

Calculus AB/BC
Summer Assignment
Mrs. Dailey

Calculus AB Summer '26 Expectations

- Complete all problems in this packet. This should be done without the use of a calculator. Completed packet will be turned in on 1st day of school, 9/2/26.
- No calculator quiz on assignment will be Friday 9/4/26.

Calculus BC Summer '26 Expectations

- Complete all problems in this packet. This should be done without the use of a calculator. Completed packet will be turned in on 1st day of school, 9/2/26.

Additional BC requirements

- Watch the first 7 lessons of Unit 1 on flippedmath.com (1.1-1.7)
- <https://calculus.flippedmath.com/version-1.html>
 - 1.1 Can Change Occur at an Instant? (even practice & test prep)
 - 1.2 Defining Limits and Using Limit Notation (even practice & test prep)
 - 1.3 Estimating Limit Values from Graphs (even practice & test prep)
 - 1.4 Estimating Limit Values from Tables (even practice & test prep)
 - 1.5 Determining Limits Using Algebraic Properties (even practice & test prep)
 - 1.6 Determining Limits Using Algebraic Manipulation (Practice # 5-9,27)
 - 1.7 Selecting Procedures for Determining Limits (Practice # 1,3,10)
- For each video,
 - Complete the class notes section
 - Practice problems as stated above.
 - Each section should take about 45 minutes to complete.
- These topics were covered in Pre-Calculus and it is assumed this is review.
- Unit 1 Class notes Packet (Notes and practice problems) will be collected on 1st day
- Test on Unit 1 will be 2nd week of school.

See you in September!

Mrs. Dailey

Email me at jdailey@pmschools.org if you have any questions.

Name _____

Summer 2026

AB/BC Calc

Getting Ready!

All of the following should be completed without the use of a calculator.

I. Algebra Skills:

Solve for x . Answers may be left in terms of e as appropriate.

1. $x^2 - 4x + 3 = 0$

6. $5^{x-3} = 125^x$

2. $x^2 + 6x = 1$

7. $16^{x-5} = 1024^{286}$

3. $6x^2 + 11x - 10 = 0$

8. $\ln x = 5$

4. $10 = \frac{100}{1 + e^{5x}}$

9. $\log_2 3x - \log_2 3 = 8$

5. $2^x = 64$

10. $4\ln(x+3) = 20$

Solve for x . Answers may be left in logarithmic form as appropriate.

$$11. 5^x = 128$$

$$12. e^{2x} = 144$$

$$13. 3^{x^2} = 18$$

II. Piecewise Functions

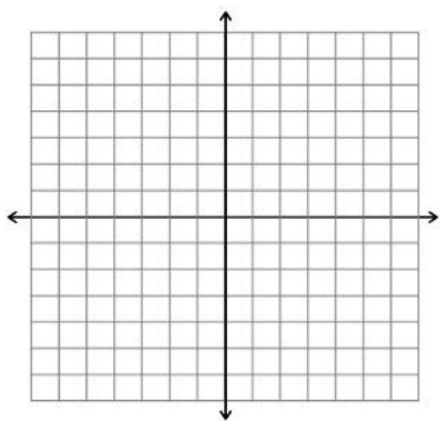
$$14. \text{ Given the function } f(x) = \begin{cases} -x^2 - 1, & x \leq 0 \\ 2, & 0 < x < 4, \\ \sqrt{x}, & x \geq 4 \end{cases}$$

a. Find $f(-2)$.

b. Find $f(9)$.

c. Find $f(0)$.

d. Graph $y = f(x)$.



e. Is $f(x)$ continuous at $x=4$? Why or why not?

f. Is $f(x)$ continuous at $x=0$? Why or why not?

15. Find the value of k that will make the following piecewise function continuous at

$$x = -1. \quad f(x) = \begin{cases} kx^2 + 1, & x \leq -1 \\ 2x - k, & x > -1 \end{cases}$$

16. Find the value of k that will make the following piecewise function continuous at

$$x = 2. \quad f(x) = \begin{cases} 2x - 3, & x \leq 2 \\ x^2 + k, & x > 2 \end{cases}$$

17. Given $f(x) = \begin{cases} 2x+10, & x < 0 \\ x^2+1, & x \geq 0 \end{cases}$, solve for x if $f(x) = 17$.

An absolute value function can be thought of as a piecewise function: $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$.

Here's another example: $|2x+6|$.

$2x+6$ will change signs when $x = -3$.

If $x < -3$, $2x+6$ comes out negative so to find the absolute value, it must be negated.

If $x \geq -3$, $2x+6$ comes out positive and its absolute value does not need to be changed.

Thus, $|2x+6| = \begin{cases} -2x-6, & x < -3 \\ 2x+6, & x \geq -3 \end{cases}$.

18. Express the absolute value function in piecewise form.

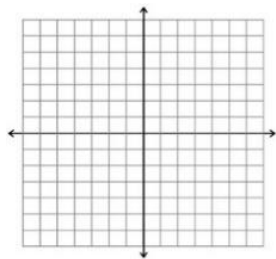
a. $f(x) = |x-1|$

b. $f(x) = |2x+3|$

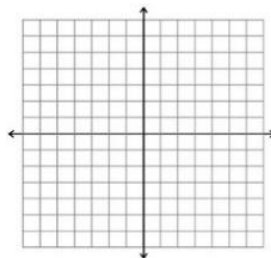
III. Common Functions

19. Graph each of the following functions. Show a few important points for each.

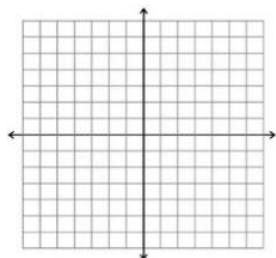
a. $y = \sqrt{x}$



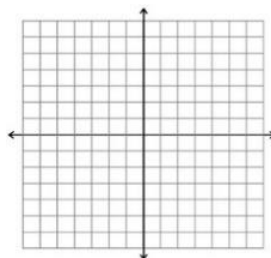
e. $y = e^x$



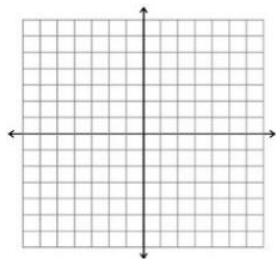
b. $y = \sqrt{x+5}$



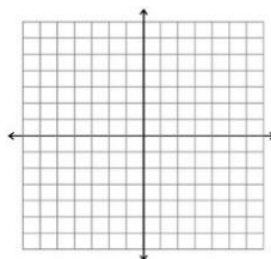
f. $y = e^{-x}$



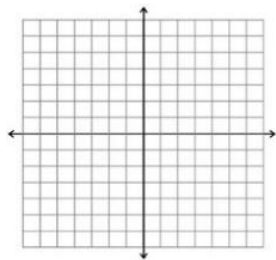
c. $y = 3 + \sqrt{x}$



g. $y = \ln x$



d. $y = 4 - x^2$



20. Match the function to its graph.

Equation	A	B	C	D	E	F	G
Graph							

A. $y = x^2$

B. $y = e^x$

C. $y = \frac{1}{x}$

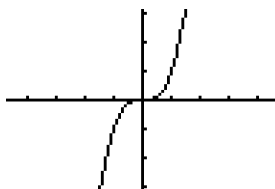
D. $y = \ln x$

E. $y = \sqrt{x}$

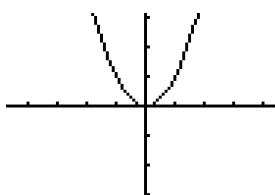
F. $y = x^3$

G. $y = |x|$

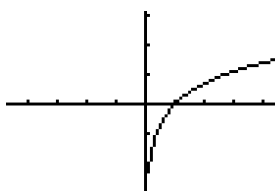
i.



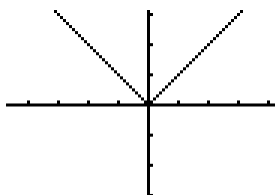
ii.



iii.



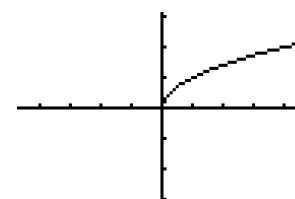
iv.



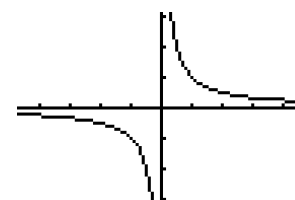
v.



vi.



vii.



IV. Odd & Even Functions

State whether the function is odd, even, or neither.

21. $y = x^4$

25. $y = \frac{x^3}{x^2 - 1}$

22. $y = x^3$

26. $y = \frac{1}{x-1}$

23. $y = x^3 + x^2$

27. $y = \cos x$

24. $y = \sqrt{x^2 + 2}$

28. $y = \sin x$

VI. Linear Equations

Write the equation of the line.

29. Passing through the point $(-2, 3)$ with slope $\frac{2}{3}$.

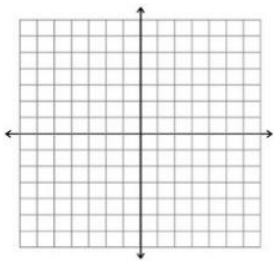
30. Parallel to the line $y = -2x + 3$ and passing through the point $(4, 0)$.

31. Passing through the point $(0, -3)$ with slope $\frac{3}{4}$.

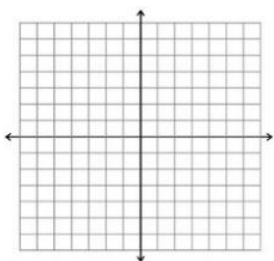
32. Through the points $(-2, 3)$ and $(2, -5)$.

Graph the line. State the coordinates of the x - and y -intercepts.

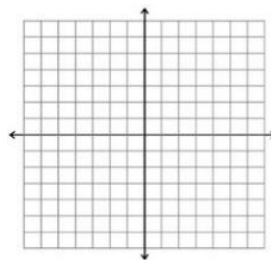
33. $y = \frac{2}{3}x - 6$



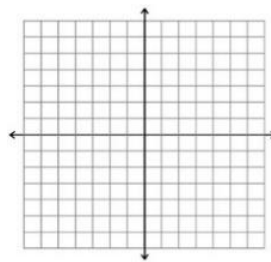
34. $y = -\frac{1}{3}x + 2$



35. $y - 2 = 3(x + 3)$



36. $y + 1 = \frac{2}{3}(x - 5)$



VII. Greatest Integer Function

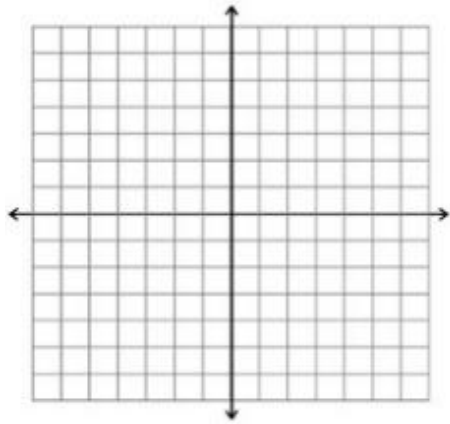
The Greatest Integer Function, also called a Floor Function, can be written in several ways:

$f(x) = \text{int}(x) = \lfloor x \rfloor = x$. It is defined to mean “the greatest integer less than or equal to x .”

For example, $\lfloor 2.3 \rfloor = 2$ $\lfloor 2.7 \rfloor = 2$ $\lfloor 2.9 \rfloor = 2$ $\lfloor 3 \rfloor = 3$

And also, $\lfloor -4.2 \rfloor = -5$ $\lfloor -4.7 \rfloor = -5$ $\lfloor -5 \rfloor = -5$ $\lfloor -5.2 \rfloor = -6$

37. Graph the function $y = \text{int}(x)$.



VIII. Rational Expressions

38. Rewrite as one fraction.

$$6x^2\sqrt{3x-1} + \frac{3x^3}{\sqrt{3x-1}}$$

39. Simplify (without a calculator; show every step)

$$\left[-\frac{1}{3} - \frac{1}{2} + 2 \right] - \left[\frac{8}{3} - 2 - 4 \right]$$

40. Given $f(x) = e^x \cos x - e^x(\sin x)$, find $f\left(\frac{3\pi}{2}\right)$. Simplify to one term.

IX. Algebra Skills

Simplify each. No calculator.

41. $\frac{\sqrt{x}}{x}$

45. $e^{\ln x}$

49. $\ln e^{2x}$

42. $e^{\ln x}$

46. $(5)^{-1}$

50. $\frac{2x-1}{2}$

43. $e^{1+\ln x}$

47. $(27)^{\frac{2}{3}}$

44. $\ln 1$

48. $\ln 6 - \ln 2$

X. Compositions

Find the composite of the following functions with $f(x)=x+5$ and $g(x)=x^2-3$.

51. $f(g(x))$

54. $g(f(0))$

57. $f(f(x))$

52. $g(f(x))$

55. $g(g(-2))$

53. $f(g(0))$

56. $f(g(4))$

58. Given the function $f(x) = x^2 + x$, solve the equation $f(\sqrt{x+5}) = 0$.

59. Given $f(x) = \frac{1}{x} + x^2$, find $f(x^{-2})$.

XI. Trigonometry

60. Fill in the chart with the exact values:

Degrees	30	45	60
Radians			
sin			
cos			
tan			

61. Evaluate each of the following using exact values.

a. $\tan \frac{2\pi}{3}$

d. $\sec \frac{\pi}{3}$

b. $\sin \frac{5\pi}{6}$

e. $\csc \frac{4\pi}{3}$

c. $\cos \frac{7\pi}{4}$

f. $\sin \frac{5\pi}{4}$

62. Solve for θ on the interval $[0, 2\pi)$:

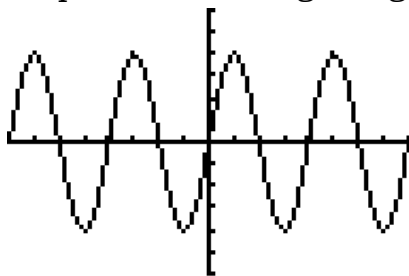
a. $2\sin\theta + 1 = 0$

b. $2\sin^2\theta + \sin\theta - 1 = 0$

c. $\sec\theta = \frac{2\sqrt{3}}{3}$

d. $2\cos^2\theta - \sqrt{2}\cos\theta = 0$

63. Which equation fits the given graph? [Window shows $-2\pi \leq x \leq 2\pi$]



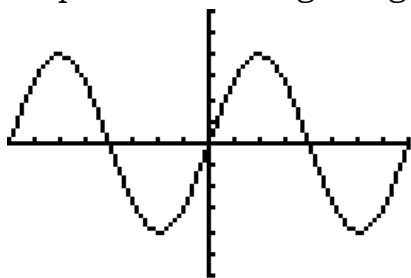
(A) $y = 4\sin x$

(B) $y = 4\sin 2x$

(C) $y = 4\cos x$

(D) $y = 4\cos 2x$

64. Which equation fits the given graph? [Window shows $-2\pi \leq x \leq 2\pi$]



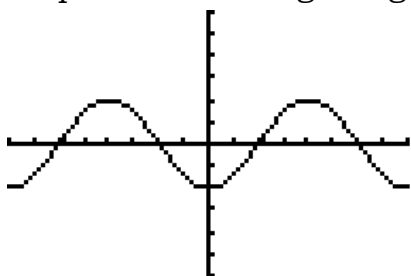
(A) $y = 4\sin x$

(C) $y = 4\cos x$

(B) $y = 4\sin 2x$

(D) $y = 4\cos 2x$

65. Which equation fits the given graph? [Window shows $-2\pi \leq x \leq 2\pi$]



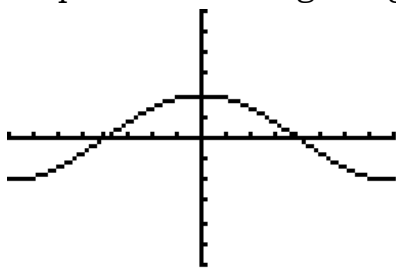
(A) $y = -2\cos x$

(C) $y = -2\sin x$

(B) $y = 2\cos \frac{1}{2}x$

(D) $y = 2\sin \frac{1}{2}x$

66. Which equation fits the given graph? [Window shows $-2\pi \leq x \leq 2\pi$]



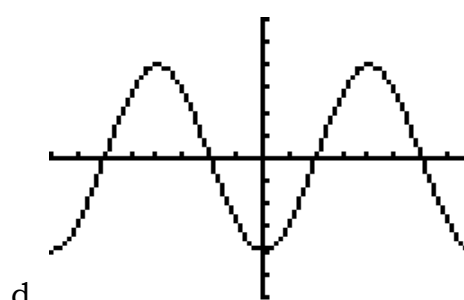
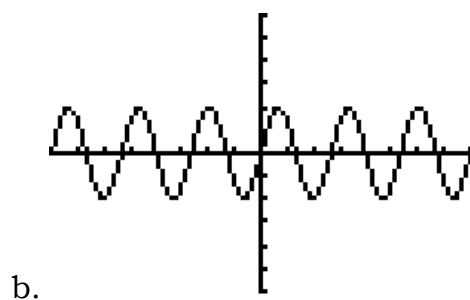
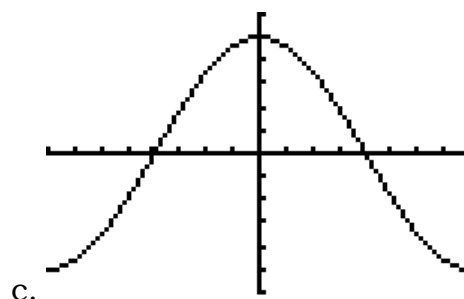
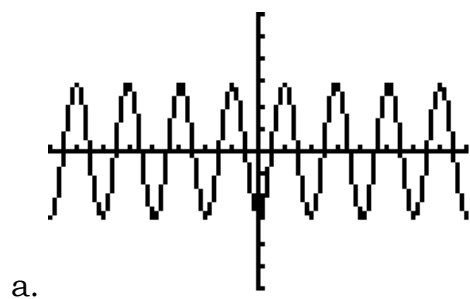
(A) $y = -2\cos x$

(C) $y = -2\sin x$

(B) $y = 2\cos \frac{1}{2}x$

(D) $y = 2\sin \frac{1}{2}x$

68. State the amplitude and frequency of the graph shown. [Window shows $-2\pi \leq x \leq 2\pi$]



69. State the amplitude and frequency of the equation:

a. $y = 4\sin 2x$

b. $y = -3\cos 2x$

c. $y = 3\sin \frac{1}{4}x$

XII. Basic Limits

Evaluate the limits.

70. $\lim_{x \rightarrow 3} \frac{x^2 - x}{2x - 2}$

72. $\lim_{x \rightarrow \infty} \frac{3x^7 - \frac{2}{3}x + 4}{x^3 - 24}$

71. $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$

73. $\lim_{x \rightarrow -\infty} \frac{5x^4 - x}{x^3 - 10}$

$$74. \lim_{x \rightarrow \infty} \frac{x^3 - 6x + 3}{x - 3}$$

$$76. \lim_{x \rightarrow \infty} \frac{6 - 5x^2}{3x + 4}$$

$$75. \lim_{x \rightarrow -\infty} \frac{9x + 7}{x^3 - 14}$$

$$77. \lim_{x \rightarrow \infty} \frac{4x^2 + 1}{2x^2 - 1}$$

XIII. Rational Functions

For each function, find all x -intercepts, Vertical Asymptotes, Holes, and Horizontal Asymptotes.

$$78. y = \frac{2x^2 - 3x + 4}{x^2 - 4}$$

$$79. y = \frac{3x^2 - 3x - 18}{x^2 - 2x - 3}$$

$$80. y = \frac{2x(x-3)(x+1)}{(x+5)(x+1)}$$

XV. Derivatives

Find the derivative.

$$81. f(t) = t^{-1}(6 + 8t^{-2})$$

$$82. y = \frac{3}{x+2}$$

$$83. f(x) = 3x^4 + 4x^2 - 2x$$

$$84. f(x) = \frac{x^2}{4x-1}$$

$$85. f(x) = \frac{x}{3} + \frac{x^2}{4}$$

$$86. y = \frac{3\sin x}{9x + \cos x}$$

$$87. f(x) = \frac{x^5}{25} - \frac{2}{x^3} + 4x^2$$

$$88. y = x^3 \sin^2(4x)$$

$$89. y = \cos(5x^2)$$

$$90. f(x) = \sqrt{6x^3 + 3}$$

91. $g(x) = \sin^3 x$

92. $f(x) = 3x^4 + 10x^3 - 36x^2 - 4$

93. $f(x) = 4\sqrt{x^3}$

94. $f(x) = 6\sqrt{x} - 3x^{-2} + 4x^3 - 7x + 5$

AP Calculus AB
Unit 1
Limits and Continuity

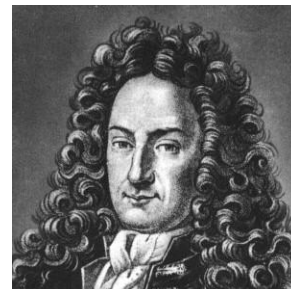
Class Notes
Packet

Name _____

Teacher _____

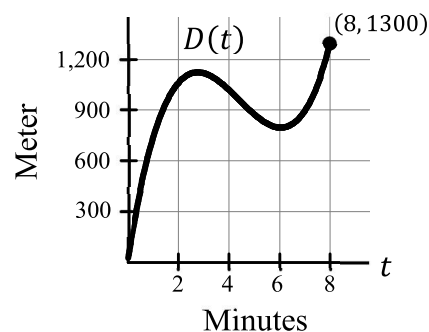
Write your questions
and thoughts here!

The Father(s) of Calculus



Can Change occur at an instant?

1. Mr. Brust's distance from his house is modeled by the function $D(t)$. While riding his bike to the store, he realizes he dropped his wallet and turns around to find it. After finding his wallet, he finishes his ride to the store.



- a. What is his average speed (rate of change) for his trip to the store if he arrives after 8 minutes?
- b. What was his average rate of change between 2 and 6 minutes?
- c. What was his average rate of change between 2 and 3 minutes?

Is it possible to know how fast he was going at an instant?

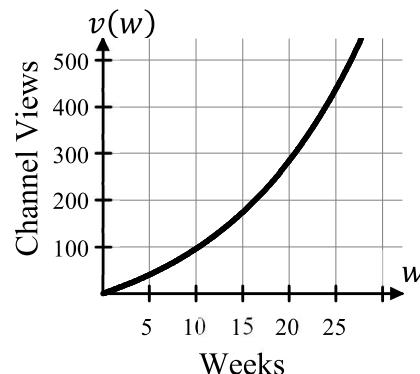
- d. Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $t = 2$.
 - e. Give a rough estimate of the instantaneous rate of change at $t = 2$.
2. $b(t)$ represents the buffalo population in the United States where t is measured in years since 1800.
 - a. What does $b(90)$ represent?
 - b. What does $\frac{b(50)-b(0)}{50-0}$ represent?
 - c. What does $\frac{b(32)-b(31.999)}{32-31.999}$ represent?

1.1 Can change occur at an instant?

Calculus

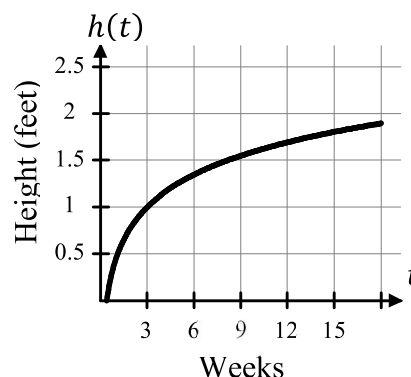
Practice

1. Mr. Kelly has decided to quit his job as a teacher and be a social influencer. The number of views on his new channel is modeled by the function v , where $v(w)$ gives the number of views and w gives the number of weeks since he started the channel for $0 \leq w \leq 26$. The graph of the function v is shown to the right.



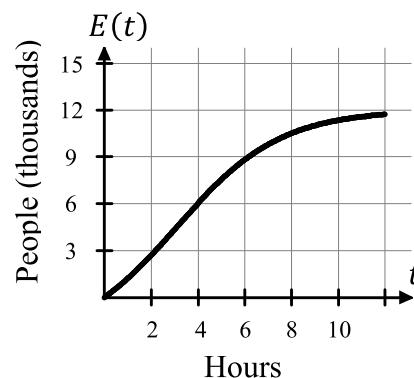
- Draw a tangent line at $w = 10$.
- Give a rough estimate of the instantaneous rate of change at $w = 10$.
- Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $w = 5$.

-
2. The height of a raspberry bush can be modeled by the function h , where $h(t)$ gives the height measured in feet and t gives the number of weeks it was planted for $0 \leq t \leq 12$. The graph of the function h is shown to the right.



- Draw a tangent line at $t = 9$.
- Give a rough estimate of the instantaneous rate of change at $t = 9$.
- Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $t = 12$.

-
3. The number of people who have entered an amusement park is modeled by the function E , where $E(t)$ gives the number of people in thousands who have entered the park and t gives the number of hours since 10:00 a.m. for $0 \leq t \leq 11$. The graph of the function E is shown to the right.



- Draw a tangent line at $t = 3$.
- Give a rough estimate of the instantaneous rate of change at $t = 3$.
- Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $t = 6$.

4. A basketball player's free throw attempts can be modeled by f , where $f(g)$ is the total number of made free throws during the season and g is the number of games for $0 \leq g \leq 82$.

- | | | |
|---------------------------------|---|---|
| a. What does $f(50)$ represent? | b. What does $\frac{f(50)-f(0)}{50-0}$ represent? | c. What does $\frac{f(50)-f(49.999)}{50-49.999}$ represent? |
|---------------------------------|---|---|

5. A monthly electric bill charges for each kilowatt-hour (kWh) used. This can be modeled by k where $k(m)$ is the kWh used for the month and m is the month for $0 \leq m \leq 12$.

- | | | |
|--------------------------------|---|---|
| a. What does $k(8)$ represent? | b. What does $\frac{k(8)-k(5)}{8-5}$ represent? | c. What does $\frac{k(2)-k(1.999)}{2-1.999}$ represent? |
|--------------------------------|---|---|

6. In a country, the number of deaths in a year can be modeled by d , where $d(t)$ is the number of deaths and t is the number of years since 1950 for $0 \leq t \leq 50$.

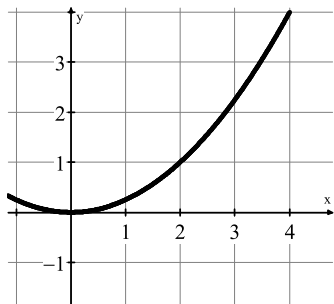
- | | | |
|---------------------------------|---|---|
| a. What does $d(40)$ represent? | b. What does $\frac{d(20)-d(10)}{20-10}$ represent? | c. What does $\frac{d(49)-d(48.999)}{49-48.999}$ represent? |
|---------------------------------|---|---|

7. A dam has a "dam release" that releases water. The amount of water released can be modeled by V , where $V(t)$ is the volume of cubic liters of water and t is the seconds since opening the dam release for $0 \leq t \leq 3600$.

- | | | |
|----------------------------------|---|---|
| a. What does $V(100)$ represent? | b. What does $\frac{V(100)-V(0)}{100-0}$ represent? | c. What does $\frac{V(100)-V(99.999)}{100-99.999}$ represent? |
|----------------------------------|---|---|

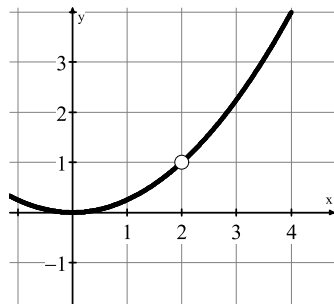
Write your questions
and thoughts here!

Limits

As x approaches ____, $f(x)$ approaches ____.

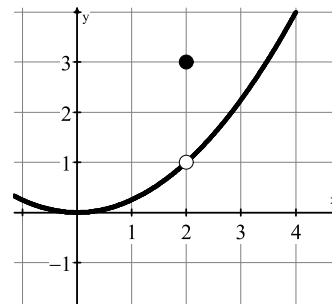
$$\lim_{x \rightarrow 2} f(x) =$$

$$f(2) =$$



$$\lim_{x \rightarrow 2} f(x) =$$

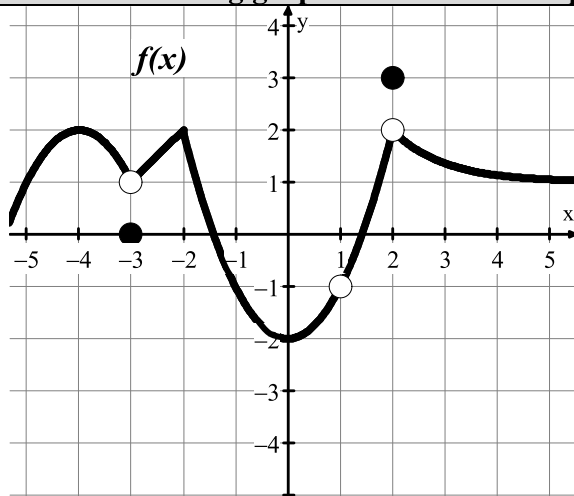
$$f(2) =$$



$$\lim_{x \rightarrow 2} f(x) =$$

$$f(2) =$$

Use the following graph to evaluate each problem.



1. $\lim_{x \rightarrow 1} f(x) =$

2. $f(-3) =$

3. $\lim_{x \rightarrow 2} f(x) =$

4. $f(2) =$

5. $f(1) =$

6. $f(-2) =$

7. $\lim_{x \rightarrow 0} f(x) =$

8. $\lim_{x \rightarrow -3} f(x) =$

9. Give an interpretation of the statement $\lim_{x \rightarrow 7} f(x) = 10$ **A limit does NOT tell us the value of $f(x)$.** It just tells us what the function approaches!True or false? $f(1) = \lim_{x \rightarrow 1} f(x)$ in all cases.True or false? $f(1) \neq \lim_{x \rightarrow 1} f(x)$ in all cases.

1.2 Defining Limits

Calculus

Practice

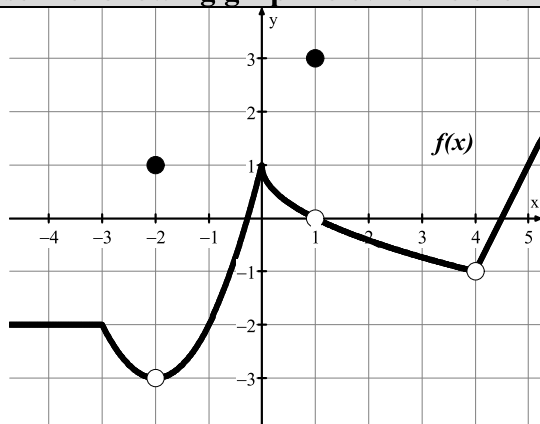
Give an interpretation of each statement.

1. $\lim_{x \rightarrow 1} f(x) = 9$

2. $\lim_{x \rightarrow -2} f(x) = 3$

3. $\lim_{x \rightarrow 4} f(x) = -8$

Use the following graph to evaluate each problem.



4. $f(-2) =$

5. $\lim_{x \rightarrow 1} f(x) =$

6. $\lim_{x \rightarrow -2} f(x) =$

7. $\lim_{x \rightarrow 0} f(x) =$

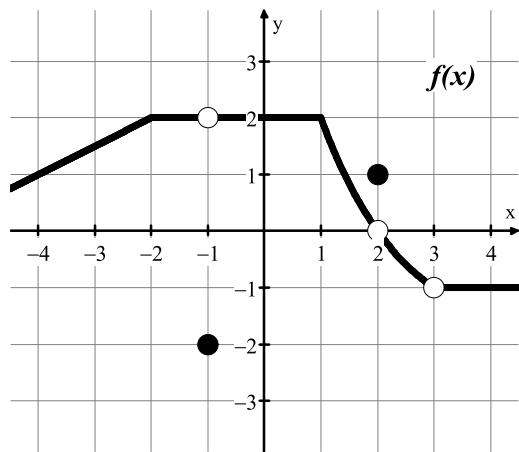
8. $f(4) =$

9. $\lim_{x \rightarrow 4} f(x) =$

10. $\lim_{x \rightarrow -4} f(x) =$

11. $f(1) =$

Use the following graph to evaluate each problem.



12. $\lim_{x \rightarrow -1} f(x) =$

13. $\lim_{x \rightarrow 3} f(x) =$

14. $f(2) =$

15. $\lim_{x \rightarrow -2} f(x) =$

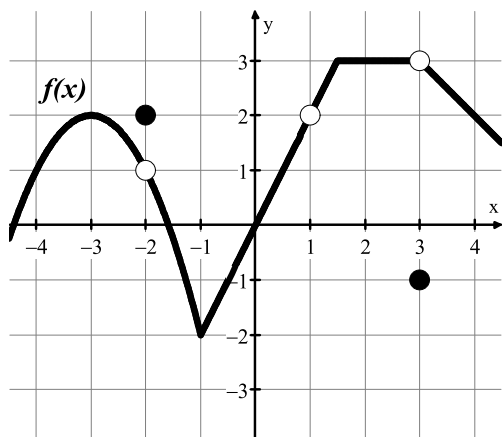
16. $\lim_{x \rightarrow 1} f(x) =$

17. $f(3) =$

18. $f(-1) =$

19. $\lim_{x \rightarrow 2} f(x) =$

Use the following graph to evaluate each problem.



20. $\lim_{x \rightarrow 2} f(x) =$

21. $f(1) =$

22. $\lim_{x \rightarrow 3} f(x) =$

23. $\lim_{x \rightarrow -2} f(x) =$

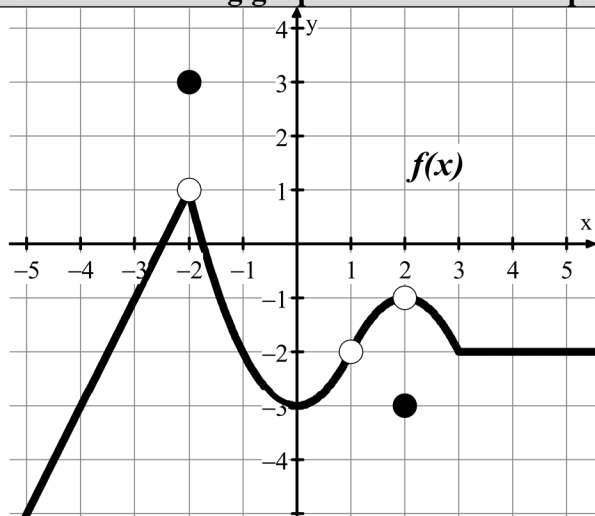
24. $\lim_{x \rightarrow 1} f(x) =$

25. $f(-2) =$

26. $\lim_{x \rightarrow -3} f(x) =$

27. $f(3) =$

Use the following graph to evaluate each problem.



28. $\lim_{x \rightarrow -2} f(x) =$

29. $\lim_{x \rightarrow 1} f(x) =$

30. $\lim_{x \rightarrow 2} f(x) =$

31. $f(-2) =$

32. $f(1) =$

33. $\lim_{x \rightarrow 0} f(x) =$

34. $\lim_{x \rightarrow -4} f(x) =$

35. $f(2) =$

1.2 Defining Limits

Test Prep

36. Let f be a function that is defined for all real numbers x . Of the following, which is the best interpretation of the statement $\lim_{x \rightarrow 4} f(x) = 8$.

- (A) The value of the function f at $x = 4$ is 8.
- (B) The value of the function f at $x = 8$ is 4.
- (C) As x approaches 4, the values of $f(x)$ approach 8.
- (D) As x approaches 8, the values of $f(x)$ approach 4.

37. Let f be a function that is defined for all real numbers x . Of the following, which is the best interpretation of the statement $\lim_{x \rightarrow -1} f(x) = 2$.

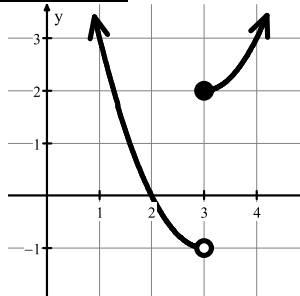
- (A) As x approaches 2, the values of $f(x)$ approach -1
- (B) The value of the function f at $x = -1$ is 2.
- (C) The value of the function f at $x = 2$ is -1 .
- (D) As x approaches -1 , the values of $f(x)$ approach 2.

Write your questions
and thoughts here!

What is a *one-sided limit*?

A *one-sided limit* is the _____ a function approaches as you approach a given _____ from either the _____ or _____ side.

Example 1



The limit of f as x approaches 3 from the left side is -1 .

$\lim_{x \rightarrow 3^-} f(x) =$

The limit of f as x approaches 3 from the right side is 2 .

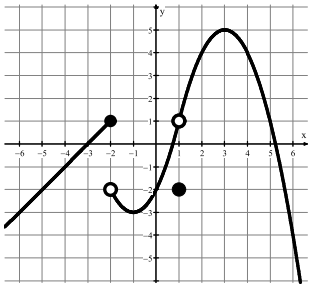
$\lim_{x \rightarrow 3^+} f(x) =$

If the two sides are different?

$\lim_{x \rightarrow 3} f(x) =$

Example 2

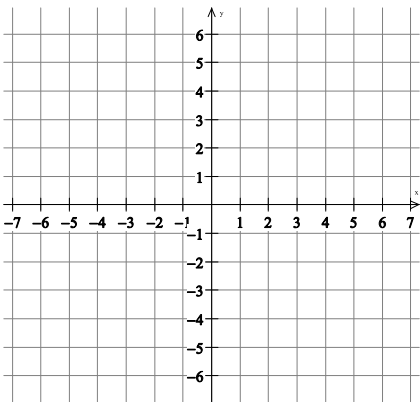
a. $\lim_{x \rightarrow -2^-} f(x) =$	b. $\lim_{x \rightarrow -2^+} f(x) =$	c. $\lim_{x \rightarrow -2} f(x) =$
d. $\lim_{x \rightarrow 1} f(x) =$	e. $\lim_{x \rightarrow 0} f(x) =$	f. $\lim_{x \rightarrow 3^-} f(x) =$
g. $\lim_{x \rightarrow -1} f(x) =$	h. $f(1) =$	i. $f(-2) =$



Example 3

Sketch a graph of a function g that satisfies all of the following conditions.

- a. $g(3) = -1$
- b. $\lim_{x \rightarrow 3} g(x) = 4$
- c. $\lim_{x \rightarrow -2^+} g(x) = 1$
- d. g is increasing on $-2 < x < 3$
- e. $\lim_{x \rightarrow -2^-} g(x) > \lim_{x \rightarrow -2^+} g(x)$



1.3 Finding Limits from Graphs

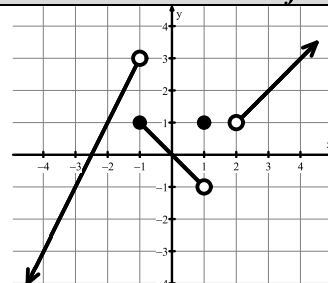
Calculus

Practice

For 1-3, give the value of each statement. If the value does not exist, write “does not exist” or “undefined.”

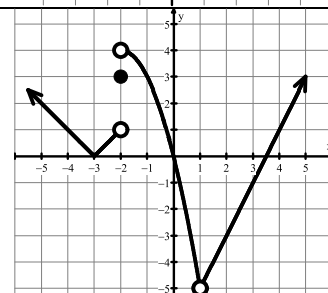
1.

- a. $\lim_{x \rightarrow -1^-} f(x) =$ b. $f(1) =$ c. $\lim_{x \rightarrow 0} f(x) =$
 d. $\lim_{x \rightarrow 2^+} f(x) =$ e. $f(-1) =$ f. $f(2) =$
 g. $\lim_{x \rightarrow -1^+} f(x) =$ h. $\lim_{x \rightarrow 1^-} f(x) =$ i. $\lim_{x \rightarrow 2} f(x) =$



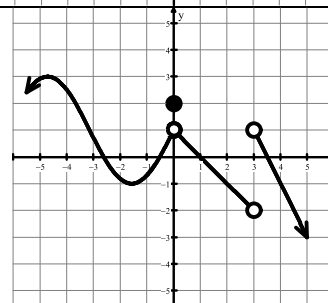
2.

- a. $\lim_{x \rightarrow -3} f(x) =$ b. $f(1) =$ c. $\lim_{x \rightarrow 1} f(x) =$
 d. $\lim_{x \rightarrow -2^+} f(x) =$ e. $f(3) =$ f. $\lim_{x \rightarrow -2^-} f(x) =$
 g. $\lim_{x \rightarrow -2} f(x) =$ h. $f(-2) =$ i. $f(4) =$



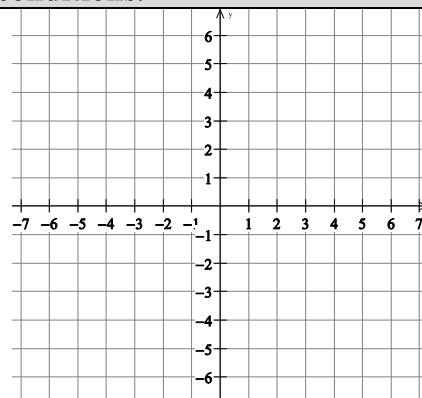
3.

- a. $\lim_{x \rightarrow 3^+} f(x) =$ b. $f(3) =$ c. $\lim_{x \rightarrow 0} f(x) =$
 d. $\lim_{x \rightarrow 3} f(x) =$ e. $f(0) =$ f. $\lim_{x \rightarrow 3^-} f(x) =$
 g. $\lim_{x \rightarrow 0^+} f(x) =$ h. $f(1) =$



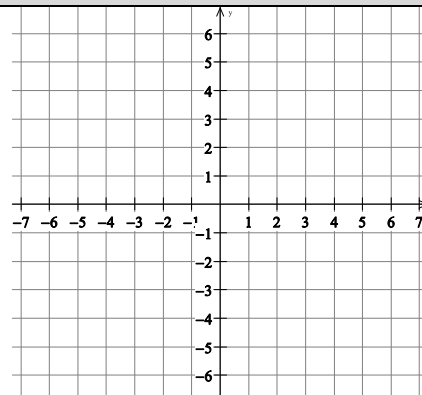
4. Sketch a graph of a function f that satisfies all of the following conditions.

- a. $f(-2) = 5$
 b. $\lim_{x \rightarrow -2} f(x) = 1$
 c. $\lim_{x \rightarrow 4^+} f(x) = 3$
 d. f is increasing on $x < -2$
 e. $\lim_{x \rightarrow 4^-} f(x) < \lim_{x \rightarrow 4^+} f(x)$



5. Sketch a graph of a function g that satisfies all of the following conditions.

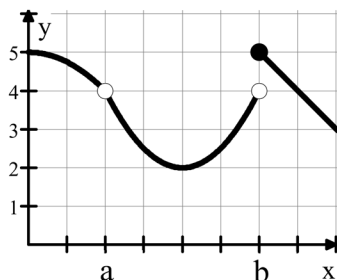
- a. $g(1) = 3$
 b. $\lim_{x \rightarrow 1} g(x) = -2$
 c. $\lim_{x \rightarrow -3^+} g(x) = 5$
 d. g is increasing only on $-5 < x < -3$ and $x > 1$
 e. $\lim_{x \rightarrow -3^-} g(x) > \lim_{x \rightarrow -3^+} g(x)$



1.3 Finding Limits from Graphs

Test Prep

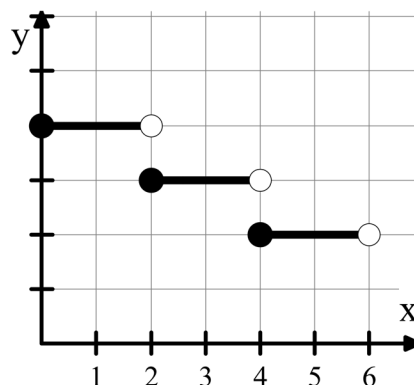
6. The graph of the function f is shown. Which of the following statements about f is true?



- (A) $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow b} f(x)$ (B) $\lim_{x \rightarrow a} f(x) = 4$
 (C) $\lim_{x \rightarrow b} f(x) = 4$ (D) $\lim_{x \rightarrow b} f(x) = 5$
 (E) $\lim_{x \rightarrow a} f(x)$ does not exist.

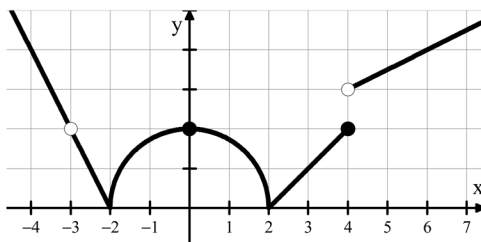
7. The figure below shows the graph of a function f with domain $0 \leq x < 6$. Which of the following statements are true?

- I. $\lim_{x \rightarrow 4^-} f(x)$ exists.
 II. $\lim_{x \rightarrow 4^+} f(x)$ exists.
 III. $\lim_{x \rightarrow 4} f(x)$ exists.



- (A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

8. The graph of a function f is shown below. For which of the following values of c does $\lim_{x \rightarrow c} f(x) = 2$?



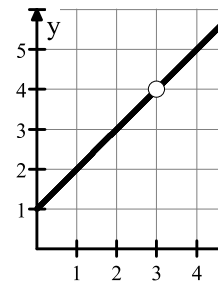
- (A) 0 only (B) 0 and 4 only (C) -3 and 0 only
 (D) -3 and 4 only (E) -3, 0, and 4

Write your questions
and thoughts here!**Calculator required for part of this lesson!**

If we have the graph, it is easy to see the value of $\lim_{x \rightarrow 3} f(x) =$

Without the graph, we could use a table of values.

x	2.9	2.99	3.01	3.1
$f(x)$	3.9	3.99	4.01	4.1



1. According to the table, what is the value of $\lim_{x \rightarrow -4} f(x)$?

x	-4.4	-4.001	-3.999	-3.5
$f(x)$	2.43	2.499	2.501	2.68

2. If $f(x) = \frac{x^3 - 4x^2 - 7x + 10}{x + 2}$, create your own table of values to help you evaluate $\lim_{x \rightarrow -2} f(x)$.

x					
$f(x)$					

$$\lim_{x \rightarrow -2} f(x) =$$

Several ways to find values of a function on a calculator. Here are two:

- Table values (not as accurate, but fast)
- Function Notation

3. The function f is continuous and increasing for $x \geq 1$. The table gives values of f at selected values of x . Approximate the value of $\lim_{x \rightarrow 2} \cos(f(x))$.

x	1.99	1.999	2.001	2.01
$f(x)$	4.85	4.999	5.001	5.15

$$\lim_{x \rightarrow 2} \cos(f(x)) =$$

1.4 Finding Limits from Tables

Calculus

Practice

Use the table for each problem to evaluate the limit.

1. $\lim_{x \rightarrow 9} f(x) =$

x	8.7	8.999	9.001	9.8
$f(x)$	-5.8	-5.001	-4.999	-4

2. $\lim_{x \rightarrow -7} f(x) =$

x	-7.5	-7.001	-6.999	-6.5
$f(x)$	3.8	3.501	3.499	3.2

3. $\lim_{x \rightarrow -2} f(x) =$

x	-2.1	-2.001	-1.999	-1.9
$f(x)$	-8.7	-8.999	-9.001	-9.4

4. $\lim_{x \rightarrow 11} f(x) =$

x	10.7	10.99	11.01	11.3
$f(x)$	10.3	10.001	9.999	9.6

For each function, create your own table of values to evaluate the limit.

5. $f(x) = \frac{x^2 - 2x - 35}{x + 5}$

x					
$f(x)$					

$\lim_{x \rightarrow -5} f(x) =$

6. $f(x) = \frac{x^2 + 5x + 6}{x + 2}$

x					
$f(x)$					

$\lim_{x \rightarrow -2} f(x) =$

7. $f(x) = \frac{x^2 + 4x - 12}{x - 2}$

x					
$f(x)$					

$\lim_{x \rightarrow 2} f(x) =$

8. $f(x) = \frac{5x^3 + 2x^2 - 13x + 6}{x - 1}$

x					
$f(x)$					

$\lim_{x \rightarrow 1} f(x) =$

Use the information given for each problem to evaluate the limit. Always round (or truncate) answers to three decimal places!

9. The function f is continuous and increasing $x \geq 0$. The table gives values of f at selected values of x .

x	6.9	6.999	7.001	7.1
$f(x)$	3.7	3.999	4.001	4.16

Approximate the value of $\lim_{x \rightarrow 7} 2 \cos(f(x))$.

10. The function f is continuous and decreasing for $x \geq 3$. The table gives values of f at selected values of x .

x	4.9	4.999	5.001	5.1
$f(x)$	2.2	2.001	1.999	1.75

Approximate the value of $\lim_{x \rightarrow 5} e^{3f(x)}$.

11. The function f is continuous and decreasing for $x \geq -5$. The table gives values of f at selected values of x .

x	-3.1	-3.01	-2.99	-2.8
$f(x)$	-3.4	-3.499	-3.501	-3.8

Approximate the value of $\lim_{x \rightarrow -3} \ln(-f(x))$.

12. The function f is continuous and increasing for $x \geq -7$. The table gives values of f at selected values of x .

x	-5.1	-5.001	-4.999	-4.8
$f(x)$	3.7	3.999	4.001	4.2

Approximate the value of $\lim_{x \rightarrow -5} \sqrt[5]{f(x)}$.

1.4 Finding Limits from Tables

Test Prep

13. The table below shows values of the function f at selected values of x . Which of the following is true based on the data from the table?

x	8.9	8.99	8.999	9.001	9.01	9.1
$f(x)$	0.7	0.8	0.999	2.001	2.01	2.3

(A) $\lim_{x \rightarrow 9} f(x) = 1$

(B) $\lim_{x \rightarrow 9} f(x) = 2$

(C) $\lim_{x \rightarrow 9^-} f(x) = 2$ and $\lim_{x \rightarrow 9^+} f(x) = 1$

(D) $\lim_{x \rightarrow 9^-} f(x) = 1$ and $\lim_{x \rightarrow 9^+} f(x) = 2$

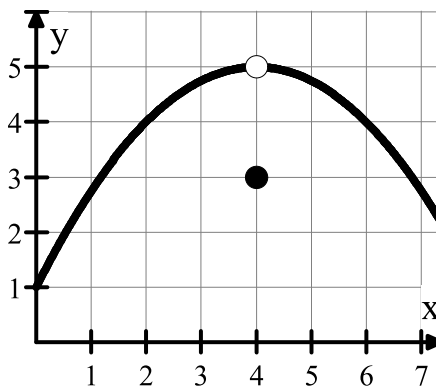
14. The graph of the function f is shown to the right. The value of $\lim_{x \rightarrow 4} 2 \cos(f(x))$ is

(A) 0.567

(B) -1.307

(C) -1.979

(D) Does not exist



15. If $[x]$ represents the greatest integer that is less than or equal to x , then $\lim_{x \rightarrow 0^-} \frac{2}{[x]} =$

(A) -2

(B) -1

(C) 0

(D) 2

(E) the limit does not exist

Write your questions and thoughts here!



$x + x =$

$\lim_{x \rightarrow c} [f(x) + f(x)] =$

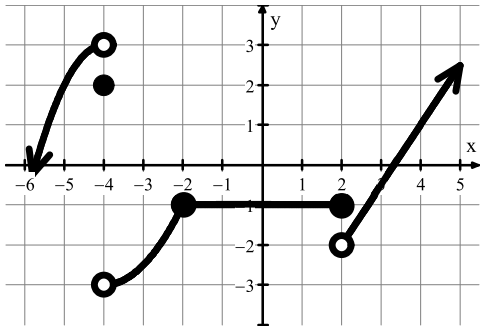
Example 1:

$\lim_{x \rightarrow -1} f(x) = 2$	$\lim_{x \rightarrow 1} f(x) = 4$	$\lim_{x \rightarrow 1} g(x) = 6$
------------------------------------	-----------------------------------	-----------------------------------

The table above gives selected limits of the functions f and g . What is $\lim_{x \rightarrow 1} \left(f(-x) + \frac{g(x)}{2} \right)$?

Example 2:

The graph of the function f is shown on the right. What is $\lim_{x \rightarrow 4} f(f(x))$?



Graph of f

Example 3:

$f(5) = 1$	$\lim_{x \rightarrow 5} f(x) = 6$
$g(5) = 2$	$\lim_{x \rightarrow 5} g(x) = -1$
$h(5) = 3$	$\lim_{x \rightarrow 5} h(x) = 5$

The table above gives selected values and limits of the functions f , g , and h .

What is $\lim_{x \rightarrow 5} \left(h(x)(f(x) + 2g(x)) \right) - h(5)$?

1.5 Algebraic Properties of Limits

Calculus

Notes

Example 4: Piecewise Functions

Piecewise defined functions and limits

$$f(x) = \begin{cases} \sqrt{11-x}, & x < -5 \\ \frac{x+3}{5-x^2}, & x \geq -5 \end{cases}$$

a. $\lim_{x \rightarrow -5^-} f(x) =$ b. $\lim_{x \rightarrow -5^+} f(x) =$

c. $\lim_{x \rightarrow -5} f(x) =$

$$g(x) = \begin{cases} \sqrt{10-x^2}, & x < -1 \\ \frac{26-5x^2}{7}, & -1 < x \leq e \\ \ln x^3, & x > e \end{cases}$$

a. $\lim_{x \rightarrow -1} g(x) =$ b. $\lim_{x \rightarrow e^+} g(x) =$

c. $\lim_{x \rightarrow e} g(x) =$

Practice

Use the table for each problem to find the given limits.

1.

$\lim_{x \rightarrow 3} f(x) = 4$	$\lim_{x \rightarrow -3} f(x) = 2$	$\lim_{x \rightarrow 3} g(x) = 1$	$\lim_{x \rightarrow -3} g(x) = 5$
-----------------------------------	------------------------------------	-----------------------------------	------------------------------------

a. $\lim_{x \rightarrow 3} (2f(x) + g(-x))$ b. $\lim_{x \rightarrow -3} \left(\frac{g(x)}{f(-x)} \right)$

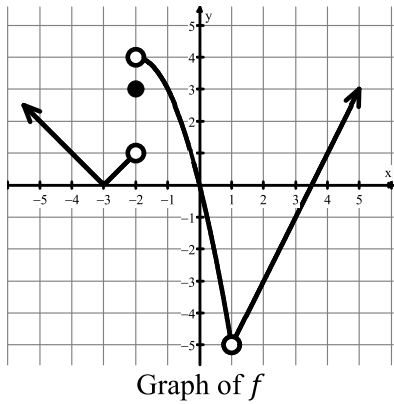
2.

$\lim_{x \rightarrow 2} f(x) = -1$	$\lim_{x \rightarrow 1} f(x) = 6$	$\lim_{x \rightarrow 4} f(x) = 2$	$\lim_{x \rightarrow -2} f(x) = -3$
------------------------------------	-----------------------------------	-----------------------------------	-------------------------------------

a. $\lim_{x \rightarrow 2} \left(f(2x) - f\left(\frac{x}{2}\right) \right)$ b. $\lim_{x \rightarrow 2} \left(\frac{f\left(\frac{x}{2}\right)}{f(-x)} \right)$

Use the graph for each problem to find the given limits.

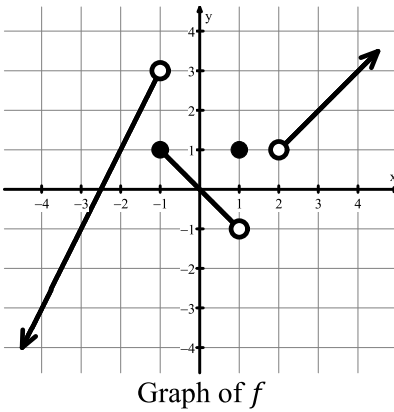
3.



a. $\lim_{x \rightarrow 3} f(f(x)) =$

b. $\lim_{x \rightarrow 1} f(f(x)) =$

4.



a. $\lim_{x \rightarrow -2} f(f(x)) =$

b. $\lim_{x \rightarrow 4} f(f(x)) =$

Use the table for each problem to find the given limits.

5.

$f(1) = 4$	$\lim_{x \rightarrow 1} f(x) = -1$
$g(1) = -2$	$\lim_{x \rightarrow 1} g(x) = 3$
$h(1) = -3$	$\lim_{x \rightarrow 1} h(x) = 6$

a. $\lim_{x \rightarrow 1} ((f(x))^2 - h(x)) - g(1)$

b. $f(1) + \lim_{x \rightarrow 1} (-g(x))$

6.

$f(-2) = 7$	$\lim_{x \rightarrow -2} f(x) = 2$
$g(-2) = 1$	$\lim_{x \rightarrow -2} g(x) = -1$
$h(-2) = -4$	$\lim_{x \rightarrow -2} h(x) = -3$

a. $\lim_{x \rightarrow -2} (h(x)(2f(x))) + h(-2)$

b. $f(-2) \lim_{x \rightarrow -2} (g(x) - h(x))$

Use the piecewise functions to find the given values.

7. $g(x) = \begin{cases} \sqrt{5-x}, & x < -4 \\ x^2 - 5, & -4 \leq x < 2 \\ x - 3, & x \geq 2 \end{cases}$

a. $\lim_{x \rightarrow 2^-} g(x) =$

b. $\lim_{x \rightarrow -4^+} g(x) =$

c. $g(2) =$

d. $\lim_{x \rightarrow -4^-} g(x) =$

e. $\lim_{x \rightarrow 2^+} g(x) =$

f. $\lim_{x \rightarrow 2} g(x) =$

g. $\lim_{x \rightarrow -4} g(x) =$

h. $g(-4) =$

8. $h(x) = \begin{cases} -|x|, & x \leq -5 \\ 20 - x^2, & -5 < x \leq 3 \\ 4x - 1, & x > 3 \end{cases}$

a. $\lim_{x \rightarrow -5^+} h(x) =$

b. $\lim_{x \rightarrow -5} h(x) =$

c. $h(3) =$

d. $\lim_{x \rightarrow -5^-} h(x) =$

e. $\lim_{x \rightarrow 3^+} h(x) =$

f. $\lim_{x \rightarrow 3} h(x) =$

g. $h(-5) =$

h. $\lim_{x \rightarrow 3^-} h(x) =$

$$9. w(\theta) = \begin{cases} \sin \theta, & \theta \leq \pi \\ \cos \theta, & \pi < \theta < 2\pi \\ \tan \theta, & \theta > 2\pi \end{cases}$$

$$a. \lim_{x \rightarrow \pi^-} w(\theta) =$$

$$b. w(\pi) =$$

$$c. \lim_{x \rightarrow \pi^+} w(\theta) =$$

$$d. \lim_{x \rightarrow 2\pi^-} w(\theta) =$$

$$e. \lim_{x \rightarrow \pi} w(\theta) =$$

$$f. \lim_{x \rightarrow 2\pi^+} w(\theta) =$$

$$g. \lim_{x \rightarrow 2\pi} w(\theta) =$$

$$h. w(2\pi) =$$

$$10. f(x) = \begin{cases} \frac{1}{x+6}, & x < -2 \\ 2^x, & -2 \leq x < 0 \\ x^2 - 4, & x \geq 0 \end{cases}$$

$$a. \lim_{x \rightarrow -2} f(x) =$$

$$b. \lim_{x \rightarrow -2^-} f(x) =$$

$$c. \lim_{x \rightarrow -2^+} f(x) =$$

$$d. \lim_{x \rightarrow 0} f(x) =$$

$$e. \lim_{x \rightarrow 0^-} f(x) =$$

$$f. \lim_{x \rightarrow 0^+} f(x) =$$

$$g. f(-2) =$$

$$h. f(0) =$$

1.5 Algebraic Properties of Limits

Test Prep

11. If f is a continuous function such that $f(3) = 7$, which of the following statements must be true?

(A) $\lim_{x \rightarrow 3} f(3x) = 9$

(B) $\lim_{x \rightarrow 3} f(2x) = 14$

(C) $\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3} = 7$

(D) $\lim_{x \rightarrow 3} f(x^2) = 49$

(E) $\lim_{x \rightarrow 3} (f(x))^2 = 49$

12. If $f(x) = \begin{cases} \ln 3x, & 0 < x \leq 3 \\ x \ln 3, & 3 < x \leq 4 \end{cases}$, then $\lim_{x \rightarrow 3} f(x)$ is

(A) $\ln 9$

(B) $\ln 27$

(C) $3 \ln 3$

(D) $3 + \ln 3$

(E) nonexistent

13. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4 \end{cases}$, then $\lim_{x \rightarrow 2} f(x)$ is

(A) $\ln 2$

(B) $\ln 8$

(C) $\ln 16$

(D) 4

(E) nonexistent

Write your questions
and thoughts here!



Direct Substitution		Factor and Cancel	
1. $\lim_{x \rightarrow -1} (x^2 + 2x - 4)$	2. $\lim_{x \rightarrow 2} 6$	3. $\lim_{x \rightarrow 0} \frac{4x^2 - 5x}{x}$	4. $\lim_{x \rightarrow -7} \frac{2x^2 + 13x - 7}{x + 7}$

Limit Does Not Exist

5. $\lim_{x \rightarrow -6} \frac{x^2 + 4x + 3}{x + 6}$

Special Trig Limits:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \quad \text{or} \quad \lim_{x \rightarrow 0} \frac{x}{\sin x} =$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = \quad \text{or} \quad \lim_{x \rightarrow 0} \frac{\cos x - 1}{x} =$$

6. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$

7. $\lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 9x}$

8. $\lim_{x \rightarrow 0} \frac{\cos^2 x - 1}{x(\cos x + 1)}$

1.6 Algebraic Manipulation of Limits

Calculus

Practice

Evaluate each limit.

1. $\lim_{x \rightarrow 2} (x - x^2)$	2. $\lim_{x \rightarrow 5} (x + 1)^2$	3. $\lim_{x \rightarrow 1} \frac{x^2 - 5x}{x - 1}$	4. $\lim_{x \rightarrow 1} \frac{x^2 + x - 30}{x - 1}$
5. $\lim_{x \rightarrow 0} \frac{3x}{\sin x}$	6. $\lim_{x \rightarrow 0} \frac{\sin(2x)}{3x}$	7. $\lim_{x \rightarrow -2} (3x^2 - x + 1)$	8. $\lim_{x \rightarrow 3} (2x^2 + 5x - 6)$
9. $\lim_{x \rightarrow -7} \frac{2x^3 + 11x^2 - 21x}{x^2 + 7x}$	10. $\lim_{x \rightarrow 8} \frac{x^2 + 2x - 80}{x - 8}$	11. $\lim_{x \rightarrow \frac{1}{3}} \frac{6x^2 + 13x - 5}{3x - 1}$	12. $\lim_{x \rightarrow 0} \frac{7x^2 + x}{x}$
13. $\lim_{x \rightarrow -3} 14$	14. $\lim_{x \rightarrow 0} \frac{x^2 + 2x - 8}{x - 4}$	15. $\lim_{x \rightarrow -2} \frac{x^2 - 4x - 10}{x}$	16. $\lim_{x \rightarrow 0} \frac{3x^2 + 5x}{x}$

$$17. \lim_{x \rightarrow 4} \frac{5x^2 - 21x + 4}{x - 4}$$

$$18. \lim_{x \rightarrow \frac{1}{2}} \frac{1 - x - 2x^2}{2x - 1}$$

$$19. \lim_{x \rightarrow \pi} \cos x$$

$$20. \lim_{x \rightarrow \frac{\pi}{8}} \sin(4x)$$

$$21. \lim_{x \rightarrow 2} \frac{x^2 + 6x - 16}{2 - x}$$

$$22. \lim_{x \rightarrow 5} \frac{2x^2 - 17x + 35}{5 - x}$$

$$23. \lim_{x \rightarrow 0} \frac{(1 - \cos^2 x) \sin x}{x^2}$$

1.6 Algebraic Manipulation of Limits

Test Prep

24. Evaluate $\lim_{x \rightarrow 1} \frac{\ln x}{3x}$ is

- (A) 0 (B) $\frac{3}{e}$ (C) e (D) 3 (E) The limit does not exist.

25. $\lim_{x \rightarrow 0} \frac{\sin x \cos x - \sin x}{x^2}$ is

- (A) 2 (B) $\frac{40}{3}$ (C) ∞ (D) 0 (E) undefined

26. $\lim_{x \rightarrow a} \frac{x^2 - 2ax + a^2}{x - a} =$

- (A) $-\infty$ (B) a (C) 0 (D) ∞ (E) The limit does not exist.

27. $\lim_{x \rightarrow 0} \left(\frac{3x^2 + 5\cos x - 5}{2x} \right) =$

- (A) 0 (B) $\frac{5}{2}$ (C) 3 (D) 4 (E) Does not exist

Write your questions
and thoughts here!

**Rationalize Fractions with Radicals**

1. $\lim_{x \rightarrow 5} \frac{\sqrt{x+4}-3}{x-5}$

2. $\lim_{x \rightarrow 10} \frac{x-10}{3-\sqrt{x-1}}$

Complex Fractions

3. $\lim_{x \rightarrow 0} \frac{x}{\frac{1}{x-4} + \frac{1}{4}}$

4. $\lim_{x \rightarrow 0} \frac{\frac{1}{(x+3)^2} - \frac{1}{9}}{x}$

1.7 Selecting Procedures for Determining Limits

Practice

Calculus

Evaluate each limit.

1. $\lim_{x \rightarrow 0} \frac{\sqrt{x+7} - \sqrt{7}}{x}$

2. $\lim_{x \rightarrow 1} \sqrt{x+4}$

3. $\lim_{x \rightarrow 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x}$

4. $\lim_{x \rightarrow -3} \frac{x-2}{x^2-3x+2}$

5. $\lim_{x \rightarrow 7} \frac{\sqrt{x+9}-4}{x-7}$

6. $\lim_{x \rightarrow 0} \frac{2x^5+3x^4}{x^4}$

$$7. \lim_{x \rightarrow 6} \frac{x}{\frac{1}{x+6} - \frac{1}{6}}$$

$$8. \lim_{x \rightarrow 5} \frac{x^2 - 5x}{x - 5}$$

$$9. \lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x}$$

$$10. \lim_{x \rightarrow 1} \frac{\frac{1}{3} - \frac{1}{3x}}{x - 1}$$

$$11. \lim_{x \rightarrow 0} \frac{\sqrt{x+11} - \sqrt{11}}{x}$$

$$12. \lim_{x \rightarrow 3} \frac{\sqrt{2x-6}}{x}$$

$$13. \lim_{x \rightarrow 0} \frac{\frac{1}{(x+2)^2} - \frac{1}{4}}{x}$$

1.7 Selecting Procedures for Determining Limits**Algebraic Manipulation of Limits**

14. $\lim_{x \rightarrow b} \frac{b-x}{\sqrt{x}-\sqrt{b}}$ is

(A) $-2\sqrt{b}$

(B) $-\sqrt{b}$

(C) $2b$

(D) $\sqrt{b}$

(E) $2\sqrt{b}$

