AP Calculus AB/BC Summer Assignment

Due Date: The first day of the second week of school.

Welcome to AP Calculus! It's been a long time coming, but you're finally here! We will be covering a lot of college level material at a very fast pace next year. In order to do that as effectively as possible, a strong foundation of fundamental skills acquired through your previous high school math classes is required. The purpose of this assignment is to practice important prerequisite skills from Algebra II and Precalculus; this includes some of the beginning of Calculus that was covered in Precalculus H.

When completing this assignment, you may use reference materials to assist and refresh your memory. This may include old notes, textbooks, online resources, etc. These problems were designed to be completed without use of a calculator. For further assistance, please seek assistance from your classmates or from me via email.

Submitting work generated by AI as your own is academic dishonesty. Students must be able to explain and reproduce all submitted work. Failure to do so may result in a zero and further disciplinary consequences.

Your assignment is to do the following:

- **Section I through III: at least the odd numbered problem. Do the even numbers if you feel that you need the practice**
- **Section IV: all problems**

All work must be shown in order to earn credit for this assignment.

The expectation is that you fully understand the contents of this packet on the first day of class in August. Due to the pace of this course, we will need to hit the ground running, and class time will not be available to re-teach prerequisite skills from previous classes. This assignment will be collected and graded **on the first day of the second week of school.** Additionally, there may be a quiz on this material during the first marking period.

I can't wait to get started with you next year! The course is very demanding, but incredibly rewarding. Please reach out if you have any questions about this assignment, or about the course in general.

- Mr. O'Brien (email: kobrien@shsd.org)

"Success is the sum of small efforts, repeated day in and day out." – Robert Collier

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Show all work – no credit will be awarded for answers missing appropriate work.

Do <u>not</u> use a calculator.

Section 1: Algebra Review

Identify the following statements as true or false.

1. $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$ _____

2. $\frac{1}{p+1} = \frac{1}{p} + \frac{1}{q}$ _____

3. $\frac{2k}{2x+h} = \frac{k}{x+h}$ _____

4. $3 \cdot \frac{a}{b} = \frac{3a}{b}$ _____

5. $3 \cdot \frac{a+b}{c} = \frac{3a+b}{c}$ _____

6. $\sqrt{a^2 + b^2} = a + b$ _____

Identify the following statements as true or false over the set of real numbers. Give a counter example for any false statement.

7. $x^3 + 1 > x^3$ _____

8. $x^3 + x > x^3$ _____

9. $x^2 \geq 0$ _____

10. $x^2 \geq x$ _____

11. $2x \geq x$ _____

12. $\sqrt{x} \geq 0$ _____

13. $-x \leq 0$ _____

14. $\frac{1}{x} \leq x$ _____

15. $x \leq |x|$ _____

Solve each of the following equations for y.

16. $xy + y + 1 = y$

17. $\ln y = kt$

Fully factor each of the following expressions:

18. $y^3 + 27$

19. $x^2(x-1) - 4(x-1)$

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Simplify each expression

20. $\frac{(x^2)^3 x}{x^7}$

21. $\sqrt{x} \cdot \sqrt[3]{x} \cdot x^{\frac{1}{6}}$

22. $\frac{5(x+h)^3-5x^3}{h}$

23. $\frac{3(x+h)^2-3x^2}{h}$

24. $\frac{\frac{x^2-1}{x}}{\frac{x+1}{x^3}}$

25. $\frac{\frac{1}{x}+\frac{4}{x^2}}{3-\frac{1}{x}}$

26. $\frac{\frac{a}{2x+h}-\frac{a}{2x}}{h}$

27. $\frac{1}{1-2a} - \frac{2}{1+2a} + \frac{6a+2}{4a^2+1}$

Simplify by rationalizing the numerator.

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x\left(\sqrt{x+4}+2\right)} = \frac{x}{x\left(\sqrt{x+4}+2\right)} = \frac{1}{\sqrt{x+4}+2}$$

28. $\frac{\sqrt{x+9}-3}{x}$

29. $\frac{\sqrt{x+h}-\sqrt{x}}{h}$

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Simplify using factoring of binomial expressions. Leave answers in factored form.

Example:

$$\frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)} = \frac{(x+1)^2[(x+1)(4x-9)-(16x+9)]}{(x-6)(x+1)}$$
$$= \frac{(x+1)^2(4x^2-5x-9-16x-9)}{(x-6)(x+1)}$$
$$= \frac{(x+1)^2(4x^2-21x-18)}{(x-6)(x+1)}$$
$$= \frac{(x+1)^2(4x+3)(x-6)}{(x-6)(x+1)}$$
$$= (x+1)(4x+3)$$

30. $(x-1)^3(2x-3)-(2x+12)(x-1)^2$

31. $\frac{(x-1)^2(3x-1)-2(x-1)\cdot 3}{(x-1)^4}$

32. $\frac{(x-1)^3(2x-3)-(4x-1)(x-1)^2}{(x-1)^2(2x-1)}$

Solve each system of equations.

33. $x+y=8$
$2x-y=7$

34. $y=x^2-3x$
$y=2x-6$

Solve each equation or inequality for x over the set of real numbers

35. $2x^4 + 3x^3 - 2x^2 = 0$

36. $\frac{2x-7}{x+1} = \frac{2x}{x+4}$

37. $\frac{3x+5}{(x-1)(x^4+7)} = 0$

38. $\sqrt{x^2 - 9} = x - 1$

39. $|2x - 3| = 14$

40. $x^2 - 2x - 8 < 0$

Section 2: Precalculus Review

Use the Unit Circle to evaluate each of the following. All answers should be in simplest radical form.
Note: Try these from memory. The Unit Circle **must** be memorized for Calculus.

41. $\sin 30°$ _____

42. $\cos\frac{2\pi}{3}$ _____

43. $\tan 45°$ _____

44. $\sin -\frac{\pi}{6}$ _____

45. $\tan \pi$ _____

46. $\csc\frac{5\pi}{6}$ _____

47. $\cos 90°$ _____

48. $\cos\frac{3\pi}{4}$ _____

49. $\tan\frac{\pi}{6}$ _____

50. $\cos^{-1}\frac{1}{2}$ _____

51. $\sin^{-1}\frac{\sqrt{2}}{2}$ _____

52. $\tan^{-1} 1$ _____

Solve each trigonometric equation for $0 \leq x < 2\pi$.

53. $\sin x = \frac{\sqrt{3}}{2}$

54. $\tan^2 x = 1$

55. $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

56. $2\sin^2 x + \sin x - 1 = 0$

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For each trigonometric function, identify the amplitude and any horizontal or vertical shifts from the parent function.

57. $y = \frac{1}{2}\cos\frac{x}{2} - 3$

58. $y = 2\sin(2x - \pi)$

59. $y = \tan 3x$

Complete each of the following using trigonometric identities and formulas.

60. $\sin(\frac{\pi}{2} - x) =$_____

61. $\sin^2 x + \cos^2 x =$_____

62. $\sin 2u =$_____

63. $\tan x =$_____

64. $1 + \cot^2 x =$_____

65. $1 - \cos^2 x =$_____

66. A right triangle has a base of 5 and a hypotenuse of 7. Find its height in simplest radical form.

Solve each exponential or logarithmic equation.

67. $5^x = 125$

68. $8^{x+1} = 16^x$

69. $81^{\frac{3}{4}} = x$

70. $8^{-\frac{2}{3}} = x$

71. $\log_2 32 = x$

72. $\log_x \frac{1}{9} = -2$

73. $\log_4 x = 3$

74. $\log_3(x + 7) = \log_3(2x - 1)$

Expand each of the following using properties of logarithms.

75. $\log_3(5x^2)$

76. $\ln\left(\frac{5x}{y^2}\right)$

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Section III: Graphing Review

Sketch the following functions. State the domain and range of each. Draw and label your own axis.

77. $f(x) = 2$

78. $f(x) = x$

79. $f(x) = x^2$

80. $f(x) = x^3$

81. $f(x) = |x|$

82. $f(x) = [x]$ (greatest integer)

83. $f(x) = \frac{1}{x}$

84. $f(x) = \sqrt{x}$

85. $f(x) = \sqrt[3]{x}$

86. $f(x) = e^x$

87. $f(x) = \ln x$

88. $f(x) = \sqrt{9 - x^2}$

89. $f(x) = \sin x$

90. $f(x) = \cos x$

91. $f(x) = \tan x$

92. $f(x) = \csc x$

93. $f(x) = \sec x$

94. $f(x) = \cot x$

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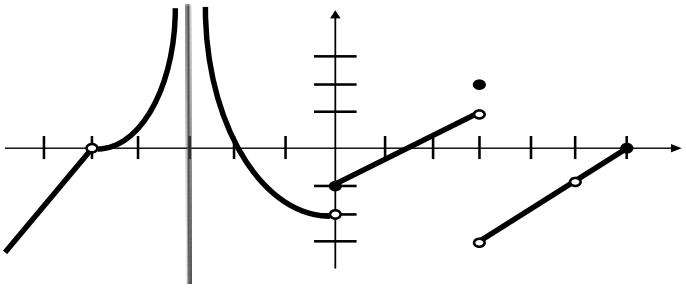
Section IV: Intro to Calculus Review – Limits/Continuity and Derivatives

95. Answer the following questions using the graph of $f(x)$ given below.



a. $f(0)$ _____ b. $\lim_{x\to 0^-} f(x)$ _____ c. $\lim_{x\to 0^+} f(x)$ _____ d. $\lim_{x\to 0} f(x)$ _____

e. $f(-5)$ _____ f. $\lim_{x\to -5} f(x)$ _____ g. $f(-3)$ _____ h. $\lim_{x\to -3} f(x)$ _____

g. List all x-values for which $f(x)$ has a removable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

h. List all x-values for which $f(x)$ has a nonremovable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

Find the limit if it exists analytically.

96. $\lim_{x\to -2} \frac{3x^2+21x+30}{x^3+8}$

97. $\lim_{x\to \frac{\pi}{6}} \frac{1-\cos^2 x}{4x}$

98. $\lim_{x\to 4} \frac{\sqrt{x-3}-1}{x-4}$

99. $\lim_{t\to 0} \frac{\sin^2 3t^2}{t^3}$

100. Let $f(x) = \begin{cases} \frac{\sqrt{2x+1}-\sqrt{3}}{x-1} & , \quad x \geq 0 \\ 4x^2 + k & , \quad x < 0 \end{cases}$. For what value of k will $f(x)$ be continuous at $x = 0$? Explain using the definition of continuity.

101. Let $f(x) = x^3 + x - 3$ and let c be a real number such that $f(c) = 0$. Select the interval on which c must exist. Justify your reasoning using the Intermediate Value Theorem.

a. $[-2, -1]$

b. $[-1, 0]$

c. $[0, 1]$

d. $[1, 2]$

e. $[2, 3]$

102. Use the limit definition to find the derivative of $f(x) = 4x^2 + 3x - 5$. **No shortcuts allowed.**

Find the derivative of each function using the power, product, quotient, and/or chain rules.

103. $f(x) = 3x^4 - 6x^3 + 16x^2 - 14x + 21$

104. $f(x) = \sqrt{x}\sin x$

105. $f(x) = \frac{x^2+x-1}{x^2-1}$

106. $f(x) = \csc 3x + \tan^2 x$

107. $f(x) = 7x^3 e^x \sec x$

108. $f(x) = \ln(x^2 - 4x + 4)$

109. Find the equation of the tangent line to the graph of $f(x) = \frac{1+\cos x}{1-\cos x}$ at $(\frac{\pi}{2}, 1)$.

For all remaining problems, circle the correct answer and explain why your answer is correct.

110. $\lim_{\Delta x \to 0} \frac{\cos\left(\frac{\pi}{6}-\Delta x\right)-\cos\left(\frac{\pi}{6}\right)}{\Delta x}$

a. $\frac{1}{2}$

b. $-\frac{1}{2}$

c. $\frac{\sqrt{3}}{2}$

d. $-\frac{\sqrt{3}}{2}$

e. The limit does not exist

111. Let f and g be differentiable functions. The table below displays select outputs for these functions and their derivatives.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	-4	12	-8
2	5	1	-6	4

Find $\frac{d}{dx}[f(x) \cdot g(x)]$ at $x = 1$.

a. -96

b. -80

c. -48

d. -32

e. 0

112. Let $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ 1 & x = 2 \end{cases}$. Which of the following statements about f are true?

I. $\lim_{x \to 2} f(x)$ exists

II. f is continuous at $x = 2$

III. f is differentiable at $x = 2$

a. I only

b. II only

c. III only

d. I and II only

e. I, II, and III

113. Which of the following represents an equation of a line *normal* to the curve $y = \sqrt[3]{x^2 - 1}$ at $x = 3$?

HINT: A normal line to a curve at a point is perpendicular to the tangent line at that same point.

a. $y + 12x = 38$

b. $y - 4x = 10$

c. $y + 2x = 4$

d. $y + 2x = 8$

e. $y - 2x = -4$