

# LIU College Calculus Summer Assignment

- Complete all the problems in this packet without use of a calculator. Packet is due on the first day of classes.
- No calculator quiz on this packet will be on Friday  
9.4.26

Name \_\_\_\_\_

All of the following should be completed without the use of a calculator.

I. Algebra Skills:

Solve for  $x$ . Answers may be left in terms of  $e$  as appropriate.

1.  $x^2 - 4x + 3 = 0$

6.  $5^{x-3} = 125^x$

2.  $x^2 + 6x = 1$

7.  $16^{x-5} = 1024^{286}$

3.  $6x^2 + 11x - 10 = 0$

8.  $\ln x = 5$

4.  $10 = \frac{100}{1 + e^{5x}}$

9.  $\log_2 3x - \log_2 3 = 8$

5.  $2^x = 64$

10.  $4\ln(x+3) = 20$

Solve for  $x$ . Answers may be left in logarithmic form as appropriate.

11.  $5^x = 128$

12.  $e^{2x} = 144$

13.  $3^{x^2} = 18$

## II. Piecewise Functions

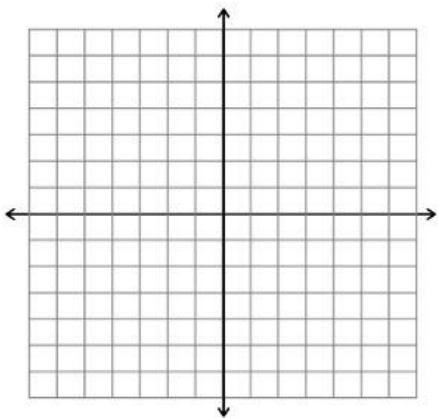
14. Given the function  $f(x) = \begin{cases} -x^2 - 1, & x \leq 0 \\ 2, & 0 < x < 4, \\ \sqrt{x}, & x \geq 4 \end{cases}$

a. Find  $f(-2)$ .

b. Find  $f(9)$ .

c. Find  $f(0)$ .

d. Graph  $y = f(x)$ .



e. Is  $f(x)$  continuous at  $x=4$ ? Why or why not?

f. Is  $f(x)$  continuous at  $x=0$ ? Why or why not?

15. Find the value of  $k$  that will make the following piecewise function continuous at

$$x = -1. \quad f(x) = \begin{cases} kx^2 + 1, & x \leq -1 \\ 2x - k, & x > -1 \end{cases}$$

16. Find the value of  $k$  that will make the following piecewise function continuous at

$$x = 2. \quad f(x) = \begin{cases} 2x - 3, & x \leq 2 \\ x^2 + k, & x > 2 \end{cases}$$

17. Given  $f(x) = \begin{cases} 2x+10, & x < 0 \\ x^2+1, & x \geq 0 \end{cases}$ , solve for  $x$  if  $f(x)=17$ .

An absolute value function can be thought of as a piecewise function:  $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$ .

Here's another example:  $|2x+6|$ .

$2x+6$  will change signs when  $x=-3$ .

If  $x < -3$ ,  $2x+6$  comes out negative so to find the absolute value, it must be negated.

If  $x \geq -3$ ,  $2x+6$  comes out positive and its absolute value does not need to be changed.

Thus,  $|2x+6| = \begin{cases} -2x-6, & x < -3 \\ 2x+6, & x \geq -3 \end{cases}$ .

18. Express the absolute value function in piecewise form.

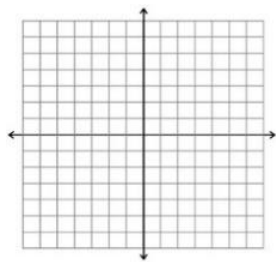
a.  $f(x) = |x-1|$

b.  $f(x) = |2x+3|$

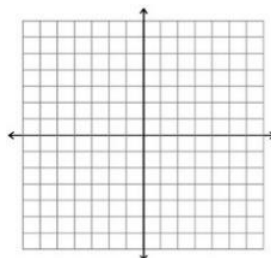
### III. Common Functions

19. Graph each of the following functions. Show a few important points for each.

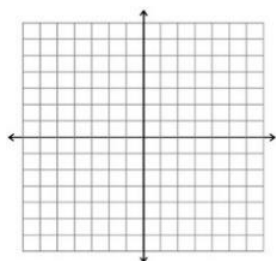
a.  $y = \sqrt{x}$



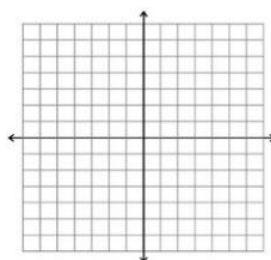
e.  $y = e^x$



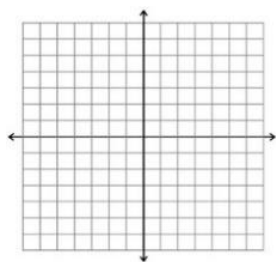
b.  $y = \sqrt{x+5}$



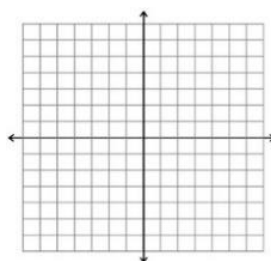
f.  $y = e^{-x}$



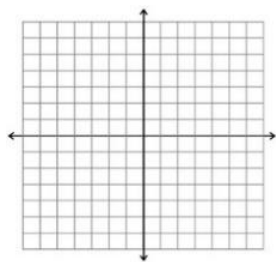
c.  $y = 3 + \sqrt{x}$



g.  $y = \ln x$



d.  $y = 4 - x^2$



20. Match the function to its graph.

| Equation | A | B | C | D | E | F | G |
|----------|---|---|---|---|---|---|---|
| Graph    |   |   |   |   |   |   |   |

A.  $y = x^2$

B.  $y = e^x$

C.  $y = \frac{1}{x}$

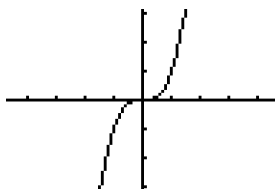
D.  $y = \ln x$

E.  $y = \sqrt{x}$

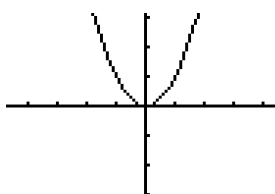
F.  $y = x^3$

G.  $y = |x|$

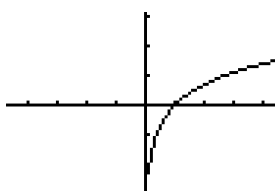
i.



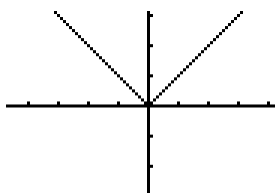
ii.



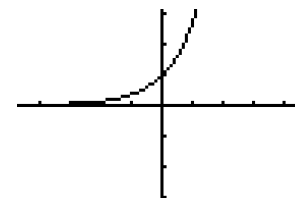
iii.



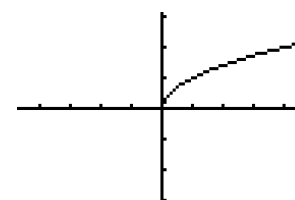
iv.



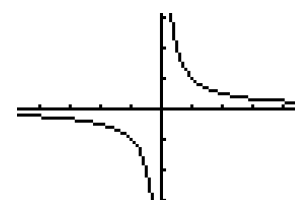
v.



vi.



vii.



#### IV. Odd & Even Functions

State whether the function is odd, even, or neither.

21.  $y = x^4$

25.  $y = \frac{x^3}{x^2 - 1}$

22.  $y = x^3$

26.  $y = \frac{1}{x-1}$

23.  $y = x^3 + x^2$

27.  $y = \cos x$

24.  $y = \sqrt{x^2 + 2}$

28.  $y = \sin x$

## VI. Linear Equations

Write the equation of the line.

29. Passing through the point  $(-2, 3)$  with slope  $\frac{2}{3}$ .

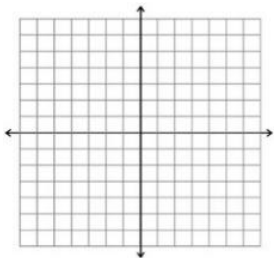
30. Parallel to the line  $y = -2x + 3$  and passing through the point  $(4, 0)$ .

31. Passing through the point  $(0, -3)$  with slope  $\frac{3}{4}$ .

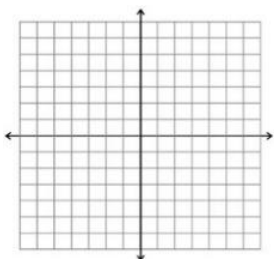
32. Through the points  $(-2, 3)$  and  $(2, -5)$ .

Graph the line. State the coordinates of the  $x$ - and  $y$ -intercepts.

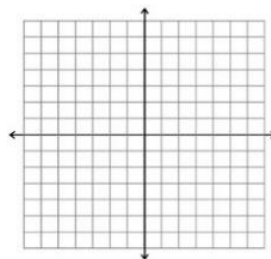
33.  $y = \frac{2}{3}x - 6$



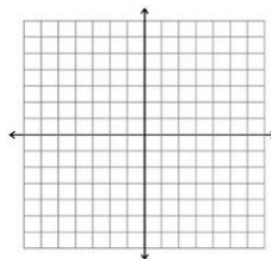
34.  $y = -\frac{1}{3}x + 2$



35.  $y - 2 = 3(x + 3)$



36.  $y + 1 = \frac{2}{3}(x - 5)$





## VII. Greatest Integer Function

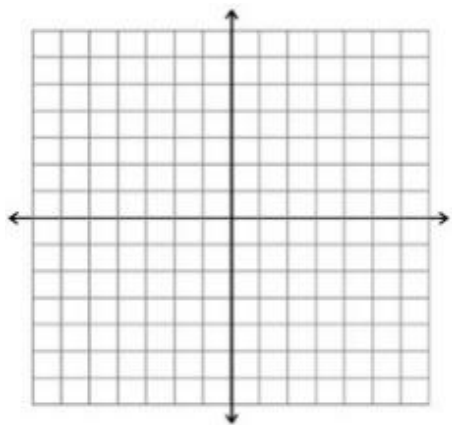
The Greatest Integer Function, also called a Floor Function, can be written in several ways:

$f(x) = \text{int}(x) = \lfloor x \rfloor = x$ . It is defined to mean “the greatest integer less than or equal to  $x$ .”

For example,  $\lfloor 2.3 \rfloor = 2$      $\lfloor 2.7 \rfloor = 2$      $\lfloor 2.9 \rfloor = 2$      $\lfloor 3 \rfloor = 3$

And also,  $\lfloor -4.2 \rfloor = -5$      $\lfloor -4.7 \rfloor = -5$      $\lfloor -5 \rfloor = -5$      $\lfloor -5.2 \rfloor = -6$

37. Graph the function  $y = \text{int}(x)$ .



## VIII. Rational Expressions

38. Rewrite as one fraction.

$$6x^2\sqrt{3x-1} + \frac{3x^3}{\sqrt{3x-1}}$$

39. Simplify (without a calculator; show every step)

$$\left[ -\frac{1}{3} - \frac{1}{2} + 2 \right] - \left[ \frac{8}{3} - 2 - 4 \right]$$

40. Given  $f(x) = e^x \cos x - e^x(\sin x)$ , find  $f\left(\frac{3\pi}{2}\right)$ . Simplify to one term.

## IX. Algebra Skills

Simplify each. No calculator.

41.  $\frac{\sqrt{x}}{x}$

45.  $e^{\ln x}$

49.  $\ln e^{2x}$

42.  $e^{\ln x}$

46.  $(5)^{-1}$

50.  $\frac{2x-1}{2}$

43.  $e^{1+\ln x}$

47.  $(27)^{\frac{2}{3}}$

44.  $\ln 1$

48.  $\ln 6 - \ln 2$

## X. Compositions

Find the composite of the following functions with  $f(x)=x+5$  and  $g(x)=x^2-3$ .

51.  $f(g(x))$

54.  $g(f(0))$

57.  $f(f(x))$

52.  $g(f(x))$

55.  $g(g(-2))$

53.  $f(g(0))$

56.  $f(g(4))$

58. Given the function  $f(x) = x^2 + x$ , solve the equation  $f(\sqrt{x+5}) = 0$ .

59. Given  $f(x) = \frac{1}{x} + x^2$ , find  $f(x^{-2})$ .

## XI. Trigonometry

60. Fill in the chart with the exact values:

|         |    |    |    |
|---------|----|----|----|
| Degrees | 30 | 45 | 60 |
| Radians |    |    |    |
| sin     |    |    |    |
| cos     |    |    |    |
| tan     |    |    |    |

61. Evaluate each of the following using exact values.

a.  $\tan \frac{2\pi}{3}$

d.  $\sec \frac{\pi}{3}$

b.  $\sin \frac{5\pi}{6}$

e.  $\csc \frac{4\pi}{3}$

c.  $\cos \frac{7\pi}{4}$

f.  $\sin \frac{5\pi}{4}$

62. Solve for  $\theta$  on the interval  $[0, 2\pi)$ :

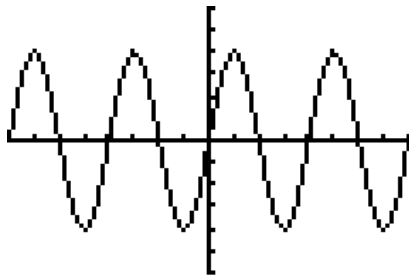
a.  $2\sin\theta + 1 = 0$

b.  $2\sin^2\theta + \sin\theta - 1 = 0$

c.  $\sec\theta = \frac{2\sqrt{3}}{3}$

d.  $2\cos^2\theta - \sqrt{2}\cos\theta = 0$

63. Which equation fits the given graph? [Window shows  $-2\pi \leq x \leq 2\pi$ ]



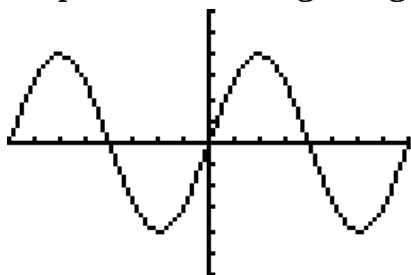
(A)  $y = 4\sin x$

(B)  $y = 4\sin 2x$

(C)  $y = 4\cos x$

(D)  $y = 4\cos 2x$

64. Which equation fits the given graph? [Window shows  $-2\pi \leq x \leq 2\pi$ ]



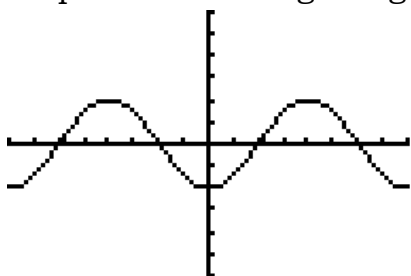
(A)  $y = 4 \sin x$

(C)  $y = 4 \cos x$

(B)  $y = 4 \sin 2x$

(D)  $y = 4 \cos 2x$

65. Which equation fits the given graph? [Window shows  $-2\pi \leq x \leq 2\pi$ ]



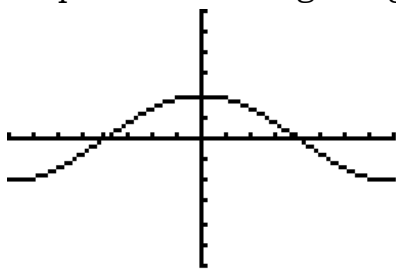
(A)  $y = -2 \cos x$

(C)  $y = -2 \sin x$

(B)  $y = 2 \cos \frac{1}{2}x$

(D)  $y = 2 \sin \frac{1}{2}x$

66. Which equation fits the given graph? [Window shows  $-2\pi \leq x \leq 2\pi$ ]



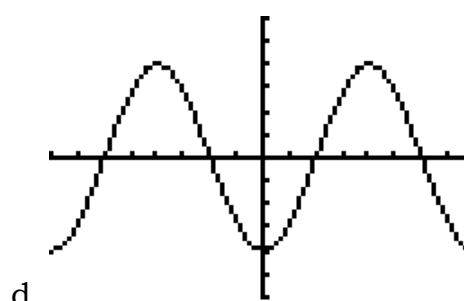
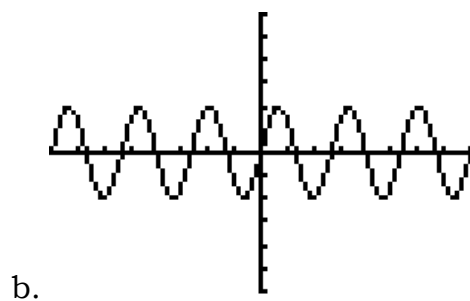
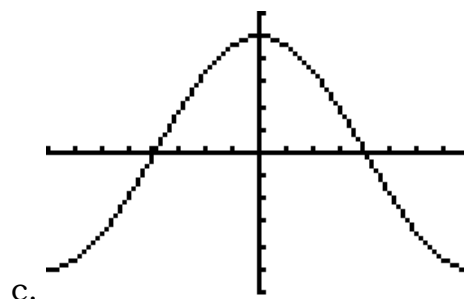
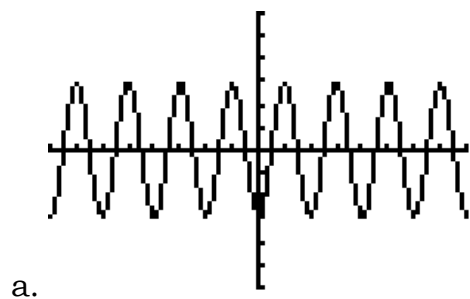
(A)  $y = -2 \cos x$

(C)  $y = -2 \sin x$

(B)  $y = 2 \cos \frac{1}{2}x$

(D)  $y = 2 \sin \frac{1}{2}x$

68. State the amplitude and frequency of the graph shown. [Window shows  $-2\pi \leq x \leq 2\pi$ ]



69. State the amplitude and frequency of the equation:

a.  $y = 4\sin 2x$

b.  $y = -3\cos 2x$

c.  $y = 3\sin \frac{1}{4}x$

## XII. Basic Limits

Evaluate the limits.

70.  $\lim_{x \rightarrow 3} \frac{x^2 - x}{2x - 2}$

72.  $\lim_{x \rightarrow \infty} \frac{3x^7 - \frac{2}{3}x + 4}{x^3 - 24}$

71.  $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$

73.  $\lim_{x \rightarrow -\infty} \frac{5x^4 - x}{x^3 - 10}$

$$74. \lim_{x \rightarrow \infty} \frac{x^3 - 6x + 3}{x - 3}$$

$$76. \lim_{x \rightarrow \infty} \frac{6 - 5x^2}{3x + 4}$$

$$75. \lim_{x \rightarrow -\infty} \frac{9x + 7}{x^3 - 14}$$

$$77. \lim_{x \rightarrow \infty} \frac{4x^2 + 1}{2x^2 - 1}$$

### XIII. Rational Functions

For each function, find all  $x$ -intercepts, Vertical Asymptotes, Holes, and Horizontal Asymptotes.

$$78. y = \frac{2x^2 - 3x + 4}{x^2 - 4}$$

$$79. y = \frac{3x^2 - 3x - 18}{x^2 - 2x - 3}$$

$$80. y = \frac{2x(x-3)(x+1)}{(x+5)(x+1)}$$

## XV. Derivatives

Find the derivative.

$$81. f(t) = t^{-1}(6 + 8t^{-2})$$

$$86. y = \frac{3 \sin x}{9x + \cos x}$$

$$82. y = \frac{3}{x+2}$$

$$87. f(x) = \frac{x^5}{25} - \frac{2}{x^3} + 4x^2$$

$$83. f(x) = 3x^4 + 4x^2 - 2x$$

$$88. y = x^3 \sin^2(4x)$$

$$84. f(x) = \frac{x^2}{4x-1}$$

$$89. y = \cos(5x^2)$$

$$85. f(x) = \frac{x}{3} + \frac{x^2}{4}$$

$$90. f(x) = \sqrt{6x^3 + 3}$$



91.  $g(x) = \sin^3 x$

92.  $f(x) = 3x^4 + 10x^3 - 36x^2 - 4$

93.  $f(x) = 4\sqrt{x^3}$

94.  $f(x) = 6\sqrt{x} - 3x^{-2} + 4x^3 - 7x + 5$