

AP Calculus Summer Packet

Congratulations! You made it to AP Calculus AB

Once the school year starts, we have a lot of calculus to cover before the AP exam. It is important that you come into class with a solid memory of the topics you have learned in your previous math courses. Each section has a quick refresher on the topic along with some example problems for you to complete. Please make sure to have each example in this packet done for the first day of school. ***There will be a test on the concepts the first week of school!*** Once we review the packet, we are jumping right into the calculus concepts and ***will not be relearning any of these concepts.***

Rules of Exponents

Product Rule when multiplying powers with the same base, add the exponents $x^4 \cdot x^2 = x^6$	Quotient Rule when dividing powers with the same base, subtract the exponents $\frac{x^5}{x^2} = x^3$	Power Rule when raising powers to another power, multiply the exponents $(x^3)^2 = x^6$
** Don't forget that the coefficients of any power do not follow the exponent rules!		
Zero Exponents Anything to the power of zero equals 1 $x^0 = 1$	Negative Exponents Moving a power from the denominator to the numerator or vice versa, changes the sign of the exponent $x^{-3} = \frac{1}{x^3}$	Rational Exponents Radical expressions can be rewritten as powers with rational exponents $\sqrt{x} = x^{\frac{1}{2}}$ $\sqrt[3]{x^4} = x^{\frac{4}{3}}$

1. Simplify the following expressions using the exponent rules. Your answer should contain only positive exponents.

a. $4x^2y^3 \cdot 7x^3y^8$

b. $(2x^4y^{-2})^3$

c. $\left(\frac{6x^4y^2}{3x^5y}\right)^2$

2. Rewrite each of the following expressions as a power in the numerator.

a. $\frac{2}{x^5}$

b. $\sqrt[5]{x^3}$

c. $\frac{3}{4\sqrt{x}}$

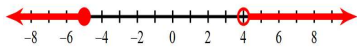
Interval Notation

Interval notation is the most efficient way to represent an interval. In Calculus, we represent intervals in many of the units. The following are things you need to know:

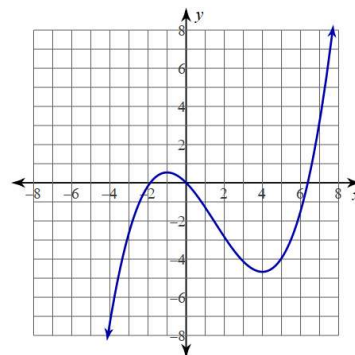
Interval notation is not a coordinate point and will represent a range of either x or y values (we mostly use it to represent a range of x values).

- Use $[$ to include an endpoint in an interval
- Use $($ to not include an endpoint in an interval

3. Complete the table below with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		

4. Use the graph below and determine the intervals that the function is increasing and the intervals that the function is decreasing.



Composition of Functions

To evaluate a function for a given value, simply plug the value into the function for x .

Recall: $f(g(x))$, read " f of g of x ", means to plug the inside function ($g(x)$) in for x in the outside function ($f(x)$)

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$

$$\begin{aligned}
 f(g(x)) &= f(x - 4) \\
 &= 2(x - 4)^2 + 1 \\
 &= 2(x^2 - 8x + 16) + 1 \\
 &= 2x^2 - 16x + 32 + 1 \\
 &= 2x^2 - 16x + 33
 \end{aligned}$$

5. Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each.

a. $f(t + 1)$

b. $f(g(-2))$

c. $g(f(m + 2))$

6. Find $\frac{f(x+h)-f(x)}{h}$ for the given function f . (This formula should look familiar!)

a. $f(x) = 9x + 3$

b. $f(x) = 5 - 2x$

Inverse Functions

Recall that inverse functions are functions in which the x and y variable have swapped. You do not need to know much about inverse functions for calculus but you should know the following.

How to find a function's inverse: Switch the x and y variables and solve for y

Original Function: $y = 2x + 6$

Inverse Function: $x = 2y + 6$
 $x - 6 = 2y$
 $y = \frac{1}{2}x - 3$

You will need to know the notation for inverse functions so you know when a problem is telling you that two functions are inverses.

$g(x) = f^{-1}(x)$	g is the inverse of f
$g(f(x)) = x$ and $f(g(x)) = x$	Recall that when two functions are plugged into one another and they simplify to x , they are inverses. You may see this notation when describing inverse functions

7. Find the inverse functions of the following.

a. $y = 3x - 5$

b. $y = \frac{1}{x+2}$

Factoring – Finding Zeroes

While you learned many types of factoring in Algebra 2 and Precalculus, the main types of factoring you need to be able to do are listed below. You will also need to be able to find the zeros of functions using factoring.

Quadratics	Greatest Common Factor (of x or some degree of x)	Greatest Common Factor (of some other function)
$x^2 - 6x - 7 = 0$ $(x - 7)(x + 1) = 0$ $x = 7 \quad x = -1$	$3x^3 - 6x^2 = 0$ $3x^2(x - 2) = 0$ $x = 0 \quad x = 2$	$x^2e^x - xe^x = 0$ $xe^x(x - 1) = 0$ $x = 0 \quad x = 1$ $(e^x \text{ can never equal } 0)$

8. Solve the following equations using factoring.

a. $x^2 - 7x + 12 = 0$

b. $2x^3 - 2x = 0$

c. $xe^x + 2e^x = 0$

Rational Functions – Vertical Asymptotes, Holes, Horizontal Asymptote Rules

Vertical Asymptote	Hole
A value of x that makes only the denominator of a rational function equal to zero	A value of x that makes both the denominator and numerator equal to zero (the factors cancel!)
Horizontal Asymptote Rules Top Degree > Bottom Degree - No HA Top Degree < Bottom Degree - $y = 0$ Top Degree = Bottom Degree - $y = \frac{\text{leading coefficient of top}}{\text{leading coefficient of bottom}}$	

9. Find the Vertical Asymptotes, Holes and Horizontal Asymptotes for each of the following functions.

a. $y = \frac{2x+6}{x+1}$

b. $y = \frac{x^2-x-2}{x-2}$

c. $y = \frac{x+3}{(x-5)(x+3)}$

Complex Fractions

When simplifying complex fractions, change the top portion into one fraction and change the bottom portion into one fraction. Once you have a fraction over a fraction you can KEEP CHANGE FLIP and simplify the multiplication.

Example:

$$\frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} = \frac{\frac{-7(x+1)}{(x+1)} - \frac{6}{x+1}}{\frac{5}{x+1}} = \frac{\frac{-7x-7}{x+1} - \frac{6}{x+1}}{\frac{5}{x+1}} = \frac{\frac{-7x-13}{x+1}}{\frac{5}{x+1}} \\ \frac{-7x-13}{x+1} \cdot \frac{x+1}{5} = \frac{-7x-13}{5}$$

10. Simplify the complex fractions below.

a. $\frac{\frac{25}{a} - a}{a}$

b. $\frac{4 - \frac{12}{2x-3}}{5 + \frac{15}{2x-3}}$

Properties of logarithms

Sometimes on the AP Calculus Exam, the multiple choice answers will be in simplified log form. You will need to know the product and quotient properties in order to rewrite your answers in the correct form. You also may need to be able to evaluate some simple logs by hand so you will also need to know the power property.

Product Property	Quotient Property	Power Property
$\log_b m + \log_b n = \log_b mn$	$\log_b m - \log_b n = \log_b \frac{m}{n}$	$\log_b m^n = n \log_b m$

11. Write each of the following expressions with a single logarithm

a. $\log x + \log(x+2)$

b. $\ln(x-4) - \ln(x+1)$

12. Evaluate the following logarithmic expressions without a calculator.

a. $\ln e^2$

b. $\ln \sqrt{e}$

Solving Exponential Equations

The exponential equations that you may see in Calculus are usually in the form of $y(t) = Pe^{kt}$. Remember that exponential equations can be solved by taking the *log* or *ln* of both sides. Remember to isolate the power first.

Example

$$\begin{aligned}200 &= 400e^{0.4t} \\0.5 &= e^{0.4t} \\\ln(0.5) &= 0.4t \cdot \ln e \\\ln(0.5) &= 0.4t \\t &= \frac{\ln(0.5)}{0.4}\end{aligned}$$

13. Solve the following exponential equations using logarithms.

a. $15 = 3e^t$

b. $100 = 50e^{0.2t}$

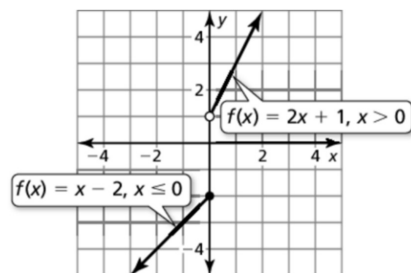
Piecewise Functions

A piecewise function is a function defined by two or more equations. Each “piece” of the function applies to a different part of its domain.

Example:

$$f(x) = \begin{cases} x - 2, & \text{if } x \leq 0 \\ 2x + 1, & \text{if } x > 0 \end{cases}$$

- The expression $x - 2$ represents the value of f when x is less than or equal to 0
- The expression $2x + 1$ represents the value of f when x is greater than 0



14. Evaluate the function $f(x) = \begin{cases} 3x - 1, & \text{if } x \leq 1 \\ 1 - 2x, & \text{if } x > 1 \end{cases}$ for the following values of x .

a. $f(0)$

b. $f(1)$

c. $f(5)$

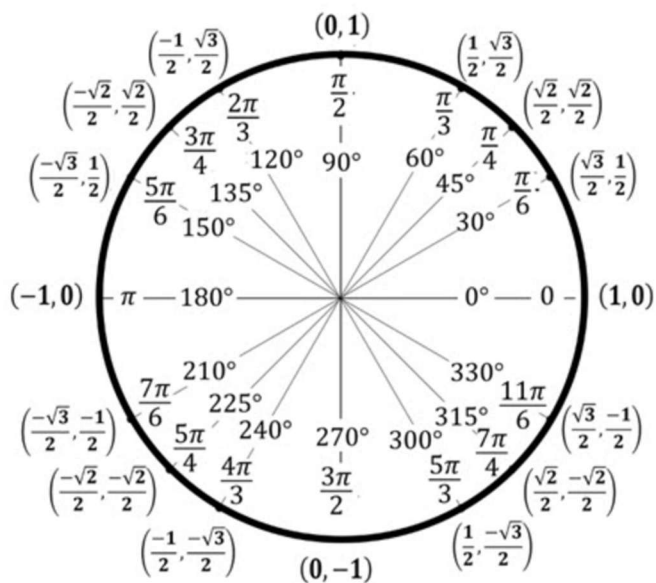
Trigonometry

In Calculus you mostly need to be able to evaluate trigonometric expressions and solve simple trigonometric equations.

Values you need to know from the Unit Circle

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Unit Circle



$$\begin{aligned}\sin \theta &= y \text{ coordinate} \\ \cos \theta &= x \text{ coordinate} \\ \tan \theta &= \frac{y \text{ coordinate}}{x \text{ coordinate}}\end{aligned}$$

15. Evaluate the following trig expressions.

a. $\sin \pi$

b. $\cos 2\pi$

c. $\tan \frac{3\pi}{2}$

d. $\sin \frac{7\pi}{6}$

e. $\cos \frac{2\pi}{3}$

f. $\tan \frac{3\pi}{4}$

16. Solve each of the equations for $0 \leq x \leq 2\pi$.

a. $\sin x = -\frac{1}{2}$

b. $2 \cos x = \sqrt{3}$

Limits from a Graph

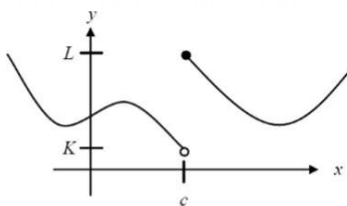
Remember that the limit describes the y value of a function as x gets closer to a given value but not necessarily the actual y value at that value of x .

Definition of a Limit

If the y -value of a function $f(x)$ becomes close to a single value L as x gets closer and closer to a point c from both the left and the right side, then the **limit of $f(x)$ as x approaches c is L** , which can be written using mathematical notation as:

$$\lim_{x \rightarrow c} f(x) = L$$

One Sided Limits:

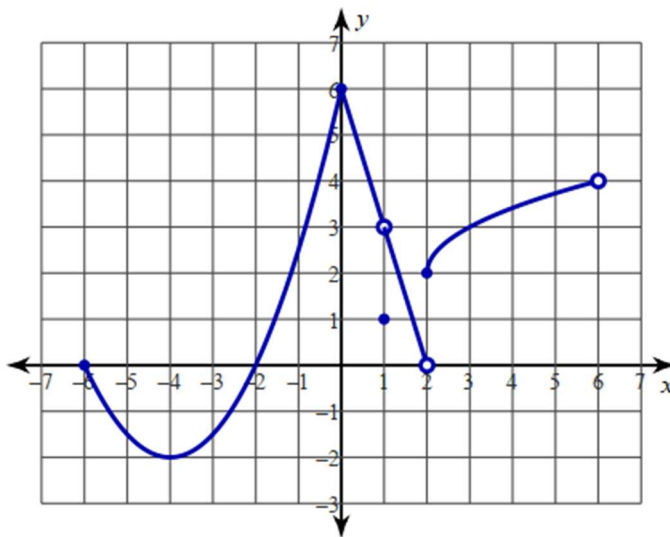


$\lim_{x \rightarrow c^+} f(x) = L$... "the limit of $f(x)$ as x approaches c from the right is L ."
 $\lim_{x \rightarrow c^-} f(x) = K$... "the limit of $f(x)$ as x approaches c from the left is K ."

$$\lim_{x \rightarrow c} f(x) \text{ exists} \iff \lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^-} f(x)$$

17. Use the graph to evaluate the following limits.

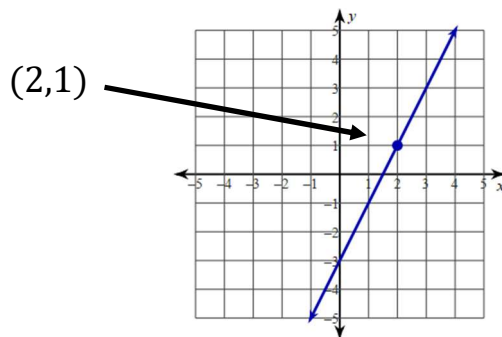
- A) $\lim_{x \rightarrow 0} f(x)$
- B) $\lim_{x \rightarrow 6^-} f(x)$
- C) $\lim_{x \rightarrow 2^-} f(x)$
- D) $\lim_{x \rightarrow -6} f(x)$
- E) $\lim_{x \rightarrow 1} f(x)$
- F) $f(1)$
- G) $f(2)$
- H) $\lim_{x \rightarrow 2} f(x)$



Limits -Direct Substitution

When you are asked to evaluate a limit, the first thing you should always try is direct substitution. This means plug the value of x directly into the function. If there is not a hole, jump or asymptote at that point in the function, the limit will be the same value as the value of the function at that point.

$$\lim_{x \rightarrow 2} (2x - 3) = 2(2) - 3 = 1$$



18. Use direct substitution to evaluate each limit.

a. $\lim_{x \rightarrow 4} (-3x + 10)$

b. $\lim_{x \rightarrow 3} \frac{x^2 + x - 6}{x + 3}$

Properties of Limits

Let b and c be real numbers and let f and g be functions with the following limits:

$$\lim_{x \rightarrow c} f(x) = L \text{ and } \lim_{x \rightarrow c} g(x) = K$$

Rule	Rule
$\lim_{x \rightarrow c} b = b$	$\lim_{x \rightarrow c} [b \cdot g(x)] = bL$
$\lim_{x \rightarrow c} x = c$	$\lim_{x \rightarrow c} [f(x)]^{\frac{r}{s}} = L^{\frac{r}{s}}$ provided r and s are integers and $s \neq 0$.
$\lim_{x \rightarrow c} [f(x) \pm g(x)] = L \pm K$	$\lim_{x \rightarrow c} \left[\frac{f(x)}{g(x)} \right] = \frac{L}{K}$ provided $K \neq 0$.
$\lim_{x \rightarrow c} [f(x) \cdot g(x)] = L \cdot K$	

19. Use the given information to evaluate the limits: $\lim_{x \rightarrow c} f(x) = 2$ and $\lim_{x \rightarrow c} g(x) = 3$

a) $\lim_{x \rightarrow c} [5g(x)]$

b) $\lim_{x \rightarrow c} [f(x) + g(x)]$

c) $\lim_{x \rightarrow c} [f(x)g(x)]$

d) $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$