum, such as exploring advanced topics, independent research projects, or problem-solving challenges.
- Inquiry-Based Learning: Encourage students to ask questions, explore concepts independently, and develop their solutions.
- Higher-Order Thinking Skills: Challenge students with complex problems that require critical thinking, analysis, and synthesis.
- Acceleration: Consider accelerating gifted students to higher-level math courses if they demonstrate readiness.
- Mentorship: Pair gifted students with mentors who can guide and support their academic pursuits.

Computer Sci Design Thinking

CS.9-12.DA	Data & Analysis
CS.9-12.ED	Engineering Design
CS.9-12.IC	Impacts of Computing

Career Readiness, Life Literacies and Key Skills Practice

TECH.9.4.12.CI	Creativity and Innovation
TECH.9.4.12.CT	Critical Thinking and Problem-solving
TECH.9.4.12.DC	Digital Citizenship
TECH.9.4.12.GCA	Global and Cultural Awareness
TECH.9.4.12.IML	Information and Media Literacy

Unit 3 : Trigonometric Functions and the Unit Circle

Content Area: **Mathematics**
Course(s):
Time Period: **3rd Marking Period**
Length: **8 weeks**
Status: **Published**

Summary of the Unit

In this unit, students extend their understanding of geometry and functions through the study of trigonometry. The unit begins with right triangle trigonometry, where students apply their knowledge of trigonometric ratios to solve for unknown side lengths and angles in right triangles. Students use these relationships to solve real-

world problems involving angles of elevation and depression and develop an understanding of how trigonometry can be uses to model and analyze practical situations. Students then transition from right triangle trigonometry to a broader understanding of angles through the development of radian measure. They explore the relationship between arc length and angle measure, sketching angles in standard position, properties of positive and negative angles, and the concept of coterminal angles. Through these investigations, students develop a deeper understanding of angles as rotational quantities and establish connections between geometric and algebraic representations. Building on this foundation, students construct and analyze the unit circle as a tool for finding exact values of the six trigonometric functions. Students will connect the unit circle to the coordinate plane and use it to explain the periodic behavior of trigonometric functions. The unit concludes with the study of trigonometric graphs. Students will analyze and graph the sine, cosine, and tangent functions and will identify key features such as amplitude, period, phase shift, vertical shifts, domain, and range. They will investigate transformations of trigonometric functions and connect graphical behavior to the unit circle and real-world periodic phenomena. Throughout the unit, students use multiple representations, mathematical reasoning, and technology to model, analyze, and communicate relationships involving angles and trigonometric functions.

Enduring Understandings

- Trigonometric ratios provide powerful tools for determining unknown side lengths and angle measures in right triangles.
- Angles can be measured in both degrees and radians.
- Radians provide a natural connection between angles and circular motion.
- Arc length, radius, and radian measure are directly related and can be used to solve geometric and real-world problems.
- The unit circle extends the definition of trigonometric functions beyond right triangles and allows trigonometric values to be determined for any angle.
- Symmetry, reference angles, and patterns within the unit circle can be used to evaluate trigonometric functions efficiently.
- The coordinates of points on the unit circle correspond directly to the values of the cosine and sine functions.
- Trigonometric functions are periodic and can be used to model cyclical and repetitive phenomena.
- The graphs of sine, cosine and tangent functions are connected to the behavior of points moving around the unit circle.
- Transformations such as changes in amplitude, period, phase shift, and vertical shift affect the behavior of trigonometric graphs.
- Mathematical models based on trigonometric functions can be used to represent and analyze real-world situations involving waves, motion, seasonal patterns, and other periodic phenomena.

Essential Questions

- How can trigonometric relationships be used to solve real-world problems involving distance, height,

and angle measurement?
- Why is radian measure a more natural way to measure angles in advanced mathematics?
- How are angles, arc length, and radian measure related?
- What advantages does the unit circle provide over right triangle trigonometry?
- How does the unit circle connect geometric representations of angles to trigonometric function values?
- How can patterns and symmetry in the unit circle be used to determine exact trigonometric values?
- What do sine, cosine, and tangent functions reveal about periodic behavior?
- How do the graphs of trigonometric functions relate to the coordinate of points on the unit circle?
- How can trigonometric functions be used to model and interpret real-world periodic phenomena?
- Why do trigonometric functions exhibit repeating patterns, and how can these patterns be analyzed mathematically?
- How are multiple representations connected in trigonometry?

Unit Summative Assessment and Alternate Assessment Options

Suggested Formative/Summative Classroom Assessments

- Complete classwork Learning vertically at whiteboard
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Teacher-Student and Student-Student Conferencing
- WebAssign Problem Sets WebAssign Instructor Help
- Error analysis tasks
- Task based station assessments
- Homework
- Students will take formal assessments, such as tests and quizzes, to assess knowledge of concepts learned throughout the unit.
- Unit 3 summative assessment taken at the end of the unit
- Students will also demonstrate mastery through various assessment criteria included in the unit such as

do nows, exit slips, graded classwork activities and assignments, and/or projects.

Resources

- Classpad.net (Casio): https://classpad.net/us/
- Desmos: https://www.desmos.com/
- Geogebra: https://www.geogebra.org/?lang=en
- Math Open Reference: https://www.mathopenref.com/
- TI Education (Texas Instruments): https://education.ti.com/en
- WolframAlpha: https://www.wolframalpha.com/
- Wolfram MathWorld: https://mathworld.wolfram.com/
- Khan Academy: https://www.khanacademy.org/math/precalculus
- Digital Mathematics Word Wall: http://www.mathwords.com/index_adv_alg_precal.htm
- Extra Notes for Pre-Calculus Content: https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates
- Review Documents for Pre-Calculus: https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets
- Pre-Calculus IXL Topics and Resources: https://www.ixl.com/math/precalculus
- Classroom Challenges to Support Teachers in Formative Assessments: http://map.mathshell.org/materials/lessons.php?gradeid=24
- Solving Exponential Equations: https://flexbooks.ck12.org/cbook/ck-12-precalculus-concepts-2.0/section/3.6/primary/lesson/exponential-equations-pcalc/
- Statistics Education Web (STEW). http://www.amstat.org/education/STEW/
- The Data and Story Library (DASL). http://lib.stat.cmu.edu/DASL/
- WebAssign Instructor Resources
- WebAssign Student Resources
- PreCalculus by Robert Blitzer 5th edition

Unit Plan

Topic/Selection Timeframe	General Objectives	Instructional Activities	Benchmarks/Assessments

3.1 - Right Triangle Trigonometry (6 days)	SWBAT use trigonometry to solve for missing sides and angles in a right triangle. SWBAT apply trigonometric functions to right triangle models to solve application problems involving angles of elevation and depression.	Students will identify the sides of a right triangle relative to a given acute angle. Students will solve right triangles by finding unknown side lengths using a trigonometric ratio. Students will find unknown angle measures by setting up a trigonometric ratio and applying inverse trigonometric functions to find the angle measure. Students will model and solve real-world problems involving right triangle trigonometry and interpret and communicate solutions within the context of the problem.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to solve right triangles for missing side lengths and angles and solve real-world application problems involving angles of elevation and depression. Closure Quiz 3.1
3.2 - Sine, Cosine, and Tangent Function Values (2 days)	SWBAT find trigonometric ratios of any angle in standard position in a coordinate plane.	Students will define and evaluate the six trigonometric functions on a circle of radius r.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to find the ratios of the six trigonometric functions by sketching a right triangle. Closure

3.3 - Angles and Radian Measure (7 days)	SWBAT use radian measure. SWBAT sketch angles in standard position in both degrees and radians. SWBAT find the length of a circular arc. SWBAT find coterminal angles. SWBAT convert between degrees and radians.	Students will determine that a radian is an angle measure with an arc length of one radius. Students will determine how to measure angles in standard position on the coordinate plane and their properties. Students will understand properties of positive and negative angles. Students will determine arc length using angle measure and radius. Students will find coterminal angles in both degrees and radians and use them to describe equivalent angles in standard position. Students will convert between degrees and radians. Students will identify reference angles for angles in standard position.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will develop an understanding of angle measurement in both radian and degrees. Emphasis is placed on sketching angles in standard position in both degrees and radians, identifying coterminal angles, and applying radian measure to determine arc length. Closure Quiz 3.2 - 3.3

3.4 - Unit Circle (10 days)	SWBAT Determine the sine, cosine, and tangent of an angle using the unit circle.	Students will label the angles on the unit circle in radians using proportional reasoning. Students will use special right triangles to determine the coordinates at key points on the unit circle. Students will evaluate sine, cosine, and tangent for key angles on the unit circle. Students will determine that in a unit circle, the sine and cosine ratios correspond to the y-value and x-value, respectively, of the point where the terminal ray intersects the circle. Students then will be able to use symmetry to identify relationships between the sine and cosine values of angles in all four quadrants. Students will find coordinates of points on circles where r≠1. Students will find the exact values of the six trigonometric functions.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessmnet Through questioning students will be able to evaluate exact values of sine, cosine, and tangent for key angles on the unit circle. Closure Quiz 3.4

3.5 Periodic Phenomena (1 day)	SWBAT Construct graphs of periodic relationships based on verbal representations. SWBAT Describe key characteristics of a periodic function based on a verbal representation.	Students will understand periodic behavior and identify periodic functions. Students will find the amplitude and period for one cycle of a periodic function. Students will sketch periodic functions based on verbal descriptions. Students will describe key characteristics of periodic functions.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Peer/self assessments Through questioning students will be able to determine if a function is periodic or not and identify the length of one cycle of a periodic function. Closure
3.6 Sine and Cosine Function Graphs (2 days)	Construct representations of the sine and cosine functions using the unit circle.	Students would be able to construct graphs of the sine and cosine functions by observing that as the input values, or angle measures, of the sine function increase, the output values of sine and and cosine oscillate between −1 and 1, taking every value in between and tracking the vertical distance of points on the unit circle from the x-axis.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to construct graphs of the sine and cosine functions using values from the unit circle.

			Closure
3.7 Sinusoidal Functions (3 days)	SWBAT Identify key characteristics of the sine and cosine functions.	Students would be able to determine that a sinusoidal function is any function that involves additive and multiplicative transformations of sin x. The sine and cosine functions are both sinusoidal functions. The period and frequency of a sinusoidal function are reciprocals. The amplitude of a sinusoidal function is half the difference between its maximum and minimum values. The midline of the graph of a sinusoidal function is determined by the average, or arithmetic mean, of the maximum and minimum values of the function. The midline of the graphs of y = sinθ and y = cosθ is y = 0. The graph of y = sinx has rotational symmetry about the origin and is therefore an odd function. The graph of y = cosx has reflective symmetry over the y-axis and is therefore an even function.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to Identify key characteristics for the parent functions y=sin x and y=cos x, including domain, amplitude, midline, period, and symmetry. Closure Quiz 3.5 - 3.7
3.8 Sinusoidal Function Transformations (4 days)	SWBAT Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.	Students will determine how the amplitude, period, domain, range, and midline of sinusoidal functions are affected by transformations. Students would be able relate additive transformations with horizontal/vertical transformation and and multiplicative transformation with dilation and use these transformations to graph transformed sine and cosine functions.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to graph transformed sine and cosine functions given an

			equation . Closure Quiz 3.8
3.9 The Tangent Function (2 days)	SWBAT Construct representations of the tangent function using the unit circle. SWBAT Describe key characteristics of the tangent function. SWBAT Describe additive and multiplicative transformations involving the tangent function.	Students would be able to construct graphs of the tangent functions by observing that as the input values, or angle measures, of the tangent function increase. The asymptote equations are found when the function in the denominator is zero.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to evaluate tangent for key angles on the unit circle. Closure

Standards for Course Content Area and Cross Content Standards Addressed

MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
MA.G-SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Suggested Modifications for Students with Disabilities, 504 eligible, Multilingual Learners, At Risk Students and Gifted Students

- Special Education Students:
    - Visual Aids and Manipulatives: Provide visual aids such as graphic organizers, anchor charts, and manipulatives like algebra tiles or graphing boards to help students visualize concepts and relationships.
    - Simplified Instructions: Break down complex concepts into smaller, more manageable steps with clear and concise instructions.
    - Extra Practice and Scaffolding: Offer additional practice problems with varying difficulty levels, along with scaffolding (hints, prompts, worked examples) to support students as they progress.
    - Alternative Assessments: Provide alternative assessment options, such as oral presentations, projects, or portfolios, to demonstrate understanding.
    - Individualized Instruction: Tailor instruction to meet individual needs, considering learning

styles, strengths, and weaknesses.

Multilingual Learners:

- Visual Support: Incorporate visuals, such as diagrams, graphs, and real-world examples, to enhance understanding of vocabulary and concepts.
- Simplified Language: Use clear and concise language, avoiding jargon and idioms. Provide a glossary of key terms with definitions and translations.
- Bilingual Resources: Offer bilingual dictionaries, textbooks, or online resources to support students' understanding.
- Collaborative Learning: Encourage peer-to-peer interaction and group work to provide opportunities for language practice and collaborative problem-solving.
- Differentiated Instruction: This method tailors instruction to meet individual language proficiency levels, providing additional support for students who need it.

Gifted Students:

- Extension Activities: Provide opportunities for enrichment and extension beyond the standard curriculum, such as exploring advanced topics, independent research projects, or problem-solving challenges.
- Inquiry-Based Learning: Encourage students to ask questions, explore concepts independently, and develop their solutions.
- Higher-Order Thinking Skills: Challenge students with complex problems that require critical thinking, analysis, and synthesis.
- Acceleration: Consider accelerating gifted students to higher-level math courses if they demonstrate readiness.
- Mentorship: Pair gifted students with mentors who can guide and support their academic pursuits.

Computer Sci Design Thinking

CS.9-12.8.1.12.DA.1	Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
CS.9-12.8.1.12.DA.5	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
CS.9-12.8.1.12.IC.1	Evaluate the ways computing impacts personal, ethical, social, economic, and cultural practices.
CS.9-12.8.2.12.ED.6	Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

Career Readiness, Life Literacies and Key Skills Practice

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or

	practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
TECH.9.4.12.IML.1	Compare search browsers and recognize features that allow for filtering of information.

Unit 4 : Advanced Trigonometry and Introduction to Limits

Content Area: **Mathematics**
Course(s):
Time Period: **4th Marking Period**
Length: **8 weeks**
Status: **Published**

Summary of the Unit

In this unit, students extend their understanding of the reciprocal trigonometric functions by exploring the graphs of secant, cosecant, and cotangent. Students will analyze the graphs of these functions and identify key characteristics such as domain, range, asymptotes, periodic behavior and will compare them to their sine, cosine, and tangent counterparts. Students then deepen their understanding of trigonometric relationships by evaluating inverse trigonometric functions. They will determine the domain and range constraints. Building on these concepts, students will develop fluency with trigonometric identities. Students will simplify trigonometric identities and verify identities using a variety of algebraic techniques such as factoring, substitution, and manipulation of known Pythagorean identities. Students will then apply these skills to solve trigonometric equations, using identities and algebraic strategies to find all solutions within a given interval. The unit concludes with an introduction to limits and continuity. Students will estimate limits graphically by analyzing function behavior near specific points and will determine limits algebraically by substitution, factoring, and rationalizing techniques. Students will extend their understanding to limits at infinity and use these ideas to describe end behavior and asymptotic behavior of functions. Finally, students explore the definition of continuity by verifying that the function value, limit, and left and right handed behavior all agree. Throughout the unit, students will use multiple representations - graphical, algebraic, numerical, and contextual - to analyze functions, justify reasoning, and communicate mathematical ideas clearly.

Enduring Understandings

- Reciprocal trigonometric functions are defined in relation to sine, cosine, and tangent and exhibit unique behavior including asymptotes and restricted domains.
- Inverse trigonometric functions and their reciprocals share deep structural relationships that can be analyzed through graphs, identities, and algebraic manipulation.
- Trigonometric identities are powerful tools that allow expressions to be rewritten, simplified, and verified as equivalent.
- Solving trigonometric equations often requires a combination of algebraic techniques, identities, and interval analysis to capture all of the solutions.
- A functions behavior can be understood through multiple representations such as algebraic structure, graphical representation, and numerical representation.
- Limits describe the behavior of function near a point, even when the function is not defined at that point.
- Algebraic techniques such as factoring, rationalizing, and simplification, are essential tools for evaluating limits.
- Continuity at a point requires that the function value and limit value agree, and that the function

behaves predictably from both sides.
- Discontinuities can be identified and classified by examining both algebraic structure and graphical behavior.
- Limits at infinity describe end behavior and help model long term trends in real world contexts.
- Understanding functions requires integrating multiple representations - graphical, algebraic, numerical, and contextual - to fully interpret behavior and relationships.

Essential Questions

- How do reciprocal trigonometric functions extend our understanding of sine, cosine, and tangent ?
- What key features distinguish secant, cosecant, and cotangent from their counterparts?
- How do inverse trigonometric functions allow use to undo trigonometric relationships?
- How can trigonometric identities be used to simplify expressions and verify equivalence?
- When solving trigonometric equations, how do we ensure that all solutions are found within a given interval?
- What strategies can be used to determine whether two trigonometric expressions are equivalent?
- How can limits be interpreted both graphically and algebraically to describe function behavior near a point?
- What does it mean for a function to approach a value without reaching it?
- How does algebraic manipulation such as factoring and rationalizing help evaluate limits?
- What is continuity at a point and how is it connected to limits and function values?
- How can discontinuities be identified and classified using graphical and algebraic methods?
- How do limits at infinity describe the long-term behavior of functions?

Unit Summative Assessment and Alternate Assessment Options

Suggested Formative/Summative Classroom Assessments

- Complete classwork Learning vertically at whiteboard
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Observation
- Graphic Organizers/ Concept Mapping

- Presentations
- Teacher-Student and Student-Student Conferencing
- WebAssign Problem Sets WebAssign Instructor Help
- Error analysis tasks
- Task based station assessments
- Homework
- Students will take formal assessments, such as tests and quizzes, to assess knowledge of concepts learned throughout the unit.
- Unit 4 summative assessment taken at the end of the unit
- Students will also demonstrate mastery through various assessment criteria included in the unit such as do nows, exit slips, graded classwork activities and assignments, and/or projects.

Resources

- Classpad.net (Casio): https://classpad.net/us/
- Desmos: https://www.desmos.com/
- Geogebra: https://www.geogebra.org/?lang=en
- Math Open Reference: https://www.mathopenref.com/
- TI Education (Texas Instruments): https://education.ti.com/en
- WolframAlpha: https://www.wolframalpha.com/
- Wolfram MathWorld: https://mathworld.wolfram.com/
- Khan Academy: https://www.khanacademy.org/math/precalculus
- Digital Mathematics Word Wall: http://www.mathwords.com/index_adv_alg_precal.htm
- Extra Notes for Pre-Calculus Content: https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates
- Review Documents for Pre-Calculus: https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets
- Pre-Calculus IXL Topics and Resources: https://www.ixl.com/math/precalculus
- Classroom Challenges to Support Teachers in Formative Assessments:

http://map.mathshell.org/materials/lessons.php?gradeid=24

- Solving Exponential Equations: https://flexbooks.ck12.org/cbook/ck-12-precalculus-concepts-2.0/section/3.6/primary/lesson/exponential-equations-pcalc/
- Statistics Education Web (STEW). http://www.amstat.org/education/STEW/
- The Data and Story Library (DASL). http://lib.stat.cmu.edu/DASL/
- WebAssign Instructor Resources
- WebAssign Student Resources
- PreCalculus by Robert Blitzer 5th edition

Unit Plan

Topic/Selection Timeframe	General Objectives	Instructional Activities	Benchmarks/Assessments
4.1 The Secant, Cosecant, and Cotangent Functions (5 days)	SWBAT Identify key characteristics of functions that involve quotients of the sine and cosine fand tangent functions.	Students will define the secant, cosecant, and cotangent functions as the reciprocal of the cosine, sine, and tangent functions, respectively. Students will understand how the zeros, vertical asymptotes, and range are related for a trigonometric function and its reciprocal function. Students will graph and analyze graphs of secant, cosecant, and cotangent	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessment Through questioning students will be able to Identify key characteristics for the parent functions y=csc x and y=sec x and cotangent including domain, period, asymptotes and symmetry. Closure

			Quiz 4.1
4.2 Inverse Trigonometric Functions (5 days)	SWBAT Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain.	Students will understand that inverse trigonometric functions input ratios and output angles. The input and output values are switched from their corresponding trigonometric functions. Students will understand through graphical representation why and how the domains of sine, cosine, and tangent must be restricted to create an inverse function. Students will evaluate inverse trig expressions.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to evaluate inverse of sine, cosine, and tangent for key points on the unit circle. Closure Quiz 4.2
4.3 Equivalent Representations of Trigonometric Functions (7 days)	SWBAT Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity. SWBAT apply fundamental trigonometric identities to simplify trigonometric expressions. SWBAT verify trigonometric	Students will apply the fundamental trigonometric identities including reciprocal, quotient, Pythagorean, even-odd, and cofunction identities to simplify trigonometric expressions. Students will use algebraic techniques such as factoring, finding common denominators, and simplifying complex fractions to simplify and verify trigonometric identities. Students will verify trigonometric identities by rewriting one side using equivalent expressions.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to simplify and verify trigonometric relationships. Closure

	identities.		Quiz 4.3
4.4 Trigonometric Equations (5 days)	SWBAT solve equations involving trigonometric functions. SWBAT Solve equations using equivalent analytic representations of trigonometric functions.	SWBAT extend the process of inverse operations to trigonometric equations. SWBAT understand that using the unit circle will give infinite solutions to a trigonometric equation which may need to be restricted based on context and that an inverse trig function gives only one solution that may need to be expanded using symmetry.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to find solutions to trigonometric equations. Closure Quiz 4.4
4.5 - Defining Limits and Using Limit Notation (1day)	SWBAT represent limits analytically using correct notation. SWBAT interpret limits expressed in analytic notation.	Explain the concept of a limit using an informal "destination point" analogy: a limit is simply the value that the outputs of a function f(x) get closer and closer to as the inputs x get closer and closer to a specific target number c, without ever needing to actually reach that target. Introduce the concept of one-sided limits both graphically and analytically.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to the limit as x approaches a given value by analyzing a graph. Closure

4.6 - Estimating Limit Values from Graphs. (4 days)	SWBAT estimate Limits of Functions from graphs.	Use graphing software like Desmos or graphing calculators to visually demonstrate how a function approaches different values from the left versus the right (e.g., using piecewise or rational functions). Students will estimate the limit of a function from its graph by analyzing the behavior of the function as x approaches a given value. Students will distinguish the difference between the limit value and function value recognizing that a limit may exist even if the function is undefined at that point. Students will identify one-sided limits from a graph, including left and right hand limits, and determine when they are equal. Students will determine whether a limit exists by analyzing continuity, jumps, holes, and vertical asymptotes on a graph.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to find the limit and one sided limit as x approaches a given value by analyzing a graph. Closure Quiz 4.5 - 4.6
4.7 -Estimating Limit Values from Tables. 1 day	SWBAT estimate limits of functions from tables.	Provide students with numerical tables where the input values (x) systematically close in on a specific number from both sides. Guide them to look for the overall trend in the corresponding output values (f(x)) to estimate the limit. Teach students how to configure the table settings on a graphing utility (like a TI-calculator or Desmos) to "Ask" mode. Have them manually input values increasingly closer to a point of discontinuity to observe the behavior of indeterminate forms. Present tables where the left and right outputs approach two entirely different numbers, or grow without bound (approaching $\pm\infty$), to model how to identify	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Students are given multiple numerical tables representing different types of functions (continuous, removable holes, asymptotes). Students must state the estimated limit or write "DNE"

		when a limit does not exist (DNE) from numerical data.	with a brief justification. Short-response question where students are given an equation, must generate their own small table of values to estimate the limit, and then verify their guess using a graph.
4.8 - Determining Limits Using Algebraic Properties of Limits (9 days)	SWBAT determine the limits of functions using limit theorems.	Students will evaluate limits algebraically by direct substitution when possible. Students will recognize indeterminate forms and understand why the require simplification. Students will use factoring techniques to simplify rational expressions before evaluating limits. Students will use rationalizing techniques to eliminate radicals in numerators or denominators when evaluating limits. Students will simplify complex rational expressions to remove discontinuities and determine limit values. Students will evaluate one sided limits algebraically. Students will determine when a limit does not exist based on algebraic behavior.	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to find the limit and one sided limit as x approaches a given value by using different algebraic techniques. Closure Quiz 4.7 - 4.8
4.9 - Limits at Infinity 3 days	SWBAT determine the end behavior of functions by evaluating limits both graphically and algebraically.	Students will analyze the end behavior of function graphically, identifying functions approach as x-values become very large or very small. Students will understand that as x values become very large or very small that rational functions are approaching their horizontal asymptotes. Students will determine horizontal asymptotes of rational functions by comparing the degrees of the	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board.

		numerator and denominator. Students will recognize when limits at infinity do not exist or are unbounded and interpret infinite limits appropriately. Students will connect algebraic results to graphical representations.	Peer/self assessments Through questioning students will be able to find the limit and one sided limit as x values get very small or very large. Closure
4.10 - Defining Continuity at a Point. 3 days	SWBAT justify conclusions about continuity at a point.	Students will explore a single graph containing all three types of discontinuities (a hole, a jump, and a vertical asymptote). Lead a class discussion to define and classify each type based on visual behavior. Students will connect visual features to algebraic conditions by checking the three requirements for continuity at a point x=c: ○ Is f(c) defined? ○ Does limit as x approaches c, f(x) exist? ○ Does limit as x approaches c f(x) = f(c)?	Circulate and monitor student progress as they are working on classwork. Have students complete problems vertically at the board. Peer/self assessments Through questioning students will be able to identify different discontinuities graphically and verify any discontinuities algebraically. Closure Quiz 4.9 - 4.10

Standards for Course Content Area and Cross Content Standards Addressed

MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MATH.9-12.F.IF.C.7.d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MATH.9-12.F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
MATH.9-12.F.IF.C.7.f	Graph trigonometric functions, showing period, midline, and amplitude.
MATH.9-12.F.TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MATH.9-12.F.TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MATH.9-12.F.TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MATH.9-12.F.TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MATH.9-12.F.TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
MATH.9-12.F.TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
MA.9-12.I	Functions, Graphs, and Limits
MA.9-12.I.B.1	An intuitive understanding of the limiting process
MA.9-12.I.B.2	Calculating limits using algebra
MA.9-12.I.B.3	Estimating limits from graphs or tables of data
MA.9-12.I.C.1	Understanding asymptotes in terms of graphical behavior
MA.9-12.I.C.2	Describing asymptotic behavior in terms of limits involving infinity
MA.9-12.I.D.1	An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
MA.9-12.I.D.2	Understanding continuity in terms of limits

Suggested Modifications for Students with Disabilities, 504 eligible, Multilingual Learners, At Risk Students and Gifted Students

Special Education Students:

- Visual Aids and Manipulatives: Provide visual aids such as graphic organizers, anchor charts, and manipulatives like algebra tiles or graphing boards to help students visualize concepts and relationships.
- Simplified Instructions: Break down complex concepts into smaller, more manageable steps with clear and concise instructions.
- Extra Practice and Scaffolding: Offer additional practice problems with varying difficulty levels, along with scaffolding (hints, prompts, worked examples) to support students as they progress.
- Alternative Assessments: Provide alternative assessment options, such as oral presentations, projects, or portfolios, to demonstrate understanding.
- Individualized Instruction: Tailor instruction to meet individual needs, considering learning styles, strengths, and weaknesses.

Multilingual Learners:

- Visual Support: Incorporate visuals, such as diagrams, graphs, and real-world examples, to enhance understanding of vocabulary and concepts.
- Simplified Language: Use clear and concise language, avoiding jargon and idioms. Provide a glossary of key terms with definitions and translations.
- Bilingual Resources: Offer bilingual dictionaries, textbooks, or online resources to support students' understanding.
- Collaborative Learning: Encourage peer-to-peer interaction and group work to provide opportunities for language practice and collaborative problem-solving.
- Differentiated Instruction: This method tailors instruction to meet individual language proficiency levels, providing additional support for students who need it.

Gifted Students:

- Extension Activities: Provide opportunities for enrichment and extension beyond the standard curriculum, such as exploring advanced topics, independent research projects, or problem-solving challenges.
- Inquiry-Based Learning: Encourage students to ask questions, explore concepts independently, and develop their solutions.
- Higher-Order Thinking Skills: Challenge students with complex problems that require critical thinking, analysis, and synthesis.
- Acceleration: Consider accelerating gifted students to higher-level math courses if they demonstrate readiness.
- Mentorship: Pair gifted students with mentors who can guide and support their academic pursuits.

Computer Sci Design Thinking

CS.9-12.8.1.12.DA.1	Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
CS.9-12.8.1.12.DA.5	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
CS.9-12.8.1.12.IC.1	Evaluate the ways computing impacts personal, ethical, social, economic, and cultural practices.
CS.9-12.8.2.12.ED.5	Evaluate the effectiveness of a product or system based on factors that are related to its

	requirements, specifications, and constraints (e.g., safety, reliability, economic considerations, quality control, environmental concerns, manufacturability, maintenance and repair, ergonomics).
CS.9-12.8.2.12.ED.6	Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

Career Readiness, Life Literacies and Key Skills Practice

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
TECH.9.4.12.IML.1	Compare search browsers and recognize features that allow for filtering of information.