Algebra 2/Trig Summer Packet

The following questions are to be completed NEATLY on a separate sheet of paper. Any questions with a "V" in front of it means there is a video solution posted on the MathCamp321 YouTube channel under the ALGEBRA 2 playlist.

Solving Equations with Fractional Coefficients

Solve each **equation** with fractional coefficients. Video Link

1V. $\frac{5}{2}x+\frac{2}{3}=\frac{1}{6}(x-3)$

2V. $\frac{2}{3}-\frac{4}{5}x=\frac{3}{2}x-\frac{3}{5}$

3. What is the LCD of the three fractions? $\frac{?}{4},\frac{?}{5},\frac{?}{3}$

4. Solve: $\frac{3}{4}+\frac{2}{5}x=\frac{1}{3}(x-2)$

5. Solve: $\frac{3}{2}(6x-4)=\frac{13}{2}x-6+\frac{5x}{2}$

6. Solve: $\frac{5}{3}-\frac{1}{4}x=\frac{5}{6}x-\frac{3}{4}$

Solving Literal Equations

1V. Solve the following literal equation for z using a 2-column proof format. Video Link

Statements	Reasons
1. $w=\frac{x+y+z}{z+a+b}$	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

For questions 2-5, solve the **literal equation** for the indicated variable. Include any restrictions.

2. Solve for b_1: $A = \frac{1}{2}h(b_1 + b_2)$

3. Solve for r: $V = \frac{4}{3}\pi r^3$

4. Solve for t: $P = \sqrt{\frac{5}{t}}$

5. Solve for w: $z = \frac{2x - w}{3x + w}$

Solving Absolute Value Equations

Solve each **absolute value equation** showing work as per the video. Video Link

1V. $2|x - 3| + 1 = 15$

2V. $|x + 5| + 12 = 10$

3V. $|2x - 4| = x - 8$

4. Solve: $|x + 7| = 3$

5. Solve: $-|5x| = 10$

6. The equation, $|x - 2| = 2x - 1$, yields the solutions $x = -1$ and $x = 1$. Show the check for each of these solutions and decide which, if any, is the solution to the equation.

7. What is an **extraneous solution**?

For questions 8 and 9, solve, clearly showing the two cases.

8. $-2|8 - x| = -6$

9. $|x + 3| = 3 - x$

10. Find the solutions to the absolute value equation: $-3|x - 8| = 6x - 12$
Identify any **extraneous** solutions.

Interval Notation

Rules: Use (,) for ∞, -∞ and non-included boundaries. Use [,] for included boundaries.

1V. Interval Notation Practice – Fill out this worksheet as you watch the corresponding video on MathCamp321. Video Link

	Algebraic inequality	*Number line*	*Interval notation*	*Set-builder notation*
25V.	$x < 5$			
26V.	$x \geq -3$			
27V.	$-4 < x \leq 4$			
28V.	$x < -2$ or $x > 3$			
29V.				$\{x \mid 0 \leq x < 10\}$
30V.			$(-6, \infty)$	
31V.			$(-\infty, 4) \cup [5, \infty)$	
32V.				

2. **Multiple choice**: Give the interval over which the inequality is true: $10 < -x$

A. $(-\infty, -10)$ B. $(-10, \infty)$ C. $(-\infty, 10)$ D. $(10, \infty)$

3. **Application**: Use interval notation to describe the solution set for the range in temperature in a certain city on July 4th: $71° \leq t \leq 94°$

4. Use interval notation to describe the solution set: $x < -4$ or $x > 20$

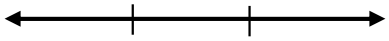
5. **Multiple choice**: Which grouping symbol will always be adjacent to the infinity symbol?

A. brace {,} B. bracket [,] C. parenthesis (,)

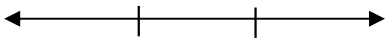
Solving Compound Inequalities

Using interval notation, find the solution set to each compound inequality. Video Link

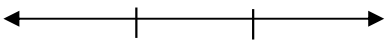
1V. $x < 3$ and $x \geq 0$



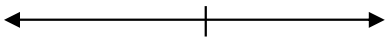
2V. $x \leq -1$ or $x > 1$



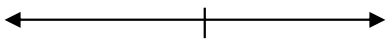
3V. $4 \leq x \leq 6$



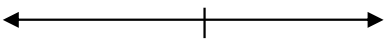
4V. $5 > x$



5V. $x < 0$ or $x > 0$



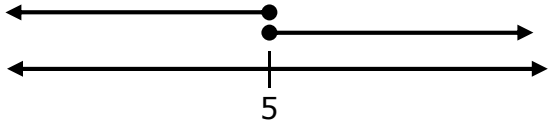
6V. $x < 0$ and $x > 0$



7. Classify the following compound inequality as an "and" or "or" problem: $-10 \leq x \leq 10$

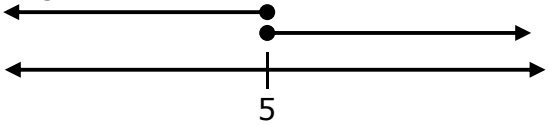
8. For which classification "and" or "or" do you look for a presence of shading?

9. Does the shading on the number line and the solution suggest an "and" or "or" problem?



solution set: $(-\infty, \infty)$

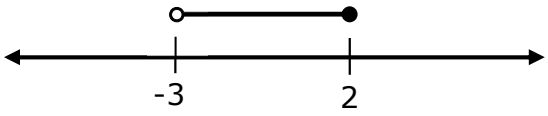
10. Does the shading on the number line and the solution suggest an "and" or "or" problem?



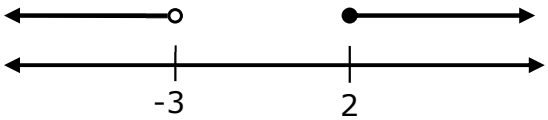
solution set: 5 only

Match the solution set shown on the number line with the corresponding solution set in interval notation:

_____11.



_____12.



A. $[-3, 2)$

B. $(-\infty, -3) \cup [2, \infty)$

C. $(-3, 2]$

D. $(-\infty, -3] \cup (2, \infty)$

For questions 13-16, find the interval over which the compound inequality is true. Show the word "**and**" or "**or**" in addition to your shaded number line *before* you state the solution interval.

13. $2 - x < 3$ and $x + 7 < 10$

14. $5 \le x - 1 < 8$

15. $6 - 2x < 20$ or $x > 1$

16. $-x + 4 \le -3 \le 3x$

Solving Absolute Value Inequalities

Solve each absolute value inequality. Clearly show the 2 cases. Include the word "and"/ "or".
Video Link

1V. $|x + 1| < 5$

2V. $2|x + 1| - 3 \ge 9$

3V. $|x + 1| \le 2x - 3$

For questions 4 and 5, classify the absolute value inequality as "and" or "or". Do not actually solve.

4. $|x - 3| < 5$

5. $-4|x + 3| < -20$

6. **True or false**: writing the word "and" or "or" in your solution to an absolute value inequality is *optional*.

_____7. **Multiple choice**: When rewriting two new inequalities from an absolute value inequality, the second inequality should:

A. have the same inequality symbol, but negated right-hand side

B. have the opposite inequality symbol, and negated right-hand side

C. have the same inequality symbol, and same right-hand side

D. have the opposite inequality symbol, but same right-hand side

For questions 8-11, match the solution set shown on the number line with the corresponding compound inequality:

8. $|x| > 2$

9. $|x| < 2$

10. $|x| \ge 0$

11. $|x| < 0$

A. $(-2, 2)$

B. $(-\infty, -2) \cup (2, \infty)$

C. no solution

D. all real numbers

E. 0 only

For questions 12-15, find the interval over which the absolute value inequality is true. Show the word "and" or "or" in addition to your shaded number line before you state the solution interval.

12. $|x-1| \geq 2$

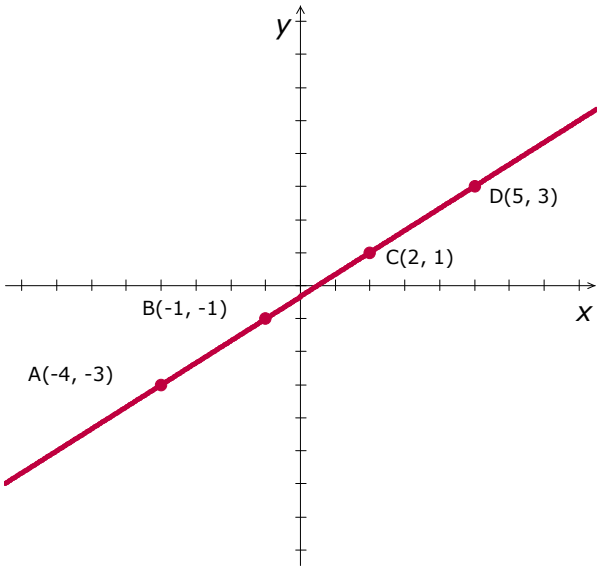
13. $3|x-2|-3 < 9$

14. $|6-x| < 2$

15. $|x| > 2x+1$

Working With Point-Slope Form

1V. Show the calculation to find the slope of $\overleftrightarrow{CD}$.



2V. Write the **point-slope** form using point **A**.

3V. Write the **point-slope** form using point **B**.

4V. Write the **point-slope** form using point **D**.

Working with General Form

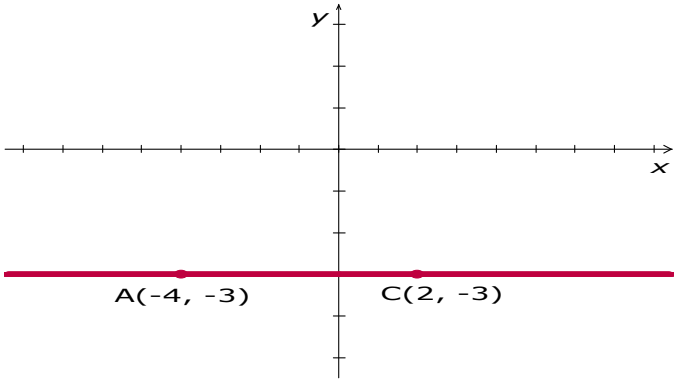
1V. Write the **general** form of the line passing through the points (5, 2) and (1, 12).

2V. Write the **general** form of the line passing through the points (-6, 1) and (-14, -5).

3. **True or false**: the template $AX + BY = C$ illustrates **general** form.

4. Manipulate $\frac{-2}{3}x + \frac{5}{2}y = \frac{7}{6}$ into **general** form.

5. Write the equation of the line shown in **general** form.



6. Write the **general** form of the line passing through the points (-2, -10) and (6, -26).

7. Johnny got a deduction on his quiz for writing the answer to a general form equation as, $6x - 4y + 18 = 0$. Why do you think he got a deduction? What would be a better answer?

Writing Equations of Lines in Various Forms

Slope-Intercept Form: $y = mx + b$
Standard Form: $ax + by = c$, where *a*, *b*, and *c* are integers and $a \geq 0$.
General Form: $ax + by + c = 0$, where *a*, *b*, and *c* are integers and $a \geq 0$.
Point-Slope Form: $y - y_1 = m(x - x_1)$
Equation of a Vertical Line: $x = c$, where *c* is any real number.
Equation of a Horizontal Line: $y = c$, where *c* is any real number.

Neatly show all work. Use the indicated form. Box in your final answers.

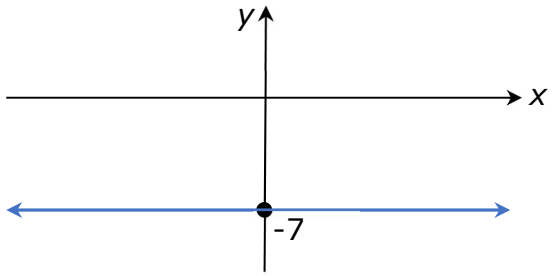
1. Write the following in **slope-intercept form**: $2x - 5y = 8$

2. Transform the following into **standard form**: $y = \frac{5}{4}x - \frac{3}{2}$

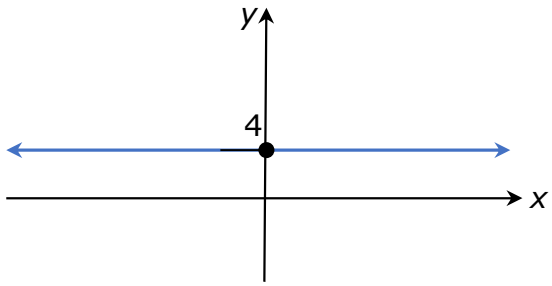
3. Manipulate the following into **general form**: $-3x + 4y = \frac{1}{2}$

4. Write the line with the following conditions into **point-slope form**: passes through $\left(-7, \frac{1}{2}\right)$ and has a slope of 9.

5. Write the equation of the line shown in **standard form**:



6. In **general form**, write the equation of the line that is <u>perpendicular</u> to the line shown and passes through the point (-5, 8).



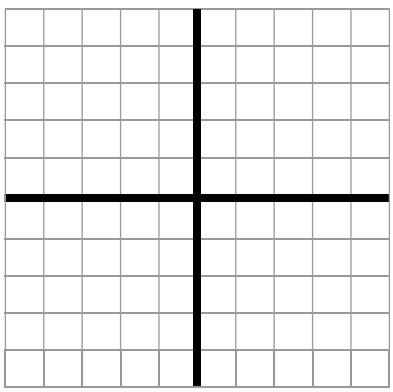
7. In **general form**, write the equation of the line that passes through the points (-5, -1) and (4, 5).

8. Write the equation of the line whose *y*-intercept is -6 and whose *x*-intercept is 2 in **standard form**.

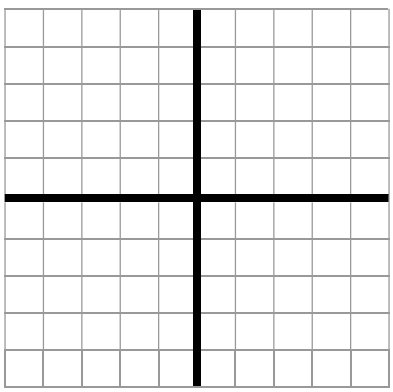
9. If line *a*||*b* and line *a* has the equation $5x - 6y - 10 = 0$, determine the equation of line *b,* in **point-slope form**, if *b* passes though (1, -2).

Graphing Inequalities

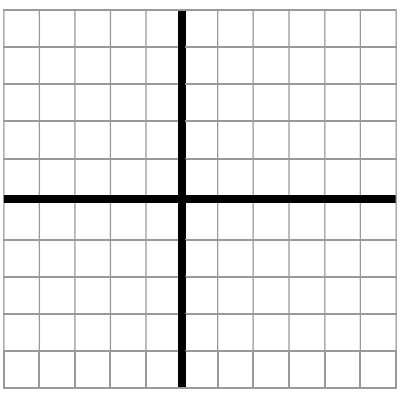
1. Sketch: $x - 2y < 4$



2. Sketch: $x < 4$



3. Sketch: $y < x + 1$



Domain and Range from a Graph

1. When determining the <u>domain</u> by analyzing a sketch, you must scan the sketch from

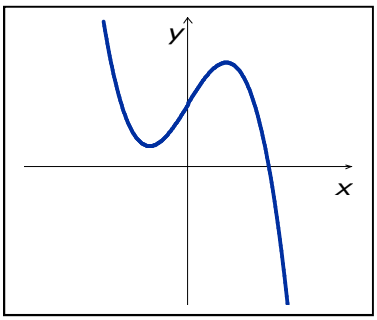
_____ to _____ to see where the _____-values exist.

2. When determining the <u>range</u> by analyzing a sketch, you must scan the sketch from

_____ to _____ to see where the _____-values exist.

Using interval notation, give the **domain** and **range** by analyzing the sketch.

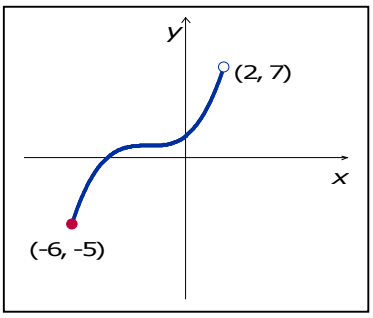
3.



D:_____

R:_____

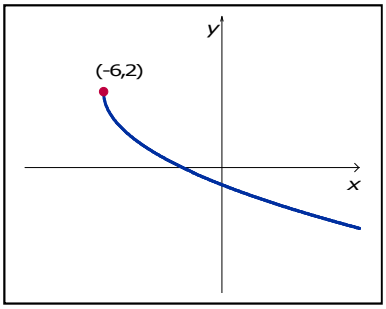
4.



D:_____

R:_____

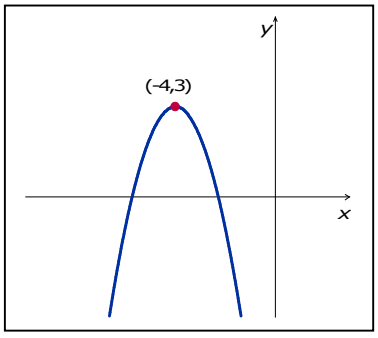
5.



D:_____

R:_____

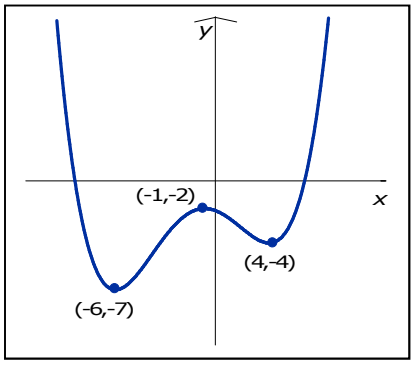
6.



D:_____

R:_____

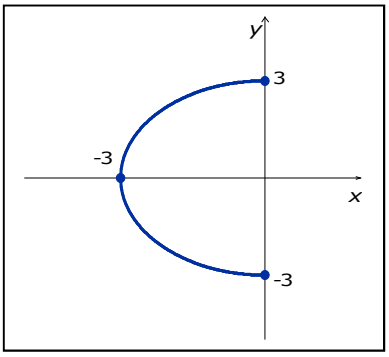
7.



D:_____

R:_____

8.



D:_____

R:_____

2x2 Systems of Equations

I. Solve each system by **<u>elimination</u>**. Express your solution as an ordered pair [i.e. (-3, 6)]

1. $4x - 5y = 17$
   $3x + 4y = 5$

2. $2x + 3y = 9$
   $3x - 2y = -19$

II. Solve each system by **substitution**. Express your solution as an ordered pair.

3. $\begin{aligned} 2x + y &= 11 \\ 6x - 2y &= -2 \end{aligned}$

4. $\begin{aligned} 6x + 3y &= 12 \\ 2x &= 8 - y \end{aligned}$

Monomials – Rules of Exponents

RULES OF EXPONENTS

$x^0 = 1$	Anything raised to the power of zero equals 1. ($x \neq 0$)
$x^a \cdot x^b = x^{a+b}$	When multiplying powers of the *same* base, retain the base and **add** the exponents.
$\frac{x^a}{x^b} = x^{a-b}$	When dividing powers of the *same* base, retain the base **subtract** the exponents.
$\left(x^a\right)^b = x^{a \cdot b}$	When raising a power to a new power, **multiply** the exponents.
$(xy)^a = x^a \cdot y^a$	When several factors are raised to a power, each factor will feel the effect of the exponent.
$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$	When a fraction is raised to a power, both the numerator and denominator will feel the effect of the exponent.
$x^{-a} = \frac{1}{x^a}; \; x^a = \frac{1}{x^{-a}}$	When a term is raised to a negative exponent, the term is sent to the denominator, and the exponent changes sign (from negative to positive) and vice-versa.
$\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$	When a fraction is raised to a negative exponent, the fraction flips and the exponent changes sign.

For #s 1-10, simplify on your own. Check your answers by watching the video link on MATHCamp321.

1V. **[video 1]** $\left(-2a^3b\right)\left(-5ab^4\right)$

2V. **[video 1]** $\frac{s^2}{s^{10}}$

3V. **[video 1]** $\frac{10^{(x-3)(x+3)}}{10^{x^2-6}}$

4V. **[video 1]** $\left(b^2\right)^3$

5V. **[video 2]** $\left(-3c^2d^5\right)^3$

6V. **[video 2]** $\left(\frac{x}{3}\right)^{-4}$

7V. **[video 2]** $\left(\frac{-3a^{5y}}{a^{6y}b^4}\right)^5$

8V. **[video 2]** $\left(\frac{-4a^{-3}b^6}{7c^{-3}a^4}\right)^{-2}$

9V. **[video 2]** $\frac{x^0 + 20y^0 - 36}{-2\left(x^0y^0\right)}$

10V. $\left(5a^4\right)\left(-4a^6\right)\left(-a^3\right)^2 - \left(-2a\right)^4\left(-a^2\right)^6$

11. Express in **scientific notation**:
    a. 4,560,000
    b. 0.000092

12. Evaluate using **scientific notation**:
    a. $(5\times10^{3})(7\times10^{8})$
    b. $(1.8\times10^{-4})(4\times10^{7})$

Operations on Polynomials

Simplify. Video Link

1V. $(3x^2+4)-(5x-2)$

3V. $-(y^2+2y-3)+(5y^2+3y+4)$

5V. $2x(x^2-x+3)$

7V. $(2-x-3x^2)(5x)$

9V. $(2x+3)(4x+1)$

11V. $(3x-y)(3x+y)$

13V. $(3-5x)^2$

17V. $(2u-v)^3$

FACTORING POLYNOMIALS

Factoring Methods
① GCF – ★*greatest common factor*
② DOTS – *difference of two squares*
③ SOC/DOC – *sum of cubes / difference of cubes*
④ FAST – *factoring a simple trinomial - trinomial factoring w/ leading coefficient = 1*
⑤ Nobes' – *trinomial factoring w/ leading coefficient ≠1*
⑥ PST – *perfect square trinomial*
⑦ *grouping*

Factor completely. Check your answers by watching the video link on MATHCamp321.

Video Link 1

1. $25a^6b^5c^7-35a^4b^5c^3+15a^5b^6c^4$

3V. $4a^2-81b^2$

9V. $x^2-9x-36$

11V. $z^2+25z+100$

13V. $2x^2+x-6$

15V. $12y^2-13y+3$

17V. $x^2-8x+16$

21V. xy^2+x-2y^2-2

Video Link 2

2V. $a^2(x-2y)-b(x-2y)-(x-2y)$

4. $16x^4-25y^2$

10. $y^2+3y-28$

12. $a^2-5ab+6b^2$

14. $10x^2+21x-10$

16. $12x^2+23xy+10y^2$

18. $y^2+10y+25$

22. $a^3+3a^2b-2ab-6b^2$

Simplest Radical Form

Product Property of Square Roots
For any nonnegative real numbers *a* and *b*: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

Procedure:
1. To find simplest radical form, start by making an organized list of factor pairs.
2. Pick the pair that contains the **LARGEST** perfect square.
3. Write this pair under the radical with the perfect square written first.
4. Take the square root of each factor.

$\sqrt{24}$

①	24
2	12
3	8
④	6

$$\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$$

Find each square root. Use simplest radical form for non-perfect squares.

9. $\sqrt{75}$
10. $\sqrt{32}$
11. $\sqrt{48}$
12. $\sqrt{72}$
13. $6\sqrt{45}$
14. $5\sqrt{28}$

Multiplying and Dividing Radicals

Rule: When multiplying radicals, multiply the numbers on the **OUTSIDE** together, then multiply the numbers on the **INSIDE** of the radical together and simplify if possible.

Multiply.

19. $3\sqrt{2} \cdot 5\sqrt{2}$
20. $-\sqrt{3} \cdot 2\sqrt{3}$
21. $\sqrt{2} \cdot \sqrt{50}$
22. $2\sqrt{5} \cdot 6\sqrt{8}$
23. $\left(2\sqrt{3}\right)^2$

When is a radical expression simplified?
1) no number under the radical can be simplified
2) no fractions are under a radical sign, and
3) no radicals are in a denominator.

The process by which we eliminate the radical in the denominator is called **rationalizing the denominator**.

Simplify, leaving no radical in the denominator. Rationalize the denominator.

24. $\frac{4}{\sqrt{6}}$

25. $\frac{1}{\sqrt{3}}$

26. $\frac{2}{\sqrt{10}}$

27. $\frac{5}{\sqrt{5}}$

28. $\sqrt{\frac{5}{12}}$

29. $\sqrt{\frac{3}{8}}$

Combining Radicals

Rule: Make sure each radical is in ***simplest form*** and only combine those radicals whose radicands are '***like***.'

Combine.

30. $20\sqrt{2} - 5\sqrt{2} - 3\sqrt{2}$

31. $\sqrt{16} + 2\sqrt{25}$

32. $\sqrt{28} + \sqrt{63}$

33. $\sqrt{45} + \sqrt{80}$

Solving Basic Quadratic Equations

Rule: For any real numbers a and b:
if $a^2 = b^2$, then $a = \pm b$

Solve (be sure to isolate the squared term first):

15. $x^2 = 16$

16. $3a^2 = 75$

17. $2c^2 - 1 = 23$

18. $d^2 - 121 = 0$

Graphing Quadratic Functions

Defn.: A **quadratic function** has the form $f(x) = ax^2 + bx + c$.

ax^2 is called the **quadratic term**, bx is called the **linear term** and c is the **constant**.

- The graph of a quadratic function is called a **parabola**.
- The x-coordinate of the vertex can be found using the formula: $\frac{-b}{2a}$.

1V. Determine a, b, and c for $f(x) = x^2 + 3x - 1$. Video Link

Then determine the vertex and create a table of 5 values.
Find the y-intercept and identify the axis of symmetry. Sketch.

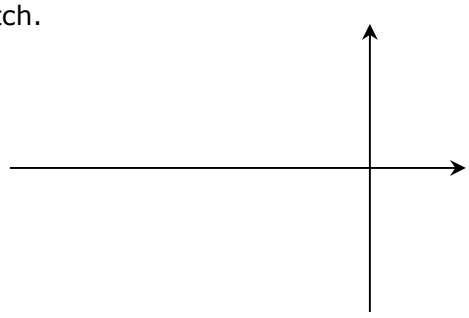
$a =$

$b =$

$c =$

x-coordinate of vertex

x	y



2. Determine a, b, and c for $f(x) = 2 - 4x + x^2$.

Then determine the vertex and create a table of 5 values.
Find the y-intercept and identify the axis of symmetry. Sketch.

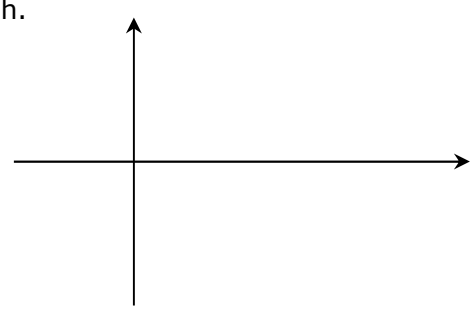
$a =$

$b =$

$c =$

x-coordinate of vertex

x	y



3. Determine a, b, and c for $f(x) = -x^2 - 4x - 1$.

Then determine the vertex and create a table of 5 values.
Find the y-intercept and identify the axis of symmetry. Sketch.

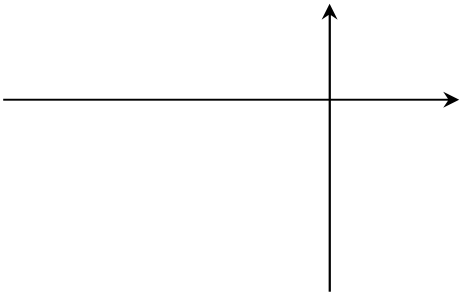
$a =$

$b =$

$c =$

x-coordinate of vertex

x	y



Solving Quadratic Equations by Factoring

Zero Product Property: for any real numbers *a* and *b*, if *ab* = 0, then either *a* = 0, *b* = 0, or both *a* and *b* equal 0.

Use **factoring** to solve to solve each quadratic equation.

1. $x(x-6)=0$
2. $x^2+10x=0$
3. $x^2=x$
4. $x^2=9$
5. $x^2-25=0$
6. $4x^2-9=0$
7. $x^2-x-6=0$
8. $x^2-8x+15=0$
9. $x^2+14x=32$
10. $3x^2-5x-2=0$
11. $6x^2+7x+2=0$
12. $2x^2-x=3$

Solving Quadratic Equations by Completing the Square

Procedure for completing the square:

❶ Manipulate the equation so that the leading coefficient is positive one.
❷ Isolate the constant.
❸ Take ½ of the coefficient of the linear term.
❹ Square the result of step 3 and add to both sides of equation.
❺ Factor the left side as a PST and square root both sides.

Use **completing the square** to solve each quadratic equation.

1. $x^2+2x-24=0$
2. $x^2-6x-1=0$
3. $x^2+2x-7=0$

Solving Quadratic Equations Using the Quadratic Formula

If $ax^2+bx+c=0$, then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Procedure: (1) set equation = 0, (2) identify *a*, *b*, *c* and (3) use formula.

Use the **quadratic formula** to solve to solve each quadratic equation.

1. $x^2+2x-8=0$
2. $2x^2-9x+10=0$
3. $x^2+2x=2$
4. $4x^2-x=0$