AP Calculus BC Summer Work

AP Calculus BC is a challenging, rewarding, and fast-paced course that will build on many of the mathematical concepts you have learned throughout Geometry, Algebra 2, and Precalculus.

As part of your summer work, you will review key topics from these courses. These are skills that you are expected to know coming into AP Calculus BC and that we will use regularly throughout the year. While we will spend the first few days of class revisiting some of these concepts, there will not be extensive review beyond that point.

Your success in AP Calculus BC depends heavily on your fluency with these prerequisite skills. The stronger your foundation in Precalculus, the more confidently you will be able to focus on the new ideas and problem-solving techniques that make calculus so interesting. If you encounter topics that feel unfamiliar or challenging, use this summer as an opportunity to strengthen those areas.

To help ensure that everyone begins the course with the necessary foundation, there will be an assessment on these prerequisite skills during the first week of school. The assessment is designed to help identify strengths and areas that may need additional attention as we begin our study of calculus.

Approach this review thoughtfully and honestly. The goal is not simply to complete the assignment, but to ensure that you are prepared to begin the year with confidence. The effort you invest now will pay dividends throughout the course.

I look forward to working with you and exploring calculus together this year. Have a wonderful summer, and I will see you in the fall!

ST. SEBASTIAN'S SCHOOL

ACADEMIC RIGOR | SPIRITUAL DEPTH

You are expected to know the general shape, domain, and range of each parent function in the table.

Please draw in each function.

"Parent" functions mean no transformations have been applied. Transformation (shifting, stretching, compressing, or reflecting) may change the domain or range.

Linear functions (x) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$	**Quadratic** functions (x^2) **D:** $(-\infty, \infty)$ **R:** $[0, \infty)$
Odd-degree polynomials (x^n for **odd** n) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$	**Even-degree** polynomials (x^n for **even** n) **D:** $(-\infty, \infty)$ **R:** $[0, \infty)$
Exponential functions (a^x, e^x) **D:** $(-\infty, \infty)$ **R:** $(0, \infty)$ Horizontal asymptote: $y = 0$	**Logarithmic** functions $(\log_a x, \ln x)$ **D:** $(0, \infty)$ **R:** $(-\infty, \infty)$ Vertical asymptote: $x = 0$
Sinusoidal functions $(a \sin x, a \cos x)$ **D:** $(-\infty, \infty)$ **R:** $[-a, a]$	**Tangent** function $(\tan x)$ **D:** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{(2n+1)\pi}{2}, \frac{(2n+3)\pi}{2}\right)$ **R:** $(-\infty, \infty)$ Vertical asymptotes: $x = \frac{(2n+1)\pi}{2}$
Reciprocal functions $\left(\frac{1}{x}\right)$ **D:** $(-\infty, 0) \cup (0, \infty)$ **R:** $(-\infty, 0) \cup (0, \infty)$ Vertical asymptote: $x = 0$ Horizontal asymptote: $y = 0$	**Absolute value** functions $(\lvert x \rvert)$ **D:** $(-\infty, \infty)$ **R:** $[0, \infty)$
Even root functions $(\sqrt{x}, \sqrt[n]{x}$ for even $n)$ **D:** $[0, \infty)$ **R:** $[0, \infty)$	**Odd root** functions $(\sqrt[3]{x}, \sqrt[n]{x}$ for odd $n > 1)$ **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x + 2) =$

3. $f(x + h) =$

Use the graph $f(x)$ to answer the following.

4. $f(0) =$

$f(4) =$

$f(-1) =$

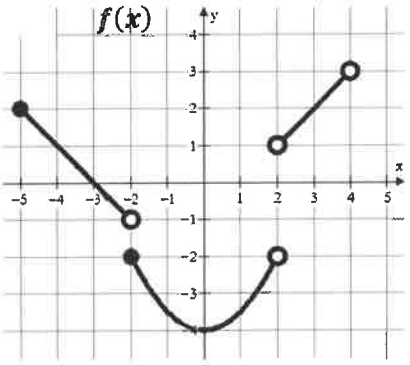
$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2$ when $x =$?

$f(x) = -3$ when $x =$?



Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

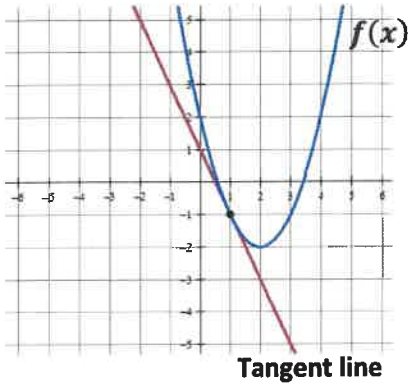
5. slope $= 3$ and $(4, -2)$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

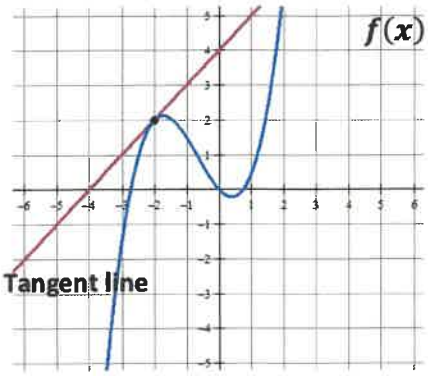
7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$

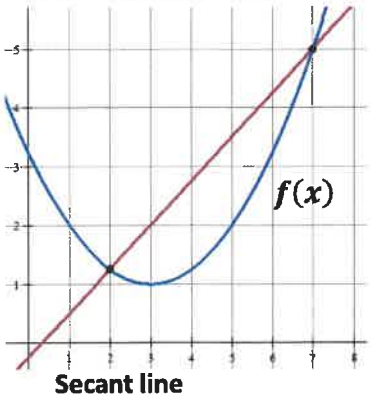


9. The line tangent to $f(x)$ at $x = -2$



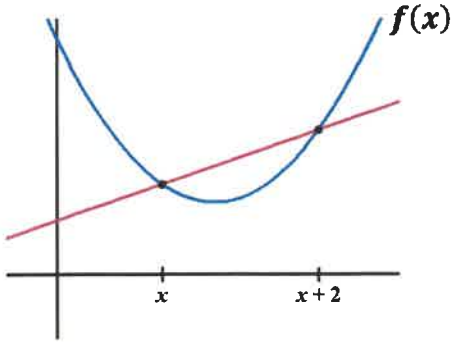
MULTIPLE CHOICE! Remember slope $= \frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?



A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$

11. Which choice represents the slope of the secant line shown?

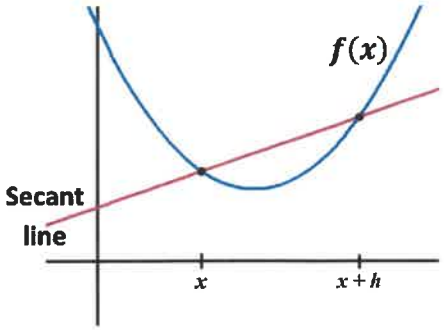


Secant line

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$

12. Which choice represents the slope of the secant line shown?



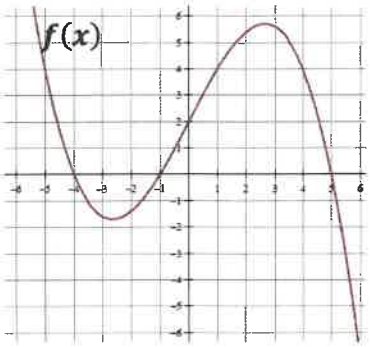
A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

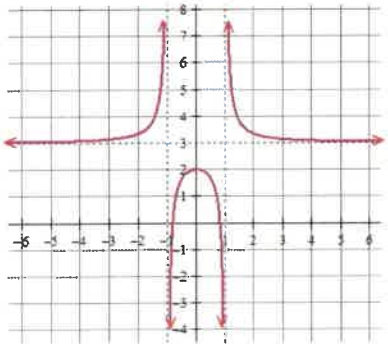
I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only



Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



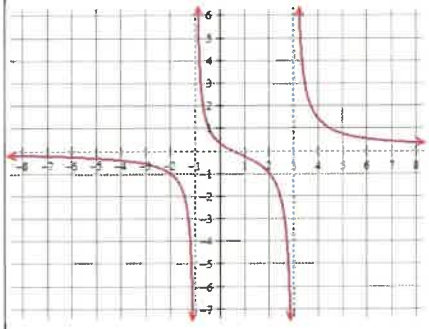
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



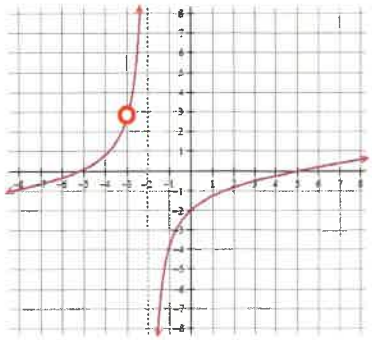
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$. Leave in calc ready form if necessary.

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$	70. $e^{\ln x}$	71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

76. $\ln \frac{1}{2}$

77. $27^{\frac{2}{3}}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $\left(4a^{5/3}\right)^{3/2}$

If $f(x) = \{(3,5),(2,4),(1,7)\}$ $g(x) = \sqrt{x-3}$
$h(x) = \{(3,2),(4,3),(1,6)\}$ $k(x) = x^2 + 5$, **then determine each of the following.**

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f\big(h(3)\big)$

84. $g\big(k(7)\big)$

85. $h(3)$

86. $g\big(g(9)\big)$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k\big(g(x)\big)$

90. $g(f(2))$

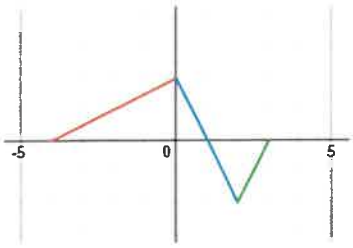
If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:

92. $f(x) - 4$

93. $-f(x+2)$

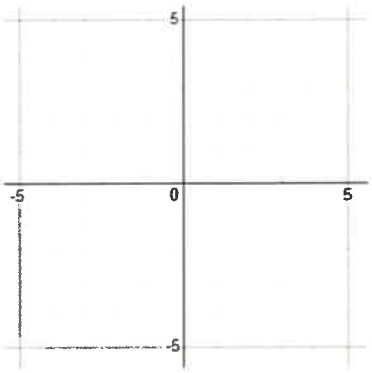
94. $\frac{1}{2}f(-x)$

Given the graph of $f(x)$, sketch the following graphs.

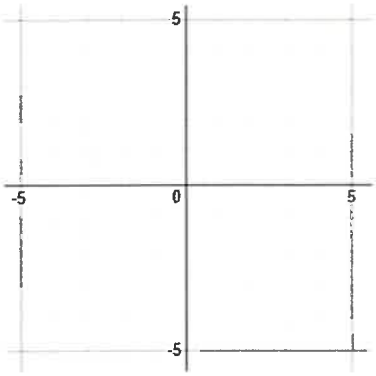


$y = f(x)$

95. $y = f(-x-1)$



96. $y = f(2x) + 1$



Simplify each complex fraction

97. $\frac{2a-\frac{1}{8a}}{4+\frac{1}{a}}$

98. $\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}}$

99. $\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

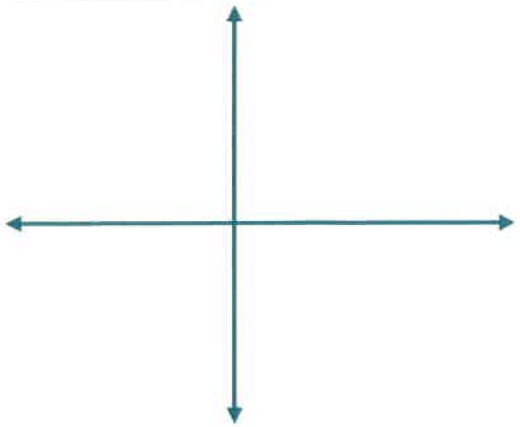
Rationalize each expression by multiplying by the conjugate.

100. $\frac{2}{5+\sqrt{3x}}$

101. $\frac{3-\sqrt{x}}{x-9}$

102. $\frac{\sqrt{x+7}-2}{-x+3}$

Sketch the graph and find the stated limits. $f(x) = \frac{2x}{x-3}$

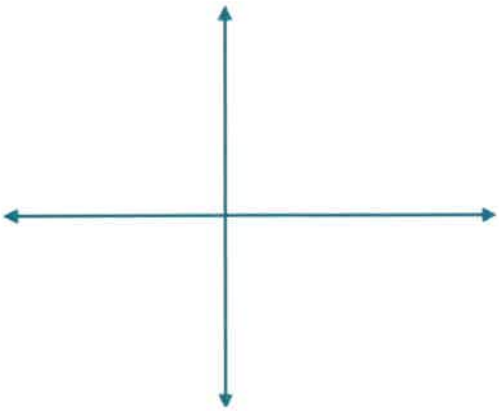


103. $\lim_{x \to 3^+} f(x)$

104. $\lim_{x \to 3} f(x)$

105. $\lim_{x \to \infty} f(x)$

Sketch the graph and find the stated limits. $f(x) = \frac{1}{(x+1)^2}$



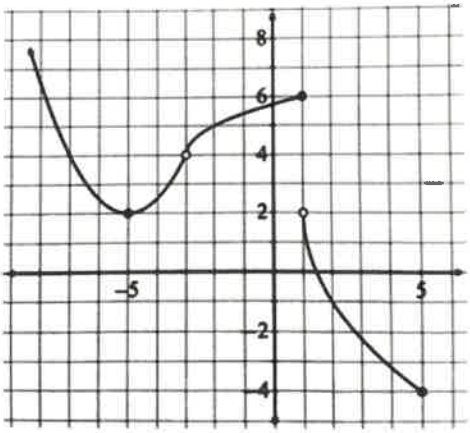
106. $\lim_{x \to -1} f(x)$

107. $\lim_{x \to \infty} f(x)$

Given the graph of $y = f(x)$ at the right, answer the following:

108. $f(1) =$

109. $\lim_{x \to -3} f(x) =$



110. $\lim_{x \to 1} f(x) =$

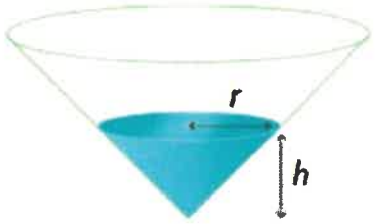
111. $\lim_{x \to -\infty} f(x) =$

112. For what values of x is $f(x)$ discontinuous?

113. An inverted conical reservoir has a height of 10 inches and a base diameter of 12 inches. It is slowly being filled with water. Write an expression for the volume of the water in terms of its …

a) radius, so $V(r) =$

b) height, $V(h) =$



Use similar triangles.

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114. A rectangular sheet of tin measures 20 inches by 12 inches. Suppose you cut a square out of each corner and fold up the sides to make an open-topped box. What size square should you cut out in order to maximize the volume of the box? You may use your calculator but only after you have set up a function.