Algebra 2
High School

Unit #1
Sequences and Functions

Essential Question
How can sequences be used to represent and model patterns of change, and how do they connect to linear and exponential functions?

Unit Summary
In this unit, students revisit functions through the study of sequences, recognizing arithmetic and geometric sequences as linear and exponential functions with domains restricted to the integers. Students describe, represent, and analyze sequences using tables, graphs, expressions, and function notation, and develop recursive and explicit formulas to model relationships. They apply sequences to real-world situations, select appropriate models and domains, and explore sums of finite sequences, building connections to future work with series.

Guiding Questions
Content

- What distinguishes an arithmetic sequence from a geometric sequence, and how can each be identified from terms, tables, or graphs?
- How do sequences function as mathematical rules that assign a term to each position, and how can they be represented in multiple ways?
- How can sequences be used to model situations and determine meaningful quantities such as missing terms or the sum of terms?

Process
- How can technology, such as spreadsheets and graphing tools, be used to generate, visualize, and analyze sequences efficiently?
- How can recursive definitions and explicit equations be written and interpreted using function notation?

- How can questions about a situation help determine the information needed to represent a sequence accurately in different forms?

Reflective

- How can I give an example of a sequence that models a pattern or situation from my own life?
- How can I use sequences to predict future values or totals in real-world situations?
- How has representing the same sequence in multiple ways helped me better understand patterns and change?

Power Standards

- **F.LQE.2. (11)** Construct exponential functions, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Supporting Standards

- **F.IF.3. (9/10/11)** Recognize patterns in order to write functions whose domain is a subset of the integers. **(9/10)** Limited to linear and quadratic. *For example, find the function given* {(-1,4), (0,7), (1,10), 2,13)}.
- **F.IF.5. (all)** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- **F.BF.1b. (11)** Determine an explicit expression , a recursive function, or steps for calculation from a context.
- **F.BF.2. (+)** Write arithmetic and geometric sequences and series both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

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Unit #2
Polynomials

Essential Question
How do different representations of polynomial functions help us understand, predict, and explain the key features and behavior of their graphs?

Unit Summary
This unit extends students' understanding of linear and quadratic functions to polynomial functions of higher degree, emphasizing how different forms of polynomials reveal key features of their graphs. Students analyze zeros, intercepts, multiplicity, degree, and leading coefficient to understand end behavior and sketch polynomial graphs written in factored form. The unit concludes with polynomial division and the Remainder Theorem, connecting zeros of a polynomial to its linear factors and preparing students for future work with rational functions.

Guiding Questions
Content
- How do polynomial expressions and functions represent real-world situations, such as volume, area, or changing quantities?
- How do features of a polynomial—such as zeros, multiplicities, degree, and leading term—determine the shape and behavior of its graph?
- Why do operations on polynomials (addition, subtraction, multiplication, and division) result in new polynomials, and how does this help analyze functions?

Process
- How can standard form, factored form, and division be used strategically to identify zeros, intercepts, and end behavior of a polynomial?
- How can long division and the Remainder Theorem be used to rewrite polynomials and determine points of intersection or factors?
- How can graphs, equations, and algebraic reasoning be connected to sketch polynomial functions and verify key features?

Reflective

- How can I create and interpret an expression that models the volume of a box or another real-world situation?
- How has understanding polynomial behavior helped me predict how quantities change or interact?
- How can I use polynomial models to make sense of complex situations that involve multiple factors or constraints?

Power Standards

- **A.SSE.3.** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **F.IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.8.** Write a function in different but equivalent forms to reveal and explain different properties of the function.

Supporting Standards

- **A.SSE.1. (all)** Interpret expressions that represent a quantity in terms of its context.
- **A.SSE.1a. (all)** Interpret parts of an expression, such as terms, factors, and coefficients.
- **A.APR.1. (9/10)** Add, subtract, and multiply polynomials.
- **A.APR.3. (11)** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number c, the remainder on division by $(x - c)$ is $p(c)$, so $p(c) = 0$ if and only if $(x - c)$ is a factor of $p(x)$.
- **A.CED.2. (all)** Apply and extend previous understanding to create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **A.REI.9. (9/10/11)** Solve an equation $f(x) = g(x)$ by graphing $y = f(x)$ and $y = g(x)$ *and finding the x-value of the intersection point. Include cases where* $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. For **(9/10)** focus on linear, quadratic, and absolute value.
- **F.IF.2. (all)** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.4. (all)** For a function that models a relationship between two quantities, interpret

key features of expressions, graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

- **F.IF.5. (all)** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- **F.IF.7e. (11)** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **F.IF.9. (all)** Compare properties of two functions using a variety of representations (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, a quantity increasing exponentially eventually exceeds a quantity increasing linearly.*

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Unit #3
Rational Functions and Equations

Essential Question
How can rational expressions and functions be used to model situations, analyze behavior, and determine when algebraic reasoning produces valid or invalid solutions?

Unit Summary
In this unit, students extend their work with polynomials to rational functions, rational equations, and identities. They model real-world situations with rational functions, analyze asymptotic and end behavior using structure and polynomial division, and solve rational equations while identifying and explaining extraneous solutions. Students also prove and use algebraic identities.

Guiding Questions
Content
- How do the structure of a rational function and its equation determine vertical asymptotes, horizontal asymptotes, and end behavior?
- How do rational expressions and equations model real-world situations involving averages, rates, or constraints?
- What distinguishes an identity from an equation?

Process
- How can rational functions be rewritten using polynomial division to analyze end behavior and asymptotes?
- How can rational equations be solved carefully to identify valid solutions and eliminate extraneous solutions?
- How can algebraic structure and identities be used to justify equivalence and derive formulas?

Reflective

- How can I write a rational function to model different properties of cylinders or other real-world objects?
- How has learning to check for extraneous solutions changed the way I verify my answers in algebra?
- How can I use identities and formulas to solve practical problems efficiently?

Power Standards

- **A.REI.3.** Solve equations in one variable and give examples showing how extraneous solutions may arise.
- **F.IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Supporting Standards

- **A.SSE.1. (all)** Interpret expressions that represent a quantity in terms of its context.
- **A.SSE.2. (all)** Use the structure of an expression to identify ways to rewrite it.
- **A.APR.4. (9/10/11)** Generate polynomial identities from a pattern. *For example, difference of squares, perfect square trinomials, (emphasize sum and difference of cubes in grade 11).*
- **A.APR.6. (+)** Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) = \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **A.CED.1. (all)** Apply and extend previous understanding to create equations and inequalities in one variable and use them to solve problems.
- **A.CED.2. (all)** Apply and extend previous understanding to create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **A.CED.4. (all)** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
- **A.REI.1. (all)** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution

method.

- **A.REI.2. (all)** Apply and extend previous understanding to solve equations, inequalities, and compound inequalities in one variable, including literal equations and inequalities.
- **A.REI.9. (9/10/11)** Solve an equation $f(x) = g(x)$ by graphing $y = f(x)$ and $y = g(x)$ *and finding the x-value of the intersection point. Include cases where* $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. For **(9/10)** focus on linear, quadratic, and absolute value.
- **F.IF.2. (all)** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.4. (all)** For a function that models a relationship between two quantities, interpret key features of expressions, graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

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Unit #4
Complex Numbers and Rational Exponents

Essential Question
How do extending exponent rules and introducing complex numbers expand the kinds of equations we can solve and the situations we can model?

Unit Summary
In this unit, students extend exponent rules to include rational exponents and use radicals to solve equations involving squares and cube roots. They develop an understanding of imaginary and complex numbers, including how to represent, add, and multiply them, and apply this knowledge to solve quadratic equations with complex solutions. Throughout the unit, students connect algebraic reasoning to graphical and real-world contexts.

Guiding Questions
Content

- How are integer, fractional, and negative exponents connected to roots, radicals, and powers?
- Why do some equations have solutions that are not real numbers, and how are complex numbers used to represent those solutions?
- How do quadratic equations connect to complex numbers when real solutions do not exist?

Process

- How can rewriting roots as exponents and exponents as radicals help simplify expressions and solve equations?
- How can operations with complex numbers be used to solve equations and verify results?
- How can completing the square or using the quadratic formula be used strategically to find real and complex solutions?

Reflective

- How can I evaluate expressions with integer, fractional, and negative exponents to make sense of mathematical models?
- How can I use complex numbers to solve equations that do not have real solutions?
- How has learning multiple methods for solving quadratic equations helped me choose efficient strategies and check my work?

Power Standards

- **A.SSE.3.** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **A.REI.3.** Solve equations in one variable and give examples showing how extraneous solutions may arise.
- **A.REI.4. (11)** Solve radical and rational exponent equations and inequalities in one variable, and give examples showing how extraneous solutions may arise.
- **A.REI.5.** Solve quadratic equations and inequalities.
- **F.IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.8.** Write a function in different but equivalent forms to reveal and explain different properties of the function.

Supporting Standards

- **N.RN.2. (11)** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* 5^1/3 *to be the cube root of 5 because we want* (5^1/3)^3 = 5^(1/3 · 3) *to hold, so* (5^1/3)^3 *must equal 5.*
- **N.RN.3. (11)** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- **N.CN.1. (11)** Know there is a complex number *i* such that *i*^2 = -1 , and every complex number has the form *a* + *bi* with *a* and *b* real.
- **N.CN.2. (11)** Use the relation *i*^2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- **N.CN.8. (11)** Solve quadratic equations with real coefficients that have complex solutions.

- **A.CED.4. (all)** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*
- **A.REI.1. (all)** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **A.REI.2. (all)** Apply and extend previous understanding to solve equations, inequalities, and compound inequalities in one variable, including literal equations and inequalities.
- **A.REI.5a. (9/10)** Solve quadratic equations by inspection *(e.g. for $x^2 = 49$),* taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives no real solutions.
- **A.REI.5c. (11)** Use the method of completing the square to transform and solve any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.
- **A.REI.9. (9/10/11)** Solve an equation $f(x) = g(x)$ by graphing $y = f(x)$ and $y = g(x)$ *and finding the x-value of the intersection point. Include cases where* $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. For **(9/10)** focus on linear, quadratic, and absolute value.
- **F.IF.7b. (11)** Graph square root, cube root, and exponential functions.

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Unit #5
Exponential Functions and Equations

Essential Question
How do exponential and logarithmic functions help us model growth and decay, and how do logarithms allow us to solve problems involving unknown exponents?

Unit Summary
In this unit, students connect geometric sequences to exponential functions and model growth and decay using functions of the form $f(x) = ab^x$. They extend their understanding to include rational and negative exponents, explore the constant *e*, and compare different exponential models across varying time intervals. Students then develop an understanding of logarithms as inverses of exponential functions, apply logarithm rules, and solve exponential and logarithmic equations graphically and algebraically while interpreting solutions in context.

Guiding Questions

Content
- What does it mean for an exponential function to change by a constant factor over equal intervals, including intervals that are not whole numbers?
- How are exponential functions and logarithmic functions related, and how does each help represent unknown exponents?
- Why is the constant *e* used in some exponential models, and how does continuous growth differ from growth applied at fixed intervals?

Process
- How can equations for exponential functions be written using two input–output pairs that are not one unit apart?
- How can logarithm rules, the change of base rule, and technology be used to evaluate logarithms and solve exponential equations?
- How can graphs and algebraic methods be used together to find where two exponential functions intersect and interpret that intersection?

Reflective

- How do I understand how to calculate values that are changing exponentially in real-world situations such as population growth or radioactive decay?
- How can I use logarithms to solve problems involving unknown exponents and make sense of results in context?
- How has learning about exponential and logarithmic models helped me interpret data and predict long-term behavior in everyday situations?

Power Standards

- **A.SSE.3.** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **F.IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.8.** Write a function in different but equivalent forms to reveal and explain different properties of the function.
- **F.LQE.2. (11)** Construct exponential functions, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Supporting Standards

- **N.RN.2. (11)** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 5^1/3 to be the cube root of 5 because we want (5^1/3)^3 = 5^(1/3 · 3) to hold, so (5^1/3)^3 must equal 5.*
- **A.SSE.1. (all)** Interpret expressions that represent a quantity in terms of its context.
- **A.SSE.1b. (all)** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1 + 2)^n as the product of P and (1 + r)^n .*
- **A.SSE.3c. (11)** Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as (1.15^1/12)^12t ≈1.012^12t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
- **A.REI.9. (9/10/11)** Solve an equation *f(x) = g(x)* by graphing *y = f(x)* and *y = g(x) and finding the x-value of the intersection point. Include cases where f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

For **(9/10)** focus on linear, quadratic, and absolute value.

- **F.IF.2. (all)** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.4. (all)** For a function that models a relationship between two quantities, interpret key features of expressions, graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- **F.IF.7b. (11)** Graph square root, cube root, and exponential functions.
- **F.IF.7c. (11)** Graph logarithmic functions, emphasizing the inverse relationship with exponentials and showing intercepts and end behavior.
- **F.IF.8c. (11)** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)^t, y = (0.97)^t, y = (1.01)^12t, y = (1.2)^t/10,* and classify them as representing exponential growth or decay.
- **F.BF.1b. (11)** Determine an explicit expression , a recursive function, or steps for calculation from a context.
- **F.BF.4.** Find inverse functions.
- **F.BF.4a. (11)** Write an expression for the inverse of a function**.**
- **F.LQE.1a. (11)** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **F.LQE.1b. (11)** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **F.LQE.1c. (11)** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

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Unit #6
Transformations of Functions

Essential Question
How can transforming functions help us model, analyze, and adapt mathematical relationships to better represent real-world situations?

Unit Summary
In this unit, students develop a unified understanding of function transformations and how they can be used to model real-world situations. They analyze and describe the effects of translations, reflections, and scaling on graphs and equations of various function types, including identifying functions as even, odd, or neither. Students apply these transformation concepts across multiple representations to adapt functions to fit real data, strengthening their ability to model with mathematics.

Guiding Questions

Content
- How do translations, reflections, and scale factors change the appearance and behavior of a function's graph?
- How are equations and graphs connected when describing horizontal and vertical transformations of functions?
- What characteristics identify even and odd functions, and how do symmetry and equations reveal these properties?

Process
- How can function notation and equations be used to represent and predict transformations of a given graph?
- How can scaling the input of a function be distinguished from scaling the output, and how does each affect the graph differently?
- How can multiple transformations be combined and applied to create a function that models a given data set?

Reflective

- How can I describe how a graph is transformed using precise mathematical language?
- How can I use transformations to adjust a function so it better fits real-world data?
- How has understanding transformations helped me see connections among different types of functions and their graphs?

Power Standards

- **F.IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.8.** Write a function in different but equivalent forms to reveal and explain different properties of the function.
- **F.BF.1.** Use functions to model real-world relationships.
- **F.LQE.2. (11)** Construct exponential functions, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Supporting Standards

- **F.IF.2. (all)** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.4. (all)** For a function that models a relationship between two quantities, interpret key features of expressions, graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- **F.IF.7a. (9/10)** Graph linear, quadratic and absolute value functions and show intercepts, maxima, minima and end behavior.
- **F.IF.7e. (11)** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **F.IF.8b. (11)** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- **F.IF.9. (all)** Compare properties of two functions using a variety of representations (algebraically, graphically, numerically in tables, or by verbal descriptions). *For*

example, a quantity increasing exponentially eventually exceeds a quantity increasing linearly.

- **F.BF.1.** Use functions to model real-world relationships.
- **F.BF.1a. (9/10)** Combine multiple functions to model complex relationships. *For example, p(x) = r(x) - c(x); (profit = revenue - cost).*
- **F.BF.3. (9/10/11)** Transform parent functions $(f(x))$ by replacing $f(x)$ with $f(x) + k,\ kf(x),\ f(kx),\ and\ f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. For **(9/10)** focus on linear, quadratic, and absolute value functions.
- **G.GPE.2. (+)** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; graph the circle in the coordinate plane;
- **S.ID.5b. (9/10)** Fit a linear function to data and use it to solve problems in the context of the data.