

AP® Calculus BC



Summer Assignment

Overview:

Welcome to AP® Calculus BC! As with most AP® courses, the breadth and depth of the material covered on the AP® Calculus BC exam tends to exceed the amount of class time available. To that end, I would like you to get a head start on the content (specifically, the review of the AP® Calculus AB subtopics) to ensure we have enough time to learn the necessary material and review for the AP® exam next May. This will be considered your first graded assignment for AP® Calculus BC. If you have any questions, please contact Mr. Waltrich twaltrich@gmahs.org.

Instructions:

On lined paper, show all work as you complete each of the exercises from the attached assignment. Indicate only your final answers on the answer sheets provided. You will submit both your answers and supporting work on the first day of class.

Section 1: Multiple-Choice, Part A	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	
29	
30	
Section 1: Multiple-Choice, Part B	
31	
32	
33	
34	
35	
36	
37	
38	
39	
40	
41	
42	
43	
44	
45	

Section 2: Free Response, Part A	
1(a)	
1(b)	
1(c)	
2(a)	
2(b)	
2(c)	
Section 2:Free Response, Part B	
3(a)	
3(b)	
3(c)	
3(d)	
4(a)	
4(b)	
4(c)	
4(d)	
5(a)	
5(b)	
5(c)	
5(d)	
6(a)	
6(b)	
6(c)	

Section 1: Multiple Choice, Part A

A calculator may not be used for Part A.

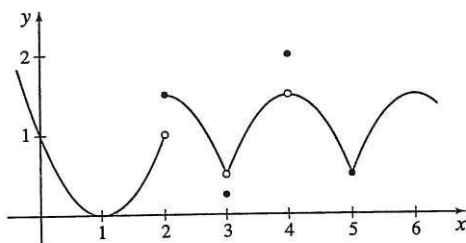
1. If
- $f(x) = e^{4x} + \sin(2x)$
- , then
- $f'(0) =$

(A) 1 (B) 2 (C) 4 (D) 6

2. If
- $\lim_{x \rightarrow 1} f(x) = 3$
- , then
- $\lim_{x \rightarrow 1} \frac{(x^2 - 1)f(x)}{x - 1} =$

(A) 0 (B) 4 (C) 6 (D) Does not exist.

3. The graph of the function
- f
- is shown below. Which of the following statements is false?

(A) $\lim_{x \rightarrow 2^-} f(x) = f(2)$ (B) The function f is discontinuous at $x = 3$.(C) $\lim_{x \rightarrow 4} f(x)$ exists.(D) The function f is continuous at $x = 5$.

4. Let
- f
- be the function
- $f(x) = x^3 - 15x^2 - 1800x + 2000$
- . On which interval is the function
- f
- both decreasing and concave down?

(A) $x < -20$ (B) $-20 < x < 5$ (C) $5 < x < 30$ (D) $x > 30$

5. If
- $y = e^{2x} \cos(3x)$
- , then
- $\frac{dy}{dx} =$

(A) $-6e^{2x} \sin(3x)$ (B) $e^{2x}(2 \cos(3x) - 3 \sin(3x))$ (C) $e^{2x}(2 \cos(3x) + 3 \sin(3x))$ (D) $e^{2x}(\cos(3x) - \sin(3x))$

6. Let
- f
- be the function defined below. For what value of
- k
- is
- f
- continuous at
- $x = 0$
- ?

$$f(x) = \begin{cases} \frac{\sin(7x)}{2x} & \text{for } x < 0 \\ k + 2 \ln(x + e^{x+1}) & \text{for } x \geq 0 \end{cases}$$

(A) $-\frac{3}{2}$ (B) -1 (C) $\frac{3}{2}$ (D) $\frac{7}{2}$

7. Let
- f
- be the function defined by
- $f(x) = \sqrt[5]{(x-3)^4}$
- for all
- x
- . Which of the following statements is true?

(A) f is continuous and differentiable at $x = 3$.(B) f is continuous but not differentiable at $x = 3$.(C) f is differentiable but not continuous at $x = 3$.(D) f is not continuous and not differentiable at $x = 3$.

8. The function
- f
- is differentiable and its derivative is continuous on the interval
- $(-4, 5)$
- . The table below lists several values of
- f
- and
- f'
- in the interval.

x	-3	-1	0	3
$f(x)$	-16	$\frac{2}{3}$	2	2
$f'(x)$	3	-1	0	15

Then $\int_{-1}^3 f'(x) dx =$ (A) -16 (B) $\frac{4}{3}$ (C) 12 (D) 16

9. If
- $y = \sqrt{4x + 6e^{\tan x}}$
- , then
- y'
- equals

(A) $\sqrt{4 + 6e^{\tan x} \sec^2 x}$ (B) $\frac{1}{2\sqrt{4x + 6e^{\tan x}}}$ (C) $\frac{2 + 3e^{\tan x}}{\sqrt{4x + 6e^{\tan x}}}$ (D) $\frac{2 + 3e^{\tan x} \sec^2 x}{\sqrt{4x + 6e^{\tan x}}}$

10. The function
- f
- is continuous on
- $1 \leq x \leq 5$
- and differentiable on
- $1 < x < 5$
- . If
- $f(1) = 10$
- and
- $f(5) = 50$
- , then the Mean Value Theorem guarantees that

(A) f is linear on the interval $1 \leq x \leq 5$.(B) $f'(c) = 10$ for at least one c between 1 and 5.(C) $f'(c) = 0$ for at least one c between 1 and 5.(D) $f(c) = 30$ for at least one c between 1 and 5.

11. The Riemann sum
- $\frac{1}{20} (e^{1/20} + e^{2/20} + e^{3/20} + \cdots + e^2)$
- is an approximation for which integral?

(A) $\int_0^2 e^{x/20} dx$ (B) $\int_0^2 e^x dx$ (C) $\frac{1}{20} \int_0^2 e^x dx$ (D) $\frac{1}{20} \int_0^2 e^{x/20} dx$

12. What is the area of the region in the first quadrant bounded by the graph of
- $y = 1 + e^{-2x}$
- and the line
- $x = 3$
- ?

(A) $2 - 2e^{-6}$ (B) $3 - \frac{1}{2}e^{-6}$ (C) $\frac{7}{2} - \frac{1}{2}e^{-6}$ (D) $5 - 2e^{-6}$

13. Using the substitution
- $u = \sqrt{x}$
- ,
- $\int_1^4 \frac{1+x}{1+\sqrt{x}} dx$
- is equal to which of the following?

(A) $\int_1^4 \frac{1+u^2}{1+u} du$ (B) $\int_1^2 \frac{1+u^2}{1+u} du$ (C) $2 \int_1^4 \frac{u+u^3}{1+u} du$ (D) $2 \int_1^2 \frac{u+u^3}{1+u} du$

14. If $f(x) = \sqrt{25 - x^2}$, then the derivative of $f(f(x))$ at $x = -3$ is

(A) -3 (B) -1 (C) 1 (D) 3

15. Let f be the function defined below. What is the value of $\int_{-1}^1 f(x) dx$?

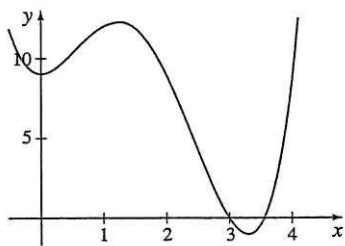
$$f(x) = \begin{cases} e^{2x} & \text{for } x < 0 \\ e^{-4x} & \text{for } x \geq 0 \end{cases}$$

(A) $e^2 - e^{-4}$ (B) $\frac{1}{4} - \frac{1}{2}e^2 - \frac{1}{4}e^{-4}$
 (C) $\frac{1}{4} - \frac{1}{2}e^2 + \frac{1}{4}e^{-4}$ (D) $\frac{3}{4} - \frac{1}{2}e^{-2} - \frac{1}{4}e^{-4}$

16. If $f(x) = \sin(2x)$, which of the following is equal to $f'\left(\frac{\pi}{4}\right)$?

(A) $\lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{4} + 2h\right) - \sin\left(\frac{\pi}{4}\right)}{h}$
 (B) $\lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} + 2h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$
 (C) $\lim_{h \rightarrow 0} \frac{2\sin\left(\frac{\pi}{4} + h\right) - 2\sin\left(\frac{\pi}{4}\right)}{h}$
 (D) $\lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} + 2h\right) - 2\sin\left(\frac{\pi}{2}\right)}{h}$

17. The graph of a differentiable function f is shown below. Which of the following is true?



(A) $f(3) < f'(3) < f''(3)$ (B) $f(3) < f''(3) < f'(3)$
 (C) $f'(3) < f(3) < f''(3)$ (D) $f''(3) < f(3) < f'(3)$

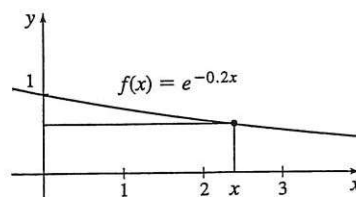
18. $\lim_{x \rightarrow \infty} \frac{4x^3 + 3x^2 + 5}{5x^3 + x^2 - x} =$

(A) 0 (B) $\frac{4}{5}$ (C) $\frac{8}{5}$ (D) ∞

19. A particle moves along the x -axis so that at any time $t > 0$, its acceleration is given by $a(t) = 3$. If the velocity of the particle at time $t = 3$ is -3 and the position at time $t = 2$ is 5 , what is the position of the particle when the particle is at rest?

(A) -49 (B) -47 (C) -1 (D) 3.5

20. Let f be the function defined by $f(x) = e^{-0.2x}$. See the figure. A rectangle with two sides along the x -axis and y -axis is inscribed using the graph of f . Find x so that the area of the rectangle is a maximum.



(A) 1.0 (B) 1.2 (C) 2.5 (D) 5.0

21. An equation of the line tangent to the graph of $y = \frac{x+2}{2x-6}$ at the point $(4, 3)$ is

(A) $y - 3 = -\frac{5}{2}(x + 4)$ (B) $y - 3 = -\frac{5}{2}(x - 4)$
 (C) $y - 4 = -\frac{5}{2}(x - 3)$ (D) $y - 3 = \frac{5}{2}(x - 4)$

22. Let f be the function $f(x) = e^{kx^2}$, where k is a constant. For what value of k does f have a point of inflection at $x = -2$ and $x = 2$?

(A) $-\frac{1}{4}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$

23. $\frac{d}{dt} \left(t^2 \int_2^t \ln(u+3) du \right) =$

(A) $2t \ln(t+3)$
 (B) $2t(\ln(t+3) - \ln(5))$
 (C) $t^2 \left(\frac{1}{t+3} \right) + 2t \int_2^t \ln(u+3) du$
 (D) $t^2 \ln(t+3) + 2t \int_2^t \ln(u+3) du$

24. If $\frac{dy}{dx} = y \cos(2x)$ and $y = 3$ when $x = 0$, then $y =$

(A) $3e^{-0.5 \sin(2x)}$ (B) $3e^{0.5 \sin(2x)}$
 (C) $e^{0.5 \sin(2x)} + 3$ (D) $e^{-\sin(2x)} + 3$

25. Let f and g be continuous functions with values given in the table below.

x	$f(x)$	$g(x)$
1	2	1
3	4	5

$$\int_1^3 (3f'(x) + g(x)g'(x) - 6) dx =$$

(A) 6 (B) 10 (C) 14 (D) 18

26. At any time $t > 0$, the rate at which a person can memorize a list of M words is proportional to the product of the number of words memorized and the number of words that have not been memorized. If x denotes the number of words memorized at time t , which differential equation models this situation? Assume k is a positive constant.

(A) $\frac{dx}{dt} = kx$ (B) $\frac{dx}{dt} = kx(x - M)$
 (C) $\frac{dx}{dt} = kx(M - x)$ (D) $\frac{dx}{dt} = kt(M - t)$

27. For what value of x does the function $f(x) = (x - 3)^2(x - 5)$ have a relative minimum?

(A) 3 (B) $\frac{7}{3}$ (C) 5 (D) $\frac{13}{3}$

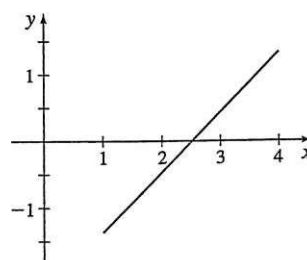
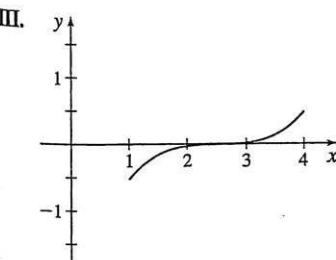
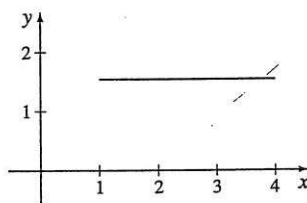
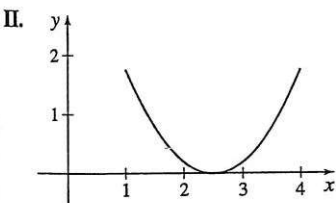
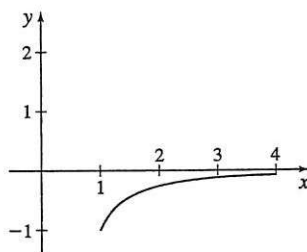
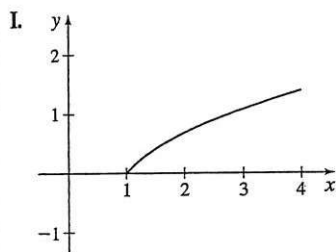
28. If $x^2 + 3y^2 = 12$, what is $\frac{d^2y}{dx^2}$ at the point $(3, 1)$?

(A) $-\frac{3}{2}$ (B) $-\frac{4}{3}$ (C) $-\frac{1}{2}$ (D) $-\frac{1}{3}$

29. $\lim_{x \rightarrow 0} \frac{3x - \sin(3x)}{1 - \cos(2x)} =$

(A) $-\frac{3}{2}$ (B) 0 (C) $\frac{9}{4}$ (D) Does not exist.

30. Which of the following pairs of graphs could represent the graph of a function and the graph of its second derivative?



- (A) I and II (B) I and III (C) II and III (D) I, II, and III

Section 1: Multiple Choice, Part B

A graphing calculator is required for some of the questions in Part B.

31. If $f(x) = \int_0^x (t - 2) \sin\left(\frac{\pi t}{2}\right) dt$, which of the following must be true?

I. $f(x) < 0$ on the interval $0 < x < 2$.

II. $f'(x) < 0$ on the interval $0 < x < 2$.

III. $f''(x) < 0$ on the interval $0 < x < 2$.

- (A) I only (B) I and II (C) II and III (D) I and III

32. Let $f(x) = 2x^2 + \sqrt[3]{x^4 - 8}$ be defined for $x \geq 0$ and let g be the inverse of f . What is the value of $g'(10)$?

(A) -0.094 (B) -0.023 (C) 0.023 (D) 0.094

33. What is the slope of the line tangent to the graph of

$$f(x) = e^x - x^e - e$$

at the point where the graph crosses the x -axis?

(A) 0.110 (B) 3.471 (C) 9.106 (D) 55.238

34. Let f be the function given by $f(x) = \ln(x + 1)$ and let g be the function given by $g(x) = \frac{1}{\sqrt{x}}$. At what value of x do the graphs of f and g have perpendicular tangent lines?

(A) 0.484 (B) 1.000 (C) 1.358 (D) 2.065

35. A table of values for a continuous function f is shown below. If five subintervals of $[0, 2]$ of equal width are used, what is the approximation of $\int_0^2 f(x) dx$ using trapezoidal sums?

x	0	0.4	0.8	1.2	1.6	2.0
$f(x)$	10	12	13	16	19	20

(A) 18 (B) 30 (C) 36 (D) 60

36. At time $t \geq 0$, the acceleration of a particle moving on the x -axis is $a(t) = 3 - \sqrt{t}$. If the velocity of the particle at time $t = 0$ is 2, what is the velocity at time $t = 4$?

(A) 2 (B) $\frac{20}{3}$ (C) $\frac{26}{3}$ (D) $\frac{58}{3}$

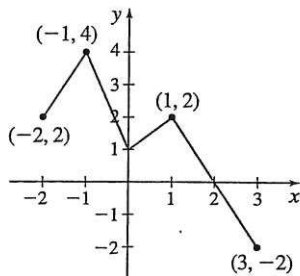
37. What is the area of the region in the first quadrant enclosed by the graphs of $y = \tan x$, $y = 3 - x$, and the y -axis?

(A) 1.905 (B) 2.595 (C) 3.442 (D) 4.132

38. At time $t = 0$, a small animal weighs 3 pounds and grows at the rate of $r(t) = 0.16e^{\sqrt{t}}$ pounds per day, where t is measured in days. How many pounds did the animal gain from $t = 0$ days to $t = 5$ days?

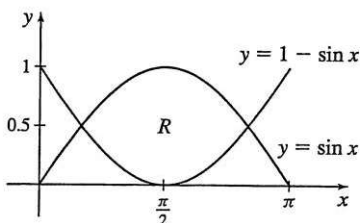
(A) 1.5 (B) 4.0 (C) 4.5 (D) 7.0

39. Let g be the function $g(x) = \frac{x \ln x - x}{x + 1}$. Which of the following is a critical number of g ?
- (A) -1 (B) 0.368 (C) 0.567 (D) 2.718
40. The graph of the function f is shown below.



Suppose $g(x) = \int_{-2}^x f(t) dt$. For which number x is $g(x)$ the greatest?

- (A) -1 (B) 1 (C) 2 (D) 3
41. The area of a regular hexagon is $A = \frac{3\sqrt{3}}{2}s^2$ where s is the length of each side. The perimeter of the hexagon is increasing at a constant rate of 30 cm per minute. What is the rate of increase in the area of the hexagon at the instant when the perimeter of the hexagon is 120 cm?
- (A) 259.808 (B) 519.615 (C) 1558.846 (D) 3117.691
42. For a continuous function f , let g be the function defined by $g(x) = \int_0^x (3f(t) + \sqrt{t^3 + 1}) dt$. If $f(2) = 4$ and $\int_0^2 f(x) dx = 10$, what is $g(2) + g'(2)$?
- (A) 30.000 (B) 45.000 (C) 48.241 (D) 78.241
43. The density of a metal rod is given by the function $f(x) = \sqrt{10 - x^3} \frac{\text{kg}}{\text{m}}$ for $0 \leq x \leq 2$ meters. What is the average density across the length of the rod?
- (A) 0.874 (B) 2.288 (C) 2.792 (D) 5.584
44. A particle travels on the x -axis so its velocity at time t is given by $v(t) = e^{0.2t} - 3$ for $0 \leq t \leq 10$. What is the total distance traveled by the particle from $t = 0$ to $t = 10$?
- (A) 1.945 (B) 2.389 (C) 11.945 (D) 14.904
45. Let R be the region bounded by $y = \sin x$ and $y = 1 - \sin x$ as shown below. R is the base of a solid whose cross sections perpendicular to the x -axis are triangles. The height of each triangle is one-half the length of the base. What is the volume of the solid?



- (A) 0.264 (B) 0.272 (C) 0.528 (D) 0.544

Section 2: Free Response, Part A

A graphing calculator is required for Part A.

1. During the workday, which lasts from 7 a.m. to 3 p.m. ($0 \leq t \leq 8$ hours), raw material is piped into a processing plant at a rate R , where $R = R(t) = 15 \sin \frac{t^2}{22}$ kiloliters per hour. The raw material is processed at a rate P , where $P = P(t) = -0.02t^3 + 0.2t^2 + t$ kiloliters per hour. At time $t = 0$, there are 20 kiloliters of raw material already in storage and ready for use.
- (a) How many kiloliters of raw material are piped during the workday?
- (b) Determine the amount of raw material processed in the first hour of operation.
- (c) At what time t , $0 \leq t \leq 8$, is the raw material available for processing the least? At what time is the amount raw material available the greatest? Justify your answers.
2. Let R be the region in the first quadrant bounded above by the graph of $y = \frac{4x}{x^2 + 7}$ and below by the horizontal line $y = \frac{1}{2}$.
- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = -2$.
- (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a rectangle with a height equal to five times the length of the base. Find the volume of the solid.

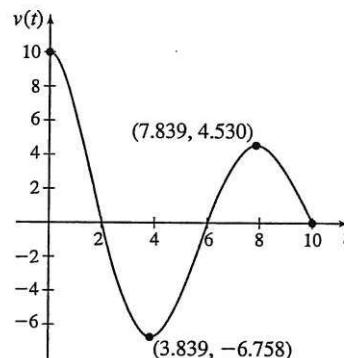
Section 2: Free Response, Part B

A calculator may not be used for Part B.

3. An object moves along the x -axis with initial position $x(0) = 4$. The velocity of the object at time $t \geq 0$ is given by

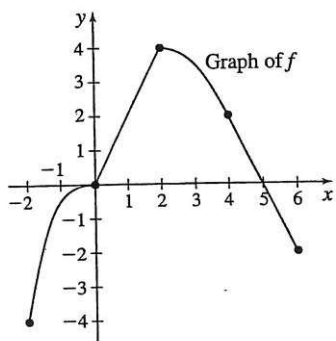
$$v(t) = 10e^{-0.1t} \cos\left(\frac{\pi t}{4}\right)$$

The graph of $v(t)$ is shown below.



- (a) What is the acceleration of the object at time $t = 3.839$?
- (b) Write, but do not evaluate, an integral that gives the position of the object at time $t = 5$.
- (c) On the interval $0 < t < 6$, when is the object moving to the left? Give a reason for your answer.
- (d) Write, but do not evaluate, an integral that gives the total distance traveled by the object over the interval $0 \leq t \leq 6$.

4. The graph of the function f below consists of polynomials on the intervals $-2 < x < 0$ and $2 < x < 4$ and line segments on the intervals $0 < x < 2$ and $4 < x < 6$.



- (a) Let g be the function given by $g(x) = \int_0^x f(t) dt$. Find $g(2)$, $g'(2)$, and $g''(2)$, or state that it does not exist.
- (b) For the function g defined in part (a), determine where g is increasing.
- (c) Let h be the function given by $h(x) = \int_{-2}^x f(t) dt$. Find the value of x for which $h(x)$ has a relative maximum value. Justify your answer.
- (d) For the function h defined in part (c), if $h(0) = -2$ and $h(6) = \frac{26}{3}$, find the value of $\int_2^4 f(t) dt$.
5. The elevation of a 6-mile hiking trail, in thousands of feet, is modeled by a twice-differentiable function E of hiking distance x , where x is measured in miles. For $0 < x < 6$, $E'(x) > 0$ and the graph of E is concave up. The table below gives selected values of the elevation, $E(x)$, of the trail over the hiking distance $0 \leq x \leq 6$.

x (hiking distance)	0	1	3	5	6
E (elevation)	1.0	1.3	2.1	4.1	5.6

- (a) Use a trapezoidal sum with the four subintervals given in the table to approximate the value of

$$\frac{1}{6} \int_0^6 E(x) dx$$

- (b) Using correct units, explain the meaning of

$$\frac{1}{6} \int_0^6 E(x) dx$$

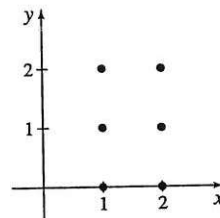
- (c) For $3 < x < 5$, must there be a distance x for which the elevation increases at a rate of 1000 feet per mile? Justify your answer.

- (d) Does a Left Riemann sum under approximate or over

approximate $\frac{1}{6} \int_0^6 E(x) dx$? Give a reason for your answer.

6. Consider the differential equation $\frac{dy}{dx} = \frac{2(y-1)^2}{\sqrt{x}}$.

- (a) Sketch a slope field for the differential equation at the six points indicated in the graph below.



- (b) Let $y = f(x)$ be the particular solution of the differential equation with the initial condition $f(1) = 2$. Find an equation for the tangent line to the graph of f at the point $(1, 2)$.
- (c) Find the particular solution $y = f(x)$ of the differential equation with the initial condition $f(1) = 2$.