



A Level Further Maths Bridging Course

Further Maths is designed to enable you to study a wide range of options in pure mathematics, mechanics, statistics, and decision mathematics. You will develop methods that will help you solve problems across a wide range of contexts, including Engineering, Computer Science and Economics. If you are passionate about maths and have strong algebraic skills, this course is for you.

Further Mathematics fits well with almost any combination of subjects and offers progression opportunities to a range of degrees or careers. This is an excellent course if you are considering a degree in Mathematics, Physics, Engineering or Computer Science as well as Economics at a highly competitive university.

Further Mathematics is becoming an increasingly recommended fourth subject at Russell Group universities.

We have two topics for you to familiarise yourself with here. We will study both of these subjects to a much greater depth. If you get stuck then Jack Brown has some excellent explanatory videos on YouTube which are linked below.

You may find the following Jack Brown links useful:

[Complex Numbers](#)

[Solving Quadratics with Complex Numbers](#)

[Dividing Complex Numbers](#)

[Introducing Matrices](#)

[Multiplying Matrices](#)

[Linear Transformations using Matrices](#)

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Hand in all work – self marked

The answers are given but you must show working out

During your first Further Maths lesson in September, the work completed from this booklet must be handed to Mr Richmond.

Any pupil who has not completed this work over the summer, having had the opportunity to do so, is unlikely to have the level of commitment required in order to succeed in A-level Further Maths.

We hope that you will use this introduction to give you a taste of what is a rewarding and enjoyable course.

Complex Numbers

1. Solve the equations:

(a) $x^2 + 9 = 0$

(b) $9x^2 + 25 = 0$

(c) $x^2 + 2x + 2 = 0$

(d) $x^2 + x + 1 = 0$

(e) $2x^2 + 3x + 2 = 0$

2. Find the quadratic equation which has roots $2 \pm \sqrt{3}i$.

3. Write the following complex numbers in the form $x + yi$.

(a) $(3 + 2i) + (2 + 4i)$

(b) $(4 + 3i) - (2 + 5i)$

(c) $(4 + 3i) + (4 - 3i)$

(d) $(2 + 7i) - (2 - 7i)$

(e) $(3 + 2i)(4 - 3i)$

(f) $(3 + 2i)^2$

(g) $(1 + i)(1 - i)(2 + i)$

4. Find the value of the real number y such that $(3 + 2i)(1 + iy)$ is (a) real (b) imaginary.

5. Simplify:

(a) i^3 (b) i^4 (c) $\frac{1}{i}$ (d) $\frac{1}{i^2}$ (e) $\frac{1}{i^3}$

6. If $z = 1 + 2i$, find

(a) z^2 (b) $\frac{1}{z}$ (c) $\frac{1}{z^2}$

7. Write in the form $x + yi$:

(a) $\frac{2+3i}{1+i}$

(b) $\frac{-4+3i}{-2-i}$

(c) $\frac{4i}{2-i}$

(d) $\frac{1}{2+3i}$

(e) $\frac{3-2i}{i}$

(f) $\frac{p+qi}{r+si}$

8. Simplify:

(a) $\frac{(2+i)(3-2i)}{1+i}$

(b) $\frac{(1-i)^3}{(2+i)^2}$

(c) $\frac{1}{3+i} - \frac{1}{3-i}$

9. Solve for z when

(a) $z(2+i) = 3-2i$ (b) $(z+i)(1-i) = 2+3i$

(c) $\frac{1}{z} + \frac{1}{2-i} = \frac{3}{1+i}$

10. Find the values of the real numbers x and y in each of the following:

(a) $\frac{x}{1+i} + \frac{y}{1-2i} = 1$

(b) $\frac{x}{2-i} + \frac{yi}{i+3} = \frac{2}{1+i}$

Complex Numbers – Answers

- | | | |
|-----|----|--|
| 1) | a) | $x = \pm 3i$ |
| | b) | $\pm \frac{5}{3}i$ |
| | c) | $-1 \pm i$ |
| | d) | $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ |
| | e) | $-\frac{3}{4} \pm \frac{\sqrt{7}}{4}i$ |
| 2) | | $x^2 - 4x + 7 = 0$ |
| 3) | a) | $5 + 6i$ |
| | b) | $2 - 2i$ |
| | c) | 8 |
| | d) | $14i$ |
| | e) | $18 - i$ |
| | f) | $5 + 12i$ |
| | g) | $4 + 2i$ |
| 4) | a) | $y = -\frac{2}{3}$ |
| | b) | $y = \frac{3}{2}$ |
| 5) | a) | $-i$ |
| | b) | 1 |
| | c) | $-i$ |
| | d) | -1 |
| | e) | i |
| 6) | a) | $-3 + 4i$ |
| | b) | $\frac{1}{5} - \frac{2}{5}i$ |
| | c) | $-\frac{3}{25} - \frac{4}{25}i$ |
| 7) | a) | $\frac{5}{2} + \frac{1}{2}i$ |
| | b) | $1 - 2i$ |
| | c) | $-\frac{4}{5} + \frac{8}{5}i$ |
| | d) | $\frac{2}{13} - \frac{3}{13}i$ |
| | e) | $-2 - 3i$ |
| | f) | $\frac{pr+qs}{r^2+s^2} + \frac{qr-ps}{r^2+s^2}i$ |
| 8) | a) | $\frac{7}{2} - \frac{9}{2}i$ |
| | b) | $-\frac{14}{25} + \frac{2}{25}i$ |
| | c) | $-\frac{1}{5}i$ |
| 9) | a) | $z = \frac{4}{5} - \frac{7}{5}i$ |
| | b) | $z = -\frac{1}{2} + \frac{3}{2}i$ |
| | c) | $z = \frac{11}{41} + \frac{17}{41}i$ |
| 10) | a) | $x = \frac{4}{3}, y = \frac{5}{3}$ |
| | b) | $x = 4, y = -6$ |

Matrix Manipulation

Simplify these. Write **undefined** if the expression isn't defined.

$$1) \begin{bmatrix} -5d & 0 \\ -4 & 2d \\ 3 & -d \end{bmatrix} \cdot \begin{bmatrix} 5 & -2d \\ -2h & -1 \end{bmatrix}$$

$$2) 3 \left\{ \begin{bmatrix} -3 & -5 \end{bmatrix} + \begin{bmatrix} 0 & 4 \end{bmatrix} \right\}$$

$$3) \begin{bmatrix} -2 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$4) \begin{bmatrix} -5g + 5 \\ -2g + 6 \\ 4 \end{bmatrix} - \begin{bmatrix} 5 \\ g + 2 \\ 6g \end{bmatrix} - \begin{bmatrix} 3 \\ -5g \\ g + 4 \end{bmatrix}$$

$$5) \begin{bmatrix} 0 & 6r \\ -3y & -5 \end{bmatrix} \cdot \begin{bmatrix} -5r & 2 \\ 4 & -2r \\ -1 & -3r \end{bmatrix}$$

$$6) 4 \begin{bmatrix} -4 & 4 & -5 \\ -6 & -3 & -6 \end{bmatrix} - 5 \begin{bmatrix} -6 \\ -3 \\ -6 \end{bmatrix}$$

Matrix Manipulation Answers

1)

$$\begin{bmatrix} -25d & 10d^2 \\ -20 - 4dh & 6d \\ 15 + 2dh & -5d \end{bmatrix}$$

2)

$$\begin{bmatrix} -9 & -3 \end{bmatrix}$$

3)

$$\begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

4)

$$\begin{bmatrix} -5g - 3 \\ 2g + 4 \\ -7g \end{bmatrix}$$

5) Undefined

6) Undefined

Matrix Transformations

Exercise 1

Question 1

$$\begin{pmatrix} -2 & a \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 22 \\ 9 \end{pmatrix}$$

Work out the value of a .

Question 2

a) Work out $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} 0 & b \\ a & c \end{pmatrix}$

Give your answer in terms of a, b, c .

b) If $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} 0 & b \\ a & c \end{pmatrix} = I$ where I is the identity matrix, work out the values of a, b, c .

Question 3

Matrix $P = \begin{pmatrix} 2 & 3 \\ a & b \end{pmatrix}$ Matrix $Q = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

You are given that $PQ = QP$

Work out the values of a and b .

Question 4

$$\begin{pmatrix} 2 & a \\ 1 & -3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Work out **all** possible pairs of values of a and b .

Exercise 2

1. [Jan 2013 Paper 2 Q15] Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
2. [Set 2 Paper 1 Q4] The transformation matrix $\begin{pmatrix} a & 2 \\ -1 & 1 \end{pmatrix}$ maps the point $(3,4)$ onto the point $(2, b)$. Work out the values of a and b .
3. [Set 3 Paper 1 Q6] The matrix $\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}$ maps the point $(5,4)$ onto the point $(1,17)$. Work out the values of a and b .
4. [Worksheet 2 Q5] Work out the image of the point $D(-1, 2)$ after transformation by the matrix $\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$
5. [Worksheet 2 Q6] The point $A(m, n)$ is transformed to the point $A'(-2, 0)$ by the matrix $\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$. Work out the values of m and n .
6. [Worksheet 2 Q8] Describe fully the transformation given by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Matrix Transformations – Answers

Exercise 1 - answers

1) $-6 + 7a = 22$

$$a = 4$$

2) a) $\begin{pmatrix} -a & 2b - c \\ 0 & \frac{1}{3}b \end{pmatrix}$

b) $a = -1, \quad b = 3, \quad c = 6$

3) $PQ = \begin{pmatrix} 2 & 5 \\ a & a + b \end{pmatrix}$

$$QP = \begin{pmatrix} 2 + a & 3 + b \\ a & b \end{pmatrix}$$

$$\therefore a = 0, b = 2$$

4) $2a + ab = -1$

$$a - 3b = 2$$

$$a = 2 + 3b$$

$$\therefore 2(2 + 3b) + (2 + 3b)b = -1$$

$$3b^2 + 8b + 5 = 0$$

$$(3b + 5)(b + 1) = 0$$

$$b = -1 \text{ or } b = -\frac{5}{3}$$

$$a = -1 \text{ or } a = -3$$

Exercise 2 - answers

1) Rotation 90° anticlockwise about the origin

2) $a = -2, \quad b = 1$

3) $a = -1, \quad b = 1.5$

4) Solution: $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

5) $m = 2, \quad n = -2$

6) Reflection in the line $y = -x$