NEW YORK STATE EDUCATION DEPARTMENT

New York State Next Generation

Mathematics Learning Standards

2017

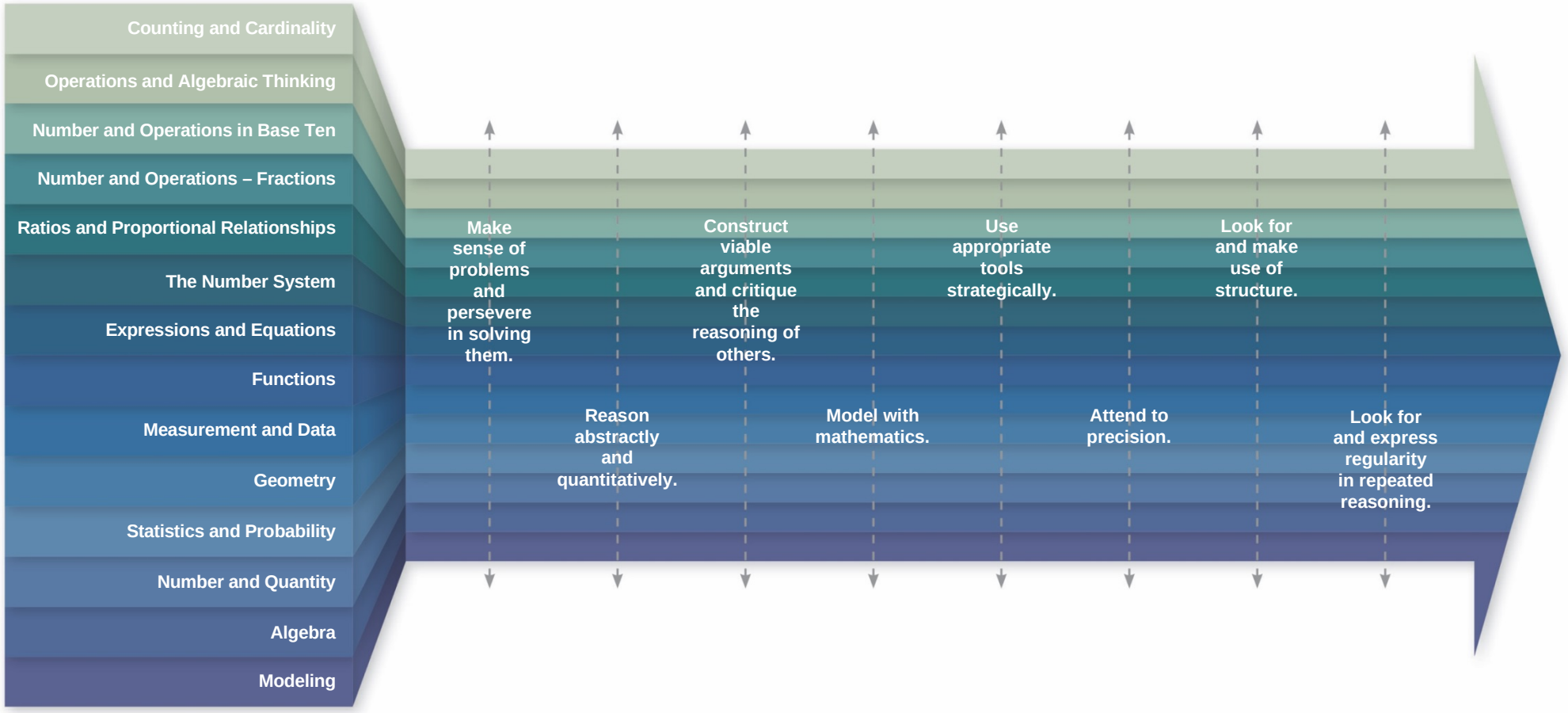


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Introduction

In 2015, New York State (NYS) began a process of review and revision of its current mathematics standards adopted in January of 2011. Through numerous phases of public comment, virtual and face-to-face meetings with committees consisting of NYS educators (Special Education, Bilingual Education and English as a New Language teachers), parents, curriculum specialists, school administrators, college professors, and experts in cognitive research, the *New York State Next Generation Mathematics Learning Standards (2017)* were developed. These revised standards reflect the collaborative efforts and expertise of all constituents involved.

The *New York State Next Generation Mathematics Learning Standards (2017)* reflect revisions, additions, vertical movement, and clarifications to the current mathematics standards. The Standards are defined as the knowledge, skills and understanding that individuals can and do habitually demonstrate over time because of instruction and learning experiences. These mathematics standards, collectively, are focused and cohesive—designed to support student access to the knowledge and understanding of the mathematical concepts that are necessary to function in a world very dependent upon the application of mathematics, while providing educators the opportunity to devise innovative programs to support this endeavor. As with any set of standards, they need to be rigorous; they need to demand a balance of conceptual understanding, procedural fluency and application and represent a significant level of achievement in mathematics that will enable students to successfully transition to post-secondary education and the workforce.

Context for Revision of the *NYS Next Generation Mathematics Learning Standards (2017)*

Changing expectations for mathematics achievement

Today's children are growing up in a world very different from the one even 15 years ago. Seismic changes in the labor market mean that we are living and working in a knowledge-based economy—one that demands advanced literacy and Science, Technology, Engineering and Mathematics (STEM) skills, whether for application in the private or public sector. Today, information moves through media at lightning speeds and is accessible in ways that are unprecedented; technology has eliminated many jobs while changing and creating others, especially those involving mathematical and conceptual reasoning skills. One characteristic of these fast-growing segment of jobs is that the employee needs to be able to solve unstructured problems while working with others in teams. At the same time, migration and immigration rates around the world bring diversity to schools and neighborhoods. The exponential growth in interactions and information sharing from around the world means there is much to process, communicate, analyze and respond to in the everyday, across all settings. For a great majority of jobs, conceptual reasoning and technical writing skills are integral parts to the daily routine.

To prepare students for the changes in the way we live and work, and to be sure that our education system keeps pace with what it means to be mathematically literate and what it means to collaboratively problem solve, we need a different approach to daily teaching and learning. We need content-rich standards that will serve as a platform for advancing children's 21st-century mathematical skills —their abstract reasoning, their collaboration skills, their ability to learn from peers and through technology, and their flexibility as a learner in a dynamic learning environment. Students need to be engaged in dialogue and learning experiences that allow complex topics and ideas to be explored from many angles and perspectives. They also need to learn how to think and solve problems for which there is no one solution—and learn mathematical skills along the way.

Increasingly Diverse Learner Populations

The need for a deeper, more innovative approach to mathematics teaching comes at a time when the system is already charged with building up language skills among the increasingly diverse population. Students who are English Language Learners (ELLs)/Multilingual Learners (MLLs) now comprise over 20% of the school-age population, which reflects significant growth in the past several decades. Between 1980 and 2009, this population increased from 4.7 to 11.2 million young people, or from 10 to 21% of the school-age population. This growth will likely continue in U.S. schools; by 2030, it is anticipated that 40% of the school-age population in the U.S. will speak a language other than English at home.[(1)] Today, in schools and districts across the U.S., many students other than those classified as ELLs are learning English as an additional language, even if not in the initial stages of language development—these children are often described as "language minority learners." Likewise, many students, large numbers of whom are growing up in poverty, speak a dialect of English that is different from the academic English found in school curriculum.[(2)] [(3)] [(4)]

Each of these groups—ELLs/MLLs, language minority learners, and students acquiring academic English—often struggle to access the language, and therefore the knowledge that fills the pages of academic texts, despite their linguistic assets. Therefore, the context for this new set of Mathematics Standards is that there is a pressing need to provide instruction that not only meets, but exceeds standards, as part of system-wide initiative to promote equal access to math skills for all learners while capitalizing on linguistic and cultural diversity.

All academic work does, to some degree, involve the academic language needed for success in school. For many students, including ELLs/MLLs, underdeveloped academic language affects their ability to comprehend and analyze texts, limits their ability to write and express their mathematical reasoning effectively, and can hinder their acquisition of academic content in all academic areas in which learning is demonstrated and assessed through oral and written language. If there isn't sufficient attention paid to building academic language across all content areas, students, including ELLs/MLLs, will not reach their potential and we will continue to perpetuate achievement gaps. The challenge is to design instruction that acknowledges the role of language; because language and knowledge are so inextricable.

In summary, today's children live in a society where many of their peers are from diverse backgrounds and speak different languages; one where technology is ubiquitous and central to daily life. They will enter a workforce and economy that demands critical thinking skills, and strong communication and social skills for full participation in society. This new society and economy has implications for today's education system—especially our instruction to foster a deeper and different set of communication and critical thinking skills, with significant attention to STEM.

Students with Disabilities and the Standards

One of the fundamental tenets guiding educational legislation (the *No Child Left Behind Act*, and *Every Student Succeeds Act*), and related policies over the past 15-years, is that all students, including students with disabilities, can achieve high standards of academic performance. A related trend is the increasing knowledge and skill expectations for PreK-Grade 12 students, especially in the area of reading and language arts, required for success in postsecondary education and 21st Century careers. Indeed, underdeveloped literacy skills have profound academic, social, emotional, and economic consequences for students, families, and society.

At the same time, the most recently available federal data[(5)] presents a portrait of the field reflecting both challenges and opportunities.

- *Students served under IDEA, Part B*: During the 2012-13 school year, there was a total of 5.83 million students with disabilities, ages 6-21; an increase from 5.67 million in 2010-11.
- *Access to the general education program*: More than 60 percent (62.1%) of students, ages 6 through 21 served under IDEA, Part B, were educated in the regular classroom 80% or more of the day, up from 60.5% in 2010-11.
- *Participation in state assessments*: Between 68.1 and 84.1 percent of students with disabilities in each of grades 3 through 8 and high school participated in the regular state assessment in reading based on grade-level academic achievement standards with or without accommodations.
- *English language arts proficiency*: The median percentages of students with disabilities in grades 3 through 8 and high school who were administered the 2012-13 state assessment in reading based on grade-level academic achievement standards who were proficient ranged from 25.4 to 37.3 percent.
- *Graduation*: Over sixty percent (65.1%) of students with disabilities graduated with a regular high school diploma.

Overall, the number of students with disabilities is increasing nationwide, as is their access to the general education curriculum, and participation in the state ELA and mathematics assessments. Attaining proficiency and graduating with a regular high school diploma are areas where significant improvements are needed.

Therefore, each student's individualized education program (IEP) must be developed in consideration of the State learning standards and should include information for teachers to effectively provide supports and services to address the individual learning needs of the student as they impact the student's ability to participate and progress in the general education curriculum. In addition to supports and services, special education must include specially designed instruction, which means adapting, as appropriate, the content, methodology or delivery of instruction to address the unique needs that result from the student's disability. By so doing, the teacher ensures each student's access to the general education curriculum so that he or she can meet the learning standards that apply to all students. The *Blueprint for Improved Results for Students with Disabilities* focuses on seven core evidence-based principles for students with disabilities to ensure they have the opportunity to benefit from high quality instruction and to

reach the same academic standards as all students. For additional information, please see the Office of Special Education's field advisory: *Blueprint for Improved Results for Students with Disabilities*.

Understanding the *NYS Next Generation Mathematics Learning Standards (2017)*

The *NYS Next Generation Mathematics Learning Standards (2017)* define what students should understand and be able to do as a result of their study of mathematics. To assess progress on the Standards, a teacher must assess whether the student has understood what has been taught and provide opportunities where a student can independently use and apply this knowledge to solve mathematical problems in similar or new contexts. While procedural skills are relatively straightforward to assess, teachers often ask: what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is accurate or where a mathematical rule comes from. Correctly using language to articulate mathematical understanding plays a part in this justification. Making the distinction between mathematical understanding and procedural skill is critical when designing curriculum and assessment; both are important for the mastery of these standards. That is, there is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain what the mnemonic represents as a process for systematically approaching algebraic problems. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task, such as expanding $(a + b + c)(x + y)$.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English Language Learners (ELLs)/Multilingual Learners (MLLs) and for Students with Disabilities. However, the department ensured that teachers of English Language Learners (ELLs)/Multilingual Learners (MLLs) and Students with Disabilities participated in the revision of the standards. The New York State Education Department (NYSED) has created two statewide frameworks, the *Blueprint for Improved Results for Students with Disabilities* and the *Blueprint for English Language Learner Success*, aimed to clarify expectations and to provide guidance for administrators, policymakers, and practitioners to prepare ELLs/MLLs and Students with Disabilities for success. These principles therein the frameworks are intended to enhance programming and improve instruction that would allow for students within these populations to reach the same standards as all students and leave school prepared to successfully transition to post school learning, living and working.

No set of grade-specific standards can fully reflect the variation in learning profiles, rates, and needs, linguistic backgrounds, and achievement levels of students in any given classroom. When designing and delivering mathematics instruction, educators must consider the cultural context and prior academic experiences of all students while bridging prior knowledge to new knowledge and ensuring that content is meaningful and comprehensible. In addition, as discussed above, educators must consider the relationship of language and content, and the vital role that language plays in obtaining and expressing mathematics content knowledge. The standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate adaptations to ensure equitable access and maximum participation of all students.

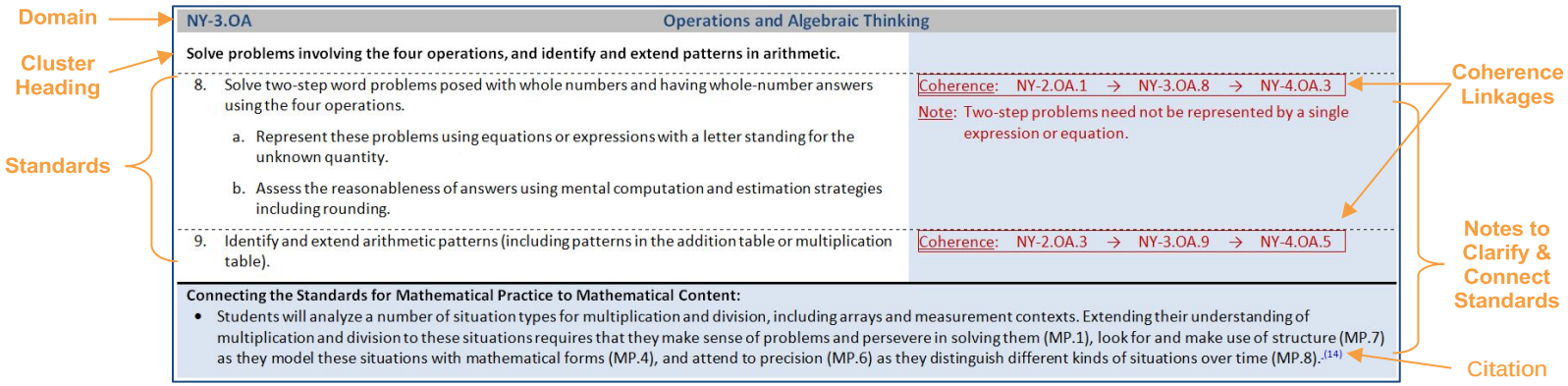
How to Read the P-8 Standards for Mathematical Content

**See High School – Introduction for how to read the High School Standards for Mathematical Content.*

The standards are organized by grade level from Prekindergarten through grade eight.

Standards	define what students should understand and be able to do.
Clusters	summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
Domains	are larger groups of related standards. Standards from different domains may sometimes be closely related.
Coherence Linkages	connect standards one grade level forward and/or back when there are very direct linking standards in those grades. For a more thorough analysis of how standards link to one another, see http://achievethecore.org/coherence-map/.
Citations	are indicated by a blue number when information was taken or adapted from another source. The number will match the source number in the *Works Cited* section at the end of this document. When viewing these standards electronically, the source information (including page number) will appear as hover-over text.

Prekindergarten through Grade Eight



The order in which the standards are presented is not necessarily the order in which the standards need to be taught. Standards from various domains are connected, and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain grade-level expectations and the knowledge articulated in the standards. That is, the standards do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information.

The Standards for Mathematical Practice

The Standards for each grade level and course begin with eight Standards for Mathematical Practice. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections.[(6)] The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding it Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that

these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards, which set an expectation of understanding, are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

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Pre-Kindergarten Overview

In Pre-Kindergarten, instructional time should focus on two areas: (1) developing a good sense of numbers using concrete objects including concepts of correspondence, counting, cardinality, and comparison; (2) describing shapes in their everyday environment. More learning time in Pre-Kindergarten should be devoted to exploring* and developing the sense of numbers than any other topic. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Counting and Cardinality*** domain, students:
   - develop a sense of numbers and count to determine the number of objects;
   - understand that number words refer to quantity;
   - use 1:1 correspondence to solve problems by matching sets and comparing number amounts and in counting objects to 10 through a variety of experiences; and
   - understand that the last number name said tells the number of objects counted (cardinality) and they count to determine number amounts and compare quantities (using language such as more than, fewer than, or equal to (the same as) the number of objects in another group).
2. Through their learning in the ***Geometry*** and ***Measurement and Data*** domains, students:
   - describe the position of objects in space based on the relations of those objects (e.g., shape and special relations) using appropriate vocabulary;
   - identify and name basic two-dimensional shapes, such as triangles, rectangles, squares, and circles; and
   - use basic shapes and spatial reasoning to model objects in their everyday environment.

*Note: *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

Mathematical Practices

1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

NY-PK.CC	Counting and Cardinality	
Know number names and the count sequence.		
1. Count to 20.		Coherence: NY-PK.CC.1 → NY-K.CC.1
2. Represent a number of objects (0 - 5), with a written numeral 0–5 (with 0 representing a count of no objects).		Coherence: NY-PK.CC.2 → NY-K.CC.3 Note: Students can select the corresponding number card and/or write the numeral.

Note on Number Reversals:

- Learning to write numerals is generally more difficult than learning to read them. It is common for students to reverse numbers at this stage (e.g., writing Ɛ for 3).[8]

NY-PK.CC	Counting and Cardinality	
Count to tell the number of objects.		
3. Understand the relationship between numbers and quantities to 10; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (1:1 correspondence) b. Explore and develop the concept that the last number name said tells the number of objects counted, (cardinality). The number of objects is the same regardless of their arrangement or the order in which they were counted.		Coherence: NY-PK.CC.3 → NY-K.CC.4
4a. Answer counting questions using as many as 10 objects arranged in a line, a rectangular array, and a circle. Answer counting questions using as many as 5 objects in a scattered configuration. 4b. Given a number from 1–10, count out that many objects.		Coherence: NY-PK.CC.4 → NY-K.CC.5 e.g., "How many _____ are there?"

Note on the Word *Explore*:

- *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

Within-Grade Connections:

- Much of the learning in prekindergarten—NY-PK.CC.5, all of NY-PK.OA.1, and NY-PK.MD.2—depends on the foundational ability to count to answer "how many?" (NY-PK.CC.4), which itself is grounded in NY-PK.CC.3. Therefore, work on NY-PK.CC.3 & 4 should likely begin at or near the beginning of the year.[9]

NY-PK.CC	Counting and Cardinality	
Compare numbers.		
5. Recognize whether the number of objects in one group is more than, fewer than, or equal to (the same as) the number of objects in another group. Note: Include groups with up to five objects.		Coherence: NY-PK.CC.5 → NY-K.CC.6 e.g., using matching and counting strategies
6. Identify "first" and "last" related to order or position.		Coherence: NY-PK.CC.6 → NY-K.CC.4d

NY-PK.OA Operations and Algebraic Thinking

Understand addition as adding to and understand subtraction as taking from.

1. Explore addition and subtraction by using objects, fingers, and responding to real world situations.

In the chart to the right, *Pre-Kindergarten students explore the two unshaded (white) subtypes for **Add To** and **Take From** situations.*

All four unshaded (white) subtypes are expectations in Kindergarten.

Grade 1 and 2 students work with all subtypes. Darker shading indicates the four difficult subtypes that students should work with in Grade 1 but need not master until Grade 2.

Coherence: NY-PK.OA.1 → NY-K.OA.1

e.g., If we have 3 apples and add two more, how many apples do we have all together?

Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	*A* bunnies were on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first *A* bunnies? $A + \square = C$	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? $C - B = \square$	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown**	**Addend Unknown**
Put Together / Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version:* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy?	*Version with "More":* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have?	*Version with "More":* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have?
	"How many fewer?" version: Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*Version with "Fewer":* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? $A + B = \square$	*Version with "Fewer":* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Note on the Word *Explore*:

- *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students progress from drawing realistic (artistic) pictures of situations to diagramming addition and subtraction situations using circles or other symbols, and making connections between them, they are relating the concrete to the abstract (MP.2) and making their first mathematical models (MP.4).[9]
- A student choosing to use objects, fingers, or a math drawing to represent and solve a word problem is an example of the student using an appropriate tool strategically (MP.5).[9]

NY-PK.OA	Operations and Algebraic Thinking	
Understand simple patterns.		
2. Duplicate and extend simple patterns using concrete objects.		Coherence: NY-PK.OA.2 → NY-K.OA.6 e.g., What comes next?

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students duplicate and extend patterns (NY-PK.OA.2), they are noticing regularity and repeated reasoning (MP.8).

NY-PK.MD	Measurement and Data
Describe and compare measurable attributes.	
1. Identify measurable attributes of objects, such as length or weight, and describe them using appropriate vocabulary.	Coherence: NY-PK.MD.1 → NY-K.MD.1 e.g., small, big, short, tall, empty, full, heavy, and light

NY-PK.MD	Measurement and Data
Sort objects and count the number of objects in each category.	
2. Sort objects and shapes into categories; count the objects in each category. Note: Limit category counts to be less than or equal to 10.	Coherence: NY-PK.MD.2 → NY-K.MD.3

Within-Grade Connections:

- Sorting objects into categories and counting them (NY-PK.MD.2) offers a context for cardinal counting (NY-PK.CC.4) and for comparing numbers (NY-PK.CC.5).[9]

NY-PK.G	Geometry	
Identify and describe shapes (squares, circles, triangles, and rectangles).		
1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as top, bottom, up, down, above, below, in front of, behind, over, under, and next to.		Coherence: NY-PK.G.1 → NY-K.G.1
2. Name shapes regardless of size.		Coherence: NY-PK.G.2 → NY-K.G.2

NY-PK.G	Geometry	
Explore and create two- and three-dimensional objects.		
3. Explore two- and three-dimensional objects and use informal language to describe their similarities, differences, and other attributes.		Coherence: NY-PK.G.3 → NY-K.G.4
4. Create and build shapes from components.		Coherence: NY-PK.G.4 → NY-K.G.5 e.g., sticks and clay balls

Note on the Word *Explore*:

- *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

Kindergarten Overview

In Kindergarten, instructional time should focus on two areas: (1) developing a sound sense of numbers by representing and comparing numbers, initially using sets of objects; (2) recognizing and describing shapes and using spatial relations. More learning time in Kindergarten should be devoted to number than to any other topic. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Counting and Cardinality*** and ***Operations and Algebraic Thinking*** domains, students:
   - develop a more formal sense of numbers;
   - use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. *Note: Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required; and*
   - choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.
2. Through their learning in the ***Geometry*** and ***Measurement and Data*** domains, students:
   - describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and appropriate vocabulary;
   - identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres;
   - use basic shapes and spatial reasoning to model objects in their everyday environment to create and compose more complex shapes; and
   - explore* coins and begin identifying of pennies and dimes.

*Note: *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NY-K.CC	Counting and Cardinality	
Know number names and the count sequence.		
1.	Count to 100 by ones and by tens.	Coherence: NY-PK.CC.1 → NY-K.CC.1 → NY-1.NBT.1
2.	Count to 100 by ones beginning from any given number (instead of beginning at 1).	Coherence: NY-K.CC.2 → NY-1.NBT.1
3.	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).	Coherence: NY-PK.CC.2 → NY-K.CC.3 → NY-1.NBT.1

Note on Number Reversals:

- Learning to write numerals is generally more difficult than learning to read them. It is common for students to reverse numbers at this stage (e.g., writing Ɛ for 3).[8]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Kindergarten students say the number names by ones and by tens ("ten, twenty, thirty, . . ."") all the way to 100 (NY-K.CC.1). The structure of a number name such as "thirty-two" reflects the underlying system of place value. Attending to and using that structure (MP.7) is an important foundation for place value. For more information about how patterns in the number names affect learning the teen numbers, see *K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking*.[9]
- As students count by tens (NY-K.CC.1), they may make sense (MP.1) of these numbers by reciting each new number in the sequence "ten, twenty, thirty . . ." as a new child joins the children already standing in front of the classroom and showing all their fingers. The patterns in the place-value system—the structure of numbers (MP.7)—become more apparent when children say "six tens is sixty, seven tens is seventy, eight tens is eighty," etc. Children can also flash ten fingers as they count by tens to feel the ten that are added on with each count.[9]

NY-K.CC	Counting and Cardinality	
Count to tell the number of objects.		
4. Understand the relationship between numbers and quantities up to 20; connect counting to cardinality.		Coherence: NY-PK.CC.3 → NY-K.CC.4
a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (1:1 correspondence)		
b. Understand that the last number name said tells the number of objects counted, (cardinality). The number of objects is the same regardless of their arrangement or the order in which they were counted.		
c. Understand the concept that each successive number name refers to a quantity that is one larger.		
d. Understand the concept of ordinal numbers (first through tenth) to describe the relative position and magnitude of whole numbers.		Coherence: NY-PK.CC.6 → NY-K.CC.4d
5a. Answer counting questions using as many as 20 objects arranged in a line, a rectangular array, and a circle. Answer counting questions using as many as 10 objects in a scattered configuration.		Coherence: NY-PK.CC.4 → NY-K.CC.5 e.g., "How many _____ are there?"
5b. Given a number from 1–20, count out that many objects.		

Within-Grade Connections:

- Much of the learning in kindergarten—NY-K.CC.6, all of NY-K.OA and NY-K.NBT, and NY-K.MD.3—depends on the foundational ability to count to answer "how many?" (NY-K.CC.5), which itself is grounded in NY-K.CC.4. Therefore, work on NY-K.CC.4 and NY-K.CC.5 should likely begin near the beginning of the year.[9]

NY-K.CC	Counting and Cardinality	
Compare numbers.		
6. Identify whether the number of objects in one group is greater than (more than), less than (fewer than), or equal to (the same as) the number of objects in another group. Note: Include groups with up to ten objects.		Coherence: NY-PK.CC.5 → NY-K.CC.6 e.g., using matching and counting strategies
7. Compare two numbers between 1 and 10 presented as written numerals.		Coherence: NY-K.CC.7 → NY-1.NBT.3 e.g., 6 is greater than 2

Within-Grade Connections:

- If students are less than fluent in number comparisons (NY-K.CC.7) by the end of kindergarten, then they may not have mastered early number concepts (NY-K.CC.1 - 4). Note that NY-K.CC.6 is a precursor to NY-K.CC.7 and portrays the kind of concrete work that students should be doing en route to mastering numeral-based comparisons.[9]

NY-K.OA Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction using objects, fingers, pennies, drawings, sounds, acting out situations, verbal explanations, expressions, equations, or other strategies.

   Note: Drawings need not show details, but should show the mathematics in the problem.

Coherence: NY-PK.OA.1 → NY-K.OA.1

2a. Add and subtract within 10.

2b. Solve addition and subtraction word problems within 10.

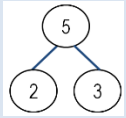
Coherence: NY-K.OA.2 → NY-1.OA.1

e.g., using objects or drawings to represent the problem

In the chart below, *the four unshaded (white) subtypes are expectations in Kindergarten*. Grade 1 and 2 students work with all subtypes. Darker shading indicates the four difficult subtypes that students should work with in Grade 1 but need not master until Grade 2.

Common Addition and Subtraction Situations

		Result Unknown	Change Unknown	Start Unknown
Add To		A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From		C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
		Total Unknown	**Both Addends Unknown**	**Addend Unknown**
Put Together / Take Apart		A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
		Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare		*"How many more?" version:* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy?	*Version with "More":* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have?	*Version with "More":* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have?
		"How many fewer?" version: Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*Version with "Fewer":* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A + B = \square$	*Version with "Fewer":* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

3. Decompose numbers less than or equal to 10 into pairs in more than one way. Record each decomposition with a drawing or equation.	Coherence: NY-K.OA.3 → NY-1.OA.6 e.g., using objects or drawings e.g., 5 = 2 + 3 and 5 = 4 + 1, or 
4. Find the number that makes 10 when given a number from 1 to 9. Record the answer with a drawing or equation.	Coherence: NY-K.OA.4 → NY-1.OA.6 e.g., using objects or drawings
5. Fluently add and subtract within 5.	Coherence: NY-K.OA.5 → NY-1.OA.6

Note on *Fluency* with Facts:

- *Fluently* adding and subtracting within 5 in kindergarten (NY-K.OA.5) means students can find sums and differences within 5 reasonably quickly, and say or write it. Fluency involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.[10] In kindergarten, some students may still need to use fingers or make drawings. Students grow in fluency throughout the year as they work with addition and subtraction situations.[9] For more on how children develop fluency, see *Grade K–5, Counting and Cardinality and Operations and Algebraic Thinking*, pp. 18-19 and *Adding it Up*, pp. 182-195.

Note on Manipulatives in Grades K-2:

- Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.[9]

Within-Grade Connections:

- Understanding that 18 is ten ones and eight more ones (NY-K.NBT.1) requires and supports understanding what it means to combine 10 and 8 or to take apart 18 (NY-K.OA.3 and NY-K.OA.4).[9]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students progress from drawing realistic (artistic) pictures of situations to diagramming addition and subtraction situations using circles or other symbols, and making connections between them, they are relating the concrete to the abstract (MP.2) and making their first mathematical models (MP.4). Equations to describe these situations (such as 8 + 2 = 10) are also mathematical models.[9]
- A student choosing to use objects, fingers, or a math drawing to represent and solve a word problem is an example of the student using an appropriate tool strategically (MP.5).[9]

NY-K.OA Operations and Algebraic Thinking

Understand simple patterns.

6. Duplicate, extend, and create simple patterns using concrete objects.	Coherence: NY-PK.OA.2 → NY-K.OA.6

NY-K.NBT	Number and Operations in Base Ten
Work with numbers 11–19 to gain foundations for place value.	
1. Compose and decompose the numbers from 11 to 19 into ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	Coherence: NY-K.NBT.1 → NY-1.NBT.2 e.g., using objects or drawings

Within-Grade Connections:

- Understanding that 18 is ten ones and eight more ones (NY-K.NBT.1) requires and supports understanding what it means to combine 10 and 8 or to take apart 18 (NY-K.OA.3 and NY-K.OA.4).[9]
- Working with numbers 11–19 (NY-K.NBT.1) provides opportunities for cardinal counting beyond 10 (NY-K.CC.5) and for writing two-digit numbers (NY-K.CC.3). Ten frames, strips with ten ones and some loose ones, and math drawings can be helpful for this work.[9]

NY-K.MD	Measurement and Data	
Describe and compare measurable attributes.		
1.	Describe measurable attributes of an object(s), such as length or weight, using appropriate vocabulary.	Coherence: NY-PK.MD.1 → NY-K.MD.1 e.g., small, big, short, tall, empty, full, heavy, and light
2.	Directly compare two objects with a common measurable attribute and describe the difference.	Coherence: NY-K.MD.2 → NY-1.MD.1

NY-K.MD	Measurement and Data	
Classify objects and count the number of objects in each category.		
3.	Classify objects into given categories; count the objects in each category and sort the categories by count. Note: Limit category counts to be less than or equal to 10.	Coherence: NY-PK.MD.2 → NY-K.MD.3 → NY-1.MD.4
4.	Explore coins (pennies, nickels, dimes, and quarters) and begin identifying pennies and dimes.	Coherence: NY-K.MD.4 → NY-1.MD.3b NY-1.MD.3c

Note on the Word *Explore*:

- *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

Within-Grade Connections:

- Classifying objects into different categories and counting them (NY-K.MD.3) offers a context for cardinal counting (NY-K.CC.5), for comparing numbers (NY-K.CC.6), and for decomposing numbers less than or equal to 10 in more than one way (NY-K.OA.3).[9]

NY-K.G	Geometry
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	
1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	Coherence: NY-PK.G.1 → NY-K.G.1
2. Name shapes regardless of their orientation or overall size.	Coherence: NY-PK.G.2 → NY-K.G.2
3. Understand the difference between two-dimensional (lying in a plane, "flat") and three-dimensional ("solid") shapes.	

NY-K.G	Geometry
Analyze, compare, sort, and compose shapes.	
4. Analyze, compare, and sort two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts, and other attributes.	Coherence: NY-PK.G.3 → NY-K.G.4 → NY-1.G.1 e.g., number of sides, number of vertices/"corners," or having sides of equal length
5. Model objects in their environment by building and/or drawing shapes.	Coherence: NY-PK.G.4 → NY-K.G.5 e.g., using blocks to build a simple representation in the classroom Note on and/or: Students should be taught to model objects by building *and* drawing shapes; however, when answering a question, students can choose to model the object by building *or* drawing the shape.
6. Compose larger shapes from simple shapes.	Coherence: NY-K.G.6 → NY-1.G.2 e.g., join two triangles to make a rectangle

Grade 1 Overview

In Grade 1, instructional time should focus on three areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; and (3) developing understanding of linear measurement and measuring lengths as iterating length units. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Operations and Algebraic Thinking*** domain, students:
   - develop strategies for adding and subtracting whole numbers based on their prior work with small numbers;
   - use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations;
   - understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two);
   - use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20; and
   - build their understanding of the relationship between addition and subtraction by comparing a variety of solution strategies.
2. Through their learning in the ***Number and Operations in Base Ten*** domain, students:
   - develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10;
   - compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes;
   - think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones); and
   - understand the order of the counting numbers and their relative magnitudes through activities that build number sense.
3. Through their learning in the ***Measurement and Data*** domain, students:
   - develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.*

*Note: Students should apply the transitivity principle of indirect measurement to make comparisons, but they need not use this technical term.

Mathematical Practices

1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

NY-1.OA Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve one step word problems involving situations of adding to, taking from, putting together, taking apart, and/or comparing, with unknowns in all positions.

   Note: Problems should be *represented* using objects, drawings, *and* equations with a symbol for the unknown number. Problems should be *solved* using objects *or* drawings, and equations.

Coherence: NY-K.OA.2 → NY-1.OA.1 → NY-2.OA.1

In the chart below, the four unshaded (white) subtypes are mastered in Kindergarten. Grade 1 and 2 students work with all subtypes. *Darker shading indicates the four difficult subtypes that students should work with in Grade 1 but need not master until Grade 2.*

Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	*A* bunnies were on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first *A* bunnies? $A + \square = C$	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? $C - B = \square$	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown**	**Addend Unknown**
Put Together/ Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version:* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy?	*Version with "More":* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have?	*Version with "More":* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have?
	"How many fewer?" version: Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*Version with "Fewer":* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? $A + B = \square$	*Version with "Fewer":* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.

Coherence: NY-1.OA.2 → NY-2.NBT.6

e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem

Note on Manipulatives in Grades K-2:

- A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.[9]

NY-1.OA	Operations and Algebraic Thinking
Understand and apply properties of operations and the relationship between addition and subtraction.	
3. Apply properties of operations as strategies to add and subtract. Note: Students need not use formal terms for these properties.	Coherence: NY-1.OA.3 → NY-2.NBT.9 e.g., • If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) • To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)
4. Understand subtraction as an unknown-addend problem within 20.	Coherence: NY-1.OA.4 → NY-2.NBT.9 e.g., Subtract 10 – 8 by finding the number that makes 10 when added to 8.

Within-Grade Connections:

- When students use the *making ten* strategy (NY-1.OA.6), they are applying the Associative property of addition (NY-1.OA.3).[9]
  For example, when solving 8 + 3 by "making a ten," a student decomposes the 3 into 2 + 1 in order to re-associate the 2 with the 8, making ten + 1.
  8 + 3
  8 + (2 + 1)
  (8 + 2) + 1

Connecting the Standards for Mathematical Practice to Mathematical Content:

- All work with properties (NY-1.OA.3) and place value (e.g., NY-1.NBT.2 & 4) should be seen as an investigation and use of the structure of the number system and of arithmetic (MP.7). Students' explanations of the properties and reasoning that they used in these contexts are early beginnings of the construction of (brief) logical arguments (MP.3). Examples of brief but excellent arguments at this grade level could include:
  - I know that 7 – 3 equals 4 because 4 + 3 equals 7. (This shows NY-1.OA.4 being met.)
  - I knew that 8 + 8 = 20 was wrong because 10 + 10 equals 20 and 8 is less than 10.
  - I know that 8 + 7 equals 15 because I know that 8 + 8 equals 16. [9]

NY-1.OA Operations and Algebraic Thinking

Add and subtract within 20.

5. Relate counting to addition and subtraction.

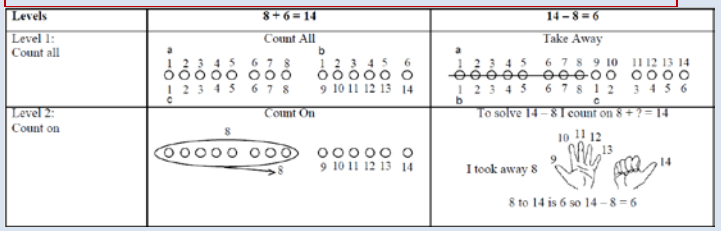
Coherence: NY-K.CC.4c → NY-1.OA.5

e.g., by counting on 2 to add 2

6a. Add and subtract within 20. Use strategies such as:

Coherence: NY-K.OA.3, NY-K.OA.4 → NY-1.OA.6 → NY-2.OA.2, NY-K.OA.5

- counting on;



- making ten;

e.g., 8 + 6 =
8 + 2 + 4 =
10 + 4 = 14

- decomposing a number leading to a ten;

e.g., 13 – 4 =
13 – 3 – 1 =
10 – 1 = 9

- using the relationship between addition and subtraction; and

e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4

- creating equivalent but easier or known sums.

e.g., adding 6 + 7 by creating the known equivalent
6 + 6 + 1 = 12 + 1 = 13

6b. Fluently add and subtract within 10.

Note on *Fluency* with Facts:

- *Fluently* adding and subtracting within 10 (NY-1.OA.6b) means students can find sums and differences within 10 reasonably quickly, and say or write it. Fluency involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.[10] Students grow in fluency throughout the year as they work with addition and subtraction situations.[9] For more on how children develop fluency, see *K–5 Progression on Counting and Cardinality and Operations and Algebraic Thinking*, pp. 18-19 and *Adding it Up*, pp. 182-195.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Grade 1 students work with some sophisticated addition and subtraction situations (NY-1.OA.1), such as "Lucy has 8 fewer apples than Julie. Julie has 12 apples. How many apples does Lucy have?" Making a math drawing or using objects to model the situation is very helpful for students (MP.5). The equations 12 – 8 = ?, 8 + ? = 12, and ? + 8 = 12 are all mathematical models of this situation (MP.4).[9]

NY-1.OA	Operations and Algebraic Thinking
Work with addition and subtraction equations.	
7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.	e.g., Which of the following equations are true and which are false? 6 = 6 7 = 8 – 1 5 + 2 = 2 + 5 4 + 1 = 5 + 2
8. Determine the unknown whole number in an addition or subtraction equation with the unknown in all positions.	e.g., Determine the unknown number that makes the equation true in each of the equations: 8 + ? = 11 __ – 3 = 5 6 + 6 = ☐

Within-Grade Connections:

- Understanding the meaning of the equal sign (NY-1.OA.7) is a crucial aspect of and solving equations (NY-1.OA.8). In order to gain understanding of the equal sign and solving equations, students must see varied equation forms – especially those with only one number on the left side of the equation. Through this work, students come to understand where the total is in addition equations (alone on one side) and in subtraction equations (before the minus sign). [9]

NY-1.NBT	Number and Operations in Base Ten	
Extend the counting sequence.		
1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.		Coherence: NY-K.CC.1, NY-K.CC.2, NY-K.CC.3 → NY-1.NBT.1 → NY-2.NBT.2, NY-2.NBT.3

Within-Grade Connections:

- Units are a connection between place value (NY-1.NBT) and measurement (NY-1.MD). Working with place value depends on having a sense of the sizes of the base-ten units and being able to see a larger unit as composed of smaller units within the system.[(11)]

NY-1.NBT	Number and Operations in Base Ten	
Understand place value.		
2. Understand that the two digits of a two-digit number represent amounts of tens and ones. a. Understand 10 can be thought of as a bundle of ten ones, called a "ten". b. Understand the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).		Coherence: NY-K.NBT.1 → NY-1.NBT.2 → NY-2.NBT.1
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.		Coherence: NY-K.CC.7 → NY-1.NBT.3 → NY-2.NBT.4

NY-1.NBT	Number and Operations in Base Ten
Use place value understanding and properties of operations to add and subtract.	
4. Add within 100, including • a two-digit number and a one-digit number, • a two-digit number and a multiple of 10. Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones, and sometimes it is necessary to compose a ten. Relate the strategy to a written representation and explain the reasoning used.	Coherence: NY-1.NBT.4 → NY-2.NBT.5, NY-2.NBT.7 Note on and/or: Students should be taught to use strategies based on place value, properties of operations, *and* the relationship between addition and subtraction; however, when solving any problem, students can choose any strategy. Note: A *written representation* is any way of showing a strategy using words, pictures, or numbers.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	Coherence: NY-1.NBT.5 → NY-2.NBT.8
6. Subtract multiples of 10 from multiples of 10 in the range 10-90 using • concrete models or drawings, and • strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy used to a written representation and explain the reasoning.	Coherence: NY-1.NBT.6 → NY-2.NBT.8 Note on and/or: Students should be taught to use concrete models and drawings; as well as strategies based on place value, properties of operations, *and* the relationship between addition and subtraction. When solving any problem, students can choose to use a concrete model *or* a drawing. Their strategy must be based on place value, properties of operations, or the relationship between addition and subtraction. Note: A *written representation* is any way of showing a strategy using words, pictures, or numbers.

Within-Grade Connections:

- Quickly finding 10 more or 10 less than a two-digit number (NY-1.NBT.5) is a good indicator of whether students have an understanding of place value for two-digit numbers (NY-1.NBT.2).[9]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- All work with properties (NY-1.OA.3) and place value (e.g., NY-1.NBT.2 & 4) should be seen as an investigation and use of the structure of the number system and of arithmetic (MP.7). Students' explanations of the properties and reasoning that they used in these contexts are early beginnings of the construction of (brief) logical arguments (MP.3). Examples of brief but excellent arguments at this grade level could include:
  - I know that 7 – 3 equals 4 because 4 + 3 equals 7. (This shows NY-1.OA.4 being met.)
  - I knew that 8 + 8 = 20 was wrong because 10 + 10 equals 20 and 8 is less than 10.
  - I know that 8 + 7 equals 15 because I know that 8 + 8 equals 16.[9]
- When students add two-digit numbers (NY-1.NBT.4), they "Look for and express regularity in repeated reasoning" (MP.8). Students will repeatedly think about the units of ten and the units of one in their concrete models or drawings and in their recorded written methods. During this work, students are also "modeling with mathematics" (MP.4).[9]

NY-1.MD	Measurement and Data
Measure lengths indirectly and by iterating length units.	
1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.	Coherence: NY-K.MD.2 → NY-1.MD.1 → NY-2.MD.4
2. Measure the length of an object using same-size "length units" placed end to end with no gaps or overlaps. Express the length of an object as a whole number of "length units."	Coherence: NY-1.MD.2 → NY-2.MD.1 Note: "Length units" could include cubes, paper clips, etc.

Within-Grade Connections:

- Units are a connection between place value (NY-1.NBT) and measurement (NY-1.MD). Working with place value depends on having a sense of the sizes of the base-ten units and being able to see a larger unit as composed of smaller units within the system.[9]
- Measurement standards NY-1.MD.1 and NY-1.MD.2 together support and provide a context for the goal of solving problems that involve comparing (NY-1.OA.1).[9]

NY-1.MD	Measurement and Data
Tell and write time and money.	
3a. Tell and write time in hours and half-hours using analog and digital clocks. Develop an understanding of common terms, such as, but not limited to, *o'clock* and *half past*.	Coherence: NY-1.MD.3a → NY-2.MD.7
3b. Recognize and identify coins (penny, nickel, dime, and quarter) and their value and use the cent symbol (¢) appropriately.	Coherence: NY-K.MD.4 → NY-1.MD.3b
3c. Count a mixed collection of dimes and pennies and determine the cent value (total not to exceed 100 cents).	Coherence: NY-1.MD.3c → NY-2.MD.8 e.g., 3 dimes and 4 pennies is the same as 3 tens and 4 ones, which is 34¢.

Within-Grade Connections:

- While students are dealing with the limited precision of only whole hours and half-hours (NY-1.MD.3a), they must distinguish the position of the hour hand. This connects to partitioning circles into halves and quarters (NY-1.G.3).[9]
- When students count a mixed collection of dimes and pennies (NY-1.MD.3c), they develop understanding of place value (NY-1.NBT.2) and practice adding within 100 (NY-1.NBT.4-5).

NY-1.MD	Measurement and Data
Represent and interpret data.	
4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	Coherence: NY-K.MD.3 → NY-1.MD.4 → NY-2.MD.10

Within-Grade Connections:

- When students ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another (NY-1.MD.4), they practice adding and subtracting numbers (NY-1.OA).[9]

NY-1.G	Geometry
Reason with shapes and their attributes.	
1. Distinguish between defining attributes versus non-defining attributes for a wide variety of shapes. Build and/or draw shapes to possess defining attributes.	Coherence: NY-K.G.4 → NY-1.G.1 → NY-2.G.1 e.g., • A defining attribute may include, but is not limited to: triangles are closed and three-sided. • Non-defining attributes include, but are not limited to: color, orientation, and overall size. Note on and/or: Students should be taught to build *and* draw shapes to possess defining attributes; however, when answering questions, students can choose to build *or* draw the shape.
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Note: Students do not need to learn formal names such as "right rectangular prism."	Coherence: NY-K.G.6 → NY-1.G.2
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of*. Describe the whole as *two of,* or *four of* the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	Coherence: NY-1.G.3 → NY-2.G.3

Within-Grade Connections:

- Composing shapes to create a new shape (NY-1.G.2) is the spatial analogue of composing numbers to create new numbers (NY-1.NBT.2). This concept is also connected to length measurement (NY-1.MD.2) since students must visualize an object that is to be measured as being built up out of equal-sized units (see also NY-1.G.3). Though assembling two congruent right triangles into a rectangle does not use the same facts or reasoning that assembling two fives into a ten uses, the idea of looking at how objects in some domain (numbers or shapes) can be combined to make other objects in that domain, and looking for new true statements one can make about these combinations, is a big idea that is common across mathematics.[(9)]

Grade 2 Overview

In Grade 2, instructional time should focus on four areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) analyzing and classifying two dimensional shapes as polygons or non-polygons. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Number and Operations in Base Ten*** domain, students:
   - extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing; and
   - understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
2. Through their learning in the ***Operations and Algebraic Thinking*** and ***Numbers and Operations in Base Ten*** domains, students:
   - use their understanding of addition to develop fluency with addition and subtraction within 100;
   - solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations; and
   - select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
3. Through their learning in the ***Measurement and Data*** domain, students:
   - recognize the need for standard units of measure (centimeter and inch) and use rulers and other measurement tools with the understanding that linear measure involves an iteration of units; and
   - recognize that the smaller the unit, the more iterations needed to cover a given length.
4. Through their learning in the ***Geometry*** domain, students:
   - describe and classify shapes as polygons or non-polygons;
   - investigate, describe, and reason about decomposing and combining shapes to make other shapes; and
   - draw, partition, and analyze two-dimensional shapes to develop a foundation for understanding area, congruence, similarity, and fractions in later grades.

Mathematical Practices

1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

NY-2.OA Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1a. Use addition and subtraction within 100 to solve one-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.

Coherence: NY-1.OA.1 → NY-2.OA.1 → NY-3.OA.8

e.g., using drawings and equations with a symbol for the unknown number to represent the problem

In the chart to the right, the four unshaded (white) subtypes are mastered in Kindergarten. Grade 1 and 2 students work with all subtypes. Darker shading indicates the four difficult subtypes that students should work with in Grade 1 but need not master until Grade 2.

Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	*A* bunnies were on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first *A* bunnies? $A + \square = C$	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? $C - B = \square$	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown**	**Addend Unknown**
Put Together/ Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version:* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy?	*Version with "More":* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have?	*Version with "More":* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have?
	"How many fewer?" version: Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*Version with "Fewer":* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? $A + B = \square$	*Version with "Fewer":* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

1b. Use addition and subtraction within 100 to develop an understanding of solving two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.

e.g., using drawings and equations with a symbol for the unknown number to represent the problem

Note on Manipulatives in Grades K-2:

- A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.[9]

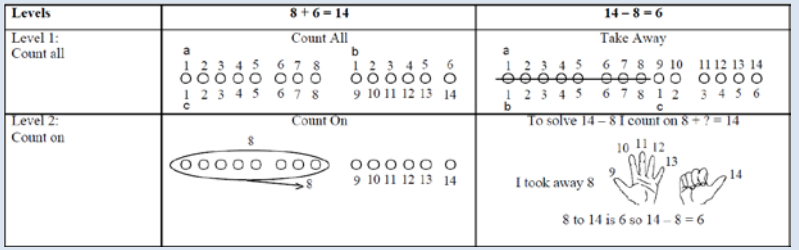
NY-2.OA Operations and Algebraic Thinking

Add and subtract within 20.

2a. Fluently add and subtract within 20 using mental strategies. Strategies could include:

- counting on;

Coherence: NY-1.OA.6 → NY-2.OA.2



- making ten;

e.g., 8 + 6 =
8 + 2 + 4 =
10 + 4 = 14

- decomposing a number leading to a ten;

e.g., 13 – 4 =
13 – 3 – 1 =
10 – 1 = 9

- using the relationship between addition and subtraction; and

e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4

- creating equivalent but easier or known sums.

e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13

2b. Know from memory all sums within 20 of two one-digit numbers.

Note on *Fluency* with Facts:

- *Fluently* adding and subtracting within 20 (NY-2.OA.2) means students can find sums and differences within 20 reasonably quickly, and say or write it. Fluency involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.[10] Reaching fluency will take much of the year for many students. For more on how children develop fluency, see *K–5 Progression on Counting and Cardinality and Operations and Algebraic Thinking*, pp. 18-19 and *Adding it Up*, pp. 182-195.

Note on *Fluency* vs. *Knowing from Memory*:

- The standards intentionally distinguish between asking for *fluency* with addition and subtraction (NY-2.OA.2a) and asking students to *know from memory* addition facts (NY-2.OA.2b). *Fluency* means students are fast, accurate, flexible, and have understanding. They use strategies efficiently.[12] By the end of the K–2 grade span, students have sufficient experience with these strategies to *know from memory* all single-digit sums.[10]

NY-2.OA	Operations and Algebraic Thinking
Work with equal groups of objects to gain foundations for multiplication.	
3a. Determine whether a group of objects (up to 20) has an odd or even number of members. 3b. Write an equation to express an even number as a sum of two equal addends.	Coherence: NY-2.OA.3 → NY-3.OA.9 e.g., by pairing objects or counting them by 2s
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. Write an equation to express the total as a sum of equal addends.	Coherence: NY-2.OA.4 → NY-3.OA.1

NY-2.NBT	Number and Operations in Base Ten
Understand place value.	
1. Understand that the digits of a three-digit number represent amounts of hundreds, tens, and ones. a. Understand 100 can be thought of as a bundle of ten tens, called a "hundred." b. Understand the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	Coherence: NY-1.NBT.2 → NY-2.NBT.1 → NY-3.NBT.4a e.g., 706 equals 7 hundreds, 0 tens, and 6 ones
2. Count within 1000; skip-count by 5s, 10s, and 100s.	Coherence: NY-1.NBT.1 → NY-2.NBT.2
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	Coherence: NY-1.NBT.1 → NY-2.NBT.3 → NY-3.NBT.4b e.g., expanded form: 237 = 200 + 30 + 7
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	Coherence: NY-1.NBT.3 → NY-2.NBT.4

NY-2.NBT	Number and Operations in Base Ten	
Use place value understanding and properties of operations to add and subtract.		
5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.		Coherence: NY-1.NBT.4 → NY-2.NBT.5 → NY-3.NBT.2 Note on and/or: Students should be taught to use strategies based on place value, properties of operations, *and* the relationship between addition and subtraction; however, when solving any problem, students can choose any strategy.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.		Coherence: NY-1.OA.2 → NY-2.NBT.6
7a. Add and subtract within 1000, using • concrete models or drawings, and • strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written representation. Note: A *written representation* is any way of showing a strategy using words, pictures, or numbers.		Coherence: NY-1.NBT.4 → NY-2.NBT.7 → NY-3.NBT.2 Note on and/or: Students should be taught to use concrete models and drawings; as well as strategies based on place value, properties of operations, *and* the relationship between addition and subtraction. When solving any problem, students can choose to use a concrete model *or* a drawing. Their strategy must be based on place value, properties of operations, and/or the relationship between addition and subtraction.
7b. Understand that in adding or subtracting up to three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones, and sometimes it is necessary to compose or decompose tens or hundreds.		
8. Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.		Coherence: NY-1.NBT.5, NY-1.NBT.6 → NY-2.NBT.8
9. Explain why addition and subtraction strategies work, using place value and the properties of operations. Note: Explanations may be supported by drawings or objects.		Coherence: NY-1.OA.3, NY-1.OA.4 → NY-2.NBT.9

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13] For more on developing procedural fluency, see *Adding it Up*, pp. 121-124.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students compose ones and tens to make tens and hundreds, and decompose hundreds and tens when they need more tens and ones they are looking for and make use of structure (MP.7). They look for and express regularity in repeated reasoning (MP.8) both as they compose and decompose such units and as they initially extend their reasoning and written methods from adding and subtracting within 100 to adding and subtracting within 1000.[9]

NY-2.MD	Measurement and Data	
Measure and estimate lengths in standard units.		
1.	Measure the length of an object to the nearest whole by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	Coherence: NY-1.MD.2 → NY-2.MD.1 → NY-3.MD.4
2.	Measure the length of an object twice, using different "length units" for the two measurements; describe how the two measurements relate to the size of the unit chosen.	
3.	Estimate lengths using units of inches, feet, centimeters, and meters.	
4.	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard "length unit."	Coherence: NY-1.MD.1 → NY-2.MD.4

NY-2.MD	Measurement and Data	
Relate addition and subtraction to length.		
5.	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units.	Coherence: NY-2.MD.5 → NY-3.MD.2 e.g., using drawings and equations with a symbol for the unknown number to represent the problem
6.	Represent whole numbers as lengths from 0 on a number line with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line.	Coherence: NY-2.MD.6 → NY-3.NF.2

Within-Grade Connections:

- A number line (NY-2.MD.6) connects numbers, lengths, and units. Number lines are first used in grade 2. A number line shows units of length; the numbers at the end points of the lengths tell how many lengths so far. Bar-graph scales (NY-2.MD.10) and rulers (NY-2.MD.1 - 4) are number lines. Length units can be added and subtracted using rulers or number-line diagrams (NY-2.MD.5 & 6).[9]

NY-2.MD	Measurement and Data	
Work with time and money.		
7. Tell and write time from analog and digital clocks in five minute increments, using a.m. and p.m. Develop an understanding of common terms, such as, but not limited to, *quarter past*, *half past*, and q*uarter to*.		Coherence: NY-1.MD.3a → NY-2.MD.7 → NY-3.MD.1
8a. Count a mixed collection of coins whose sum is less than or equal to one dollar. 8b. Solve real world and mathematical problems within one dollar involving quarters, dimes, nickels, and pennies, using the ¢ (cent) symbol appropriately.		Coherence: NY-1.MD.3c → NY-2.MD.8 → e.g., If you have 2 quarters, 2 dimes, and 3 pennies, how many cents do you have? Note: Students are not introduced to decimals, and therefore the dollar symbol, until Grade 4.

Within-Grade Connections:

- Problems involving dollars, dimes, and pennies (NY-2.MD.8) should be connected with the place-value learning of hundreds, tens, and ones (NY-2.NBT.1). A dollar is 100 cents, or a "bundle" of 10 dimes, each of which is a "bundle" of 10 pennies. Work with dollars, dimes, and pennies (without the notation) can support methods of whole-number addition (e.g., dimes are added to dimes).[9]
- Work with nickels (NY-2.MD.8) and with telling time to the nearest five minutes on analog clocks (NY-2.MD.7) should be taken together with counting by 5s (NY-2.NBT.2) as contexts for gaining familiarity with repeating groups of 5 (NY-2.OA.4).[9]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students use rulers (NY-2.MD.1 – 4), clocks (NY-2.MD.7), coins (NY-2.MD.8), and the number line (essentially an abstract ruler or measurement scale), they are using objects that remain appropriate tools (MP.5) for a lifetime. The contexts of nickels, hands, and telling time, for example, may help students to understand the usefulness of counting by 5 in different situations. Through the use of these tools across the domains, students recognize that the same sequence of numbers (0, 5, 10, 15,…) is common to all of them. That sequence of number names expresses the regularity (MP.8) of a calculation (counting five more) that recurs in many contexts.[9]

NY-2.MD Measurement and Data

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Present the measurement data in a line plot, where the horizontal scale is marked off in whole-number units.

Coherence: NY-2.MD.9 → NY-3.MD.4

10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a picture graph or a bar graph.

Coherence: NY-1.MD.4 → NY-2.MD.10 → NY-3.MD.3

	Total Unknown	Both Addends Unknown	Addend Unknown
Put Together/ Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version:* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy?	*Version with "More":* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have?	*Version with "More":* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have?
	"How many fewer?" version: Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*Version with "Fewer":* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? $A + B = \square$	*Version with "Fewer":* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Within-Grade Connections:

- Picture graphs and bar graphs (NY-2.MD.10) can add variety as contexts for posing and solving addition and subtraction problems (NY-2.OA.1 & 2).[(9)]

NY-2.G	Geometry	
Reason with shapes and their attributes.		
1. Classify two-dimensional figures as polygons or non-polygons.	Coherence: NY-1.G.1 → NY-2.G.1 → NY-3.G.1	
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	Coherence: NY-2.G.2 → NY-3.MD.6	
3. Partition circles and rectangles into two, three, or four equal shares. Describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc. Describe the whole as *two halves*, *three thirds*, *four fourths*. Recognize that equal shares of identical wholes need not have the same shape.	Coherence: NY-1.G.3 → NY-2.G.3 → NY-3.G.2 NY-3.NF.1	

Grade 3 Overview

In Grade 3, instructional time should focus on four areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing polygons based on the number of sides and vertices. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Operations and Algebraic Thinking*** domain, students:
   - develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size;
   - use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors; and
   - compare a variety of solution strategies to learn the relationship between multiplication and division.
2. Through their learning in the ***Number Sense and Operations—Fractions*** domain, students:
   - develop an understanding of fractions, beginning with unit fractions;
   - view fractions in general as being built out of unit fractions, and use fractions along with visual fraction models to represent parts of a whole;
   - understand that the size of a fractional part is relative to the size of the whole. Use fractions to represent numbers equal to, less than, and greater than one; and
   - solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
3. Through their learning in the ***Measurement and Data*** domain, students:
   - recognize area as an attribute of two-dimensional regions;
   - measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area; and
   - understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
4. Through their learning in the ***Geometry*** domain, students:
   - classify polygons by examining their sides and vertices; and
   - relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematical Practices

1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

NY-3.OA Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers.

Coherence: NY-2.OA.4 → NY-3.OA.1 → NY-4.OA.1

e.g., Interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. Describe a context in which a total number of objects can be expressed as 5 × 7.

2. Interpret whole-number quotients of whole numbers.

e.g., Interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. Describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.

Coherence: NY-3.OA.3 → NY-4.OA.2

e.g., using drawings and equations with a symbol for the unknown number to represent the problem

	Unknown Product $a \times b = ?$	Group Size Unknown ("How many in each group?" Division) $a \times ? = p$ and $p \div a = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times b = p$ and $p \div b = ?$
Equal Groups	There are a bags with b plums in each bag. How many plums are there in all? *Measurement example:* You need a lengths of string, each b inches long. How much string will you need altogether?	If p plums are shared equally into a bags, then how many plums will be in each bag? *Measurement example:* You have p inches of string, which you will cut into a equal pieces. How long will each piece of string be?	If p plums are to be packed b to a bag, then how many bags are needed? *Measurement example:* You have p inches of string, which you will cut into pieces that are b inches long. How many pieces of string will you have?
Arrays & Area	There are a rows of apples with b apples in each row. How many apples are there? *Area example*: What is the area of an a cm by b cm rectangle?	If p apples are arranged into a equal rows, how many apples will be in each row? *Area example*: A rectangle has area p square centimeters. If it is a cm long, what is its width?	If p apples are arranged into equal rows of b apples, how many rows will there be? *Area example*: A rectangle has area p square centimeters. If it is b cm wide, what is its length?

Array problems can also be stated in terms of columns, exchanging the order of a and b, so that the same array is described. For example: There are b columns of apples with a apples in each column. How many apples are there?

4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

e.g., Determine the unknown number that makes the equation true in each of the equations: 8 × ? = 48, 5 = __ ÷ 3, 6 × 6 = ?

Within-Grade Connections:

- Students should begin work with multiplication and division (NY-3.OA) at or near the beginning of the year to allow time for understanding and fluency to develop.
  - The area models for products (NY-3.MD.7) are an important part of this process because they provide a visual model for understanding the concept of multiplication and because they provide a visual model that makes the distributive property explicit. Hence, work on concepts of area (NY-3.MD.5-6) should likely begin near the beginning of the year as well.[(14)]

NY-3.OA Operations and Algebraic Thinking

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.

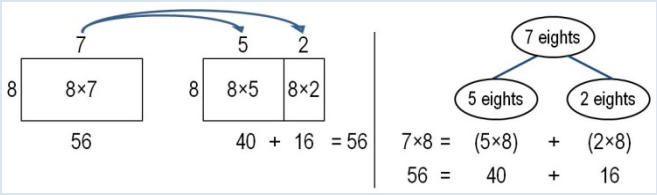
   Note: Students need not use formal terms for these properties.

Coherence: NY-3.OA.5 → NY-4.NBT.5, NY-4.NBT.6

e.g.,

- If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication)
- 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication)
- Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property)

Note: A variety of representations can be used when applying the properties of operations, which may or may not include parentheses. The area model (NY-3.MD.7c) is a multiplication/division strategy that appies the distributive property (NY-3.OA.5), e.g.,



6. Understand division as an unknown-factor problem.

Coherence: NY-3.OA.6 → NY-4.NBT.6

e.g., Find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Students learn and use strategies for finding products and quotients that are based on the properties of operations; for example, to find 4 × 7, they may recognize that 7 = 5 + 2 and compute 4 × 5 + 4 × 2. This is an example of seeing and making use of structure (MP.7). Such reasoning processes amount to brief arguments that students may construct and critique (MP.3).[14]

NY-3.OA	Operations and Algebraic Thinking

Multiply and divide within 100.

7a. Fluently solve single-digit multiplication and related divisions, using strategies such as the relationship between multiplication and division or properties of operations.

7b. Know from memory all products of two one-digit numbers.

Coherence: NY-3.OA.7 → NY-4.OA.4

e.g., Knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8.

Note on *Fluency* with Facts:

- Fluency involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.[10] Reaching fluency will take much of the year for many students.[14] For more on how children develop fluency, see *K–5 Progression on Counting and Cardinality and Operations and Algebraic Thinking*, pp. 18-19 and *Adding it Up*, pp. 182-195.

Note on *Fluency* vs. *Knowing from Memory*:

- The standards intentionally distinguish between asking for *fluency* with multiplication and division (NY-3.OA.7a) and asking students to *know from memory* multiplication facts (NY-3.OA.7b). *Fluency* means students are fast, accurate, flexible, and have understanding. They use strategies efficiently.[12] By the end of grade 3, students have sufficient experience with these strategies to *know from memory* all single-digit products.[10]

NY-3.OA	Operations and Algebraic Thinking

Solve problems involving the four operations, and identify and extend patterns in arithmetic.

8. Solve two-step word problems posed with whole numbers and having whole-number answers using the four operations.

   a. Represent these problems using equations or expressions with a letter standing for the unknown quantity.

   b. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Coherence: NY-2.OA.1 → NY-3.OA.8 → NY-4.OA.3

Note: Two-step problems need not be represented by a single expression or equation.

9. Identify and extend arithmetic patterns (including patterns in the addition table or multiplication table).

Coherence: NY-2.OA.3 → NY-3.OA.9 → NY-4.OA.5

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Students will analyze a number of situation types for multiplication and division, including arrays and measurement contexts. Extending their understanding of multiplication and division to these situations requires that they make sense of problems and persevere in solving them (MP.1), look for and make use of structure (MP.7) as they model these situations with mathematical forms (MP.4), and attend to precision (MP.6) as they distinguish different kinds of situations over time (MP.8).[14]

NY-3.NBT	Number and Operations in Base Ten	
Use place value understanding and properties of operations to perform multi-digit arithmetic.		
1.	Use place value understanding to round whole numbers to the nearest 10 or 100.	Coherence: NY-3.NBT.1 → NY-4.NBT.3
2.	Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. Note: A range of algorithms may be used.	Coherence: NY-2.NBT.5, NY-2.NBT.7 → NY-3.NBT.2 → NY-4.NBT.4 Note on and/or: Students should be taught to use strategies and algorithms based on place value, properties of operations, *and* the relationship between addition and subtraction; however, when solving any problem, students can choose any strategy.
3.	Multiply one-digit whole numbers by multiples of 10 in the range 10-90 using strategies based on place value and properties of operations.	Coherence: NY-3.NBT.3 → NY-4.NBT.5 e.g., 9 × 80, 5 × 60
4a.	Understand that the digits of a four-digit number represent amounts of thousands, hundreds, tens, and ones.	Coherence: NY-2.NBT.1 → NY-3.NBT.4a → NY-4.NBT.1 e.g., 3,245 equals 3 thousands, 2 hundreds, 4 tens, and 5 ones.
4b.	Read and write four digit numbers using base-ten numerals, number names, and expanded form.	Coherence: NY-2.NBT.3 → NY-3.NBT.4b → NY-4.NBT.2 e.g., The number 3,245 in expanded form can be written as 3,245 = 3,000 + 200 + 40 + 5.

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13] For more on developing procedural fluency, see *Adding it Up*, pp. 121-124.

NY-3.NF Number and Operations – Fractions

Develop understanding of fractions as numbers.

Note: Fractions are limited to those with denominators 2, 3, 4, 6, and 8.

1. Understand a unit fraction, $\frac{1}{b}$, is the quantity formed by 1 part when a whole is partitioned into b equal parts. Understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

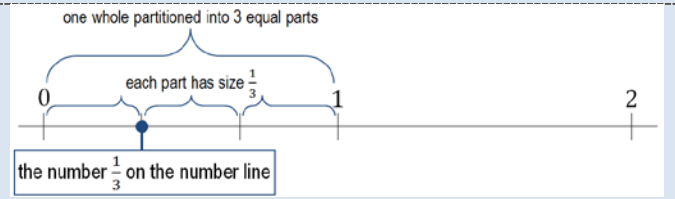
Coherence: NY-2.G.3 → NY-3.NF.1 → NY-4.NF.3

2. Understand a fraction as a number on the number line; represent fractions on a number line.

Coherence: NY-2.MD.6 → NY-3.NF.2

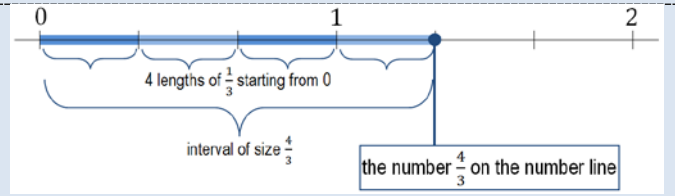
   a. Represent a fraction $\frac{1}{b}$ on a number line by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part starting at 0 locates the number $\frac{1}{b}$ on the number line.

e.g.,



   b. Represent a fraction $\frac{a}{b}$ on a number line by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

e.g.,



3. Explain equivalence of fractions and compare fractions by reasoning about their size.

Coherence: NY-3.NF.3 → NY-4.NF.1, NY-4.NF.5

   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

   b. Recognize and generate equivalent fractions.

e.g., $\frac{1}{2} = \frac{2}{4}$; $\frac{4}{6} = \frac{2}{3}$

   Explain why the fractions are equivalent.

e.g., using a visual fraction model

   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

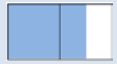
e.g., Express 3 in the form $3 = \frac{3}{1}$, recognize that $\frac{6}{3} = 2$, and locate $\frac{4}{4}$ and 1 at the same point on a number line.

   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.

Coherence: NY-3.NF.3d → NY-4.NF.2

e.g., using a visual fraction model

Note: Without specifying the whole, the shaded area could represent the fraction $\frac{3}{2}$ (if one square is the whole) or $\frac{3}{4}$ (if the entire rectangle is the whole).[15]



NY-3.MD	Measurement and Data	
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.		
1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve one-step word problems involving addition and subtraction of time intervals in minutes.		Coherence: NY-2.MD.7 → NY-3.MD.1 → NY-4.MD.2 e.g., representing the problem on a number line or other visual model Note: This includes one-step problems that cross into a new hour.
2a. Measure and estimate liquid volumes and masses of objects using grams (g), kilograms (kg), and liters (l). Note: Does not include compound units such as cm^3 and finding the geometric volume of a container.		
2b. Add, subtract, multiply, or divide to solve one-step word problems involving masses or liquid volumes that are given in the same units. Note: Does not include multiplicative comparison problems involving notions of "times as much."		Coherence: NY-2.MD.5 → NY-3.MD.2 e.g., using drawings (such as a beaker with a measurement scale) to represent the problem

Within-Grade Connections:

- Work with metric measurements (NY-3.MD.2) supports understanding of the base-ten number system (NY-3.NBT.4) and work in science.

NY-3.MD	Measurement and Data	
Represent and interpret data.		
3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in a scaled picture graph or a scaled bar graph.		Coherence: NY-2.MD.10 → NY-3.MD.3 e.g., Draw a bar graph in which each square in the bar graph might represent 5 pets.
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.		Coherence: NY-2.MD.9 → NY-3.MD.4 → NY-4.MD.4

Within-Grade Connections:

- Scaled graphs (NY-3.MD.3) can be a visually appealing context for solving one- and two-step multiplication and division problems (NY-3.OA.8).[14]

NY-3.MD Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. Recognize a square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
   b. Recognize a plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Coherence: NY-3.MD.5 → NY-5.MD.3

6. Measure areas by counting unit squares.

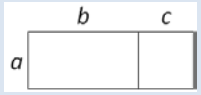
Coherence: NY-2.G.2 → NY-3.MD.6 → NY-5.MD.4

Note: Unit squares include square cm, square m, square in., square ft., and improvised units.

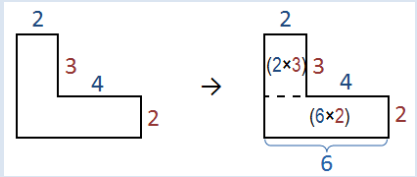
7. Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side length a and side length $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
   d. Recognize area as additive. Find areas of figures composed of non-overlapping rectangles, and apply this technique to solve real world problems.

Coherence: NY-3.MD.7 → NY-4.MD.3, NY-5.MD.5

e.g.,



e.g.,



Note: Problems include no more than one unknown side length.

Within-Grade Connections:

- Area models for products (NY-3.MD.7) make the distributive property (NY-3.OA.5) explicitly visible to students, and therefore, are an important part of the process of developing understanding and fluency with multiplication and division (NY-3.OA). In order to make this connection, students must first understand the conecpt of area (NY-3.MD.5-6). Therefore, this entire cluster should likely begin near the beginning of the year.[14]

NY-3.MD	Measurement and Data	
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.		
8a. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths or finding one unknown side length given the perimeter and other side lengths.		Coherence: NY-3.MD.8 → NY-4.MD.3
8b. Identify rectangles with the same perimeter and different areas or with the same area and different perimeters.		

NY-3.G	Geometry
Reason with shapes and their attributes.	
1. Recognize and classify polygons based on the number of sides and vertices (triangles, quadrilaterals, pentagons, and hexagons). Identify shapes that do not belong to one of the given subcategories.	Coherence: NY-2.G.1 → NY-3.G.1 → NY-4.G.2 Note: Include both regular and irregular polygons, however, students need not use formal terms "regular" and "irregular," e.g., students should be able to classify an irregular pentagon as "a pentagon," but do not need to classify it as an "irregular pentagon."
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.	Coherence: NY-2.G.3 → NY-3.G.2 e.g., Partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.

Within-Grade Connections:

- Student's work with partitioning shapes (NY-3.G.2) relates to visual fraction models (NY-3.NF).[14]

Grade 4 Overview

In Grade 4, instructional time should focus on three areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Number and Operations in Base Ten*** domain, students:
   - generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place;
   - apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers;
   - select and accurately apply appropriate methods to estimate or mentally calculate products, depending on the numbers and the context;
   - develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems;
   - apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends; and
   - select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
2. Through their learning in the ***Numbers and Operations—Fractions*** domain, students:
   - develop understanding of fraction equivalence and operations with fractions;
   - recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and develop methods for generating and recognizing equivalent fractions; and
   - extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
3. Through their learning in the ***Geometry*** domain, students:
   - deepen their understanding of properties of two-dimensional shapes (e.g., angles, parallelism, and symmetry).

Mathematical Practices

1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

NY-4.OA Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.

Coherence: NY-3.OA.1 → NY-4.OA.1 → NY-5.NF.5

e.g.,

- Interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 or 7 times as many as 5.
- Represent "Four times as many as eight is thirty two" as an equation, 4 × 8 = 32.

2. Multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison. Use drawings and equations with a symbol for the unknown number to represent the problem.

Coherence: NY-3.OA.3 → NY-4.OA.2

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$
Compare	A blue hat costs \$*b*. A red hat costs *a* times as much as the blue hat. How much does the red hat cost? *Measurement example:* A rubber band is *b* cm long. How long will the rubber band be when it is stretched to be *a* times as long?	A red hat costs \$*p* and that is *a* times as much as a blue hat costs. How much does a blue hat cost? *Measurement example:* A rubber band is stretched to be *p* cm long and that is *a* times as long as it was at first. How long was the rubber band at first?	A red hat costs \$*p* and a blue hat costs \$*b*. How many times as much does the red hat cost as the blue hat? *Measurement example:* A rubber band was *b* cm long at first. Now it is stretched to be *p* cm long. How many times as long is the rubber band now as it was at first?

Multiplicative Compare problems appear first in Grade 4, with whole-number values for *a*, *b*, and *p*, and with the "times as much" language in the table. In Grade 5, unit fractions language such as "one third as much" may be used.

3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.

   a. Represent these problems using equations or expressions with a letter standing for the unknown quantity.

   b. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Coherence: NY-3.OA.8 → NY-4.OA.3

Note: Multistep problems need not be represented by a single expression or equation.

Within-Grade Connections:

- The idea of "times as much" in multiplication (NY-4.OA.1) can be connected to the work that students do with units of measure (NY-4.MD.1–2) and with multiplication of a fraction by a whole number (NY-4.NF.4).[(14)]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- To compute and interpret remainders in word problems (NY-4.OA.3), students must reason abstractly and quantitatively (MP.2), make sense of problems (MP.1), and look for and express regularity in repeated reasoning (MP.8) as they search for the structure (MP.7) in problems with similar interpretations of remainders.[(14)]

NY-4.OA	Operations and Algebraic Thinking

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

Coherence: NY-3.OA.7 → NY-4.OA.4 → NY-6.NS.4

Within-Grade Connections:

- Work with factors and multiples (NY-4.OA.4) supports students' work with multi-digit arithmetic (NY-4.NBT.4 – 6) as well as their work with fraction equivalence (NY-4.NF.1 – 2).[14]

NY-4.OA	Operations and Algebraic Thinking

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify and informally explain apparent features of the pattern that were not explicit in the rule itself.

Coherence: NY-3.OA.9 → NY-4.OA.5 → NY-5.OA.3

e.g., Given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

NY-4.NBT	Number and Operations in Base Ten
Generalize place value understanding for multi-digit whole numbers. Note: Grade 4 expectations are limited to whole numbers less than or equal to 1,000,000.	
1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	Coherence: NY-3.NBT.4a → NY-4.NBT.1 → NY-5.NBT.1 e.g., Recognize that 70 × 10 = 700 (and, therefore, 700 ÷ 10 = 70) by applying concepts of place value, multiplication, and division.
2a. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. 2b. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Coherence: NY-3.NBT.4b → NY-4.NBT.2 → NY-5.NBT.3 e.g., 50,327 = 50,000 + 300 + 20 + 7
3. Use place value understanding to round multi-digit whole numbers to any place.	Coherence: NY-3.NBT.1 → NY-4.NBT.3 → NY-5.NBT.4

NY-4.NBT	Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic. Note: Grade 4 expectations are limited to whole numbers less than or equal to 1,000,000.	
4. Fluently add and subtract multi-digit whole numbers using a standard algorithm.	Coherence: NY-3.NBT.2 → NY-4.NBT.4
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Coherence: NY-3.NBT.3, NY-3.OA.5, NY-3.MD.7 → NY-4.NBT.5 → NY-5.NBT.5 Note on and/or: Students should be taught to use equations, rectangular arrays, *and* area models; however, when illustrating and explaining any calculation, students can choose any strategy.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Coherence: NY-3.OA.5, NY-3.OA.6, NY-3.MD.7 → NY-4.NBT.6 → NY-5.NBT.6 Notes on and/or: • Students should be taught to use strategies based on place value, the properties of operations, *and* the relationship between multiplication and division; however, when solving any problem, students can choose any strategy. • Students should be taught to use equations, rectangular arrays, *and* area models; however, when illustrating and explaining any calculation, students can choose any strategy.

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13] For more on developing procedural fluency, see *Adding it Up*, pp. 121-124.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students decompose numbers into sums of multiples of base-ten units to multiply them (NY-4.NBT.5), they are seeing and making use of structure (MP.7). As they illustrate and explain the calculation by using physical or drawn models, they are modeling (MP.4), using appropriate drawn tools strategically (MP.5) and attending to precision (MP.6) as they use base-ten units in the appropriate places.[14]

NY-4.NF Number and Operations—Fractions

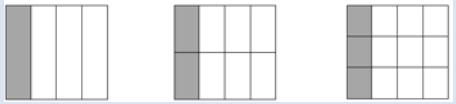
Extend understanding of fraction equivalence and ordering.

Note: Grade 4 expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

1. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{a \times n}{b \times n}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Coherence: NY-3.NF.3 → NY-4.NF.1 → NY-5.NF.1

e.g., $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{1 \times 3}{4 \times 3}$



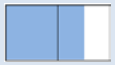
2. Compare two fractions with different numerators and different denominators.

Coherence: NY-3.NF.3d → NY-4.NF.2

e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$

Recognize that comparisons are valid only when the two fractions refer to the same whole.

Note: Without specifying the whole, the shaded area could represent the fraction $\frac{3}{2}$ (if one square is the whole) or $\frac{3}{4}$ (if the entire rectangle is the whole).[15]



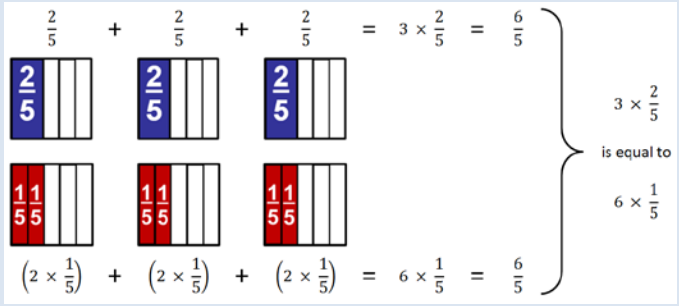
Record the results of comparisons with symbols >, =, or <, and justify the conclusions.

e.g., using a visual fraction model

NY-4.NF	Number and Operations—Fractions
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. Note: Grade 4 expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	
3. Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.	Coherence: NY-3.NF.1 → NY-4.NF.3 → NY-5.NF.1, NY-5.NF.2 Note: $\frac{1}{b}$ refers to the unit fraction for $\frac{a}{b}$.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions.	e.g., Justify decompositions by using a visual fraction model such as, but not limited to: • $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ • $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$ • $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$
c. Add and subtract mixed numbers with like denominators.	e.g., replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.	e.g., using visual fraction models and equations to represent the problem
4. Apply and extend previous understandings of multiplication to multiply a whole number by a fraction.	Coherence: NY-4.NF.4 → NY-5.NF.4 Note: This standard refers to *n* groups of a fraction (where *n* is a whole number), e.g., 4 groups of $\frac{1}{3}$; which lends itself to being thought about as repeated addition. In grade 5 (NY-5.NF.4) students will be multiplying a fraction by a whole number, e.g., $\frac{1}{3}$ of 4.
a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$.	e.g., Use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times \frac{1}{4}$, recording the conclusion with the equation $\frac{5}{4} = 5 \times \frac{1}{4}$.

b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a whole number by a fraction.

e.g., use a visual fraction model to express $3 \times \frac{2}{5}$ as $6 \times \frac{1}{5}$, recognizing this product as $\frac{6}{5}$, e.g.,



In general, $n \times \frac{a}{b} = \frac{(n \times a)}{b}$.

c. Solve word problems involving multiplication of a whole number by a fraction.

Coherence: NY-4.NF.4c → NY-5.NF.6

e.g., using visual fraction models and equations to represent the problem

e.g., If each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Within-Grade Connections:

- Addition of fractions (NY-4.NF.3) and concepts of angle measure (NY-4.MD.5a and NY-4.MD.7) are connected in that a one-degree measure is a fraction of an entire rotation and that adding angle measures together is adding fractions with a denominator of 360.[14]
- The work that students do with multiplication of a fraction by a whole number (NY-4.NF.4) can be connected to the idea of "times as much" in multiplication (NY-4.OA.1).[14]

NY-4.NF Number and Operations—Fractions

Understand decimal notation for fractions, and compare decimal fractions.

Note: Grade 4 expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.

   Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Coherence: NY-3.NF.3 → NY-4.NF.5 → NY-5.NBT.7

e.g., Express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.

6. Use decimal notation for fractions with denominators 10 or 100.

Coherence: NY-4.NF.6 → NY-5.NBT.3a

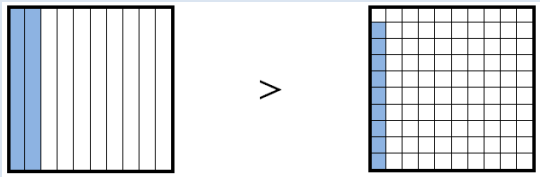
e.g.,

- Rewrite 0.62 as $\frac{62}{100}$ or $\frac{62}{100}$ as 0.62.
- Describe a length as 0.62 meters.
- Locate 0.62 on a number line.

7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.

Coherence: NY-4.NF.7 → NY-5.NBT.3b

e.g., using a visual model



0.2 > 0.09 because, when these decimals refer to the same whole, 2 out of 10 equal parts is more of that whole than 9 out of 100 equal parts. If the wholes were not the same size, this comparison would not be valid.



Within-Grade Connections:

- Students' work with decimals (NY-4.NF.5–7) depends to some extent on concepts of fraction equivalence (NY-4.NF.1-2) and elements of fraction arithmetic (NY-4.NF.3-4). Students express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100; comparisons of decimals require that students use similar reasoning to comparisons with fractions.[14]

NY-4.MD	Measurement and Data
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	
1. Know relative sizes of measurement units: ft., in.; km, m, cm	Coherence: NY-4.MD.1 → NY-5.MD.1 e.g., An inch is about the distance from the tip of your thumb to your first knuckle. A foot is the length of two dollar bills. A meter is about the height of a kitchen counter. A kilometer is 2 ½ laps around most tracks.
Know the conversion factor and use it to convert measurements in a larger unit in terms of a smaller unit: ft., in.; km, m, cm; hr., min., sec.	e.g., Know that 1 ft. is 12 times as long as 1 in. and express the length of a 4 ft. snake as 48 in.
Given the conversion factor, convert all other measurements within a single system of measurement from a larger unit to a smaller unit.	e.g., Given the conversion factors, convert kilograms to grams, pounds to ounces, or liters to milliliters.
Record measurement equivalents in a two-column table.	e.g., Generate a conversion table for feet and inches. Feet | Inches 1 | 12 2 | 24 3 | 36
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money. a. Solve problems involving fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. b. Represent measurement quantities using diagrams that feature a measurement scale, such as number lines.	Coherence: NY-3.MD.1 → NY-4.MD.2 Note: Grade 4 expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.	Coherence: NY-3.MD.7, NY-3.MD.8 → NY-4.MD.3 e.g., Find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

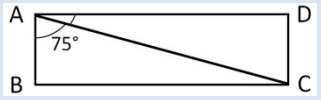
Within-Grade Connections:

- The work that students do with units of measure (NY-4.MD.1–2) can be connected to the idea of "times as much" in multiplication (NY-4.OA.1).[(14)]
- Standard NY-4.MD.2 refers to using the four operations to solve word problems involving measurement quantities such as liquid volume, mass, time, and so on. Some parts of this standard could be met earlier in the year (such as using whole-number multiplication to express measurements given in a larger unit in terms of a smaller unit — see also NY-4.MD.1), while others might be met only by the end of the year (such as word problems involving addition and subtraction of fractions or multiplication of a fraction by a whole number — see also NY-4.NF.3d and NY-4.NF.4c).[(14)]

NY-4.MD	Measurement and Data
Represent and interpret data.	
4. Make a line plot to display a data set of measurements in fractions of a unit $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$. Solve problems involving addition and subtraction of fractions by using information presented in line plots.	Coherence: NY-3.MD.4 → NY-4.MD.4 → NY-5.MD.2 e.g., Given measurement data on a line plot, find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Within-Grade Connections:

- When students use a line plot to display measurements in fractions of a unit and solve problems involving addition and subtraction of fractions presented in line plots (NY-4.MD.4), they practice adding and subtracting fractions with like denominators (NY-4.NF.3).[14]

NY-4.MD	Measurement and Data
Geometric measurement: understand concepts of angle and measure angles.	
5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. a. Recognize an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. b. Recognize an angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	
6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems.	e.g., using an equation with a symbol for the unknown angle measure; such as, in the rectangle below, angle CAD could be found by : 75 + *x* = 90 or 90 - 75 = ? 

Within-Grade Connections:

- Standard NY-4.MD.7 refers to word problems involving unknown angle measures. Before this standard can be met, students must understand concepts of angle measure (NY-4.MD.5) and, presumably, gain some experience measuring angles (NY-4.MD.6). Before that can happen, students must have some familiarity with the geometric terms that are used to define angles as geometric shapes (NY-4.G.1).[14]
- Addition of fractions (NY-4.NF.3) and concepts of angle measure (NY-4.MD.5a and NY-4.MD.7) are connected in that a one-degree measure is a fraction of an entire rotation and that adding angle measures together is adding fractions with a denominator of 360.[14]

NY-4.G	Geometry	
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.		
1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.		
2a. Identify and name triangles based on angle size (right, obtuse, acute). 2b. Identify and name all quadrilaterals with 2 pairs of parallel sides as parallelograms. 2c. Identify and name all quadrilaterals with four right angles as rectangles.		Coherence: NY-3.G.1 → NY-4.G.2 → NY-5.G.3
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.		

Grade 5 Overview

In Grade 5, instructional time should focus on three areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimals into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Number and Operations – Fractions*** and ***Operations and Algebraic Thinking*** domains, students:
   - apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators;
   - develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them; and
   - use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
2. Through their learning in the ***Operations and Algebraic Thinking*** and ***Number and Operations in Base Ten*** domains, students:
   - develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations;
   - apply understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths;
   - develop fluency with decimal computations to hundredths, and make reasonable estimates of their results; and
   - use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense.
3. Through their learning in the ***Measurement and Data*** and ***Geometry*** domains, students:
   - recognize volume as an attribute of three-dimensional space;
   - understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps;
   - understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume;
   - select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume;
   - decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes; and
   - measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Mathematical Practices

1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

NY-5.OA	Operations and Algebraic Thinking
Write and interpret numerical expressions.	
1. Apply the order of operations to evaluate numerical expressions.	Coherence: NY-5.OA.1 → NY-6.EE.2 e.g., • 6 + 8 ÷ 2 • (6 + 8) ÷ 2 Note: Exponents and nested grouping symbols are not included.
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	Coherence: NY-4.OA.1 → NY-5.OA.2 → NY-6.EE.2, NY-6.EE.3, NY-6.EE.4 e.g., Express the calculation "add 8 and 7, then multiply by 2" as (8 + 7) × 2. Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.

NY-5.OA	Operations and Algebraic Thinking
Analyze patterns and relationships.	
3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	Coherence: NY-4.OA.5 → NY-5.OA.3 → NY-6.EE.9 e.g., Given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

NY-5.NBT	Number and Operations in Base Ten
Understand the place value system.	
1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.	Coherence: NY-4.NBT.1 → NY-5.NBT.1
2. Use whole-number exponents to denote powers of 10. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.	Coherence: NY-5.NBT.2 → NY-6.EE.1
3. Read, write, and compare decimals to thousandths.	
a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.	Coherence: NY-4.NBT.2a, NY-4.NF.6 → NY-5.NBT.3a e.g., • $47.392 = 4 \times 10 + 7 \times 1 + 3 \times \frac{1}{10} + 9 \times \frac{1}{100} + 2 \times \frac{1}{1000}$ • $47.392 = (4 \times 10) + (7 \times 1) + (3 \times \frac{1}{10}) + (9 \times \frac{1}{100}) + (2 \times \frac{1}{1000})$ • $47.392 = (4 \times 10) + (7 \times 1) + (3 \times 0.1) + (9 \times 0.01) + (2 \times 0.001)$
b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Coherence: NY-4.NBT.2b, NY-4.NF.7 → NY-5.NBT.3b → NY-6.NS.7
4. Use place value understanding to round decimals to any place.	Coherence: NY-4.NBT.3 → NY-5.NBT.4

Within-Grade Connections:

- Conversions within the metric system (NY-5.MD.1) represent an important practical application of the place value system and an opportunity to develop understanding of it (NY-5.NBT.1 - 2).[(14)]
- Understanding that in a multi-digit number, a digit in one place represents $\frac{1}{10}$ of what it represents in the place to its left (NY-5.NBT.1) is an example of multiplying a quantity by a fraction (NY-5.NF.4).[(14)]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students explain patterns in the number of zeros of the product when multiplying a number by powers of 10 (NY-5.NBT.2), they have an opportunity to look for and express regularity in repeated reasoning (MP.8). When they use these patterns in division, they are making sense of problems (MP.1) and reasoning abstractly and quantitatively (MP.2).[(14)]

NY-5.NBT	Number and Operations in Base Ten
Perform operations with multi-digit whole numbers and with decimals to hundredths.	
5. Fluently multiply multi-digit whole numbers using a standard algorithm.	Coherence: NY-4.NBT.5 → NY-5.NBT.5 → NY-6.NS.3
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Coherence: NY-4.NBT.6 → NY-5.NBT.6 → NY-6.NS.2 Notes on and/or: • Students should be taught to use strategies based on place value, the properties of operations, *and* the relationship between multiplication and division; however, when solving any problem, students can choose any strategy. • Students should be taught to use equations, rectangular arrays, *and* area models; however, when illustrating and explaining any calculation, students can choose any strategy.
7. Using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations: • add and subtract decimals to hundredths; • multiply and divide decimals to hundredths. Relate the strategy to a written method and explain the reasoning used.	Coherence: NY-4.NF.5 → NY-5.NBT.7 → NY-6.NS.3 Notes on and/or: Students should be taught to use concrete models and drawings; as well as strategies based on place value, properties of operations, *and* the relationship between operations. When solving any problem, students can choose to use a concrete model *or* a drawing. Their strategy must be based on place value, properties of operations, or the relationship between operations. Note: Division problems are limited to those that allow for the use of concrete models or drawings, strategies based on properties of operations, and/or the relationship between operations (e.g., 0.25 ÷ 0.05). Problems should not be so complex as to require the use of an algorithm (e.g., 0.37 ÷ 0.05).

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13] For more on developing procedural fluency, see *Adding it Up*, pp. 121-124.

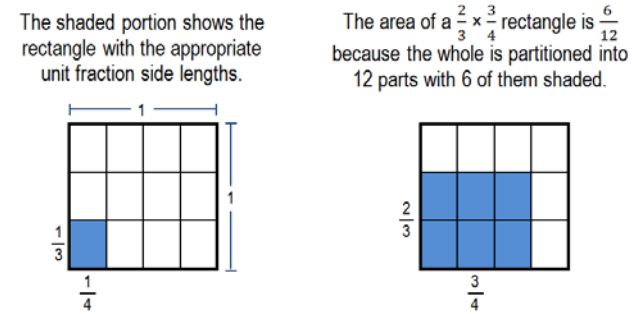
Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students break divisors and dividends into sums of multiples of base-ten units (NY-5.NBT.6), they are seeing and making use of structure (MP.7) and attending to precision (MP.6). Initially for most students, multi-digit division problems take time and effort, so they also require perseverance (MP.1) and looking for and expressing regularity in repeated reasoning (MP.8).[14]

NY-5.NF	Number and Operations—Fractions
Use equivalent fractions as a strategy to add and subtract fractions.	
1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	Coherence: NY-4.NF.1, NY-4.NF.3c → NY-5.NF.1 e.g., • $\frac{1}{3}+\frac{2}{9}=\frac{3}{9}+\frac{2}{9}=\frac{5}{9}$ • $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	Coherence: NY-4.NF.3d → NY-5.NF.2 e.g., using visual fraction models or equations to represent the problem e.g., Recognize an incorrect result $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$ by observing that $\frac{3}{7}<\frac{1}{2}$.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students use benchmark fractions and number sense to estimate mentally (NY-5.NF.2) they are reasoning abstractly and quantitatively (MP.2). When students assess the reasonableness of answers they are making sense of problems (MP.1).

NY-5.NF	Number and Operations—Fractions
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	
3. Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$).	Coherence: NY-5.NF.3 → NY-6.RP.2 e.g., Interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$.
Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.	e.g., using visual fraction models or equations to represent the problem e.g., If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.	Coherence: NY-4.NF.4 → NY-5.NF.4 → NY-6.G.1
a. Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.	e.g., Use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.
b. Find the area of a rectangle with fractional side lengths by tiling it with rectangles of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	e.g., 
5. Interpret multiplication as scaling (resizing).	Coherence: NY-4.OA.1 → NY-5.NF.5 → NY-6.RP.1
a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	e.g., In the case of $10 \times \frac{1}{2} = 5$, 5 is half of 10 and 5 is 10 times larger than $\frac{1}{2}$.
b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case). Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number. Relate the principle of fraction equivalence $\frac{a}{b} = \frac{a}{b} \times \frac{n}{n}$ to the effect of multiplying $\frac{a}{b}$ by 1.	e.g., • Explain why $4 \times \frac{3}{2}$ is greater than 4. • Explain why $4 \times \frac{1}{2}$ is less than 4. • $\frac{1}{3}$ is equivalent to $\frac{2}{6}$ because $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$.

6. Solve real world problems involving multiplication of fractions and mixed numbers.	Coherence: NY-4.NF.4c → NY-5.NF.6 e.g., using visual fraction models or equations to represent the problem
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Note: Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement until grade 6 (NY-6.NS.1).	Coherence: NY-5.NF.7 → NY-6.NS.1
a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.	e.g., Create a story context for $\frac{1}{3} \div 4$ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ because $\frac{1}{12} \times 4 = \frac{1}{3}$.
b. Interpret division of a whole number by a unit fraction, and compute such quotients.	e.g., Create a story context for $4 \div \frac{1}{5}$ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div \frac{1}{5} = 20$ because $20 \times \frac{1}{5} = 4$.
c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.	e.g., using visual fraction models and equations to represent the problem e.g., How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb. of chocolate equally? How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?

Within-Grade Connections:

- Understanding that in a multi-digit number, a digit in one place represents $\frac{1}{10}$ of what it represents in the place to its left (NY-5.NBT.1) is an example of multiplying a quantity by a fraction (NY-5.NF.4).[14]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- The understanding of multiplication as scaling (NY-5.NF.5) is an important opportunity for students to reason abstractly (MP2).[15]

NY-5.MD	Measurement and Data

Convert like measurement units within a given measurement system.

1. Convert among different-sized standard measurement units within a given measurement system when the conversion factor is given. Use these conversions in solving multi-step, real world problems.

Coherence: NY-4.MD.1 → NY-5.MD.1 → NY-6.RP.3d

Notes:

- All conversion factors will be given.
- Grade 5 expectations for decimal operations are limited to work with decimals to hundredths.

Within-Grade Connections:

- Work on this standard supports computation with decimals to hundredths (NY-5.NBT.7). For example, converting 5 cm to 0.05 m involves computation with decimals to hundredths.(14)
- Conversions within the metric system (NY-5.MD.1) are an important practical application of the place value system and an opportunity to develop understanding of it (NY-5.NBT.1 - 2).

NY-5.MD	Measurement and Data

Represent and interpret data.

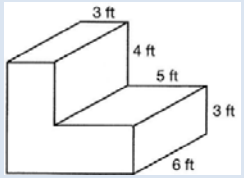
2. Make a line plot to display a data set of measurements in fractions of a unit $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$. Use operations on fractions for this grade to solve problems involving information presented in line plots.

Coherence: NY-4.MD.4 → NY-5.MD.2 → NY-6.SP.2, NY-6.SP.4

e.g., Given different measurements of liquid in identical beakers, make a line plot to display the data and find the total amount of liquid in all of the beakers.

Within-Grade Connections:

- Displaying data of measurements in fractions of a unit and solving problems involving information presented in line plots (NY-5.MD.2), provides an opportunity for solving real-world problems with operations on fractions (NY-5.NF).(14)

NY-5.MD	Measurement and Data	
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.		
3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. Recognize that a cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. Recognize that a solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.		Coherence: NY-3.MD.5 → NY-5.MD.3
4. Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.		Coherence: NY-3.MD.6 → NY-5.MD.4
5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	e.g., 	Coherence: NY-3.MD.7 → NY-5.MD.5 → NY-6.G.2, NY-6.G.5

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students show that the volume of a right rectangular prism is the same as would be found by multiplying the side lengths (NY-5.MD.5), they also have an opportunity to look for and express regularity in repeated reasoning (MP.8). They attend to precision (MP.6) as they use correct length or volume units, and they use appropriate tools strategically (MP.5) as they understand or make drawings to show these units.[(14)]

NY-5.G	Geometry	
Graph points on the coordinate plane to solve real-world and mathematical problems.		
1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond.		Coherence: NY-5.G.1 → NY-6.NS.6 e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate
2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.		Coherence: NY-5.G.2 → NY-6.NS.8, NY-6.G.3

NY-5.G	Geometry	
Classify two-dimensional figures into categories based on their properties.		
3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.		Coherence: NY-4.G.2 → NY-5.G.3 e.g., All rectangles have four right angles and squares are rectangles, so all squares have four right angles. Note: The inclusive definition of a trapezoid will be utilized, which defines a trapezoid as "A quadrilateral with *at least* one pair of parallel sides."
4. Classify two-dimensional figures in a hierarchy based on properties.		

Grade 6 Overview

In Grade 6, instructional time should focus on five areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; (4) deepening understanding of area, surface area and volume; and (5) developing understanding of simple probabilities and statistical thinking. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Ratios and Proportional Relationships*** domain, students:
   - use reasoning about multiplication and division to solve ratio and rate problems about quantities;
   - connect understanding of multiplication and division with ratios and rates by viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities; and
   - expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions.
2. Through their learning in the ***Number System*** domain, students:
   - use the meaning of fractions and relationships between multiplication and division to understand and explain why the procedures for dividing fractions make sense;
   - extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, particularly negative integers; and
   - reason about the order and absolute value of rational numbers and about the location of points on a coordinate plane.
3. Through their learning in the ***Expressions, Equations, and Inequalities*** domain, students:
   - write expressions and equations that correspond to give situations, using variables to represent an unknown and describe relationships between quantities;
   - understand that expressions in different forms can be equivalent, and use the properties of operations to rewrite and evaluate expressions in equivalent forms; and
   - use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations.
4. Through their learning in the ***Geometry*** domain, students:
   - find areas of polygons, surface areas of prisms, and use area models to understand perfect squares; and
   - extend formulas for the volume of a right rectangular prism to fractional side lengths and use volume models to understand perfect cubes.
5. Through their learning in the ***Statistics and Probability*** domain, students:
   - learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected; and
   - understand the probability of a chance event and develop probability models for simple events.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NY-6.RP	Ratios and Proportional Relationships
Understand ratio concepts and use ratio reasoning to solve problems.	
1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	Coherence: NY-5.NF.5 → NY-6.RP.1 e.g., "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received three votes."
2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship. Note: Expectations for unit rates in this grade are limited to non-complex fractions.	Coherence: NY-5.NF.3 → NY-6.RP.2 → NY-7.RP.1 e.g., "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there are ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."
3. Use ratio and rate reasoning to solve real-world and mathematical problems.	Note: Strategies may include but are not limited to the following: tables of equivalent ratios, tape diagrams, double number lines, and equations.
a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	Coherence: NY-6.RP.3a → NY-7.RP.2b
b. Solve unit rate problems.	Coherence: NY-6.RP.3b → NY-7.RP.1 e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? What is the unit rate? Note: Problems may include unit pricing and constant speed.
c. Find a percent of a quantity as a rate per 100. Solve problems that involve finding the whole given a part and the percent, and finding a part of a whole given the percent.	Coherence: NY-6.RP.3c → NY-7.RP.3 e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	Coherence: NY-5.MD.1 → NY-6.RP.3d Note: Conversion of units occur within a given measurement system, not across different measurement systems.

Within-Grade Connections:

- Plotting pairs of values on the coordinate plane (NY-6.RP.3a) supports other work with the coordinate plane in The Number System domain (NY-6.NS.6 & 8). [(14)]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Thinking about the point (1, r) in a graph of a proportional relationship with unit rate r (NY-6.RP.3) involves reasoning abstractly and quantitatively (MP.2). The graph models with mathematics (MP.4) and uses appropriate tools strategically (MP.5). [(14)]

NY-6.NS	The Number System	
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.		
1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions.		Coherence: NY-5.NF.7 → NY-6.NS.1 → NY-7.NS.2 e.g., Create a story context for $(\frac{2}{3}) \div (\frac{3}{4})$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. In general, $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$. e.g., • How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? • How many $\frac{3}{4}$ cup servings are in $\frac{2}{3}$ of a cup of yogurt? • How wide is a rectangular strip of land with length $\frac{3}{4}$ mi. and area $\frac{1}{2}$ square mi.? Note: Strategies may include but are not limited to the following: using visual fraction models, a standard algorithm, and equations to represent the problem.

NY-6.NS	The Number System	
Compute fluently with multi-digit numbers and find common factors and multiples.		
2.	Fluently divide multi-digit numbers using a standard algorithm.	Coherence: NY-5.NBT.6 → NY-6.NS.2
3.	Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	Coherence: NY-5.NBT.5, NY-5.NBT.7 → NY-6.NS.3 → NY-7.NS.3
4.	Find the greatest common factor of two whole numbers less than or equal to 100. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor other than 1. Find the least common multiple of two whole numbers less than or equal to 12.	Coherence: NY-4.OA.4 → NY-6.NS.4 e.g., Express 36 + 8 as 4(9 + 2).

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13] For more on developing procedural fluency, see *Adding it Up*, pp. 121-124.

Within-Grade Connections:

- Students use their skill in recognizing common factors (NY-6.NS.4) to rewrite expressions (NY-6.EE.3). [14]

NY-6.NS	The Number System	
Apply and extend previous understandings of numbers to the system of rational numbers.		
5.	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	Coherence: NY-6.NS.5 → NY-7.NS.1 e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge

6. Understand a rational number as a point on the number line. Use number lines and coordinate axes to represent points on a number line and in the coordinate plane with negative number coordinates.	Coherence: NY-5.G.1 → NY-6.NS.6 → NY-7.NS.1
a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line. Recognize that the opposite of the opposite of a number is the number itself, and that 0 is its own opposite.	e.g., With the number 3, – (–3) = 3.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane. Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	
c. Find and position integers and other rational numbers on a horizontal or vertical number line. Find and position pairs of integers and other rational numbers on a coordinate plane.	
7. Understand ordering and absolute value of rational numbers.	Coherence: NY-5.NBT.3b → NY-6.NS.7 → NY-7.NS.1
a. Interpret statements of inequality as statements about the relative position of two numbers on a number line.	e.g., Interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.
b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.	e.g., Write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.
c. Understand the absolute value of a rational number as its distance from 0 on the number line. Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.	e.g., For an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.
d. Distinguish comparisons of absolute value from statements about order.	e.g., Someone with a balance of $100 in their bank account has more money than someone with a balance of –$1,000, because 100 > –1,000. But, the second person's debt balance is much greater than the first person's credit balance because |–1,000| > |100|.
8. Solve real-world and mathematical problems by graphing points on a coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	Coherence: NY-5.G.2 → NY-6.NS.8

Within-Grade Connections:

- Students must be able to place rational numbers on a number line (NY-6.NS.7) before they can place ordered pairs of rational numbers on a coordinate plane (NY-6.NS.8).[(14)]
- Plotting rational numbers in the coordinate plane (NY-6.NS.8) is part of analyzing proportional relationships (NY-6.RP.3a).[(14)]

NY-6.EE	Expressions, Equations, and Inequalities
Apply and extend previous understandings of arithmetic to algebraic expressions.	
1. Write and evaluate numerical expressions involving whole-number exponents.	Coherence: NY-5.NBT.2 → NY-6.EE.1
2. Write, read, and evaluate expressions in which letters stand for numbers.	Coherence: NY-5.OA.1, NY-5.OA.2 → NY-6.EE.2
a. Write expressions that record operations with numbers and with letters standing for numbers.	e.g., Express the calculation "Subtract *y* from 5" as 5 – *y*.
b. Identify parts of an expression using mathematical terms (term, coefficient, sum, difference, product, factor, and quotient); view one or more parts of an expression as a single entity.	e.g., Describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
c. Evaluate expressions given specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order (Order of Operations).	e.g., Use the formulas $V = s^3$ and $SA = 6s^2$ to find the volume and surface area of a cube with sides of length $s = ½$. Note: Expressions may or may not include parentheses. Nested grouping symbols are not included.
3. Apply the properties of operations to generate equivalent expressions.	Coherence: NY-5.OA.2 → NY-6.EE.3 → NY-7.EE.1 e.g., Apply the distributive property to the expression 3(2 + *x*) to produce the equivalent expression 6 + 3*x*; apply the distributive property to the expression 24*x* + 18*y* to produce the equivalent expression 6(4*x* + 3*y*); apply properties of operations to *y* + *y* + *y* to produce the equivalent expression 3*y*.
4. Identify when two expressions are equivalent.	Coherence: NY-5.OA.2 → NY-6.EE.4 → NY-7.EE.2 e.g., The expressions *y* + *y* + *y* and 3*y* are equivalent because they name the same number regardless of which number *y* represents.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Reading and transforming expressions (NY-6.EE.1 – 4) involves seeing and making use of structure (MP.7).[14]

NY-6.EE Expressions, Equations, and Inequalities

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. Understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$; $x - p = q$; $px = q$; and $\frac{x}{p} = q$ for cases in which p, q, and x are all nonnegative rational numbers.

Coherence: NY-6.EE.7 → NY-7.EE.4a

Note: For the $\frac{x}{p} = q$ case, $p \neq 0$.

Analogous arithmetical and algebraic solutions

J. bought three packs of balloons. He opened them and counted 12 balloons. How many balloons are in a pack?

Arithmetical solution
If three packs have twelve balloons, then one pack has $12 \div 3 = 4$ balloons.

Algebraic solution
Defining the variable: Let b be the number of balloons in a pack.
Writing the equation:

$$3b = 12$$

Solving (mirrors the reasoning of the numerical solution):

$$3b = 12 \rightarrow \frac{3b}{3} = \frac{12}{3}$$

$$b = 4.$$

(16)

8. Write an inequality of the form $x > c$, $x \geq c$, $x \leq c$, or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of these forms have infinitely many solutions; represent solutions of such inequalities on a number line.

Coherence: NY-6.EE.8 → NY-7.EE.4b

Within-Grade Connections:

- Equations of the form $px = q$ (NY-6.EE.7) are unknown-factor problems; the solution will sometimes be the quotient of a fraction divided by a fraction (NY-6.NS.1).(14)

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Relating expressions (NY-6.EE.6), equations (NY-6.EE.7), and inequalities (NY-6.EE.8) to situations requires making sense of problems (MP.1), reasoning abstractly and quantitatively (MP.2), and modeling with mathematics (MP.4).(14)
- The sequence of steps in the solution of an equation (NY-6.EE.5 & 7) is a logical argument that students can construct and critique (MP.3). Such arguments require looking for and making use of structure (MP.7) and, over time, expressing regularity in repeated reasoning (MP.8).(14)

NY-6.EE	Expressions, Equations, and Inequalities
Represent and analyze quantitative relationships between dependent and independent variables.	
9. Use variables to represent two quantities in a real-world problem that change in relationship to one another.	Coherence: NY-5.OA.3 → NY-6.EE.9 e.g., In a problem involving motion at constant speed, list and graph ordered pairs of distances and times.
Given a verbal context and an equation, identify the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	e.g., Given the equation $d = 65t$ to represent the relationship between distance and time, identify t as the independent variable and d as the dependent variable.

Within-Grade Connections:

- Work with representing quantitative relationships between dependent and independent variables (NY-6.EE.9) can be connected with work with ratios and proportional relationships (NY-6.RP).[(14)]

NY-6.G	Geometry
Solve real-world and mathematical problems involving area, surface area, and volume.	
1. Find area of triangles, trapezoids, and other polygons by composing into rectangles or decomposing into triangles and quadrilaterals. Apply these techniques in the context of solving real-world and mathematical problems.	Coherence: NY-5.NF.4 → NY-6.G.1 → NY-7.G.6 Note: The inclusive definition of a trapezoid will be utilized, which defines a trapezoid as "A quadrilateral with *at least* one pair of parallel sides." (This definition includes parallelograms.)
2. Find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	Coherence: NY-5.MD.5 → NY-6.G.2 → NY-7.G.6
3. Draw polygons in the coordinate plane given coordinates for the vertices. Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	Coherence: NY-5.G.2 → NY-6.G.3
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	Coherence: NY-6.G.4 → NY-7.G.6 Note: Three-dimensional figures include only right rectangular prisms, right rectangular pyramids, and right triangular prisms. When finding surface areas, all necessary measurements will be given.
5. Use area and volume models to explain perfect squares and perfect cubes.	Coherence: NY-5.MD.5 → NY-6.G.5

Within-Grade Connections:

- Students work on problems with areas of triangles and volumes of right rectangular prisms (NY-6.G.1 & 2, and NY-6.G.4), connects to work in the NY-6.EE domain.[(14)]
- Writing, reading, evaluating, and transforming variable expressions (NY-6.EE.1 – 4) and solving equations and inequalities (NY-6.EE.7 & 8) can be combined with use of the volume formulas $V = lwh$ and $V = Bh$ (NY-6.G.2).[(14)]
- Drawing polygons on the coordinate plane (NY-6.G.3), supports other work with the coordinate plane in The Number System domain (NY-6.NS.6 & 8).[(14)]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Area, surface area, and volume (NY-6.G.1 & 2, and NY-6.G.4) present modeling opportunities (MP.4) and require students to attend to precision with the types of units involved (MP.6).[(14)]

NY-6.SP	Statistics and Probability
Develop understanding of statistical variability.	
1a. Recognize that a statistical question is one that anticipates variability in the data related to the question and accounts for it in the answers.	e.g., "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.
1b. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population.	Note: Students need to understand that data are generated with respect to particular contexts or situations and can be used to answer questions about those contexts or situations.
1c. Understand that the method and sample size used to collect data for a particular question is intended to reduce the difference between a population and a sample taken from the population so valid inferences can be drawn about the population. Generate multiple samples (or simulated samples) of the same size to recognize the variation in estimates or predictions.	Coherence: NY-6.SP.1c → NY-7.SP.4 Note: Examples of acceptable methods to obtain a representative sample from a population include, but are not limited to, a simple random sample for a given population or a systematic random sample for an unknown population. Examples of unacceptable methods of sampling include, but are not limited to, online polls and convenience sampling because they introduce bias and are not representative of the population.
2. Understand that a set of quantitative data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	Coherence: NY-5.MD.2 → NY-6.SP.2 → NY-7.SP.3 Notes: • Students need to determine and justify the most appropriate graph to display a given set of data (histogram or dot plot). • Students extend their knowledge of symmetric shapes, to describe data displayed in dot plots and histograms in terms of symmetry. They identify clusters, peaks and gaps, recognizing common shapes and patterns in these displays of data distributions, and ask why a distribution takes on a particular shape for the context of the variable being considered.[17]
3. Recognize that a measure of center for a quantitative data set summarizes all of its values with a single number while a measure of variation describes how its values vary with a single number.	Coherence: NY-6.SP.3 → NY-7.SP.4 Note: Measures of center are mean, median, and mode. The measure of variation is the range.

Within-Grade Connections:

- The standards in this cluster represent opportunities to apply percentages and proportional reasoning. To make inferences about a population, one needs to apply such reasoning to the sample and the entire population.[14]

NY-6.SP	Statistics and Probability	
Summarize and describe distributions.		
4.	Display quantitative data in plots on a number line, including dot plots, and histograms.	Coherence: NY-5.MD.2 → NY-6.SP.4 → NY-7.SP.1
5.	Summarize quantitative data sets in relation to their context.	
a.	Report the number of observations.	
b.	Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.	
c.	Calculate range and measures of center, as well as describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	Coherence: NY-6.SP.5c → NY-7.SP.1 Note: Measures of center are mean, median, and mode. The measure of variation is the range. The role of outliers should be discussed, but no formula is required.
d.	Relate the range and the choice of measures of center to the shape of the data distribution and the context in which the data were gathered.	Note: Measures of center are mean, median, and mode. The measure of variation is the range.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Working with data (NY-6.SP.4 & 5) gives students an opportunity to use appropriate tools strategically (MP.5). For example, spreadsheets can be powerful for working with a data set with dozens or hundreds of data points.[(14)]

NY-6.SP	Statistics and Probability
Investigate chance processes and develop, use, and evaluate probability models.	
6. Understand that the probability of a chance event is a number between 0 and 1 inclusive, that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	
7. Approximate the probability of a simple event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.	e.g., When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. Note: Compound events are introduced in grade 7.
8. Develop a probability model and use it to find probabilities of simple events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.	Coherence: NY-6.SP.8 → NY-7.SP.8
a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of simple events.	e.g., The probability of rolling a six-sided fair number cube and landing on a 2 is $\frac{1}{6}$. The probability of landing on an even number is $\frac{3}{6}$.
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.	e.g., Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

Within-Grade Connections:

- Probability models draw on proportional reasoning and should be connected to the major work in those standards.[14]

Grade 7 Overview

In Grade 7, instructional time should focus on three areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; and (3) drawing inferences about populations based on samples. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Ratios and Proportional Relationships*** domain, students:
   - extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems;
   - use their understanding of ratios and proportionality to solve a wide variety of percent problems;
   - solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects;
   - graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line; and
   - distinguish proportional relationships from other relationships.
2. Through their learning in the ***Number System*** and the ***Expressions, Equations, and Inequalities*** domains, students:
   - develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers;
   - extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division;
   - explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers by applying properties of operations, and view negative numbers in terms of everyday contexts; and
   - use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.
3. Through their learning in the ***Statistics and Probability*** domain, students:
   - build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations;
   - begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences; and
   - extend previous understandings of simple probabilities in grade 6 to calculate probabilities of compound events.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NY-7.RP	Ratios and Proportional Relationships
Analyze proportional relationships and use them to solve real-world and mathematical problems.	
1. Compute unit rates associated with ratios of fractions.	Coherence: NY-6.RP.2, NY-6.RP.3b → NY-7.RP.1 e.g., If a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour with 2 being the unit rate. Note: Problems may include ratios of lengths, areas, and other quantities measured in like or different units, including across measurement systems.
2. Recognize and represent proportional relationships between quantities.	
a. Decide whether two quantities are in a proportional relationship.	Note: Strategies include but are not limited to the following: testing for equivalent ratios in a table and/or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	Coherence: NY-6.RP.3a → NY-7.RP.2b → NY-8.EE.6, NY-8.F.2, NY-8.F.4
c. Represent a proportional relationship using an equation.	Coherence: NY-7.RP.2c → NY-8.EE.5 e.g., If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	Coherence: NY-7.RP.2d → NY-8.F.5
3. Use proportional relationships to solve multistep ratio and percent problems.	Coherence: NY-6.RP.3c → NY-7.RP.3 Note: Examples of percent problems include: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Proportional relationships (NY-7.RP.2 & 3) present opportunities for modeling (MP.4). For example, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building for modeling purposes.[14]

NY-7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers. Represent addition and subtraction on a horizontal or vertical number line.
   a. Describe situations in which opposite quantities combine to make 0.
   b. Understand addition of rational numbers; $p + q$ is the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
   d. Apply properties of operations as strategies to add and subtract rational numbers.

   Coherence: NY-6.NS.5, NY-6.NS.6, NY-6.NS.7 → NY-7.NS.1

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
   b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(\frac{p}{q}) = \frac{-p}{q} = \frac{p}{-q}$. Interpret quotients of rational numbers by describing real-world contexts.
   c. Apply properties of operations as strategies to multiply and divide rational numbers.
   d. Convert a fraction to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

   Coherence: NY-6.NS.1 → NY-7.NS.2 → NY-8.NS.1

3. Solve real-world and mathematical problems involving the four operations with rational numbers.

   Note: Computations with rational numbers extend the rules for manipulating fractions to complex fractions limited to $\frac{\frac{a}{b}}{\frac{c}{d}}$ where a, b, c, and d are integers and b, c, and $d \neq 0$.

   Coherence: NY-6.NS.3 → NY-7.NS.3

Within-Grade Connections:

- Because rational number arithmetic (NY-7.NS.1 – 3) underlies the problem solving detailed in NY-7.EE.3 as well as the solution of linear expressions and equations (NY-7.EE.1 & 2 and NY-7.EE.4), this work should likely begin at or near the start of the year.[14]

NY-7.EE	Expressions, Equations, and Inequalities	
Use properties of operations to generate equivalent expressions.		
1.	Add, subtract, factor, and expand linear expressions with rational coefficients by applying the properties of operations.	Coherence: NY-6.EE.3 → NY-7.EE.1 → NY-8.EE.7
2.	Understand that rewriting an expression in different forms in real-world and mathematical problems can reveal and explain how the quantities are related.	Coherence: NY-6.EE.4 → NY-7.EE.2 → NY-8.EE.8c e.g., $a + 0.05a$ and $1.05a$ are equivalent expressions meaning that "increase by 5%" is the same as "multiply by 1.05."

NY-7.EE	Expressions, Equations, and Inequalities	
Solve real-life and mathematical problems using numerical and algebraic expressions, equations, and inequalities.		
3.	Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate. Assess the reasonableness of answers using mental computation and estimation strategies.	e.g., • If a woman making $25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. • If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	Note: Solving equations that contain variables on both sides is not an expectation in grade 7.
a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	Coherence: NY-6.EE.7 → NY-7.EE.4a → NY-8.EE.7, NY-8.F.3 e.g., The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? Notes: • The words *leading to* in the standard may require students to simplify or combine like terms on the same side of the equation before it is in the form stated in the standard. • This standard is a fluency expectation for grade 7.
b. Solve word problems leading to inequalities of the form $px + q > r$, $px + q \geq r$, $px + q \leq r$, or $px + q < r$, where p, q, and r are rational numbers. Graph the solution set of the inequality on the number line and interpret it in the context of the problem.	Coherence: NY-6.EE.8 → NY-7.EE.4b e.g., As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions. Note: The words *leading to* in the standard may require students to simplify or combine like terms on the same side of the equation before it is in the form stated in the standard.

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

Within-Grade Connections:

- The work leading to meeting standards NY-7.EE.1 – 4 could be divided into two phases, one centered on addition and subtraction (e.g., solving $x + q = r$) in relation to rational number addition and subtraction (NY-7.NS.1) and another centered on multiplication and division (e.g., solving $px + q = r$ and $p(x + q) = r$) in relation to rational number multiplication and division (NY-7.NS.2).[14]
- Meeting standard NY-7.EE.3 in its entirety will involve using rational number arithmetic (NY-7.NS.1 – 3) and percents (NY-7.RP.3). Work leading to meeting this standard could be organized as a recurring activity that tracks the students' ongoing acquisition of new skills in rational number arithmetic and percents.[14]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students compare arithmetic and algebraic solutions to the same problem (NY-7.EE.4a), they are identifying correspondences between different approaches (MP.1).[14]
- Solving an equation such as $4 = 8(x - ½)$ requires students to see and make use of structure (MP.7), temporarily viewing $x - ½$ as a single entity.[14]

NY-7.G	Geometry
Draw, construct, and describe geometrical figures and describe the relationships between them.	
1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	
2. Draw triangles when given measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	Note: Create triangles through the use of freehand drawings, materials (scaffolds may include: pipe cleaners, Legos®, and toothpicks), rulers, protractors, and/or technology.
3. Describe the two-dimensional shapes that result from slicing three-dimensional solids parallel or perpendicular to the base.	Note: Focus of standard is on plane sections resulting from the slicing of right rectangular prisms and right rectangular pyramids.

Within-Grade Connections:

- Students use proportional reasoning (NY-7.RP) when they analyze scale drawings (NY-7.G.1). [14]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students notice when given geometric conditions determine a unique triangle, more than one triangle or no triangle (NY-7.G.2), they have an opportunity to construct viable arguments and critique the reasoning of others (MP.3). Such problems also present opportunities for using appropriate tools strategically (MP.5).[14]

NY-7.G	Geometry
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	
4. Apply the formulas for the area and circumference of a circle to solve problems.	Coherence: NY-7.G.4 → NY-8.G.9 Note: Students in grade 7 are not expected to calculate the radius of a circle given its area.
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	Coherence: NY-7.G.5 → NY-8.G.5 Note: Students in grade 7 are limited to solving equations that involve linear expressions on one side of the equation.
6. Solve real-world and mathematical problems involving area of two-dimensional objects composed of triangles and trapezoids.	Coherence: NY-6.G.1, NY-6.G.2, NY-6.G.4 → NY-7.G.6 → NY-8.G.6 Note: The inclusive definition of a trapezoid will be utilized, which defines a trapezoid as "A quadrilateral with *at least* one pair of parallel sides." (This definition includes parallelograms and rectangles.)
Solve surface area problems involving right prisms and right pyramids composed of triangles and trapezoids.	Note: Right prisms include cubes.
Find the volume of right triangular prisms, and solve volume problems involving three-dimensional objects composed of right rectangular prisms.	

NY-7.SP	Statistics and Probability
Draw informal comparative inferences about two populations.	
1. Construct and interpret box-plots, find the interquartile range, and determine if a data point is an outlier.	Coherence: NY-6.SP.4, NY-6.SP.5c → NY-7.SP.1 Note: Students in grade 7 are *not* expected to *construct* box-plots that include outliers in the data, but students *are* expected to *interpret* box-plots that may contain outliers.
3. Informally assess the degree of visual overlap of two quantitative data distributions.	Coherence: NY-6.SP.2 → NY-7.SP.3
4. Use measures of center and measures of variability for quantitative data from random samples or populations to draw informal comparative inferences about the populations.	Coherence: NY-6.SP.1c, NY-6.SP.3 → NY-7.SP.4 Note: Measures of center are mean, median, and mode. The measures of variation include range and the interquartile range.

NY-7.SP	Statistics and Probability
Investigate chance processes and develop, use, and evaluate probability models.	
8. Find probabilities of compound events using organized lists, sample space tables, tree diagrams, and simulation.	Coherence: NY-6.SP.8 → NY-7.SP.8
a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	
b. Represent sample spaces for compound events using methods such as organized lists, sample space tables, and tree diagrams. For an event described in everyday language, identify the outcomes in the sample space which compose the event.	e.g., "rolling double sixes"
c. Design and use a simulation to generate frequencies for compound events.	e.g., Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Within-Grade Connections:

- Probability models draw on proportional reasoning and should be connected to the major work in those standards.[14]
- Students use proportional reasoning and percentages (NY-7.RP) when they extrapolate from random samples and use probability (NY-7.SP.8).[14]

Grade 8 Overview

In Grade 8, instructional time should focus on three areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Number System***, the ***Expressions, Equations, and Inequalities***, and the ***Probability and Statistics*** domains, students:
   - recognize equations for proportions (y/x = m or y = mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin;
   - understand that the slope (m) of a line is a constant rate of change, as well as how the input and output change as a result of the constant rate of change;
   - interpret a model in the context of the data by expressing a linear relationship between the two quantities in question and interpret components of the relationship (such as slope and y-intercept) in terms of the situation;
   - solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line; and
   - use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to represent, analyze, and solve a variety of problems.
2. Through their learning in the ***Functions*** and the ***Expressions, Equations, and Inequalities*** domains, students:
   - grasp the concept of a function as a rule that assigns to each input exactly one output;
   - understand that functions describe situations where one quantity determines another; and
   - translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations of the function), and describe how aspects of the function are reflected in the different representations.
3. Through their learning in the ***Geometry*** domain, students:
   - use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems;
   - show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines;
   - understand the statement of the Pythagorean Theorem and its converse, and why the Pythagorean Theorem holds; and
   - apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NY-8.NS	The Number System	
Know that there are numbers that are not rational, and approximate them by rational numbers.		
1. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion eventually repeats. Know that other numbers that are not rational are called irrational.		Coherence: NY-7.NS.2 → NY-8.NS.1 → AI-N.RN.3
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions.		

Within-Grade Connections:

- Work with the number system in this grade (NY-8.NS.1 & 2) is intimately related to work with radicals (NY-8.EE.2), and both of these may be connected to the Pythagorean theorem (NY-8.G.6 – 8) as well as to volume problems (NY-8.G.9), e.g., in which a cube has known volume but unknown edge lengths.[14] Irrational numbers, radicals, the Pythagorean Theorem, and volume are nonlinear in nature.

NY-8.EE	Expressions, Equations, and Inequalities	
Work with radicals and integer exponents.		
1.	Know and apply the properties of integer exponents to generate equivalent numerical expressions.	Coherence: NY-8.EE.1 → AI-A.SSE.2 e.g., $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$
2.	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know square roots of perfect squares up to 225 and cube roots of perfect cubes up to 125. Know that the square root of a non-perfect square is irrational.	Coherence: NY-8.EE.2 → AI-A.REI.4 e.g., The $\sqrt{2}$ is irrational.
3.	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.	e.g., Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.
4.	Perform multiplication and division with numbers expressed in scientific notation, including problems where both standard decimal form and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.	Coherence: NY-8.EE.4 → AI-N.Q.3

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Scientific notation (NY-8.EE.4) presents opportunities for strategically using appropriate tools (MP.5). For example, the computation $(1.73 \times 10^{-4}) \cdot (1.73 \times 10^{-5})$ can be done quickly with a calculator by squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection.[(14)]

NY-8.EE	Expressions, Equations, and Inequalities	
Understand the connections between proportional relationships, lines, and linear equations.		
5.	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	Coherence: NY-7.RP.2c → NY-8.EE.5 e.g., Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
6.	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	Coherence: NY-7.RP.2b → NY-8.EE.6 → AI-A.REI.10, GEO-G.GPE.5

NY-8.EE	Expressions, Equations, and Inequalities	
Analyze and solve linear equations and pairs of simultaneous linear equations.		
7. Solve linear equations in one variable.		Coherence: NY-7.EE.1, NY-7.EE.4a → NY-8.EE.7 → AI-A.REI.3
a. Recognize when linear equations in one variable have one solution, infinitely many solutions, or no solutions. Give examples and show which of these possibilities is the case by successively transforming the given equation into simpler forms.		
b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms.		Note: This includes equations that contain variables on both sides of the equation.
8. Analyze and solve pairs of simultaneous linear equations.		Coherence: NY-7.EE.2 → NY-8.EE.8 → AI-A.REI.6a, AI-A.REI.7a
a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. Recognize when the system has one solution, no solution, or infinitely many solutions.		
b. Solve systems of two linear equations in two variables with integer coefficients: graphically, numerically using a table, and algebraically. Solve simple cases by inspection.		e.g., $3x + y = 5$ and $3x + y = 6$ have no solution because $3x + y$ cannot simultaneously be 5 and 6. Notes: • Solving systems algebraically will be limited to at least one equation containing at least one variable whose coefficient is 1. Algebraic solution methods include elimination and substitution. • This standard is a fluency expectation for grade 8.
c. Solve real-world and mathematical problems involving systems of two linear equations in two variables with integer coefficients.		Note: Solving systems algebraically will be limited to at least one equation containing at least one variable whose coefficient is 1.

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Solving an equation such as $3(x - ½) = x + 2$ requires students to see and make use of structure (MP.7).[14]

NY-8.F	Functions	
Define, evaluate, and compare functions. Note: Function notation is not required in Grade 8.		
1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.		Coherence: NY-8.F.1 → AI-F.IF.1, GEO-G.CO.2 Note: The terms *domain* and *range* may be introduced at this level; however, these terms are formally introduced in Algebra I (AI-F.IF.1).
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).		Coherence: NY-7.RP.2b → NY-8.F.2 → AI-F.IF.9 e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic equation, determine which function has the greater rate of change.
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line. Recognize examples of functions that are linear and non-linear.		Coherence: NY-7.EE.4a → NY-8.F.3 → AI-F.LE.1b e.g., The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line.

NY-8.F	Functions	
Use functions to model relationships between quantities. Note: Function notation is not required in Grade 8.		
4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.		Coherence: NY-7.RP.2b → NY-8.F.4 → AI-A.CED.1, AI-F.IF.6, AI-F.BF.1
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described in a real-world context.		Coherence: NY-7.RP.2d → NY-8.F.5 → AI-F.IF.4, AI-F.LE.3 e.g., where the function is increasing or decreasing or when the function is linear or non-linear

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students model linear relationships with functions (NY-8.F.4), they are modeling with mathematics (MP.4).[14]
- By repeatedly checking whether points are on a line through (1, 2) with slope 3, students might abstract the equation of the line in the form $(y – 2)/(x – 1) = 3$. In doing so, students look for and express regularity in repeated reasoning (MP.8).[14]

NY-8.G	Geometry
Understand congruence and similarity using physical models, transparencies, or geometry software.	
1. Verify experimentally the properties of rotations, reflections, and translations. a. Verify experimentally lines are mapped to lines, and line segments to line segments of the same length. b. Verify experimentally angles are mapped to angles of the same measure. c. Verify experimentally parallel lines are mapped to parallel lines.	Coherence: NY-8.G.1 → GEO-G.CO.2, GEO-G.CO.4 Note: A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector. A rotation requires knowing the center/point of rotation and the measure/direction of the angle of rotation. A line reflection requires a line and the knowledge of perpendicular bisectors.
2. Know that a two-dimensional figure is congruent to another if the corresponding angles are congruent and the corresponding sides are congruent. Equivalently, two two-dimensional figures are congruent if one is the image of the other after a sequence of rotations, reflections, and translations. Given two congruent figures, describe a sequence that maps the congruence between them on the coordinate plane.	Coherence: NY-8.G.2 → GEO-G.CO.5, GEO-G.CO.6, GEO-G.CO.7
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Coherence: NY-8.G.3 → GEO-G.SRT.1 Note: Lines of reflection are limited to both axes and lines of the form $y = k$ and $x = k$, where k is a constant. Rotations are limited to 90 and 180 degrees about the origin. Unless otherwise specified, rotations are assumed to be counterclockwise.
4. Know that a two-dimensional figure is similar to another if the corresponding angles are congruent and the corresponding sides are in proportion. Equivalently, two two-dimensional figures are similar if one is the image of the other after a sequence of rotations, reflections, translations, and dilations. Given two similar two-dimensional figures, describe a sequence that maps the similarity between them on the coordinate plane.	Coherence: NY-8.G.4 → GEO-G.CO.3, GEO-G.SRT.2 Note: With dilation, the center and scale factor must be specified.

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	Coherence: NY-7.G.5 → NY-8.G.5 → GEO-G.CO.9, GEO-G.CO.10, GEO-G.SRT.3 e.g., Arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so. Note: This standard does not include formal geometric proof. Multiple representations may be used to demonstrate understanding.

Within-Grade Connections:

- An important development takes place in grade 8 when students make connections between proportional relationships, lines, and linear equations (NY-8.EE.5 & 6). The angle-angle criterion for triangle similarity underlies the fact that a nonvertical line in the coordinate plane has equation $y = mx + b$ (see page 12 of the *6-8 Progression on Expressions and Equations*). Therefore, students must do work with congruence and similarity (NY-8.G.1 – 5) before they are able to justify the connections among proportional relationships, lines, and linear equations. Hence the indicated geometry work should likely begin at or near the very start of the year.[14]

NY-8.G Geometry

Understand and apply the Pythagorean Theorem.

6. Understand a proof of the Pythagorean Theorem and its converse.	Coherence: NY-7.G.6 → NY-8.G.6
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	Coherence: NY-8.G.7 → GEO-G.SRT.8
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Coherence: NY-8.G.8 → GEO-G.GPE.1a, GEO-G.GPE.4

Connecting the Standards for Mathematical Practice to Mathematical Content:

- The Pythagorean Theorem (NY-8.G.6 – 8) can provide opportunities for students to construct viable arguments and critique the reasoning of others (e.g., if a student in the class seems to be confusing the theorem with its converse) (MP.3).[14]

NY-8.G Geometry

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Given the formulas for the volume of cones, cylinders, and spheres, solve mathematical and real-world problems.	Coherence: NY-7.G.4 → NY-8.G.9 → GEO-G.GMD.1, GEO-G.MG.1

NY-8.SP	Statistics and Probability	
Investigate patterns of association in bivariate data.		
1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Coherence: NY-8.SP.1 → AI-S.ID.6, AI-S.ID.9	
2. Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	Coherence: NY-8.SP.2 → AI-S.ID.6a, AI-S.ID.8	
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.	Coherence: NY-8.SP.3 → AI-S.ID.6a, AI-S.ID.7 e.g., In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr. as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.	

Within-Grade Connections:

- Working with scatter plots and linear models of association in bivariate measurement data (NY-8.SP.1 – 3) supports work with proportional relationships, lines, linear equations, and linear functions (NY-8.EE.5 – 8 and NY-8.F.3 & 4).[(14)]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Students are modeling with mathematics (MP.4) when they construct and interpret scatter plots, as well as when they use linear models to describe data (NY-8.SP.1 – 3).[(14)]

High School – Introduction

Organization of the High School Standards for Mathematical Content

The standards are organized by course at the high school level. The high school courses include Algebra I, Geometry, and Algebra II. The Plus (+) Standards do not make up a course. They are additional standards that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics. The Plus (+) Standards may be dispersed throughout any high school course, including Algebra I, Geometry, and Algebra II, but are not expected to be addressed on state assessments.

The high school courses and the Plus (+) Standards are grouped by conceptual categories. These categories include Number and Quantity, Algebra, Functions, Geometry, Statistics and Probability, and Modeling. Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of courses, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

High School – Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

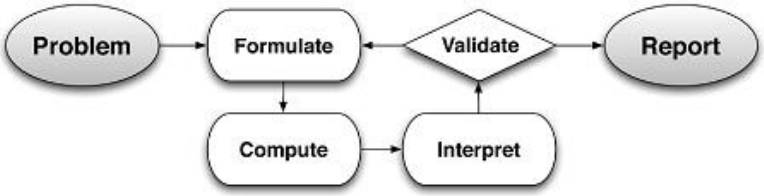
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of

our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO_2 over time.

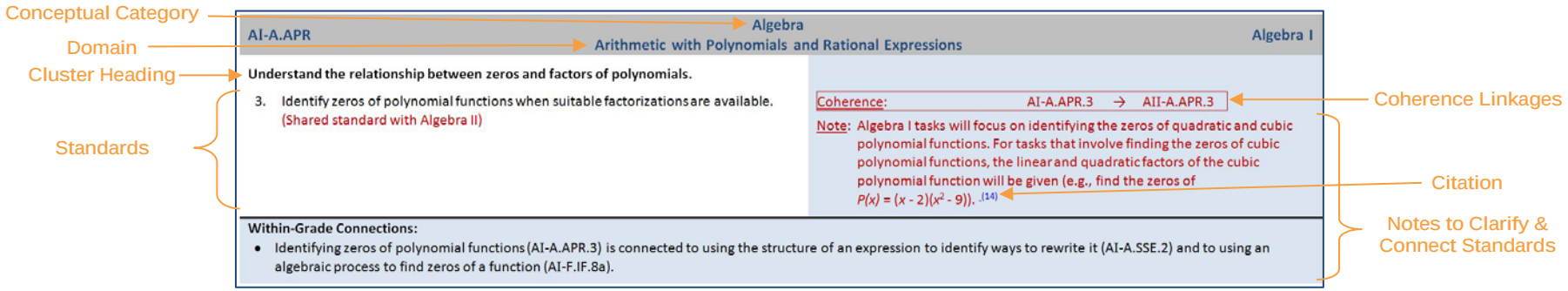
Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

How to read the High School Standards for Mathematical Content

Standards	define what students should understand and be able to do.
Clusters	summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
Domains	are larger groups of related standards. Standards from different domains may sometimes be closely related.
Conceptual Categories	provide coherence across the courses.
Coherence Linkages	connect standards one grade level forward and/or back when there are very direct linking standards in those grades. For a more thorough analysis of how standards link to one another, see http://achievethecore.org/coherence-map/.
Citations	are indicated by a blue number when information was taken or adapted from another source. The number will match the source number in the *Works Cited* section at the end of this document. When viewing these standards electronically, the source information (including page number) will appear as hover-over text.

High School Courses and Plus (+) Standards



The order in which the standards are presented is not necessarily the order in which the standards need to be taught. Standards from various domains are connected, and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain course expectations and the knowledge articulated in the standards. That is, the standards do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information.

Algebra I Overview

Algebra I is the first mathematics course in high school and the focal point is functions; specifically linear, quadratic, and exponential functions. By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. In Algebra I, students analyze and explain precisely the process of solving an equation. Students, through reasoning, develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities and make conjectures about the form that a linear equation might take in a solution to a problem. They reason abstractly and quantitatively by choosing and interpreting units in the context of creating equations in two variables to represent relationships between quantities. They master the solution of linear equations and apply related solution techniques and the properties of exponents to the creation and solution of simple exponential equations.

Students learn the terminology specific to polynomials and understand that polynomials form a system analogous to that of integers. Students learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions represented graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on their understanding of integer exponents to consider exponential functions with integer domains. They compare and contrast linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change. Students explore systems of linear and quadratic equations and linear inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions focusing in on the explicit forms of sequences written in subscript notation. In building models of relationships between two quantities, students analyze the key features of a graph or table of a function.

Students strengthen their ability to discern structure in polynomial expressions. They create and solve equations involving quadratic and cubic expressions. Students reason abstractly and quantitatively in interpreting parts of an expression that represent a quantity in terms of its context; they also learn to make sense of problems and persevere in solving them by choosing or producing equivalent forms of an expression. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They learn through repeated reasoning to anticipate the graph of a quadratic function by interpreting the structure of various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function, which may require students to write solutions in simplest radical form.

Students expand their experience with functions to include more specialized functions—linear, exponential, quadratic, square, and those that are piecewise-defined, including absolute value and step. Students select from among these functions to model phenomena using the modeling cycle.

Students build upon prior experiences with data, and are introduced to working with more formal means of assessing how a model fits data. Students display and interpret graphical representations of data, and if appropriate, choose regression techniques when building a model that approximates a linear relationship between quantities. They analyze their knowledge of the context of a situation to justify their choice of a linear model, compute and interpret the correlation coefficient, and distinguish between situations of correlation and causation.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

AI-N.RN	Number and Quantity The Real Number System	Algebra I
Use properties of rational and irrational numbers.		
3. Use properties and operations to understand the different forms of rational and irrational numbers.	Coherence: NY-8.NS.1 → AI-N.RN.3 → AII-A.APR.6	
a. Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots.	Note: Tasks include rationalizing numerical denominators of the form $\frac{a}{\sqrt{b}}$ where *a* is an integer and *b* is a natural number.	
b. Categorize the sum or product of rational or irrational numbers. • The sum and product of two rational numbers is rational. • The sum of a rational number and an irrational number is irrational. • The product of a nonzero rational number and an irrational number is irrational. • The sum and product of two irrational numbers could be either rational or irrational.		

AI-N.Q	Number and Quantity Quantities	Algebra I
Reason quantitatively and use units to solve problems. ★		
1. Select quantities and use units as a way to: i) interpret and guide the solution of multi-step problems; ii) choose and interpret units consistently in formulas; and iii) choose and interpret the scale and the origin in graphs and data displays.		
3. Choose a level of accuracy appropriate to limitations on measurement and context when reporting quantities.	Coherence: NY-8.EE.4 → AI-N.Q.3	

AI-A.SSE	Algebra Seeing Structure in Expressions	Algebra I
Interpret the structure of expressions.		
1. Interpret expressions that represent a quantity in terms of its context. ★		
a. Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient, and constant term.		
b. Interpret expressions by viewing one or more of their parts as a single entity.	e.g., Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Note: This standard is a fluency recommendation for Algebra I. Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.[14]	
2. Recognize and use the structure of an expression to identify ways to rewrite it. (Shared standard with Algebra II)	Coherence: NY-8.EE.1 → AI-A.SSE.2 → AII-A.SSE.2, AII-A.SSE.3a e.g., • $x^3 - x^2 - x = x(x^2 - x - 1)$ • $53^2 - 47^2 = (53 + 47)(53 - 47)$ • $16x^2 - 36 = (4x)^2 - (6)^2 = (4x + 6)(4x - 6) = 4(2x + 3)(2x - 3)$ or $16x^2 - 36 = 4(4x^2 - 9) = 4(2x + 3)(2x - 3)$ • $-2x^2 + 8x + 10 = -2(x^2 - 4x - 5) = -2(x - 5)(x + 1)$ • $x^4 + 6x^2 - 7 = (x^2 + 7)(x^2 - 1) = (x^2 + 7)(x + 1)(x - 1)$ Note: Algebra I expressions are limited to numerical and polynomial expressions in one variable.[14] Use factoring techniques such as factoring out a greatest common factor, factoring the difference of two perfect squares, factoring trinomials of the form $ax^2 + bx + c$ with a lead coefficient of 1, or a combination of methods to factor completely. Factoring will not involve factoring by grouping and factoring the sum and difference of cubes.	

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Writing $49x^2 + 35x + 6$ as $(7x)^2 + 5(7x) + 6$ highlights the structural similarity between this expression and $z^2 + 5z + 6$, leading to a factorization of the original: $(7x + 3)(7x + 2)$ (AI-A.SSE.1-3 and AI-A.APR.3). This is an example of looking for and making use of structure (MP.7).[14]

AI-A.SSE	Algebra Seeing Structure in Expressions	Algebra I
Write expressions in equivalent forms to reveal their characteristics. ★		
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (Shared standard with Algebra II)		
c. Use the properties of exponents to rewrite exponential expressions. (Shared standard with Algebra II)	Coherence: AI-A.SSE.3c → AII-A.SSE.3c e.g., • $3^{2x} = (3^2)^x = 9^x$ • $3^{2x+3} = 3^{2x}3^3 = 9^x \cdot 27$ Note: Exponential expressions will include those with integer exponents, as well as those whose exponents are linear expressions. Any linear term in those expressions will have an integer coefficient. Rational exponents are an expectation for Algebra II.	

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Students are reasoning abstractly and quantitatively (MP.2) when they transform abstract algebraic models via algebraic calculations (AI-A.SSE.3, AI-A.APR.1 & 3, AI-F.IF.1-2, and AI-F.IF.4-7) to reveal properties of the problems.[14]

AI-A.APR	Algebra Arithmetic with Polynomials and Rational Expressions	Algebra I

Perform arithmetic operations on polynomials.

1. Add, subtract, and multiply polynomials and recognize that the result of the operation is also a polynomial. This forms a system analogous to the integers.

Coherence: AI-A.APR.1 → AII-A.APR.2

Note: This standard is a fluency recommendation for Algebra I. Fluency in adding, subtracting and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions.[14]

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Students are reasoning abstractly and quantitatively (MP.2) when they transform abstract algebraic models via algebraic calculations (AI-A.SSE.3, AI-A.APR.1 & 3, AI-F.IF.1-2, and AI-F.IF.4-7) to reveal properties of the problems.[14]

AI-A.APR	Algebra Arithmetic with Polynomials and Rational Expressions	Algebra I

Understand the relationship between zeros and factors of polynomials.

3. Identify zeros of polynomial functions when suitable factorizations are available. (Shared standard with Algebra II)

Coherence: AI-A.APR.3 → AII-A.APR.2, AII-A.APR.3

Note: Algebra I tasks will focus on identifying the zeros of quadratic and cubic polynomial functions. For tasks that involve finding the zeros of cubic polynomial functions, the linear and quadratic factors of the cubic polynomial function will be given (e.g., find the zeros of $P(x) = (x - 2)(x^2 - 9)$).[14]

Within-Grade Connections:

- Identifying zeros of polynomial functions (AI-A.APR.3) is connected to using the structure of an expression to identify ways to rewrite it (AI-A.SSE.2) and to using an algebraic process to find zeros of a function (AI-F.IF.8).

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Writing $49x^2 + 35x + 6$ as $(7x)^2 + 5(7x) + 6$ highlights the structural similarity between this expression and $z^2 + 5z + 6$, leading to a factorization of the original: $(7x + 3)(7x + 2)$ (AI-A.SSE.1-3 and AI-A.APR.3). This is an example of looking for and making use of structure (MP.7).[14]

AI-A.CED	Algebra Creating Equations	Algebra I
Create equations that describe numbers or relationships. ★		
1. Create equations and inequalities in one variable to represent a real-world context. (Shared standard with Algebra II)	Coherence: NY-8.F.4 → AI-A.CED.1 → AII-A.CED.1 Notes: • This is strictly the development of the model (equation/inequality). • Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).[14] • Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar^{n-1}$, where a is the first term and r is the common ratio. • Inequalities are limited to linear inequalities. • Algebra I tasks do not involve compound inequalities.	
2. Create equations and linear inequalities in two variables to represent a real-world context.	Notes: • This is strictly the development of the model (equation/inequality). • Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).	
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.	
4. Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	e.g., Rearrange Ohm's law $V = IR$ to highlight resistance R.	

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Students are reasoning abstractly (MP.2) when they create abstract algebraic models of problems (AI-A.CED.1-4 and AI-F.BF.1).[14]
- The habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Attending to precision (MP.6) involves describing an idea precisely (AI-A.CED.3 and AI-A.REI.1a), which helps students understand the idea in new ways.[14]
- An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism (AI-A.CED.1-3). For example, when comparing two different text messaging plans, many students who can compute the cost for a given number of minutes have a hard time writing general formulas that express the cost of each plan for any number of minutes. Constructing these formulas can be facilitated by methodically calculating the cost for several different input values and then expressing the steps in the calculation, first in words and then in algebraic symbols. Once such expressions are obtained, students can find the break-even point for the two plans, graph the total cost against the number of messages sent, and make a complete analysis of the two plans. This is an example of looking for and expressing regularity in repeated reasoning (MP.8).[14]

AI-A.REI	Algebra Reasoning with Equations and Inequalities	Algebra I

Understand solving equations as a process of reasoning and explain the reasoning.

1a. Explain each step when solving a linear or quadratic equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Coherence: AI-A.REI.1a → AII-A.REI.1b

Connecting the Standards for Mathematical Practice to Mathematical Content:

- The habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Attending to precision (MP.6) involves describing an idea precisely (AI-A.CED.3 and AI-A.REI.1a), which helps students understand the idea in new ways.[14]

AI-A.REI	Algebra Reasoning with Equations and Inequalities	Algebra I
Solve equations and inequalities in one variable.		
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Coherence: NY-8.EE.7 → AI-A.REI.3 Note: Algebra I tasks do not involve solving compound inequalities.	
4. Solve quadratic equations in one variable.	Coherence: NY-8.EE.2 → AI-A.REI.4 → AII-A.REI.4b Note: Solutions may include simplifying radicals.	
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Understand that the quadratic formula is a derivative of this process.	Note: When utilizing the method of completing the square, the quadratic's leading coefficient will be 1 and the coefficient of the linear term will be limited to even (after the possible factoring out of a GCF). Students in Algebra I should be able to complete the square in which manipulating the given quadratic equation yields an integer value for q.	
b. Solve quadratic equations by:	Notes: • Solutions may include simplifying radicals or writing solutions in simplest radical form.	
i) inspection,	• An example for inspection would be $x^2 = 49$, where a student should know that the solutions would include 7 and -7.	
ii) taking square roots,		
iii) factoring,		
iv) completing the square,		
v) the quadratic formula, and	• When utilizing the quadratic formula, there are no coefficient limits.	
vi) graphing.		
Recognize when the process yields no real solutions. (Shared standard with Algebra II)	• The discriminant is a sufficient way to recognize when the process yields no real solutions.	

Within-Grade Connections:

- Solving quadratic equations in one variable (AI-A.REI.4) is connected to using the structure of an expression to identify ways to rewrite it (AI-A.SSE.2).
- Working with radicals while solving quadratic equations in one variable (AI-A.REI.4) is connected to working with the products of rational and irrational numbers (AI-N.RN.3).
- The method of solving graphically (AI-A.REI.4b vi) shows the relationship between the zeros of a quadratic function and its graph. This is connected to identifying zeros of polynomial functions (AI-A.APR.3 & AI-F.IF.8a) and interpreting key features of a graph (AI-F.IF.4a & AI-F.IF.7a).[14]

AI-A.REI	Algebra Reasoning with Equations and Inequalities	Algebra I
Solve systems of equations.		
6a. Solve systems of linear equations in two variables both algebraically and graphically.	Coherence: NY-8.EE.8 → AI-A.REI.6a → (+)-A.REI.6b Note: Algebraic methods include both elimination and substitution.	
7a. Solve a system, with rational solutions, consisting of a linear equation and a quadratic equation (parabolas only) in two variables algebraically and graphically. (Shared standard with Algebra II)	Coherence: NY-8.EE.8 → AI-A.REI.7a → AII-A.REI.7b	

AI-A.REI	Algebra Reasoning with Equations and Inequalities	Algebra I
Represent and solve equations and inequalities graphically.		
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.	Coherence: NY-8.EE.6 → AI-A.REI.10 Note: Graphing linear equations is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena.[14]	
11. Given the equations $y = f(x)$ and $y = g(x)$: i) recognize that each x-coordinate of the intersection(s) is the solution to the equation $f(x) = g(x)$; ii) find the solutions approximately using technology to graph the functions or make tables of values; and iii) interpret the solution in context. ★ (Shared standard with Algebra II)	Coherence: AI-A.REI.11 → AII-A.REI.11 Note: Algebra I tasks are limited to cases where $f(x)$ and $g(x)$ are linear, polynomial, absolute value, and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$). Note: Students should be taught to find the solutions approximately using technology to graph the functions *and* make tables of values. When solving any problem, students can choose either strategy.	
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	Note: Graphing linear equations is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena (including modeling using systems of linear inequalities in two variables).[14]	

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

AI-F.IF	Functions Interpreting Functions	Algebra I
Understand the concept of a function and use function notation.		
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f(x)*.	Coherence: NY-8.F.1 → AI-F.IF.1 → AII-F.IF.7e Note: Domain and range can be expressed using inequalities, set builder, verbal description, and interval notations for functions of subsets of real numbers to the real numbers.	
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.		
3. Recognize that a sequence is a function whose domain is a subset of the integers. (Shared standard with Algebra II)	Coherence: AI-F.IF.3 → AII-F.IF.3 Notes: • Sequences (arithmetic and geometric) will be written explicitly and only in subscript notation. • Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar^{n-1}$, where *a* is the first term and *r* is the common ratio.	

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Students are reasoning abstractly and quantitatively (MP.2) when they transform abstract algebraic models via algebraic calculations (AI-A.SSE.3, AI-A.APR.1 & 3, AI-F.IF.1-2, and AI-F.IF.4-7) to reveal properties of the problems.[14]

AI-F.IF	Functions Interpreting Functions	Algebra I
Interpret functions that arise in applications in terms of the context. ★		
4. For a function that models a relationship between two quantities: i) interpret key features of graphs and tables in terms of the quantities; and ii) sketch graphs showing key features given a verbal description of the relationship. (Shared standard with Algebra II)	Coherence: NY-8.F.5 → AI-F.IF.4a → AII-F.IF.4b Notes: • Algebra I key features include the following: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries. • Tasks have a real-world context and are limited to the following functions: linear, quadratic, square root, piece-wise defined (including step and absolute value), and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).[14]	
5. Determine the domain of a function from its graph and, where applicable, identify the appropriate domain for a function in context.		
6. Calculate and interpret the average rate of change of a function over a specified interval. (Shared standard with Algebra II)	Coherence: NY-8.F.4 → AI-F.IF.6 → AII-F.IF.6 Notes: • Functions may be presented by function notation, a table of values, or graphically. • Algebra I tasks have a real-world context and are limited to the following functions: linear, quadratic, square root, piece-wise defined (including step and absolute value), and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).[14]	

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Students are reasoning abstractly and quantitatively (MP.2) when they transform abstract algebraic models via algebraic calculations (AI-A.SSE.3, AI-A.APR.1 & 3, AI-F.IF.1-2, and AI-F.IF.4-7) to reveal properties of the problems.[14]

AI-F.IF	Functions Interpreting Functions	Algebra I
Analyze functions using different representations.		
7. Graph functions and show key features of the graph by hand and by using technology where appropriate. ★ (Shared standard with Algebra II)	Coherence: AI-F.IF.7 → AII-F.IF.7c, AII-F.IF.7e	
a. Graph linear, quadratic, and exponential functions and show key features.	Notes: • Algebra I key features include the following: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries. • Exponential functions are of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$). • Graphing linear functions is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena.[14]	
b. Graph square root, and piecewise-defined functions, including step functions and absolute value functions and show key features.	Note: Algebra I key features include the following: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries.	
8. Write a function in different but equivalent forms to reveal and explain different properties of the function. (Shared standard with Algebra II)	Coherence: AI-F.IF.8a → AII-A.SSE.3a	
a. For a quadratic function, use an algebraic process to find zeros, maxima, minima, and symmetry of the graph, and interpret these in terms of context.	Note: Algebraic processes include but not limited to factoring, completing the square, use of the quadratic formula, and the use of the axis of symmetry.	
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (Shared standard with Algebra II)	Coherence: NY-8.F.2 → AI-F.IF.9 → AII-F.IF.9 Note: Algebra I tasks are limited to the following functions: linear, quadratic, square root, piecewise defined (including step and absolute value), and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).[14]	

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

AI-F.BF	Functions Building Functions	Algebra I

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities. ★
   (Shared standard with Algebra II)

   a. Determine a function from context.
   Define a sequence explicitly or steps for calculation from a context.
   (Shared standard with Algebra II)

Coherence: NY-8.F.4 → AI-F.BF.1a → AII-F.BF.1a

Notes:

- Algebra I tasks are limited to linear, quadratic and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).[14]
- Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar^{n-1}$, where a is the first term and r is the common ratio.
- Sequences will be written explicitly and only in subscript notation.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Students are reasoning abstractly (MP.2) when they create abstract algebraic models of problems (AI-A.CED.1-4 and AI-F.BF.1).[14]

AI-F.BF	Functions Building Functions	Algebra I

Build new functions from existing functions.

3a. Using $f(x) + k$, $k\,f(x)$, and $f(x + k)$:

   i) identify the effect on the graph when replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, and $f(x + k)$ for specific values of k (both positive and negative);

   ii) find the value of k given the graphs;

   iii) write a new function using the value of k; and

   iv) use technology to experiment with cases and explore the effects on the graph.

(Shared standard with Algebra II)

Coherence: AI-F.BF.3a → AII-F.BF.3b

Note: Tasks are limited to linear, quadratic, square root, and absolute value functions; and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).[14]

Note on the Word *Explore*:

- *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

AI-F.LE	Functions Linear, Quadratic, and Exponential Models ★	Algebra I
Construct and compare linear, quadratic, and exponential models and solve problems.		
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.	Coherence: NY-8.F.3 → AI-F.LE.1	
a. Justify that a function is linear because it grows by equal differences over equal intervals, and that a function is exponential because it grows by equal factors over equal intervals.		
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another, and therefore can be modeled linearly.	e.g., A flower grows two inches per day.	
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another, and therefore can be modeled exponentially.	e.g., A flower doubles in size after each day.	
2. Construct a linear or exponential function symbolically given: i) a graph; ii) a description of the relationship; iii) two input-output pairs (include reading these from a table). (Shared standard with Algebra II)	Coherence: AI-F.LE.2 → AII-F.LE.2 Note: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).[14]	
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	Coherence: NY-8.F.5 → AI-F.LE.3	

AI-F.LE	Functions Linear, Quadratic, and Exponential Models ★	Algebra I
Interpret expressions for functions in terms of the situation they model.		
5. Interpret the parameters in a linear or exponential function in terms of a context. (Shared standard with Algebra II)	Coherence: AI-F.LE.5 → AII-F.LE.5 Note: Tasks have a real-world context. Exponential functions are limited to those with domains in the integers and are of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).[14]	

Within-Grade Connections:

- Interpreting the parameters in a linear or exponential function in terms of a context (AI-F.LE.5) is connected to interpreting expressions by viewing one or more of their parts as a single entity (AI-A.SSE.1b).

AI-S.ID	Statistics and Probability Interpreting Categorical and Quantitative Data	Algebra I
Summarize, represent, and interpret data on a single count or measurement variable.		
1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	Coherence: AI-S.ID.1 → AII-S.ID.4a	
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (inter-quartile range, sample standard deviation) of two or more different data sets.	Coherence: AI-S.ID.2 → AII-S.ID.4a Note: Values in the given data sets will represent samples of larger populations. The calculation of standard deviation will be based on the sample standard deviation formula $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$. The sample standard deviation calculation will be used to make a statement about the population standard deviation from which the sample was drawn.	
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).		

AI-S.ID	Statistics and Probability Interpreting Categorical and Quantitative Data	Algebra I
Summarize, represent, and interpret data on two categorical and quantitative variables.		
5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.		
6. Represent bivariate data on a scatter plot, and describe how the variables' values are related.	Coherence: NY-8.SP.1 → AI-S.ID.6 → AII-S.ID.6 Note: It's important to keep in mind that the data must be linked to the same "subjects," not just two unrelated quantitative variables; being careful not to assume a relationship between the actual variables (correlation/causation issue).	
a. Fit a function to real-world data; use functions fitted to data to solve problems in the context of the data. (Shared standard with Algebra II)	Coherence: NY-8.SP.2, NY-8.SP.3 → AI-S.ID.6a → AII-S.ID.6a Note: Algebra I emphasis is on linear models and includes the regression capabilities of the calculator.	

AI-S.ID	Statistics and Probability Interpreting Categorical and Quantitative Data	Algebra I
Interpret linear models.		
7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	Coherence: NY-8.SP.3 → AI-S.ID.7	
8. Calculate (using technology) and interpret the correlation coefficient of a linear fit.	Coherence: NY-8.SP.2 → AI-S.ID.8	
9. Distinguish between correlation and causation.	Coherence: NY-8.SP.1 → AI-S.ID.9	

Geometry Overview

Geometry is intended to be the second course in mathematics for high school students. During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions to establish the validity of geometric conjectures through deduction, proof, or mathematical arguments. Over the years, students develop an understanding of the attributes and relationships of two- and three-dimensional geometric shapes that can be applied in diverse contexts.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformations. Fundamental are the rigid motions: translations, rotations, reflections, and sequences of these, all of which are here assumed to preserve distance and angle measure reflections and rotations each explain a particular type of symmetry leading to insight into a figure's attributes. Two geometric figures are defined to be congruent if there is a sequence of rigid motions that maps one figure onto the other. For triangles, congruence means that all corresponding pairs of sides and all corresponding pairs of angles are congruent. This leads to the triangle congruence criteria ASA, SAS, SSS, AAS and Hypotenuse-Leg (HL). Once these criteria are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations define similarity as a sequence of dilations and/or rigid motions that maps one figure onto another. Students formalize the similarity ideas of "same shape" and "scale factor" developed in the middle grades by establishing that similar triangles have all pairs of corresponding angles congruent and all corresponding pairs of sides proportional. These transformations lead to the criteria AA, SSS similarity, and SAS similarity for similar triangles.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, along with the Pythagorean Theorem and are fundamental in many mathematical situations. Radian measure will be introduced in Algebra II, along with the unit circle.

Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area, and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line. They reason abstractly and quantitatively to model problems using volume formulas.

Students prove and apply basic theorems about circles and study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students explain the correspondence between the definition of a circle and the equation of a circle written in terms of the distance formula, its radius, and coordinates of its center. Given an equation of a circle, students graph the circle in the coordinate plane and apply techniques for solving quadratic equations.

Note: *For scaffolding purposes, the use of a variety of tools and methods for construction is encouraged. These scaffolds include compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena using visualization, reasoning, and geometric modeling to solve problems, in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. Students can create geometric models and ideas to solve not only problems in mathematics, but in other disciplines or everyday situations.*

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

GEO-G.CO	Geometry Congruence	Geometry
Experiment with transformations in the plane.		
1. Know precise definitions of angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc as these exist within a plane.		
2. Represent transformations as geometric functions that take points in the plane as inputs and give points as outputs. Compare transformations that preserve distance and angle measure to those that do not.	Coherence: NY-8.F.1, NY-8.G.1 → GEO-G.CO.2 Note: Instructional strategies may include drawing tools, graph paper, transparencies and software programs.	
3. Given a regular or irregular polygon, describe the rotations and reflections (symmetries) that map the polygon onto itself.	Coherence: NY-8.G.4 → GEO-G.CO.3 Note: The inclusive definition of a trapezoid will be utilized, which defines a trapezoid as "A quadrilateral with *at least* one pair of parallel sides."	
4. Develop definitions of rotations, reflections, and translations in terms of points, angles, circles, perpendicular lines, parallel lines, and line segments.	Coherence: NY-8.G.1 → GEO-G.CO.4 Notes: • Include point reflections. • A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector. • A rotation requires knowing the center/point and the measure/direction of the angle of rotation. • A line reflection requires a line and the knowledge of perpendicular bisectors.	
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another.	Coherence: NY-8.G.2 → GEO-G.CO.5 Notes: • Include point reflections. • A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector. • A rotation requires knowing the center/point and the measure/direction of the angle of rotation. • A line reflection requires a line and the knowledge of perpendicular bisectors. • Instructional strategies may include graph paper, tracing paper, and geometry software. • Singular transformations that are equivalent to a sequence of transformations may be utilized, such as a glide reflection. However, glide reflections are not an expectation of the course.	

GEO-G.CO	Geometry Congruence	Geometry
Understand congruence in terms of rigid motions.		
6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	Coherence: NY-8.G.2 → GEO-G.CO.6 Notes: • A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector. • A rotation requires knowing the center/point and the measure/direction of the angle of rotation. • A line reflection requires a line and the knowledge of perpendicular bisectors.	
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	Coherence: NY-8.G.2 → GEO-G.CO.7	
8. Explain how the criteria for triangle congruence (ASA, SAS, SSS, AAS and HL (Hypotenuse Leg)) follow from the definition of congruence in terms of rigid motions.		

Connecting the Standards for Mathematical Practice to Mathematical Content:

- One place in geometry where precision (MP.6) is necessary and useful is in the refinement of conjectures so that initial conjectures that are not correct can be salvaged — two angle measures and a side length do not determine a triangle, but a certain configuration of these parts leads to the angle-side-angle theorem (GEO-G.CO.8).[14]

GEO-G.CO	Geometry Congruence	Geometry
Prove geometric theorems.		
9. Prove and apply theorems about lines and angles.	Coherence: NY-8.G.5 → GEO-G.CO.9 Notes: • Include multi-step proofs and algebraic problems built upon these concepts. • Examples of theorems include but are not limited to: o Vertical angles are congruent. o If two parallel lines are cut by a transversal, then the alternate interior angles are congruent. o The points on a perpendicular bisector are equidistant from the endpoints of the line segment.	
10. Prove and apply theorems about triangles.	Coherence: NY-8.G.5 → GEO-G.CO.10 Notes: • Include multi-step proofs and algebraic problems built upon these concepts. • Examples of theorems include but are not limited to: o Angle Relationships: ▪ The sum of the interior angles of a triangle is 180 degrees. ▪ The measure of an exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles of the triangle. o Side Relationships: ▪ The length of one side of a triangle is less than the sum of the lengths of the other two sides. ▪ In a triangle, the segment joining the midpoints of any two sides will be parallel to the third side and half its length. o Isosceles Triangles: ▪ Base angles of an isosceles triangle are congruent.	

11. Prove and apply theorems about parallelograms.	Notes: • Include multi-step proofs and algebraic problems built upon these concepts. • The inclusive definition of a trapezoid will be utilized, which defines a trapezoid as "A quadrilateral with *at least* one pair of parallel sides." • Examples of theorems include but are not limited to: o A diagonal divides a parallelogram into two congruent triangles. o Opposite sides/angles of a parallelogram are congruent. o The diagonals of parallelogram bisect each other. o If the diagonals of quadrilateral bisect each other, then quadrilateral is a parallelogram. o If the diagonals of a parallelogram are congruent then the parallelogram is a rectangle. • Additional theorems covered allow for proving that a given quadrilateral is a particular parallelogram (rhombus, rectangle, square) based on given properties.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Abstraction (MP.2) is used in geometry when, for example, students use a diagram of a specific isosceles triangle as an aid to reason about *all* isosceles triangles (GEO-G.CO.9).[14]
- When students develop the skill of creating and presenting proofs (GEO-G.CO.9 & 10), they are constructing viable arguments and have opportunities to critique the reasoning of others (MP.3).[14]
- Seeing structure in geometric configurations (MP.7) can lead to insights and proofs. This often involves the creation of auxiliary lines not originally part of a given figure. Two classic examples are the construction of a line through a vertex of a triangle parallel to the opposite side as a way to see that the angle measures of a triangle add to 180 degrees and the introduction of a symmetry line in an isosceles triangle to see that the base angles are congruent (GEO-G.CO.9 & 10).[14]

GEO-G.CO	Geometry Congruence	Geometry
Make geometric constructions.		
12. Make, justify, and apply formal geometric constructions.	Notes: • Examples of constructions include but are not limited to: o Copy segments and angles. o Bisect segments and angles. o Construct perpendicular lines including through a point on or off a given line. o Construct a line parallel to a given line through a point not on the line. o Construct a triangle with given lengths. o Construct points of concurrency of a triangle (centroid, circumcenter, incenter, and orthocenter). o Construct the inscribed circle of a triangle. o Construct the circumscribed circle of a triangle. o Constructions of transformations. (see G.CO.A.5) • This standard is a fluency recommendation for Geometry. Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.[14]	
13. Make and justify the constructions for inscribing an equilateral triangle, a square and a regular hexagon in a circle.		

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Dynamic geometry environments can help students look for invariants in a whole class of geometric constructions (GEO-G.CO.12 & 13), and the constructions in such environments can sometimes lead to an idea behind a proof of a conjecture. This is an example of using appropriate tools strategically (MP.5).[14]

GEO-G.SRT	Geometry Similarity, Right Triangles, and Trigonometry	Geometry
Understand similarity in terms of similarity transformations.		
1. Verify experimentally the properties of dilations given by a center and a scale factor. a. Verify experimentally that dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. Verify experimentally that the dilation of a line segment is longer or shorter in the ratio given by the scale factor.	Coherence: NY-8.G.3 → GEO-G.SRT.1	
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar. Explain using similarity transformations that similar triangles have equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	Coherence: NY-8.G.4 → GEO-G.SRT.2 Notes: • The center and scale factor of the dilation must always be specified with dilation. • A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector. • A rotation requires knowing the center/point and the measure/direction of the angle of rotation. • A line reflection requires a line and the knowledge of perpendicular bisectors.	
3. Use the properties of similarity transformations to establish the AA~, SSS~, and SAS~ criterion for two triangles to be similar.	Coherence: NY-8.G.5 → GEO-G.SRT.3	

GEO-G.SRT	Geometry Similarity, Right Triangles, and Trigonometry	Geometry
Prove theorems involving similarity.		
4. Prove and apply similarity theorems about triangles.	Notes: • Include multi-step proofs and algebraic problems built upon these concepts. • Examples of theorems include but are not limited to: o If a line parallel to one side of a triangle intersects the other two sides of the triangle, then the line divides these two sides proportionally (and conversely) . o The length of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the lengths of the two segments of the hypotenuse. o The centroid of the triangle divides each median in the ratio 2:1.	
5. Use congruence and similarity criteria for triangles to: a. Solve problems algebraically and geometrically. b. Prove relationships in geometric figures.	Notes: • ASA, SAS, SSS, AAS, and Hypotenuse-Leg (HL) theorems are valid criteria for triangle congruence. AA~, SAS~, and SSS~ are valid criteria for triangle similarity. • This standard is a fluency recommendation for Geometry. Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in many geometric modeling tasks.[14]	

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students use area as a device to establish results about proportions, such as the important theorem (and its converse) that a line parallel to one side of a triangle divides the other two sides proportionally (GEO-G.SRT.4), they are making use of structure (MP.7).[14]

GEO-G.SRT	Geometry Similarity, Right Triangles, and Trigonometry	Geometry
Define trigonometric ratios and solve problems involving right triangles.		
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of sine, cosine and tangent ratios for acute angles.	Coherence: GEO-G.SRT.6 → AII-F.TF.2 (+)-G.SRT.10	
7. Explain and use the relationship between the sine and cosine of complementary angles.	Coherence: GEO-G.SRT.7 → AII-F.TF.4	
8. Use sine, cosine, tangent, the Pythagorean Theorem and properties of special right triangles to solve right triangles in applied problems. ★	Coherence: NY-8.G.7 → GEO-G.SRT.8 → (+)-G.SRT.10 Note: Special right triangles refer to the 30-60-90 and 45-45-90 triangles.	

GEO-G.SRT	Geometry Similarity, Right Triangles, and Trigonometry	Geometry
Apply Trigonometry to general triangles.		
9. Justify and apply the formula $A=\frac{1}{2}ab \sin(C)$ to find the area of any triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.		

GEO-G.C	Geometry Circles	Geometry
Understand and apply theorems about circles.		
1. Prove that all circles are similar.		
2a. Identify, describe and apply relationships between the angles and their intercepted arcs of a circle.	Note: These relationships that pertain to the circle may be utilized to prove other relationships in geometric figures, e.g., the opposite angles in any quadrilateral inscribed in a circle are supplements of each other.	
2b. Identify, describe and apply relationships among radii, chords, tangents, and secants of a circle.	Coherence: GEO-G.C.2b → (+)-G.C.4 Note: Include algebraic problems built upon these concepts.	
Connecting the Standards for Mathematical Practice to Mathematical Content: • In geometry, students conjecture about geometric phenomena that pertain to infinitely many cases at once (ex., *every* angle inscribed in a semicircle is a right angle) because it is impossible to check every case (GEO-G.C.2a & b). When they do this, they are constructing viable arguments and have opportunities to critique the reasoning of others (MP.3).[(14)]		

GEO-G.C	Geometry Circles	Geometry
Find arc lengths and area of sectors of circles.		
5. Using proportionality, find one of the following given two others; the central angle, arc length, radius or area of sector.	Note: Angle measure is in degrees.	

GEO-G.GPE	Geometry Expressing Geometric Properties with Equations	Geometry
Translate between the geometric description and the equation of a conic section.		
1a. Derive the equation of a circle of given center and radius using the Pythagorean Theorem. Find the center and radius of a circle, given the equation of the circle.	Coherence: NY-8.G.8 → GEO-G.GPE.1a → (+)-G.GPE.2 Notes: • Finding the center and radius may involve completing the square. The completing the square expectation for Geometry follows Algebra I: leading coefficients will be 1 (after possible removal of GCF) and the coefficients of the linear terms will be even. • Completing the square may yield a fractional radius.	
1b. Graph circles given their equation.	Note: For circles being graphed, the center will be an ordered pair of integers and the radius will be a positive integer.	

GEO-G.GPE	Geometry Expressing Geometric Properties with Equations	Geometry
Use coordinates to prove simple geometric theorems algebraically.		
4. On the coordinate plane, algebraically prove geometric theorems and properties.	Coherence: NY-8.G.8 → GEO-G.GPE.4 Notes: • Examples include but not limited to: o Given points and/or characteristics, prove or disprove a polygon is a specified quadrilateral or triangle based on its properties. o Given a point that lies on a circle with a given center, prove or disprove that a specified point lies on the same circle. • This standard is a fluency recommendation for Geometry. Fluency with the use of coordinates to establish geometric results and the use of geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields.[14]	
5. On the coordinate plane: a. Explore the proof for the relationship between slopes of parallel and perpendicular lines; b. Determine if lines are parallel, perpendicular, or neither, based on their slopes; and c. Apply properties of parallel and perpendicular lines to solve geometric problems.	Coherence: NY-8.EE.6 → GEO-G.GPE.5 Note: This standard is a fluency recommendation for Geometry. Fluency with the use of coordinates to establish geometric results and the use of geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields.[14]	
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Note: Midpoint formula is a derivative of this standard.	
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. ★	Note: This standard is a fluency recommendation for Geometry. Fluency with the use of coordinates to establish geometric results and the use of geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields.[14]	

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

Note on the Word *Explore*:

- *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

GEO-G.GMD	Geometry Geometric Measurement and Dimension	Geometry
Explain volume formulas and use them to solve problems.		
1. Provide informal arguments for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	Coherence: NY-8.G.9 → GEO-G.GMD.1	
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★	Coherence: GEO-G.GMD.3 → (+)-G.GMD.2	

GEO-G.GMD	Geometry Geometric Measurement and Dimension	Geometry
Visualize relationships between two-dimensional and three-dimensional objects.		
4. Identify the shapes of plane sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	Note: Plane sections are not limited to being parallel or perpendicular to the base.	

GEO-G.MG	Geometry Modeling with Geometry	Geometry
Apply geometric concepts in modeling situations.		
1. Use geometric shapes, their measures, and their properties to describe objects.	Coherence: NY-8.G.9 → GEO-G.MG.1	
2. Apply concepts of density based on area and volume of geometric figures in modeling situations.		
3. Apply geometric methods to solve design problems.	Note: Applications may include designing an object or structure to satisfy constraints such as area, volume, mass, and cost.	

Algebra II Overview

Algebra II is the capstone course of the three high school mathematics courses and is a continuation and extension of the two courses that precede it. Building on their work with linear, quadratic, and exponential functions in Algebra I, students in Algebra II extend their repertoire of functions to include polynomial, rational, radical, and trigonometric functions. Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers, and solving exponential equations using the properties of logarithms.

Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect the structure inherent in multi-digit whole number multiplication with multiplication of polynomials and similarly connect division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials. Through regularity in repeated reasoning, they make connections between zeros of polynomials and solutions of polynomial equations. Students analyze the key features of a graph or table of a polynomial function and relate those features back to the two quantities the function is modeling.

Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students extend trigonometric functions to all (or most) real numbers. To reinforce their understanding of these functions, students begin building fluency with the values of sine, cosine, and tangent at $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, etc. Students make sense of periodic phenomena as they model with trigonometric functions.

Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms as well as understanding the inverse relationship between exponential and logarithmic functions. They explore (with appropriate tools) the effects of transformations on graphs of diverse functions, including functions arising in an application. Students identify appropriate types of functions to model a situation. They adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. With repeated opportunities in working through the modeling cycle, students acquire the insight that the same mathematical or statistical structure can sometimes model seemingly different situations.

Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data, including sample surveys, observational studies, and experiments. Using simulation, randomization, and careful design, students make inferences, justify conclusions, and critique statistical claims. Students create theoretical and experimental probability models following the modeling cycle. They compute and interpret probabilities from those models for compound events, attending to mutually exclusive events, independent events, and conditional probability.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

AII-N.RN	Number and Quantity The Real Number System	Algebra II
Extend the properties of exponents to rational exponents.		
1. Explore how the meaning of rational exponents follows from extending the properties of integer exponents.	e.g., We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $5^{(1/3)3}$ must equal 5.	
2. Convert between radical expressions and expressions with rational exponents using the properties of exponents.	Note: All radical expressions involving variables assume the variables are representing positive numbers. Includes expressions with variable factors, such as $\sqrt[3]{27x^5y^3}$, being equivalent to $(27x^5y^3)^{1/3}$ which equals $3x^{5/3}y$.	

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AII-N.CN	Number and Quantity The Complex Number System	Algebra II
Perform arithmetic operations with complex numbers.		
1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	Coherence: AII-N.CN.1 → (+)-N.CN.4a, (+)-N.CN.8	
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Coherence: AII-N.CN.2 → (+)-N.CN.3 Note: Tasks include simplifying powers of *i*.	

AII-A.SSE	Algebra Seeing Structure in Expressions	Algebra II

Interpret the structure of expressions.

2. Recognize and use the structure of an expression to identify ways to rewrite it. (Shared standard with Algebra I)

Coherence: AI-A.SSE.2 → AII-A.SSE.2 → (+)-A.APR.4, (+)-A.APR.5

e.g.,

- $81x^4 - 16y^4$ is equivalent to $(9x^2)^2 - (4y^2)^2$ or $(9x^2 - 4y^2)(9x^2 + 4y^2)$ or $(3x + 2y)(3x - 2y)(9x^2 + 4y^2)$
- $\frac{x^2+4}{x^2+3}$ is equivalent to $\frac{(x^2+3)+1}{x^2+3} = \frac{x^2+3}{x^2+3} + \left(\frac{1}{x^2+3}\right) = 1 + \frac{1}{x^2+3}$
- $3x^3 - 5x^2 - 48x + 80$ is equivalent to $3x(x^2 - 16) - 5(x^2 - 16)$, which when factored completely is $(3x - 5)(x + 4)(x - 4)$

Notes:

- Includes factoring by grouping and factoring the sum and difference of cubes.
- Tasks are limited to polynomial, rational, or exponential expressions. Quadratic expressions include leading coefficients other than 1.[14]
- This standard is a fluency expectation for Algebra II. The ability to see structure in expressions and to use this structure to rewrite expressions is a key skill in everything from advanced factoring (e.g., grouping) to summing series, to rewriting of rational expressions, to examining the end behavior of the corresponding rational function.[14]

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students see $x^4 - y^4$ as the difference of squares, they are seeing and using the structure of algebraic expressions (MP.7). This habit of seeing subexpressions as single entities will serve students well in areas such as trigonometry, where, for example, the factorization of $x^4 - y^4$ described above can be used to show that the functions $\cos^4 x - \sin^4 x$ and $\cos^2 x - \sin^2 x$ are, in fact, equal (AII-A.SSE.2).[14]

AII-A.SSE	Algebra Seeing Structure in Expressions	Algebra II
Write expressions in equivalent forms to reveal their characteristics. ★		
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (Shared standard with Algebra I)		
a. Factor quadratic expressions including leading coefficients other than 1 to reveal the zeros of the function it defines.	Coherence: AI-A.SSE.2, AI-F.IF.8a → AII-A.SSE.3a	
c. Use the properties of exponents to rewrite exponential expressions. (Shared standard with Algebra I)	Coherence: AI-A.SSE.3c → AII-A.SSE.3c Note: Tasks include rewriting exponential expressions with rational coefficients in the exponent.	

AII-A.APR	Algebra Arithmetic with Polynomials and Rational Expressions	Algebra II

Understand the relationship between zeros and factors of polynomials.

2. Apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by $x - a$ is *p(a)*, so *p(a)* = 0 if and only if $(x - a)$ is a factor of *p(x)*.	Coherence: AI-A.APR.1, AI-A.APR.3 → AII-A.APR.2
3. Identify zeros of polynomial functions when suitable factorizations are available. (Shared standard with Algebra I)	Coherence: AI-A.APR.3 → AII-A.APR.3

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students make general arguments about *all* functions of a certain type (ex., the factor theorem for polynomial functions or the Remainder Theorem (AII-A.APR.2)), they are constructing viable arguments and have opportunities to critique the reasoning of others (MP.3).[14]

Within-Grade Connections:

- Identifying zeros of polynomial functions (AII-A.APR.3) is connected to solving quadratic equations (AII-A.REI.4b) and graphing polynomial functions by identifying its zeros (AII-F.IF.7c).

AII-A.APR	Algebra Arithmetic with Polynomials and Rational Expressions	Algebra II

Rewrite rational expressions.

6. Rewrite rational expressions in different forms: Write *a(x)/b(x)* in the form *q(x)* + *r(x)/b(x)*, where *a(x)*, *b(x)*, *q(x)*, and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*.	Coherence: AI-N.RN.3 → AII-A.APR.6 → (+)-A.APR.7 Note: This standard is a fluency expectation for Algebra II. This standard sets an expectation that students will divide polynomials with remainders by inspection in simple cases. For example, one can view the rational expression $\frac{x+4}{x+3}$ as $\frac{(x+3)+1}{(x+3)}$ which is $1 + \frac{1}{x+3}$. [14]

Note on *Fluency* with Procedures:

- *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure.[12],[13]

AII-A.CED	Algebra Creating Equations	Algebra II
Create equations that describe numbers or relationships. ★ 1. Create equations and inequalities in one variable to represent a real-world context. (Shared standard with Algebra I)	Coherence: AI-A.CED.1 → AII-A.CED.1 Note: This is strictly the development of the model (equation/inequality). Tasks include linear, quadratic, rational, and exponential functions.	

AII-A.REI	Algebra Reasoning with Equations and Inequalities	Algebra II
Understand solving equations as a process of reasoning and explain the reasoning.		
1b. Explain each step when solving rational or radical equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Coherence: AI-A.REI.1a → AII-A.REI.1b	
2. Solve rational and radical equations in one variable, identify extraneous solutions, and explain how they arise.	Note: Radical equations may include but are not limited to those of the form $x^{\frac{3}{5}} = 8$ and $3x^{\frac{3}{4}} + 5 = 86$.	

AII-A.REI	Algebra Reasoning with Equations and Inequalities	Algebra II
Solve equations and inequalities in one variable.		
4. Solve quadratic equations in one variable.	Note: Solutions may include simplifying radicals.	
b. Solve quadratic equations by:	Coherence: AI-A.REI.4b → AII-A.REI.4b	
i) inspection,	Notes: • An example for inspection would be $x^2 = -81$, where a student should know that the solutions would include $\pm 9i$.	
ii) taking square roots,		
iii) factoring,		
iv) completing the square,	• An example where students need to factor out a leading coefficient while completing the square would be $4x^2 + 8x - 9 = 0$.	
v) the quadratic formula, and		
vi) graphing.		
Write complex solutions in $a + bi$ form.		
(Shared standard with Algebra I)		

Within-Grade Connections:

- Solving quadratic equations (AII-A.REI.4b) is connected to identifying zeros of polynomial functions (AII-A.APR.3) and graphing polynomial functions by identifying its zeros (AII-F.IF.7c).

AII-A.REI	Algebra Reasoning with Equations and Inequalities	Algebra II

Solve systems of equations.

7b. Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
(Shared standard with Algebra I)

Coherence: AI-A.REI.7a → AII-A.REI.7b

e.g., Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Note: Conics are limited to parabolas and circles.

AII-A.REI	Algebra Reasoning with Equations and Inequalities	Algebra II

Represent and solve equations and inequalities graphically.

11. Given the equations $y = f(x)$ and $y = g(x)$:
    i) recognize that each x-coordinate of the intersection(s) is the solution to the equation $f(x) = g(x)$;
    ii) find the solutions approximately using technology to graph the functions or make tables of values;
    iii) find the solution of $f(x) < g(x)$ or $f(x) \leq g(x)$ graphically; and
    iv) interpret the solution in context. ★

(Shared standard with Algebra I)

Coherence: AI-A.REI.11 → AII-A.REI.11

Note: Tasks include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, absolute value, square root, cube root, trigonometric, exponential, and logarithmic functions.

AII-F.IF	Functions Interpreting Functions	Algebra II

Understand the concept of a function and use function notation.

3. Recognize that a sequence is a function whose domain is a subset of the integers.
(Shared standard with Algebra I)

Coherence: AI-F.IF.3 → AII-F.IF.3

Notes:

- In Algebra II, sequences will be defined/written recursively and explicitly in subscript notation.
- This standard is a fluency expectation for Algebra II. Fluency in translating between recursive definitions and closed forms is helpful when dealing with many problems involving sequences and series, with applications ranging from fitting functions to tables to problems in finance.[14]

Note on *Fluency* with Procedures:

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Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students analyze sequences (AII-F.IF.3), especially arithmetic and geometric sequences, and their associated series, they are looking for and expressing regularity (MP.8).[14]

Within-Grade Connections:

- Recognizing that a sequence is a function whose domain is a subset of the integers (AII-F.IF.3) supports writing arithmetic and geometric sequences (AII-F.BF.2).[14]

AII-F.IF	Functions Interpreting Functions	Algebra II
Interpret functions that arise in applications in terms of the context. ★		
4. For a function that models a relationship between two quantities: i) interpret key features of graphs and tables in terms of the quantities; and ii) sketch graphs showing key features given a verbal description of the relationship. (Shared standard with Algebra I)	Coherence: AI-F.IF.4a → AII-F.IF.4b Notes: • Algebra II key features include: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity. • Tasks may involve real-world context and may include polynomial, square root, cube root, exponential, logarithmic, and trigonometric functions.[14]	
6. Calculate and interpret the average rate of change of a function over a specified interval. (Shared standard with Algebra I)	Coherence: AI-F.IF.6 → AII-F.IF.6 Notes: • Functions may be presented by function notation, a table of values, or graphically. • Algebra II tasks have a real-world context and may involve polynomial, square root, cube root, exponential, logarithmic, and trigonometric functions.[14]	

AII-F.IF	Functions Interpreting Functions	Algebra II
Analyze functions using different representations.		
7. Graph functions and show key features of the graph by hand and using technology when appropriate. ★ (Shared standard with Algebra I)	Coherence: AI-F.IF.1, AI-F.IF.7a, AI-F.IF.7b → AII-F.IF.7 → (+)-F.IF.7d	
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.		
e. Graph cube root, exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.	Note: Trigonometric functions include sin(*x*), cos(*x*) and tan(*x*).	

AII-F.IF	Functions Interpreting Functions	Algebra II
Analyze functions using different representations.		
8. Write a function in different but equivalent forms to reveal and explain different properties of the function. (Shared standard with Algebra I)		
b. Use the properties of exponents to interpret exponential functions, and classify them as representing exponential growth or decay.	Note: Tasks also include real world problems that involve compounding growth/decay ($A = P(1 + (r/n))^{nt}$) and continuous compounding growth/decay ($A = Pe^{rt}$).	
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (Shared standard with Algebra I)	Coherence: AI-F.IF.9 → AII-F.IF.9 Note: Tasks may involve polynomial, square root, cube root, exponential, logarithmic, and trigonometric functions.[14]	

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Deciding whether two functions are equal on an infinite set (AII-F.IF.8) cannot be settled by looking at tables or graphs; it requires arguments of a sort (MP.3).[14]

AII-F.BF	Functions Building Functions	Algebra II
Build a function that models a relationship between two quantities. ★		
1. Write a function that describes a relationship between two quantities. (Shared standard with Algebra I)		
a. Determine a function from context. Determine an explicit expression, a recursive process, or steps for calculation from a context. (Shared standard with Algebra I)	Coherence: AI-F.BF.1a → AII-F.BF.1a Notes: • Tasks may involve linear functions, quadratic functions, and exponential functions.[14] • In Algebra II, sequences will be defined/written recursively and explicitly in subscript notation.	
b. Combine standard function types using arithmetic operations.	e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.	
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	Note: In Algebra II, sequences will be defined/written recursively and explicitly in subscript notation.	

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Developing recursive formulas for sequences (AII-F.BF.2) is facilitated by the practice of abstracting regularity (MP.8) for how you get from one term to the next and then giving a precise description of this process in algebraic symbols.[14]
- Most Computer Algebra Systems allow one to model recursive function (AII-F.BF.2) definitions in notation that is close to standard mathematical notation. And spreadsheets make natural the process of taking successive differences and running totals. Using appropriate tools (i.e., technology) strategically (MP.5) in this way reduces the computational overhead needed to investigate many classical and useful areas of algebra.[14]

AII-F.BF	Functions Building Functions	Algebra II
Build new functions from existing functions.		
3b. Using $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$: i) identify the effect on the graph when replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); ii) find the value of k given the graphs; iii) write a new function using the value of k; and iv) use technology to experiment with cases and explore the effects on the graph. Include recognizing even and odd functions from their graphs. (Shared standard with Algebra I)		Coherence: AI-F.BF.3a → AII-F.BF.3b → (+)-F.BF.3c Note: Algebra II tasks may involve polynomial, square root, cube root, exponential, logarithmic, and trigonometric functions.[14]
4a. Find the inverse of a one-to-one function both algebraically and graphically.		Coherence: AII-F.BF.4a → (+)-F.BF.4b
5a. Understand inverse relationships between exponents and logarithms algebraically and graphically.		Coherence: AII-F.BF.5a → (+)-F.BF.5b
6. Represent and evaluate the sum of a finite arithmetic or finite geometric series, using summation (sigma) notation.		
7. Explore the derivation of the formulas for finite arithmetic and finite geometric series. Use the formulas to solve problems. ★		

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Within-Grade Connections:

- Understanding inverse relationships between exponents and logarithms (AII-F.BF.5a) is connected to solving exponential equations using logs (AII-F.LE.4).

AII-F.LE	Functions Linear, Quadratic, and Exponential Models ★	Algebra II
Construct and compare linear, quadratic, and exponential models and solve problems.		
2. Construct a linear or exponential function symbolically given: i) a graph; ii) a description of the relationship; and iii) two input-output pairs (include reading these from a table). (Shared standard with Algebra I)	Coherence: AI-F.LE.2 → AII-F.LE.2	
4. Use logarithms to solve exponential equations, such as $ab^{ct} = d$ (where *a, b, c,* and *d* are real numbers and $b > 0$) and evaluate the logarithm using technology.		

AII-F.LE	Functions Linear, Quadratic, and Exponential Models ★	Algebra II
Interpret expressions for functions in terms of the situation they model.		
5. Interpret the parameters in a linear or exponential function in terms of a context. (Shared standard with Algebra I)	Coherence: AI-F.LE.5 → AII-F.LE.5 Note: Algebra II tasks have a real-world context and exponential functions are not limited to integer domains.[14]	

AII-F.TF	Functions Trigonometric Functions	Algebra II
Extend the domain of trigonometric functions using the unit circle.		
1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.		
2. Apply concepts of the unit circle in the coordinate plane to calculate the values of the six trigonometric functions given angles in radian measure.	Coherence: GEO-G.SRT.6 → AII-F.TF.2 → (+)-F.TF.3	
4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	Coherence: GEO-G.SRT.7 → AII-F.TF.4 → (+)-F.TF.6 Note: Focus of this standard is on cos(x), sin(x) and tan(x).	

AII-F.TF	Functions Trigonometric Functions	Algebra II
Model periodic phenomena with trigonometric functions. ★		
5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, horizontal shift, and midline.		

Within-Grade Connections:

- Choosing trigonometric functions to model periodic phenomena (AII-F.TF.5) is connected to (AII-F.BF.3b), specifically $y = A \sin(\omega(x-h)) + k$, $y = A \cos(\omega(x-h)) + k$ with the focus being on sin(x) and cos(x).

AII-F.TF	Functions Trigonometric Functions	Algebra II
Prove and apply trigonometric identities.		
8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$. Find the value of any of the six trigonometric functions given any other trigonometric function value and when necessary find the quadrant of the angle.	Coherence: AII-F.TF.8 → (+)-F.TF.9	

AII-S.ID	Statistics and Probability Interpreting Categorical and Quantitative Data ★	Algebra II
Summarize, represent, and interpret data on a single count or measurement variable.		
4a. Recognize whether or not a normal curve is appropriate for a given data set.	Coherence: AII-S.ID.1, AII-S.ID.2 → AII-S.ID.4a	
4b. If appropriate, determine population percentages using a graphing calculator for an appropriate normal curve.		

AII-S.ID	Statistics and Probability Interpreting Categorical and Quantitative Data ★	Algebra II
Summarize, represent, and interpret data on two categorical and quantitative variables.		
6. Represent bivariate data on a scatter plot, and describe how the variables' values are related.	Note: It's important to keep in mind that the data must be linked to the same "subjects", not just two unrelated quantitative variables. Do not assume that an association between two variables implies that one causes another to change.	
a. Fit a function to real-world data; use functions fitted to data to solve problems in the context of the data. (Shared standard with Algebra I)	Coherence: AI-S.ID.6a → AII-S.ID.6a → (+)-S.ID.6b Note: Algebra II emphasis is on quadratic, exponential, and power models, including the regression capabilities of the calculator.	

AII-S.IC	Statistics and Probability Making Inferences and Justifying Conclusions ★	Algebra II
Understand and evaluate random processes underlying statistical experiments.		
2. Determine if a value for a sample proportion or sample mean is likely to occur based on a given simulation.	Note: For the purposes of this course, if the statistic falls within two standard deviations of the mean (95% interval centered on the population parameter), then the statistic is considered likely (plausible, usual).	

AII-S.IC	Statistics and Probability Making Inferences and Justifying Conclusions ★	Algebra II
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.		
3. Recognize the purposes of and differences among surveys, experiments, and observational studies. Explain how randomization relates to each.		
4. Given a simulation model based on a sample proportion or mean, construct the 95% interval centered on the statistic (+/- two standard deviations) and determine if a suggested parameter is plausible.		
6a. Use the tools of statistics to draw conclusions from numerical summaries.		
6b. Use the language of statistics to critique claims from informational texts. For example, causation vs correlation, bias, measures of center and spread.		

AII-S.CP	Statistics and Probability Conditional Probability and the Rules of Probability ★	Algebra II
Understand independence and conditional probability and use them to interpret data.		
1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).		
4. Interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and calculate conditional probabilities.		

AII-S.CP	Statistics and Probability Conditional Probability and the Rules of Probability ★	Algebra II
Use the rules of probability to compute probabilities of compound events in a uniform probability model.		
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.	Coherence: AII-S.CP.7 → (+)-S.CP.9	

The Plus (+) Standards Overview

The Plus (+) Standards offer additional mathematics beyond Algebra II that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics. The Plus (+) Standards are not a high school course. The Plus (+) standards however, may be dispersed throughout course curriculums, including Algebra I, Algebra II and Geometry to increase rigor and support the needs of individual students and mathematical programs, but are not expected to be addressed on state assessments. Plus (+) Standards provide coherence and extend concepts previously taught in Algebra I, Geometry, and Algebra II.

In Algebra II, students extended their understanding of number to include complex numbers as they studied polynomials with complex zeros. In the Plus (+) Standards, students graph complex numbers in the complex plane and translate between rectangular and polar forms of complex numbers. Thinking of a complex number, $a + bi$, once again as a point (a,b) in the coordinate plane, students investigate how multiplying by a complex number can be thought of as a map from the coordinate plane to itself. That study, in turn, leads to matrix notation and a natural definition for multiplying a vector by a matrix. Thus, students discern structure in the operations with matrices and vectors by comparing them to arithmetic with complex numbers.

Students study matrices and vectors as objects in their own right. They interpret the properties and operations of matrices to learn multiple ways to solve problems with them, including solving systems of linear equations. They construct viable arguments using matrices to once again derive equations for conic sections, this time by translating and rotating the locus of points into a "standard" position using matrix operations. (For example, applying rigid motions to move the directrix of a parabola to one of the coordinate axes.) Students precisely define rotations, reflections and dilations in the coordinate plane using 2 × 2 matrices (and translations by vector addition). These well-defined definitions of transformations of the coordinate plane shed light on how geometry software and video games efficiently perform rigid motion calculations.

Students further study rational and exponential functions. They graph rational functions by extending what they learned about graphing polynomials functions. Students, build off their knowledge of the relationship between exponential and logarithmic functions, to solidify the meaning of inverse functions. Additionally, students learn to explicitly build composite functions to model relationships between two quantities. In particular, they analyze the composite of two functions in describing the relationship of three or more quantities in modeling situations.

Students visualize graphs of trigonometric functions with the aid of appropriate software and interpret how a family of graphs defined by varying a parameter in a given function changes based upon that parameter. They analyze symmetry and periodicity of trigonometric functions. They extend their knowledge of inverse functions to trigonometric functions by restricting domains to create the inverses, and apply inverse functions to solve trigonometric equations that arise in modeling contexts. Students also construct viable arguments to prove the Law of Sines, Law of Cosines, and the addition and subtraction formulas for the trigonometric functions.

Students consolidate their study of statistics as they analyze decisions and strategies using newly refined skills in calculating expected values to solve problems such as comparing high-deductible vs. low deductible insurance policies. Students model with probability and simulate probability outcomes using technology in order to make fair decisions as well as calculate permutations and combinations. Technology also plays an important role in statistics by making it possible to generate residual line plots to analyze regression functions

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

(+)-N.CN	Number and Quantity The Complex Number System	The Plus (+) Standards
Perform arithmetic operations with complex numbers.		
3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	Coherence: AII-N.CN.2 → (+)-N.CN.3	

(+)-N.CN	Number and Quantity The Complex Number System	The Plus (+) Standards
Represent complex numbers and their operations on the complex plane.		
4a. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and convert between rectangular and polar forms of a given complex number.	Coherence: AII-N.CN.1 → (+)-N.CN.4a	
4b. Determine whether rectangular or polar form is more efficient given the context.		
5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.		
6a. Calculate the distance between two points in the complex plane.	e.g., $(-1 + \sqrt{3}\,i)^3 = 8$ because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.	
6b. Find the midpoint of the segment whose endpoints are in the complex plane.	Note: Standard extends the distance and midpoint calculation from the Cartesian coordinate plane to the complete plane.	

(+)-N.CN	Number and Quantity The Complex Number System	The Plus (+) Standards
Use complex numbers in polynomial identities and equations.		
8. Extend polynomial identities to the complex numbers.	Coherence: AII-N.CN.1 → (+)-N.CN.8 e.g., Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.	
9. State the Fundamental Theorem of Algebra and use it to find roots of polynomials.		

(+)-N.VM	Number and Quantity Vector and Matrix Quantities	The Plus (+) Standards
Represent and model with vector quantities.		
1. Represent a vector analytically and geometrically.	e.g., rectangular form, polar form, unit form	
2. Find the magnitude and direction of a given vector.		
3. Solve problems using vectors analytically and geometrically.	e.g., velocity and forces	

(+)-N.VM	Number and Quantity Vector and Matrix Quantities	The Plus (+) Standards
Perform operations on vectors.		
4. Add and subtract vectors analytically and geometrically.		
5. Multiply a vector by a scalar analytically and geometrically.		

(+)-N.VM	Number and Quantity Vector and Matrix Quantities	The Plus (+) Standards
Perform operations on matrices and use matrices in applications.		
6. Use matrices to represent and model real world situations.	e.g., networks	
7. Multiply matrices by scalars.		
8. Add, subtract, and multiply matrices.		
9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.		
11. Use matrices to perform linear transformations in the plane.	e.g., multiplying a vector by 2 × 2 matrix	
12. Calculate and interpret the determinant of a matrix.	e.g., calculating area	

(+)-A.APR	Algebra Arithmetic with Polynomial and Rational Expressions	The Plus (+) Standards
Use polynomial identities to solve problems.		
4. Prove polynomial identities and use them to describe numerical relationships.	Coherence: AII-A.SSE.2 → (+)-A.APR.4 e.g., The polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.	
5. Use the Binomial Theorem for the expansion of $(x + y)^n$ for a positive integer n.	Coherence: AII-A.SSE.2 → (+)-A.APR.5	

(+)-A.APR	Algebra Arithmetic with Polynomial and Rational Expressions	The Plus (+) Standards
Rewrite rational expressions.		
7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	Coherence: AII-A.APR.6 → (+)-A.APR.7	

(+)-A.REI	Algebra Reasoning with Equations and Inequalities	The Plus (+) Standards
Solve systems of equations.		
6b. Solve systems of linear equations in three variables.	Coherence: AI-A.REI.6a → (+)-A.REI.6b	
8. Represent a system of linear equations as a single matrix equation in a vector variable.		
9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).		

(+)-F.IF	Functions Interpreting Functions	The Plus (+) Standards
Analyze functions using different representations. 7d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available. ★	Coherence: AII-F.IF.7c, AII-F.IF.7e → (+)-F.IF.7d	

(+)-F.BF	Functions Building Functions	The Plus (+) Standards
Build a function that models a relationship between two quantities.		
1c. Compose functions and state resulting domain. ★	e.g., if *T(y)* is the temperature in the atmosphere as a function of height, and *h(t)* is the height of a weather balloon as a function of time, then *T(h(t))* is the temperature at the location of the weather balloon as a function of time. Note: The domain of a resulting composition function could differ from the domains of the individual functions.	

(+)-F.BF	Functions Building Functions	The Plus (+) Standards
Build new functions from existing functions.		
3c. Determine algebraically whether or not a function is even or odd.	Coherence: AII-F.BF.3b → (+)-F.BF.3c	
4b. Verify by composition that one function is the inverse of another.	Coherence: AII-F.BF.4a → (+)-F.BF.4b	
4c. Given the graph or table of an invertible function, determine coordinates of its inverse.		
4d. Determine an invertible function from a non-invertible function by restricting the domain.	e.g., inverse trigonometric functions	
5b. Use inverse relationships to solve problems involving logarithms and exponents.	Coherence: AII-F.BF.5a → (+)-F.BF.5b	
5c. Apply the properties of logarithms to rewrite logarithmic expressions in equivalent forms and solve logarithmic equations.		

Within-Grade Connections:

- Determining an invertible function from a non-invertible function by restricting the domain ((+)-F.BF.6b) is connected to understanding that restricting the domain of a trigonometric function allows its inverse to be constructed ((+)-F.TF.6).

(+)-F.TF	Functions Trigonometric Functions	The Plus (+) Standards
Extend the domain of trigonometric functions using the unit circle.		
3. Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for x, π + x, and 2π – x in terms of their values for x, where x is any real number.	Coherence: AII-F.TF.2 → (+)-F.TF.3	

(+)-F.TF	Functions Trigonometric Functions	The Plus (+) Standards
Model periodic phenomena with trigonometric functions.		
6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	Coherence: AII-F.TF.4 → (+)-F.TF.6	
7. Solve trigonometric equations: • analytically with inverse functions and • graphically with technology and interpret solutions in terms of the context. ★		

(+)-F.TF	Functions Trigonometric Functions	The Plus (+) Standards
Prove and apply trigonometric identities.		
9. Prove the sum and difference formulas for sine, cosine, and tangent and use them to solve problems.	Coherence: AII-F.TF.8 → (+)-F.TF.9	

(+)-G.SRT	Geometry Similarity, Right Triangles, and Trigonometry	The Plus (+) Standards
Apply trigonometry to general triangles.		
10. Prove the Law of Sines and the Law of Cosines and apply in all cases, including the ambiguous case.	Coherence: GEO-G.SRT.6, GEO-G.SRT.8 → (+)-G.SRT.10	
11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.	e.g., surveying problems or problems that involve resultant forces	

(+)-G.C	Geometry Circles	The Plus (+) Standards
Understand and apply theorems about circles.		
4. Construct a tangent line from a point outside a given circle to the circle.	Coherence: GEO-G.C.2b → (+)-G.C.4	

(+)-G.GPE	Geometry Expressing Geometric Properties with Equations	The Plus (+) Standards
Translate between the geometric description and the equation for a conic section.		
2. Explore the relationship among the parabola, focus, and directrix and use the equation to model a real-life situation, using technology as appropriate. ★	Coherence: GEO-G.GPE.1a → (+)-G.GPE.2	
3a. Derive the equations of ellipses and hyperbolas given the foci.		
3b. Use these equations to model real life situations. ★		

Note on the Word *Explore*:

- *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

(+)-G.GMD	Geometry Geometric Measurement and Dimension	The Plus (+) Standards
Explain volume formulas and use them to solve problems.		
2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	Coherence: GEO-G.GMD.3 → (+)-G.GMD.2	

(+)-S.ID	Statistics and Probability Interpreting Categorical and Quantitative Data ★	The Plus (+) Standards
Summarize, represent, and interpret data on two categorical and quantitative variables.		
6b. Informally assess the fit of a function by plotting and analyzing residuals.	Coherence: AII-S.ID.6a → (+)-S.ID.6b	

(+)-S.CP	Statistics and Probability Conditional Probability and the Rules of Probability ★	The Plus (+) Standards
Use the rules of probability to compute probabilities of compound events in a uniform probability model.		
9. Solve problems using permutations and combinations to compute probabilities of compound events.	Coherence: AII-S.CP.7 → (+)-S.CP.9	

(+)-S.MD	Statistics and Probability Using Probability to Make Decisions ★	The Plus (+) Standards
Calculate expected values and use them to solve problems.		
1a. Define a random variable for a quantity of interest.		
1b. Graph a probability distribution for a discrete random variable based on either empirical or theoretical probabilities.		
2. Calculate and interpret the expected value of a random variable.	e.g., Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. e.g., Find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?	

(+)-S.MD	Statistics and Probability Using Probability to Make Decisions ★	The Plus (+) Standards
Use probability to evaluate outcomes of decisions.		
5. Use expected values from probability distributions to evaluate and compare the outcomes of decisions.	e.g., Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.	
6. Use probabilities to make fair decisions.	e.g., Determine if a decision-making strategy produces equally probable outcomes.	
7. Using probability concepts, evaluate decisions and strategies.	e.g., Make decisions based on the most favorable outcome.	

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