Summer Review Packet for Students Entering Pre-Calculus

Notre Dame High School

Summer Review Packet

For Students entering PRE-CALCULUS

Name: _____

1. This packet is to be handed in to your Pre-Calculus teacher on the first day of the school year.
2. All work must be shown in the packet OR on a separate sheet of paper attached to the packet.
3. Completion of this packet will be worth a quiz grade and will be recorded for the first quarter.
4. Answers to odd-numbered problems have been provided.

My signature below indicates that I have received the Pre-Calculus Summer Review Packet.

_____ _____

Student Signature Date

Radicals:

To simplify means that 1) no radicand has a perfect square factor and
2) there is no radical in the denominator (rationalize).

Recall the **Product Property** $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \times \sqrt{6}$ find the perfect square factor

$= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ multiply numerator & denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms, multiply the numerator and denominator by conjugate of the denominator (the conjugate of $3 + \sqrt{2}$ is $3 - \sqrt{2}$)

Simplify each of the following.

1. $\sqrt{32}$

2. $\sqrt{(2x)^8}$

3. $\sqrt[3]{-64}$

4. $\sqrt{49m^2n^8}$

5. $\sqrt{\frac{11}{9}}$

6. $\sqrt{60} \times \sqrt{105}$

7. $\left(\sqrt{5} - \sqrt{6}\right)\left(\sqrt{5} + \sqrt{2}\right)$

Rationalize

8. $\frac{1}{\sqrt{2}}$

9a. $\frac{2}{\sqrt{3}}$

10a. $\frac{3}{2 - \sqrt{5}}$

Complex Numbers:

Forms of complex numbers: $a + bi$

Where *a* is the real part and the *bi* is the imaginary part

Always makes these substitutions $\sqrt{-1} = i$ and $i^2 = -1$

To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5} = \sqrt{-1} \times \sqrt{5}$ Pull out $\sqrt{-1}$

$= i\sqrt{5}$ Make substitution

Example: $\left(i\sqrt{5}\right)^2 = i\sqrt{5} \times i\sqrt{5}$

Treat *i* like any other variable when +,–,×,or ÷ (but always simplify $i^2 = -1$)

Example:

$2i(3+i) = 2(3i) + 2i(i)$ Distribute

$= 6i + 2i^2$ Simplify

$= 6i + 2(-1)$ Substitute

$= -2 + 6i$ Simplify and rewrite in complex form

Since *i* = $\sqrt{-1}$, no answer can have an 'i' in the denominator. RATIONALIZE!

Simplify.

9b. $\sqrt{-49}$

10b. $6\sqrt{-12}$

11. $-6(2-8i) + 3(5+7i)$

12. $(3-4i)^2$

13. $(6-4i)(6+4i)$

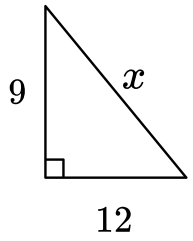
Rationalize

14. $\dfrac{1+6i}{5i}$

Geometry:
Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Find the value of x.

15.



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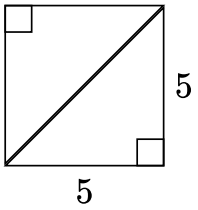

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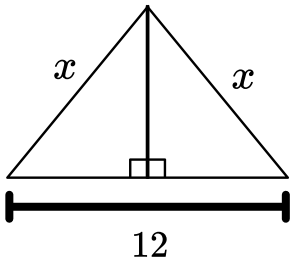
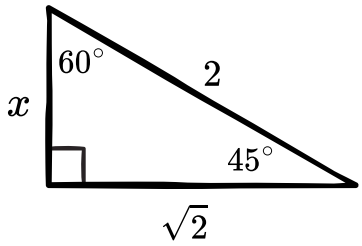
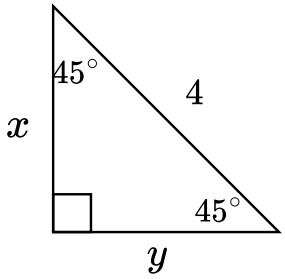


18. A square has a perimeter 12 cm. Find the length of the diagonal.

*In 30º - 60º - 90º triangles, sides are in proportional

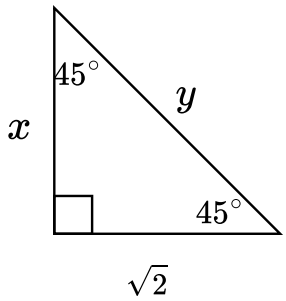


*In 45º - 45º - 90º triangles, sides are in proportional



Solve for x and y.

19.



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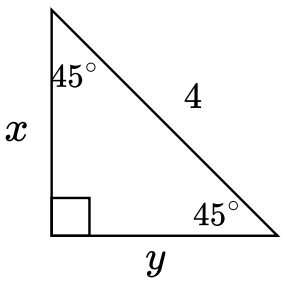
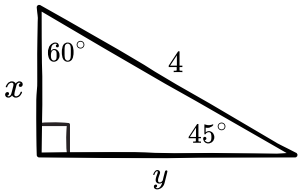
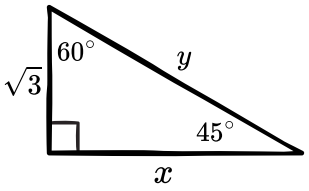


21. 

22. 

Equations fo Lines:

Slope-intercept form: $y = mx + b$ Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y_2 - y_1 = m(x_2 - x_1)$ Horizontal line: $y = c$ (slope is zero)

Standard Form: $Ax + By = C$ Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equations: 5x - 4y = 8

24. Find the x-intercept and y-intercept of the equation: 2x - y = 5

25. Write the equation in standard form: y = 7x - 5

Write the equation of the line in slope-intercept form with the following conditions:

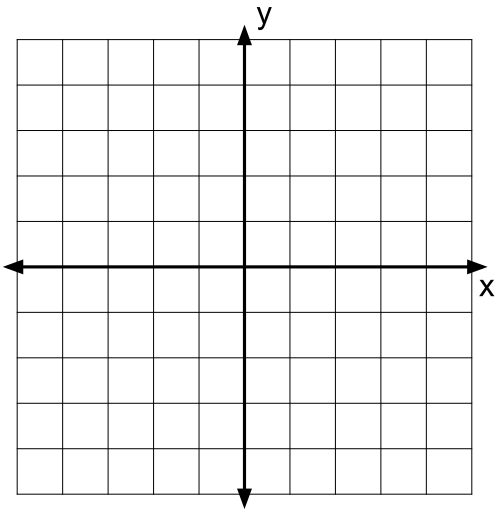
26. slope = -5 and passes through the point (-3, 8)

27. passes through the points (4, 3) and (7, -2)

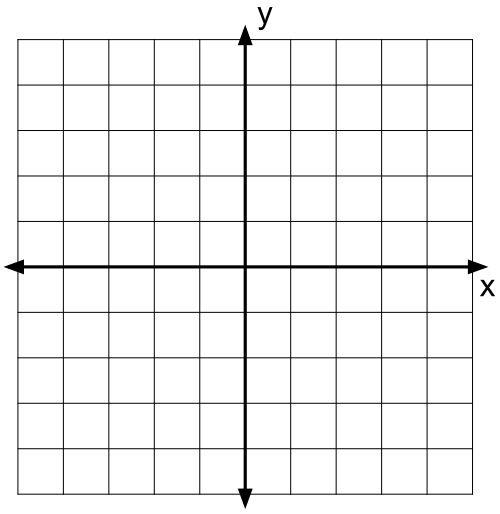
28. x-intercept = 3 and y-intercept = 2

<u>Graphing:</u> Graph each function, inequality, and/or system.

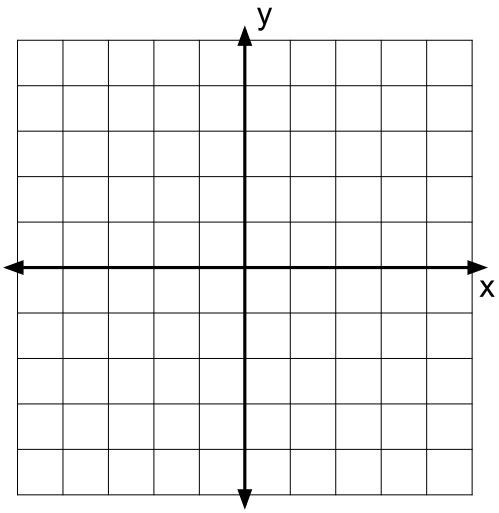
29. 3x - 4y =12



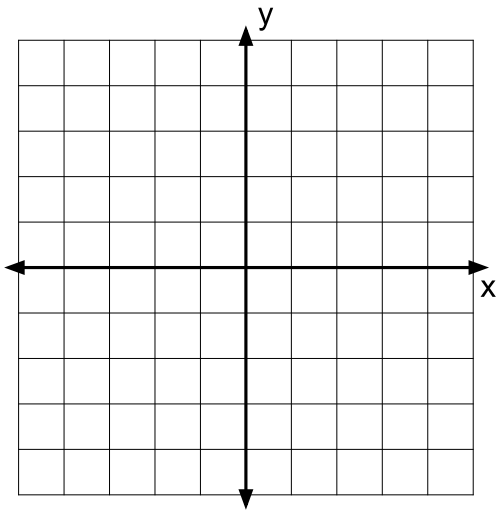
30. $\begin{cases} 2x + y = 4 \\ x - y = 2 \end{cases}$



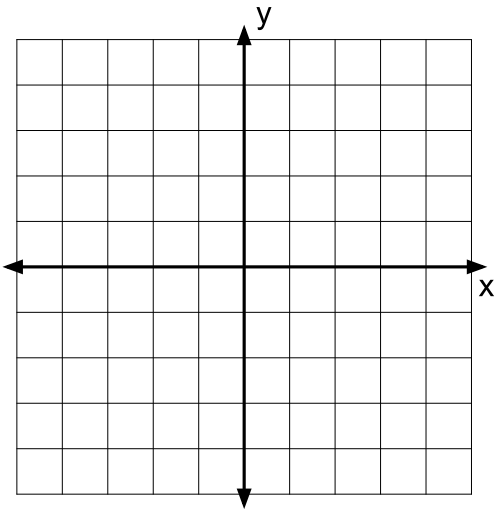
31. y < -4x - 2



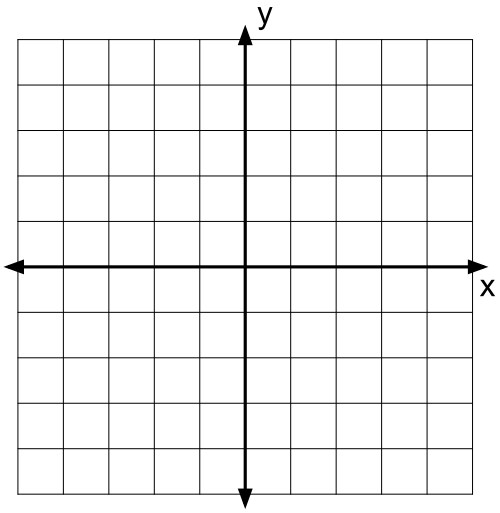
32. y + 2 = |x + 1|



33. y > |x| - 1



34. y + 4 = (x-1)^2



Systems of Equations:

$$\begin{cases} 3x + y = 6 \\ 2x - 2y = 4 \end{cases}$$

Substitution:	**Elimination:**
Solve 1 equation for 1 variable	Find opposite coefficients for 1 variable
Rearrange.	Multiply equation(s) by constants(s)
Plug into 2nd equation	Add equations together (lose 1 variable)
Solve for the other variable	Solve for variable

Then plug answers back into an original equation to solve for the 2nd variable.

y = 6 - 3x	Solve 1st equation for y	6x + 2y = 12	Multiply 1st equation by 2
2x - 2(6 - 3x) = 4	Plug into 2nd equation	2x - 2y = 4	coefficients of y are opposite
2x - 12 + 6x = 4	Distribute	8x = 16	Add
8x = 16 and x = 2	Simplify	x = 2	Simplify

Solve each system of equations, using any method.

35. $\begin{cases} 2x + y = 4 \\ 3x - 2y = 1 \end{cases}$

36. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Exponent:
Recall the following rules of exponents:

1.	$a^1 = a$	Any number raised to the power of one equals itself.
2.	$1^a = 1$	One raised to any power is one.
3.	$a^0 = 1$	Any nonzero umber raised to the power of zero is one.
4.	$a^m \times a^n = a^{m+n}$	When multiplying two powers with the same base, add the exponents.
5.	$\frac{a^m}{a^n} = a^{m-n}$	When dividing two powers with the same base, subtract the exponents.
6.	$(a^m)^n = a^{mn}$	When a power is raised to another, multiply the exponents.
7.	$a^{-n} = \frac{1}{a^n}$ **and** $\frac{1}{a^{-n}} = a^n$	Any nonzero number raised to a negative power equals its reciprocal raise to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\frac{3c}{c^{-1}}$

40. $\frac{2ef^{-1}}{e^{-1}}$

41. $\frac{(n^3p^{-1})^2}{(np)^{-2}}$

Simplify.

42. $3m^2(2m)$

43. $(a^3)^2$

44. $(-b^3c^4)^5$

45. $4m(3a^2m)$

Polynomials:

To add/subtract polynomials, combine like terms.

EX: 8x - 3y + 6 - (6y + 4x - 9) *Distribute the negative through the parentheses*
= 8x - 3y +6 - 6y - 4x + 9 *Combine like terms with similar variables.*
= 8x - 4x - 3y - 6y + 6 + 9
= 4x - 9y + 15

Simplify.

46. $3x^3 + 9 + 7x^2 - x^3$

47. $7m - 6 - (2m + 5)$

To multiply two binomials, use FOIL.

EX: $(3x - 2)(x + 4)$ *Multiply the first, outer, inner, and last terms.*

$= 3x^2 + 12x - 2x - 8$ *Combine like terms together.*

$= 3x^2 + 10x - 8$

Multiply.

48. $(3a + 1)(a - 2)$

49. $(s + 3)(s - 3)$

50. $(c - 5)^2$

51. $(5x + 7y)(5x - 7y)$

Factoring:

Follow these steps in order to factor polynomials

STEP 1: Look for a GCF in ALL of the terms.

a) If you have one (other than 1) factor it out.

b)If you don't have one move on STEP 2

STEP 2: How many terms does the polynomial have?

2 Terms a) is it the difference of two square? $a^2 - b^2 = (a+b)(a-b)$

EX: $x^2 - 25 = (x+5)(x-5)$

b) Is it the sum or difference of two cubes? $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

$a^3 + b^3 = (a-b)\left(a^2 - ab + b^2\right)$

EX: $m^3 + 64 = (m+4)\left(m^2 - 4m + 16\right)$

$m^3 + 64 = (m+4)\left(m^2 - 4m + 16\right)$

3 Terms **EX:**

$x^2 + bx + c = (x + __)(x + __)$ $x^2 + 7x + 12 = (x+3)(x+4)$

$x^2 - bx - c = (x - __)(x - __)$ $x^2 - 5x + 4 = (x-1)(x-4)$

$x^2 + bx - c = (x - __)(x + __)$ $x^2 + 6x - 16 = (x-2)(x+8)$

$x^2 - bx - c = (x - __)(x + __)$ $x^2 - 2x - 24 = (x-6)(x+4)$

4 Terms --- Factor by Grouping

a) Pair up first two terms and last two terms.

b) Factor out GCF of each pair of numbers.

c) Factor out front parentheses that the terms have in common.

d) Put leftover terms in parentheses.

Ex: $x^3 + 3x^2 + 9x + 27 = \left(x^3 + 3x^2\right) + (9x + 27)$

$= x^2(x+3) + 9(x+3)$

$= (x+3) + \left(x^2 + 9\right)$

Factor completely.

52. $z^2 + 4x - 12$

53. $6 - 5x - x^2$

54. $2k^2 + 2k - 60$

55. $-10b^4 - 15b^2$

56. $9c^2 + 30c + 25$

57. $9n^2 - 4$

58. $27z^3 - 8$

59. $2mn - 2mt - 2sb - 2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use the quadratic formula.

EX: $x^2 - 4x = 21$ *Set equal to zero FIRST.*

$x^2 - 4x - 21 = 0$ *Now factor.*

$(x + 3)(x - 7) = 0$ *Set each factor equal to zero.*

$x + 3 = 0 \quad x - 7 = 0$ *Solve for each x.*

$x = -3 \quad x = 7$

Solve each equation.

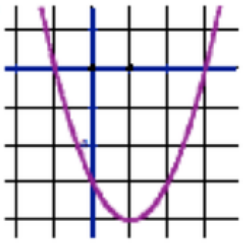
60. $x^2 - 4x - 12 = 0$

61. $x^2 + 25 = 10x$

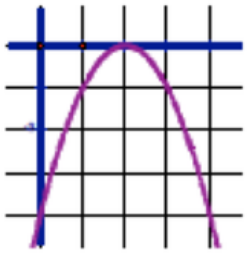
62. $x^2 - 14x + 40 = 0$

Discriminant: The number under the radical in the quadratic formula (b^2-4ac) can tell you what kind of roots you will have.

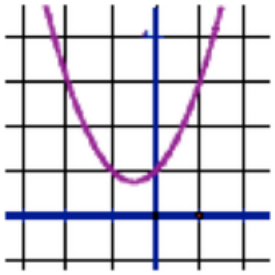
If $b^2-4ac > 0$ you have TWO real roots (touches the x-axis twice)



If b^2-4ac you will have ONE real root (touches axis once)



If $b^2-4ac < 0$ you will have TWO imaginary roots. (Function does not cross the x-axis)



QUADRATIC FORMULA – allows you to solve any quadratic for all its real and imaginary roots.

$5x^2 - 2x + 4 = 0$ $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EX: In the equation $x^2 + 2x + 3 = 0$, find the value of the discriminant describe the nature of the roots, than solve.

$x^2 + 2x + 3 = 0$ *Determine the values of a, b, and c.*

$a = 1, b = 2, c = 3$ *Find the discriminant.*

$D = 2^2 - 4(1)(3)$

$D = 4 - 12$

$D = -8$ *There are two imaginary roots.*

Solve: $x = \dfrac{-2 \pm \sqrt{-8}}{2}$

$x = \dfrac{-2 \pm 2i\sqrt{2}}{2}$

$x = -1 \pm i\sqrt{2}$

Find the value of the discriminant, describe the nature of the roots, then solve each quadratic. Use EXACT values.

63. $x^2 - 9x + 14 = 0$

64. $5x^2 - 2x + 4 = 0$

Discriminant = _____

Type of Roots: _____

Exact Value of Roots: _____

Discriminant = _____

Type of Roots: _____

Exact Value of Roots: _____

Long Division--can be used when dividing any polynomials.

Synthetic Division--can ONLY be used when dividing a polynomial by a linear polynomial.

EX: $\dfrac{2x^3+3x^2-6x+10}{x+3}$

Long Division

$$\frac{2x^3+3x^2-6x+10}{x+3}$$

$$x+3\overline{)\,2x^3+3x^2-6x+10}$$

$$2x^2-3x+3+\frac{1}{x+3}$$

$$(-)\,(2x^3+6x^2)$$

$$-3x^2-6x$$

$$(-)\,(-3x^2-9x)$$

$$3x+10$$

$$(-)\,(3x+9)$$

$$1$$

Synthetic Division

$$\frac{2x^3+3x^2-6x+10}{x+3}$$

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -6 & 10 \\ & \downarrow & -6 & 9 & -9 \\ \hline & 2 & -3 & 3 & 1 \end{array}$$

$$= 2x-3x+3+\frac{1}{x+3}$$

Divide each polynomial using long division OR synthetic division.

65. $\dfrac{c^3-3c^2+18c-16}{c^2+3c-2}$

66. $\dfrac{x^4-2x^2-x+2}{x+2}$

To evaluate a function for the given value, simply plug the value into the function for x.
Evaluate each function for the given value.

67. $f(x) = x^2 - 6x + 2$

$f(3) =$

68. $g(x) = 6x - 7$

$g(x+h) =$

69. $f(x) = 3x^2 - 4$

$5[f(x+2)] =$

Composition and Inverses of Functions:

Recall: (f g)(x) = f(g(x)) OR f[g(x)] read **"f of g of x"** means to plug the inside function in for x in the outside function.

Example: Given f(x) = $2x^2$ +1 *and* g(x) = x - 4 find f(g(x)).

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2\left(x^2 - 8x + 16\right) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ &= 2x^2 - 16x + 33 \end{aligned}$$

Suppose f(x) = 2x, g(x) = 3x - 2, *and* h(x) = x^2 - 4. **Find the following:**

70. $f[g(2)] =$

71. $f[g(x)] =$

72. $f[h(3)] =$

73. $g[f(x)] =$

To find the inverses of a function, simply switch the x and y and solve for the new "y" value.

Example:	$f(x) = \sqrt[3]{x+1}$	Rewrite *f(x)* as *y*
	$y = \sqrt[3]{x+1}$	Switch *x* and *y*
	$x = \sqrt[3]{y+1}$	Solve for your new *y*
	$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
	$x^3 = y+1$	Simplify
	$y = x^3 - 1$	Solve for *y*
	$f^{-1}(x) = x^3 - 1$	Rewrite in inverses notation

Find the inverse, $f^{-1}(x)$, if possible

74. $f(x) = 5x + 2$

75. $f(x) = \frac{1}{2}x - \frac{1}{3}$

Rational Algebraic Expressions:

Multiplying and Dividing: **Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.**

EX: $\left(\dfrac{x^2+10x+21}{5-4x-x^2}\right)\left(\dfrac{x^2+2x-15}{x^2+4x^2-21x}\right)$ *Factor everything completely.*

$= \left(\dfrac{(x+7)(x+3))}{(5+x))\,(1-x)}\right)\left(\dfrac{(x+5)\,(x-3)}{x\,(x-3)\,(x+7)}\right)$ *Cancel out common factors in the top and bottom.*

$= \dfrac{(x+3)}{x\,(1-x)}$ *Simplify.*

76. $\dfrac{5z^3+z^2-z}{3z}$

77. $\dfrac{m^2-25}{m^2+5m}$

78. $\left(\dfrac{10r^5}{21s^2}\right)\left(\dfrac{3s}{5r^3}\right)$

79. $\left(\dfrac{a^2-5a+6}{a+4}\right)\left(\dfrac{3a+12}{a-2}\right)$

80. $\dfrac{6d-9}{5d+1} \div \dfrac{6-13d+6d^2}{15d^2-7d-2}$

Addition and Subtraction

First find the least common denominator. Write each fraction with that LCD. Add/subtract numerators as indicated and leave the denominators as they are.

Example:

$$\frac{3x+1}{x^2+2x}+\frac{5x-4}{2x+4}$$ *Factor denominator completely*

$$\frac{3x+1}{x\left(x+2\right)}+\frac{5x-4}{2\left(x+2\right)}$$ *Find LCD, which is* (2x)(x+2)

$$\frac{2\left(3x+1\right)}{2x\left(x+2\right)}+\frac{x\left(5x-4\right)}{2x\left(x+2\right)}$$ *Rewrite each fraction with the LCD in the denominator*

$$\frac{6x+2+5x^2-4x}{2x\left(x+2\right)}$$ *Write as one fraction*

$$\frac{5x^2+2x+2}{2x\left(x+2\right)}$$ *Combine like terms.*

81. $\frac{2x}{5}-\frac{x}{3}$

82. $\frac{b-a}{a^2b}-\frac{a+b}{ab^2}$

83. $\frac{2-a^2}{a^2+a}+\frac{3a+4}{3a+3}$

Complex Fractions: Eliminate complex fractions by multiplyign the numerator and denominator by the LCD of each of the small fractions. Then simplify the result.

Example: $\dfrac{1+\frac{1}{a}}{\frac{2}{a^2}-1}$ *Find LCD*: a^2

$= \dfrac{\left(1+\frac{1}{a}\right)a^2}{\left(\frac{2}{a^2}-1\right)a^2}$ *Multiply top and bottom by LCD*

$= \dfrac{a^2+a}{2-a^2}$ *Factor and simplify if possible*

$= \dfrac{a\,(a+1)}{2-a^2}$

84. $\dfrac{1-\frac{1}{2}}{2+\frac{1}{4}}$

85. $\dfrac{1+\frac{1}{z}}{z+1}$

86. $\dfrac{5+\frac{1}{m}-\frac{6}{m^2}}{\frac{2}{m}-\frac{2}{m^2}}$

87. $\dfrac{2+\frac{1}{x}-\frac{1}{x^2}}{1+\frac{4}{x}+\frac{3}{x^2}}$

Solving Rational Equation:

Multiply each term by the LCD of all the fractions. This should eliminate all of our fractions. Then solve the equation as usual.

$\frac{5}{x+2}+\frac{1}{x}=\frac{5}{x}$ *Factor denominator completely*

$x(x+2)\frac{5}{x+2}+x(x+2)\frac{1}{x}=\frac{5}{x}x(x+2)$ *Find LCD, which is* (2x)(x+2)

$5x+1(x+2)=5(x+2)$ *Rewrite each fraction with the LCD in the denominator*

$5x+x+2=5x+10$ *Write as one fraction*

$6x+2=5x+10$ *Combine like terms.*

$x=8$ ← *Check your answers!*

Check: $\frac{5}{8+2}+\frac{1}{8}=\frac{5}{8}$

$\frac{5}{10}+\frac{1}{8}=\frac{5}{8}$

$\frac{5}{8}=\frac{5}{8}$

Solve each equation. Check your solutions.

88. $\frac{12}{x}+\frac{3}{4}=\frac{3}{2}$

89. $\frac{x+10}{x^2-2}=\frac{4}{x}$

90. $\frac{5}{x-5}=\frac{x}{x-5}-1$

Solutions to Odd Exercises

1. $4\sqrt{2}$ 3. -4 5. $\frac{\sqrt{11}}{3}$ 7. $5+\sqrt{10}-\sqrt{30}-2\sqrt{3}$ 9a. $\frac{2\sqrt{3}}{3}$

9b. $7i$ 11. 3 + 69i 13. 52 15. 15 17. 10 19. $\sqrt{2}$

21. x=2, $y=2\sqrt{3}$ 23. $m=\frac{5}{4}, b=-2$ 25. $7x-y=5$ 27. $y=-\frac{5}{3}x+\frac{29}{3}$

29, 31, 33 graphs 35. (7, 10) 37. $\left(\frac{89}{36},-\frac{29}{18}\right)$ 39. $3c^2$ 41. n^8

43. a^6 45. $12a^2m^2$ 47. $5m-11$ 49. s^2-9

51. $25x^2-49y^2$ 53. $-(x+6)(x-1)$ 55. $5b^2\left(2b^2+3\right)$ 57. $(3n-2)(3n+2)$

59. $2(m+s)(n-t)$ 61. $x=5$ 63. 25; 2 real; 7 and 2 65. $(c-6)+\frac{38c-28}{c^2+3c-2}$

67. -7 69. $15x^2 + 60x + 40$ 71. 6x - 4 73. 6x - 2

75. $f^{-1}(x)=2x+\frac{2}{3}$ 77. $\frac{m-5}{m}$ 79. 3(a - 3) 81. $\frac{x}{15}$ 83. $\frac{4a+6}{3a(a+1)}$

85. $\frac{1}{z}$ 87. $\frac{2x-1}{x+3}$ 89. $x=4;-\frac{2}{3}$