



## Honors Pre-Calculus Summer Packet

Welcome to the challenging world of higher-level mathematics. Included in this packet is a description of the skills that you are expected to bring with you into the Honors Pre-Calculus course. These concepts were learned in Algebra and Geometry. Please refresh your memory by doing all the exercises completely. Please read all the enclosed material and do all the problems, showing your work. Bring the completed packet with you on the first day of class for your first homework grade. There will also be a quiz on this material at some time during the first week of classes. We will spend a little bit of class time going over the packet and Activity Period will be available to you for extra help prior to this quiz.

The first set of pages are resources to help you recall the different properties and procedures. You only need to turn in the actual problems which are on the final pages of the packet.

Enjoy your summer and we look forward to working with you next school year!

## Section 1 Properties of Exponents and Radicals

**Multiply and Divide Monomials** Negative exponents are a way of expressing the multiplicative inverse of a number

|                           |                                                                                                            |
|---------------------------|------------------------------------------------------------------------------------------------------------|
| <b>Negative Exponents</b> | $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ for any real number $a \neq 0$ and any integer $n$ . |
|---------------------------|------------------------------------------------------------------------------------------------------------|

When you **simplify an expression**, you rewrite it without powers of powers, parentheses, or negative exponents. Each base appears only once, and all fractions are in simplest form. The following properties are useful when simplifying expressions.

|                             |                                                                                                                                                                                                                                                               |
|-----------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Product of Powers</b>    | $a^m \cdot a^n = a^{m+n}$ for any real number $a$ and integers $m$ and $n$ .                                                                                                                                                                                  |
| <b>Quotient of Powers</b>   | $\frac{a^m}{a^n} = a^{m-n}$ for any real number $a \neq 0$ and integers $m$ and $n$ .                                                                                                                                                                         |
| <b>Properties of Powers</b> | For $a, b$ real numbers and $m, n$ integers:<br>$(a^m)^n = a^{mn}$<br>$(ab)^m = a^m b^m$<br>$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$<br>$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}, a \neq 0, b \neq 0$ |

**Example: Simplify. Assume that no variable equals 0.**

a.  $(3m^4n^{-2})(-5mn)^2$

$$\begin{aligned} (3m^4n^{-2})(-5mn)^2 &= 3m^4n^{-2} \cdot 25m^2n^2 \\ &= 75m^4m^2n^{-2}n^2 \\ &= 75m^{4+2}n^{-2+2} \\ &= 75m^6 \end{aligned}$$

b.  $\frac{(-m^4)^3}{(2m^2)^{-2}}$

$$\begin{aligned} \frac{(-m^4)^3}{(2m^2)^{-2}} &= \frac{-m^{12}}{\frac{1}{4m^4}} \\ &= -m^{12} \cdot 4m^4 \\ &= -4m^{16} \end{aligned}$$

|                      |                                          |
|----------------------|------------------------------------------|
| <b>Zero Exponent</b> | For any nonzero number $a$ , $a^0 = 1$ . |
|----------------------|------------------------------------------|

The simplified form of an expression containing negative exponents must contain only positive exponents.

**Example: Simplify**  $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$ . **Assume that no denominator equals zero.**

$$\begin{aligned} \frac{4a^{-3}b^6}{16a^2b^6c^{-5}} &= \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right) \\ &= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5) \\ &= \frac{1}{4}a^{-5}b^0c^5 \\ &= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5 \\ &= \frac{c^5}{4a^5} \end{aligned}$$

Group powers with the same base.

Quotient of Powers and Negative Exponent Properties

Simplify.

Negative Exponent and Zero Exponent Properties

Simplify.

The solution is  $\frac{c^5}{4a^5}$ .

**Rational Exponents** For any real numbers  $a$  and  $b$  and any positive integer  $n$ , if  $a^n = b$ , then  $a$  is an  $n$ th root of  $b$ . Rational exponents can be used to represent  $n$ th roots.

|             |                                 |
|-------------|---------------------------------|
| Square Root | $b^{\frac{1}{2}} = \sqrt{b}$    |
| Cube Root   | $b^{\frac{1}{3}} = \sqrt[3]{b}$ |
| $n$ th Root | $b^{\frac{1}{n}} = \sqrt[n]{b}$ |

**Example 1:** Write  $(6xy)^{\frac{1}{2}}$  in radical form.

$(6xy)^{\frac{1}{2}} = \sqrt{6xy}$ 

Definition of  $b^{\frac{1}{n}}$

**Example 2:** Simplify  $625^{\frac{1}{4}}$ .

$625^{\frac{1}{4}} = \sqrt[4]{625}$ 

$b^{\frac{1}{n}} = \sqrt[n]{b}$   
 $625 = 5^4$   
Simplify.

$= \sqrt[4]{5 \cdot 5 \cdot 5 \cdot 5}$   
 $= 5$

| Properties of Radicals                                                                                                                                                                                                                                                                                                                 |                                                                                                          |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|
| Throughout the table: <ul style="list-style-type: none"> <li><math>a</math> and <math>b</math> can represent constants, variables, or more complicated algebraic expressions.</li> <li><math>m</math> and <math>n</math> represent natural numbers.</li> <li>It is assumed all expressions are defined and are real numbers</li> </ul> |                                                                                                          |
| Property                                                                                                                                                                                                                                                                                                                               | Example                                                                                                  |
| $\sqrt[n]{a^n} = a$ if $n$ is odd                                                                                                                                                                                                                                                                                                      | $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$                                                                 |
| $\sqrt[n]{a^n} =  a $<br>$= \pm a$ if $n$ is even                                                                                                                                                                                                                                                                                      | $\sqrt[4]{1296} = \sqrt[4]{(6)^4}$ or $\sqrt[4]{(-6)^4} =  -6  = \pm 6$                                  |
| $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$                                                                                                                                                                                                                                                                                         | $\sqrt[3]{3x^6y^2} = \sqrt[3]{3} \cdot \sqrt[3]{(x^2)^3} \cdot \sqrt[3]{y^2} = x^2 \cdot \sqrt[3]{3y^2}$ |
| $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$                                                                                                                                                                                                                                                                              | $\sqrt[4]{\frac{x^4}{16}} = \frac{\sqrt[4]{x^4}}{\sqrt[4]{2^4}} = \frac{ x }{2} = \pm \frac{x}{2}$       |
| $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$                                                                                                                                                                                                                                                                                                 | $\sqrt[3]{\sqrt[2]{64}} = \sqrt[3 \cdot 2]{64} = \sqrt[6]{2^6} = \pm 2$                                  |

## Section 2    Polynomials

Some binomial products occur so frequently that it is worth memorizing their **special product patterns**. You can verify these products by multiplying.

### Special Product Patterns

#### **Sum and Difference**

$$(a+b)(a-b) = a^2 - b^2$$

#### **Example**

$$(x+3)(x-3) = x^2 - 9$$

#### **Square of a Binomial**

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(y+4)^2 = y^2 + 8y + 16$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(3t^2 - 2)^2 = 9t^4 - 12t^2 + 4$$

#### **Cube of a Binomial**

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \quad (x+1)^3 = x^3 + 3x^2 + 3x + 1$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \quad (p-2)^3 = p^3 - 6p^2 + 12p - 8$$

| Techniques                                                                                                       | Examples                                                                  |
|------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------|
| <b>Factoring out the GCF</b><br>Factor out the greatest common factor of all the terms.                          | $15x^4 - 20x^3 + 35x^2$ $= 5x^2(3x^2 - 4x + 7)$                           |
| <b>Quadratic Trinomials</b><br>For $ax^2 + bx + c$ , find factors with product $ac$ and sum $b$ .                | $6x^2 + 11x - 10$ $= (3x - 2)(2x + 5)$                                    |
| <b>Perfect Square Trinomials</b><br>$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$                  | $x^2 + 10x + 25 = (x + 5)^2$ $x^2 - 10x + 25 = (x - 5)^2$                 |
| <b>Difference of Squares</b><br>$a^2 - b^2 = (a + b)(a - b)$                                                     | $4x^2 - 15 = (2x + \sqrt{15})(2x - \sqrt{15})$                            |
| <b>Factoring by Grouping</b><br>$ax + ay + bx + by$ $= a(x + y) + b(x + y)$ $= (a + b)(x + y)$                   | $x^3 + 2x^2 - 3x - 6$ $= x^2(x + 2) + (-3)(x + 2)$ $= (x^2 - 3)(x + 2)$   |
| <b>Sum or Difference of Cubes</b><br>$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ | $8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$ $8x^3 - 1 = (2x - 1)(4x^2 + 2x + 1)$ |

## Solving Quadratic Equations by Factoring

**Solve Quadratic Equations Using the Square Root Property** You may be able to use the **Square Root Property** below to solve certain equations. The repeated factor gives just one solution to the equation.

|                             |                                                                  |
|-----------------------------|------------------------------------------------------------------|
| <b>Square Root Property</b> | For any number $n > 0$ , if $x^2 = n$ , then $x = \pm\sqrt{n}$ . |
|-----------------------------|------------------------------------------------------------------|

**Example: Solve each equation. Check your solutions.**

**a.  $x^2 = 20$**

|                     |                      |
|---------------------|----------------------|
| $x^2 = 20$          | Original equation    |
| $x = \pm\sqrt{20}$  | Square Root Property |
| $x = \pm 2\sqrt{5}$ | Simplify.            |

The solution set is  $\{-2\sqrt{5}, 2\sqrt{5}\}$ . Since  $(-2\sqrt{5})^2 = 20$  and  $(2\sqrt{5})^2 = 20$ , the solutions check.

**b.  $(a - 5)^2 = 64$**

|                        |                      |
|------------------------|----------------------|
| $(a - 5)^2 = 64$       | Original equation    |
| $a - 5 = \pm\sqrt{64}$ | Square Root Property |
| $a - 5 = \pm 8$        | $64 = 8 \cdot 8$     |
| $a = 5 \pm 8$          | Add 5 to each side.  |

$a = 5 + 8$  or  $a = 5 - 8$  Separate into 2 equations.

$a = 13$        $a = -3$  Solve each equation.

The solution set is  $\{-3, 13\}$ . Since  $(-3 - 5)^2 = 64$  and  $(13 - 5)^2 = 64$ , the solutions check.

**Solve Equations by Factoring** Factoring and the Zero Product Property can be used to solve equations that can be written as the product of any number of factors set equal to 0.

**Example: Solve each equation. Check your solutions.**

**a.  $x^2 + 6x = 7$**

|                            |                                             |
|----------------------------|---------------------------------------------|
| $x^2 + 6x = 7$             | Original equation                           |
| $x^2 + 6x - 7 = 0$         | Rewrite equation so that one side equals 0. |
| $(x - 1)(x + 7) = 0$       | Factor.                                     |
| $x - 1 = 0$ or $x + 7 = 0$ | Zero Product Property                       |
| $x = 1$ $x = -7$           | Solve each equation.                        |

Since  $1^2 + 6(1) = 7$  and  $(-7)^2 + 6(-7) = 7$ , the solution set is  $\{1, -7\}$ .

**b.  $12x^2 + 3x = 2 - 2x$**

|                                      |                                             |
|--------------------------------------|---------------------------------------------|
| $12x^2 + 3x = 2 - 2x$                | Original equation                           |
| $12x^2 + 5x - 2 = 0$                 | Rewrite equation so that one side equals 0. |
| $(3x + 2)(4x - 1) = 0$               | Factor the left side.                       |
| $3x + 2 = 0$ or $4x - 1 = 0$         | Zero Product Property                       |
| $x = -\frac{2}{3}$ $x = \frac{1}{4}$ | Solve each equation.                        |

The solution set is  $\left\{-\frac{2}{3}, \frac{1}{4}\right\}$ .

# Solving Quadratic Equations by Using the Quadratic Formula

**Quadratic Formula** To solve the standard form of the quadratic equation,  $ax^2 + bx + c = 0$ , use the Quadratic Formula

|                          |                                                                                                                   |
|--------------------------|-------------------------------------------------------------------------------------------------------------------|
| <b>Quadratic Formula</b> | The solutions of $ax^2 + bx + c = 0$ , where $a \neq 0$ , are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . |
|--------------------------|-------------------------------------------------------------------------------------------------------------------|

**Example 1:** Solve  $x^2 + 2x = 3$  by using the Quadratic Formula.

Rewrite the equation in standard form.

$$x^2 + 2x = 3 \quad \text{Original equation}$$

$$x^2 + 2x - 3 = 3 - 3 \quad \text{Subtract 3 from each side.}$$

$$x^2 + 2x - 3 = 0 \quad \text{Simplify.}$$

Now let  $a = 1$ ,  $b = 2$ , and  $c = -3$  in the Quadratic Formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{16}}{2} \end{aligned}$$

$$\begin{aligned} x &= \frac{-2 + 4}{2} \text{ or } x = \frac{-2 - 4}{2} \\ &= 1 \quad \quad \quad = -3 \end{aligned}$$

The solution set is  $\{-3, 1\}$ .

**Example 2:** Solve  $x^2 - 6x - 2 = 0$  by using the Quadratic Formula.

For this equation  $a = 1$ ,  $b = -6$ , and  $c = -2$ .

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)} \\ &= \frac{6 \pm \sqrt{44}}{2} \end{aligned}$$

$$x = \frac{6 + \sqrt{44}}{2} \quad \text{or} \quad x = \frac{6 - \sqrt{44}}{2}$$

The solution set is  $\left\{\frac{6 + \sqrt{44}}{2}, \frac{6 - \sqrt{44}}{2}\right\}$ .

## Section 3 Rational Expressions

### Multiplying and Dividing Rational Expressions: Section 3

**Simplify Rational Expressions** A ratio of two polynomial expressions is a **rational expression**. To simplify a rational expression, divide both the numerator and the denominator by their greatest common factor (GCF).

|                                         |                                                                                                                                                               |
|-----------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Multiplying Rational Expressions</b> | For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ , $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ , if $b \neq 0$ and $d \neq 0$ .               |
| <b>Dividing Rational Expressions</b>    | For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ , $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ , if $b \neq 0$ , $c \neq 0$ , and $d \neq 0$ . |

**Example: Simplify each expression.**

a.  $\frac{24a^5b^2}{(2ab)^4}$

$$\frac{24a^5b^2}{(2ab)^4} = \frac{\overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{3} \cdot \overset{1}{a} \cdot \overset{1}{a} \cdot \overset{1}{a} \cdot \overset{1}{a} \cdot \overset{1}{a} \cdot \overset{1}{b} \cdot \overset{1}{b}}{\overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{a} \cdot \overset{1}{a} \cdot \overset{1}{a} \cdot \overset{1}{a} \cdot \overset{1}{b} \cdot \overset{1}{b} \cdot \overset{1}{b} \cdot \overset{1}{b}} = \frac{3a}{2b^2}$$

b.  $\frac{3r^2n^3}{5t^4} \cdot \frac{20t^2}{9r^3n}$

$$\frac{3r^2n^3}{5t^4} \cdot \frac{20t^2}{9r^3n} = \frac{\overset{1}{3} \cdot \overset{1}{r} \cdot \overset{1}{r} \cdot \overset{1}{n} \cdot \overset{1}{n} \cdot \overset{1}{n} \cdot \overset{1}{2} \cdot \overset{1}{2} \cdot \overset{1}{t} \cdot \overset{1}{t}}{\overset{1}{5} \cdot \overset{1}{t} \cdot \overset{1}{t} \cdot \overset{1}{t} \cdot \overset{1}{t} \cdot \overset{1}{3} \cdot \overset{1}{r} \cdot \overset{1}{r} \cdot \overset{1}{r} \cdot \overset{1}{n}} = \frac{2 \cdot 2 \cdot n \cdot n}{3 \cdot r \cdot t \cdot t} = \frac{4n^2}{3rt^2}$$

c.  $\frac{x^2+8x+16}{2x-2} \div \frac{x^2+2x-8}{x-1}$

$$\begin{aligned} \frac{x^2+8x+16}{2x-2} \div \frac{x^2+2x-8}{x-1} &= \frac{x^2+8x+16}{2x-2} \cdot \frac{x-1}{x^2+2x-8} \\ &= \frac{\overset{1}{(x+4)} \cdot \overset{1}{(x+4)} \cdot \overset{1}{(x-1)}}{\overset{1}{2} \cdot \overset{1}{(x-1)} \cdot \overset{1}{(x-2)} \cdot \overset{1}{(x+4)}} = \frac{(x+4)}{2(x-2)} \end{aligned}$$

**Add and Subtract Rational Expressions** To add or subtract rational expressions, follow these steps

- Step 1** Find the least common denominator (LCD). Rewrite each expression with the LCD.  
**Step 2** Add or subtract the numerators.  
**Step 3** Combine any like terms in the numerator.  
**Step 4** Factor if possible.  
**Step 5** Simplify if possible.

**Example: Simplify**  $\frac{6}{2x^2+2x-12} - \frac{2}{x^2-4}$

$$\begin{aligned} &\frac{6}{2x^2+2x-12} - \frac{2}{x^2-4} \\ &= \frac{6}{2(x+3)(x-2)} - \frac{2}{(x-2)(x+2)} \\ &= \frac{6(x+2)}{2(x+3)(x-2)(x+2)} - \frac{2 \cdot 2(x+3)}{2(x+3)(x-2)(x+2)} \\ &= \frac{6(x+2) - 4(x+3)}{2(x+3)(x-2)(x+2)} \\ &= \frac{6x+12-4x-12}{2(x+3)(x-2)(x+2)} \\ &= \frac{2x}{2(x+3)(x-2)(x+2)} \\ &= \frac{x}{(x+3)(x-2)(x+2)} \end{aligned}$$

Factor the denominators.

The LCD is  $2(x+3)(x-2)(x+2)$ .

Subtract the numerators.

Distribute.

Combine like terms.

Simplify.

**Simplify Complex Fractions** A **complex fraction** is a rational expression with a numerator and/or denominator that is also a rational expression. To simplify a complex fraction, first rewrite it as a division problem.

**Example:** Simplify  $\frac{\frac{3n-1}{n}}{\frac{3n^2+8n-3}{n^4}}$ .

$$\frac{\frac{3n-1}{n}}{\frac{3n^2+8n-3}{n^4}} = \frac{3n-1}{n} \div \frac{3n^2+8n-3}{n^4}$$

Express as a division problem.

$$= \frac{3n-1}{n} \cdot \frac{n^4}{3n^2+8n-3}$$

Multiply by the reciprocal of the divisor.

$$= \frac{(3n-1)\cancel{n^4}}{\cancel{n}(3n-1)\cancel{n}(n+3)}$$

Factor and eliminate.

$$= \frac{n^3}{n+3}$$

Simplify.

## Section 4 Lines and Coordinate Plane

### Midpoint of a Segment

|                                       |                                                                                                                                                                                               |
|---------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Midpoint on a Coordinate Plane</b> | If a segment has endpoints with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ , then the coordinates of the midpoint of the segment are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ . |
|---------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

**Example:** Find the coordinates of  $M$ , the midpoint of  $\overline{PQ}$ , for  $P(-2, 4)$  and  $Q(4, 1)$ .

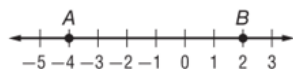
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 4}{2}, \frac{4 + 1}{2}\right) \text{ or } (1, 2.5)$$

### Distance

**Distance Between Two Points** The distance between two points on a number line is the absolute value of the difference between their coordinates. If  $P$  and  $Q$  are on the same line,  $P$  has coordinate  $x_1$ , and  $Q$  has coordinate  $x_2$ ,  $PQ = |x_2 - x_1|$  or  $|x_1 - x_2|$ .

To find the distance between two points in a coordinate plane, you can form a right triangle, and then use the Pythagorean Theorem to find the distance between the two points. If  $P$  and  $Q$  are in the same coordinate plane,  $P$  has coordinate  $(x_1, y_1)$ , and  $Q$  has coordinates  $(x_2, y_2)$ , then  $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .

**Example 1:** Find  $AD$ .



The coordinates of  $A$  and  $D$  are  $-4$  and  $2$ .

$$\begin{aligned} AD &= |x_2 - x_1| && \text{Distance Formula} \\ &= |2 - (-4)| && x_1 = -4 \text{ and } x_2 = 2 \\ &= |6| \text{ or } 6 && \text{Simplify.} \end{aligned}$$

**Example 2:** Find the distance between  $C(-1, 3)$  and  $D(3, -5)$ .

$$\begin{aligned} CD &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(3 - (-1))^2 + (-5 - 3)^2} && (x_1, y_1) = (-1, 3) \text{ and } (x_2, y_2) = (3, -5) \\ &= \sqrt{4^2 + (-8)^2} && \text{Subtract.} \\ &= \sqrt{16 + 64} && \text{Evaluate exponents.} \\ &= \sqrt{80} && \text{Add.} \end{aligned}$$

The distance between  $C$  and  $D$  is  $\sqrt{80}$  or about 8.9 units.



# Writing Equations in Standard and Slope-Intercept Form

## Forms of Linear Equations

|                             |                        |                                                                                                                      |
|-----------------------------|------------------------|----------------------------------------------------------------------------------------------------------------------|
| <b>Slope-Intercept Form</b> | $y = mx + b$           | $m$ = slope; $b$ = y-intercept                                                                                       |
| <b>Point-Slope Form</b>     | $y - y_1 = m(x - x_1)$ | $m$ = slope; $(x_1, y_1)$ is a given point on a nonvertical line                                                     |
| <b>Standard Form</b>        | $Ax + By = C$          | $A \geq 0$ , $A$ and $B$ are not both zero, and $A$ , $B$ , and $C$ are integers with a greatest common factor of 1. |

**Example 1:** Write  $y + 5 = \frac{2}{3}(x - 6)$  in standard form.

$$\begin{aligned}
 y + 5 &= \frac{2}{3}(x - 6) && \text{Original equation} \\
 3(y + 5) &= 3\left(\frac{2}{3}\right)(x - 6) && \text{Multiply each side by 3.} \\
 3y + 15 &= 2(x - 6) && \text{Distributive Property} \\
 3y + 15 &= 2x - 12 && \text{Distributive Property} \\
 3y &= 2x - 27 && \text{Subtract 15 from each side.} \\
 -2x + 3y &= -27 && \text{Add } -2x \text{ to each side.} \\
 2x - 3y &= 27 && \text{Multiply each side by } -1.
 \end{aligned}$$

Therefore, the standard form of the equation is  $2x - 3y = 27$ .

**Example 2:** Write  $y - 2 = -\frac{1}{4}(x - 8)$  in slope-intercept form.

$$\begin{aligned}
 y - 2 &= -\frac{1}{4}(x - 8) && \text{Original equation} \\
 y - 2 &= -\frac{1}{4}x + 2 && \text{Distributive Property} \\
 y &= -\frac{1}{4}x + 4 && \text{Add 2 to each side.}
 \end{aligned}$$

Therefore, the slope-intercept form of the equation is  $y = -\frac{1}{4}x + 4$ .

**Parallel and Perpendicular Lines** Use the slope-intercept or point-slope form to find equations of lines that are parallel or perpendicular to a given line. Remember that parallel lines have equal slope. The slopes of two perpendicular lines are negative reciprocals, that is, their product is  $-1$ .

**Example 1:** Write an equation of the line that passes through  $(8, 2)$  and is perpendicular to the line whose equation is  $y = -\frac{1}{2}x + 3$ .

The slope of the given line is  $-\frac{1}{2}$ . Since the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line is 2.

Use the slope and the given point to write the equation.

$$\begin{aligned}
 y - y_1 &= m(x - x_1) && \text{Point-slope form} \\
 y - 2 &= 2(x - 8) && (x_1, y_1) = (8, 2), m = 2 \\
 y - 2 &= 2x - 16 && \text{Distributive Prop.} \\
 y &= 2x - 14 && \text{Add 2 to each side.}
 \end{aligned}$$

An equation of the line is  $y = 2x - 14$ .

**Example 2:** Write an equation of the line that passes through  $(-1, 5)$  and is parallel to the graph of  $y = 3x + 1$ .

The slope of the given line is 3. Since the slopes of parallel lines are equal, the slope of the parallel line is also 3.

Use the slope and the given point to write the equation.

$$\begin{aligned}
 y - y_1 &= m(x - x_1) && \text{Point-slope form} \\
 y - 5 &= 3(x - (-1)) && (x_1, y_1) = (-1, 5), m = 3 \\
 y - 5 &= 3x + 3 && \text{Distributive Prop.} \\
 y &= 3x + 8 && \text{Add 5 to each side.}
 \end{aligned}$$

An equation of the line is  $y = 3x + 8$ .

## Section 1

Simplify each expression. Write answers in positive exponents only.

1.  $(-a)^6(2a^4)$

2.  $5x^0 + (5x)^0$

3.  $8x^3(2x^5)^2$

4.  $\frac{15b^3}{3b^6}$

5.  $(x^{-5}y^6)(x^2y^3)$

6.  $\frac{c^2 \cdot c^{4x}}{c^x \cdot c^3}$

7.  $\sqrt[3]{8x^3y^4}$

8.  $\sqrt[6]{x^6}$

9.  $\sqrt{6xy^2}\sqrt{2x}$

10.  $\frac{\sqrt[4]{16b^5}}{\sqrt[4]{b}}$

11.  $\frac{9x}{\sqrt[3]{(3x)^2}}$

12.  $\sqrt{75y} - \sqrt{3y}$

13.  $\sqrt[3]{250} + 2\sqrt[3]{16}$

14.  $\left(\frac{a^2b}{a^{-3}b^2}\right)^{-1}$

15.  $\frac{2mn^2(3m^2n)^2}{12m^3n^4}$

16. Write the expression as a single radical

$$\sqrt{18}\sqrt[3]{2}$$

## Section 2

Perform the indicated operations and write the result in standard form.

1.  $(3x - 2) - (8x - 6)$

2.  $(x - 2)(x^2 - 5)$

3.  $(3x - 7y)^2$

4.  $(x - 4)^3$

5.  $5a(4a - 6) - a(3a + 2)$

6.  $(y - z)(y + z) + 3(y - z)$

7.  $[3 - (a + b)][3 + (a + b)]$

8.  $(a + 2b)^3$

9.  $3y^2 - [2y^2 - (5y^2 + 4)]$

10.  $[(x + 2) - y]^2$

Factor completely.

11.  $6y^3 - 4y^2 + 2y$

12.  $8x^3 - 1$

13.  $x^3 + 6x^2 - 5x - 30$

14.  $2r^3 + 2$

15.  $x^2 - \frac{9}{16}$

16.  $9t^2 - 30t + 25$

17.  $4x^2 + 5x + 1$

18.  $y^5 + 3y^3 + y^2 + 3$

19.  $27a^3 + 125b^3$

20.  $4 - (a - b)^2$

21.  $c^3 + 7c^2 - 5c - 35$

22.  $2r^3 - 50r$

Solve.

$$23. 6x^2 + 5x - 4 = 0$$

$$24. 4x^2 - 3x - 2 = 0$$

$$25. 2x^3 - x^2 = 6x - 3$$

$$26. 4x^4 - 73x^2 + 144 = 0$$

$$27. 3t^6 - 48t^2 = 0$$

$$28. 4x^2 - 4x - 11 = 0$$

### Section 3

Write in reduced form.

1.  $\frac{12b^6}{60b^2}$

2.  $\frac{5w^2 - 10w}{15w - 15}$

3.  $\frac{z - 5}{25 - z^2}$

4.  $\frac{x^3 + 2x^2 - 4x - 8}{x^3 - 8}$

Perform the indicated operations and simplify.

5.  $\frac{y^2 - 9}{y^2 + y - 6} \cdot \frac{y^2 - 4y + 4}{y - 3}$

6.  $\left(\frac{a+b}{b}\right) + \left(\frac{a}{b} - \frac{b}{a}\right)$

$$7. \frac{2}{s-4} - \frac{1}{s+3}$$

$$8. \frac{3x}{x^2 - 6x + 8} + \frac{4}{x^2 - x - 12}$$

$$9. \frac{2 + \frac{2}{x}}{\frac{1}{x+2} - 1}$$

$$10. \frac{\frac{3}{\sqrt{x-1}} - \sqrt{x-1}}{4\sqrt{x-1}}$$

Write in reduced form.

$$11. \frac{t^3 - 27}{t^3 - 3t^2 - 2t + 6}$$

$$12. \frac{x^3 - 3x^2 + 2x}{x^3 - 4x}$$

Perform the indicated operations and simplify.

$$13. \frac{b^2 - 9}{b^2 - b - 6} \cdot \frac{b^2 + 8b + 9}{b + 3}$$

$$14. \frac{x + 1}{x^2 - 1} + \frac{x^2 + 1}{x - 1}$$

$$15. \left( \frac{1}{x + y} \right) \left( \frac{x}{y} + \frac{y}{x} \right)$$

$$16. \frac{2}{c - 3} + \frac{1}{c + 2}$$

$$17. \frac{3}{x^2 - 1} - \frac{4}{x^2 - 3x + 2}$$

$$18. \frac{\frac{3}{x} - 4}{1 + \frac{1}{x - 1}}$$



#### Section 4

- Find  $y$  so that the distance between  $(3,y)$  and  $(-1,1)$  is 5.
- Determine the quadrant(s) in which  $(x,y)$  is located so that the conditions are satisfied.
  - $x > 0$  and  $y < 0$
  - $y > 2$
- Determine if the lines  $L_1$  and  $L_2$  passing through the given pairs of points are parallel, perpendicular or neither.
  - $L_1 : (0, -1), (5, 9)$   
 $L_2 : (0, 3), (4, 1)$
  - $L_1 : (4, 8), (-4, 2)$   
 $L_2 : (3, -5), (-1, \frac{1}{3})$
- Find the slope and y-intercept (if possible) of the line specified by the given equation.
  - $5x - y + 3 = 0$
  - $5x - 2 = 0$
- Write an equation of the line through the given point (a) parallel to the given line and (b) perpendicular to the given line.  
  
Point  $(-6, 4)$  Line  $3x + 4y = 7$

6. Show that the points form the vertices of the indicated polygon:

Right Triangle:  $(4, 0)$ ,  $(2, 1)$ ,  $(-1, -5)$

7. Find an equation of the line that passes through the given point and has the indicated slope. Sketch the graph of the line.

a) Point  $(-3, 6)$   $m = -2$

b) Point  $(6, -1)$   $m$  is undefined

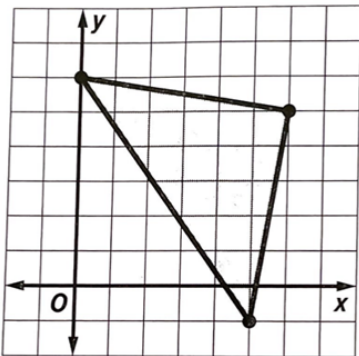
8. Determine the quadrant(s) in which  $(x, y)$  is located so that the given conditions are satisfied:

a)  $(x, -y)$  is in the second quadrant

b)  $(-x, y)$  is in the fourth quadrant

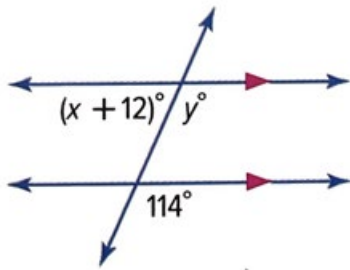
## Geometry

9. To the nearest tenth, what is the perimeter of the right triangle below?

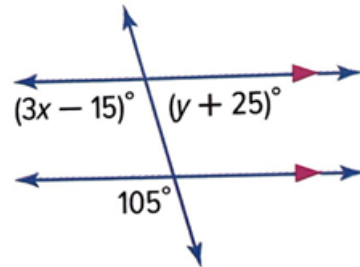


10. Find the value of the variable(s) in each figure.

a)



b)



11. A lifeguard is watching a beach from a line of sight 6 feet above the ground. She sees a swimmer at an angle of depression of  $8^\circ$ . How far away from the tower is the swimmer?