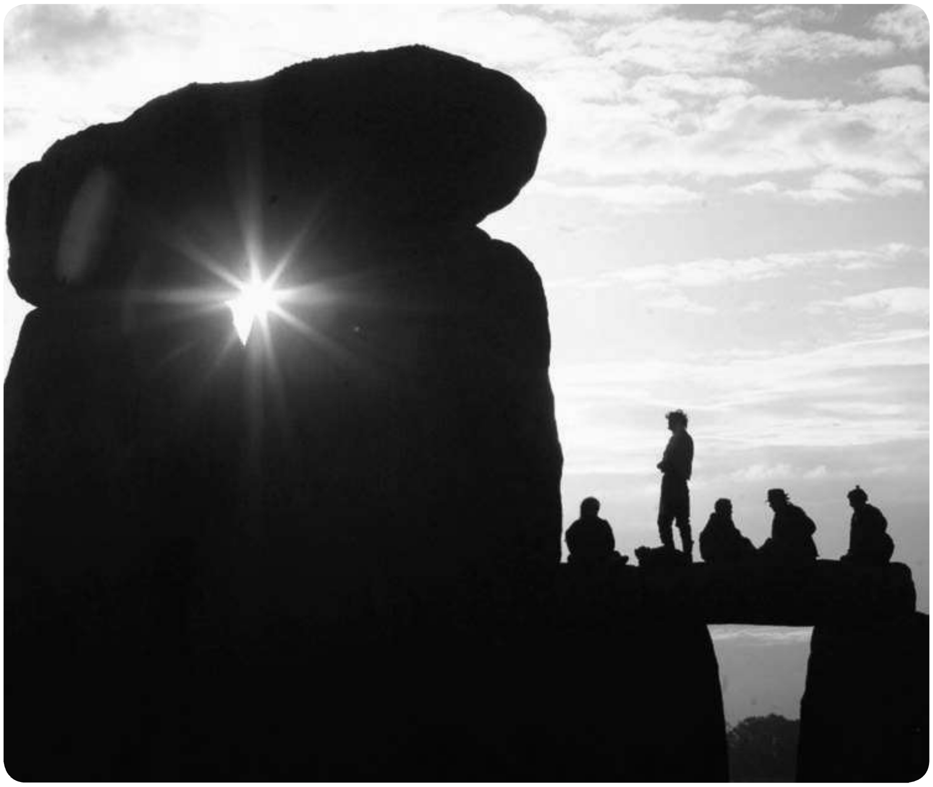

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1

Stonehenge, in southern England, was built thousands of years ago to help keep track of the seasons. At dawn on the summer solstice the sun can be seen through these giant stone slabs.

INTRODUCTION

The goal of physics is to provide an understanding of the physical world by developing theories based on experiments. A physical theory is essentially a guess, usually expressed mathematically, about how a given physical system works. The theory makes certain predictions about the physical system which can then be checked by observations and experiments. If the predictions turn out to correspond closely to what is actually observed, then the theory stands, although it remains provisional. No theory to date has given a complete description of all physical phenomena, even within a given subdiscipline of physics. Every theory is a work in progress.

The basic laws of physics involve such physical quantities as force, velocity, volume, and acceleration, all of which can be described in terms of more fundamental quantities. In mechanics, the three most fundamental quantities are **length** (L), **mass** (M), and **time** (T); all other physical quantities can be constructed from these three.

1.1 STANDARDS OF LENGTH, MASS, AND TIME

To communicate the result of a measurement of a certain physical quantity, a *unit* for the quantity must be defined. If our fundamental unit of length is defined to be 1.0 meter, for example, and someone familiar with our system of measurement reports that a wall is 2.0 meters high, we know that the height of the wall is twice the fundamental unit of length. Likewise, if our fundamental unit of mass is defined as 1.0 kilogram and we are told that a person has a mass of 75 kilograms, then that person has a mass 75 times as great as the fundamental unit of mass.

In 1960 an international committee agreed on a standard system of units for the fundamental quantities of science, called **SI** (Système International). Its units of length, mass, and time are the meter, kilogram, and second, respectively.

Length

In 1799 the legal standard of length in France became the meter, defined as one ten-millionth of the distance from the equator to the North Pole. Until 1960, the official length of the meter was the distance between two lines on a specific bar of platinum-iridium alloy stored under controlled conditions. This standard was abandoned for several reasons, the principal one being that measurements of the separation between the lines are not precise enough. In 1960 the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp. In October 1983 this definition was abandoned also, and **the meter was redefined as the distance traveled by light in vacuum during a time interval of 1/299 792 458 second**. This latest definition establishes the speed of light at 299 792 458 meters per second.

Definition of the meter →

Mass

Definition of the kilogram →

The SI unit of mass, the kilogram, is defined as the mass of a specific platinum-iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France (similar to that shown in Fig. 1.1a). As we'll see in Chapter 4, mass is a quantity used to measure the resistance to a change in the motion of an object. It's more difficult to cause a change in the motion of an object with a large mass than an object with a small mass.

TIP 1.1 No Commas in Numbers with Many Digits

In science, numbers with more than three digits are written in groups of three digits separated by spaces rather than commas; so that 10 000 is the same as the common American notation 10,000. Similarly, $\pi = 3.14159265$ is written as 3.141 592 65.

Time

Before 1960, the time standard was defined in terms of the average length of a solar day in the year 1900. (A solar day is the time between successive appearances of the Sun at the highest point it reaches in the sky each day.) The basic unit of time, the second, was defined to be $(1/60)(1/60)(1/24) = 1/86\ 400$ of the average solar day. In 1967 the second was redefined to take advantage of the high precision attainable with an atomic clock, which uses the characteristic frequency of the light emitted from the cesium-133 atom as its "reference clock." **The second is now defined as 9 192 631 700 times the period of oscillation of radiation from the cesium atom.** The newest type of cesium atomic clock is shown in Figure 1.1b.

Definition of the second →

Approximate Values for Length, Mass, and Time Intervals

Approximate values of some lengths, masses, and time intervals are presented in Tables 1.1, 1.2, and 1.3, respectively. Note the wide ranges of values. Study these tables to get a feel for a kilogram of mass (this book has a mass of about 2 kilograms), a time interval of 10^{10} seconds (one century is about 3×10^9 seconds), or two meters of length (the approximate height of a forward on a basketball

(a)

(b)

FIGURE 1.1 (a) The National Standard Kilogram No. 20, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology. (b) The nation's primary time standard is a cesium fountain atomic clock developed at the National Institute of Standards and Technology laboratories in Boulder, Colorado. This clock will neither gain nor lose a second in 20 million years.

TABLE 1.1

Approximate Values of Some Measured Lengths

	Length (m)
Distance from Earth to most remote known quasar	1×10^{26}
Distance from Earth to most remote known normal galaxies	4×10^{25}
Distance from Earth to nearest large galaxy (M31, the Andromeda galaxy)	2×10^{22}
Distance from Earth to nearest star (Proxima Centauri)	4×10^{16}
One light year	9×10^{15}
Mean orbit radius of Earth about Sun	2×10^{11}
Mean distance from Earth to Moon	4×10^{8}
Mean radius of Earth	6×10^{6}
Typical altitude of satellite orbiting Earth	2×10^{5}
Length of football field	9×10^{1}
Length of housefly	5×10^{-3}
Size of smallest dust particles	1×10^{-4}
Size of cells in most living organisms	1×10^{-5}
Diameter of hydrogen atom	1×10^{-10}
Diameter of atomic nucleus	1×10^{-14}
Diameter of proton	1×10^{-15}

TABLE 1.2

Approximate Values of Some Masses

	Mass (kg)
Observable Universe	1×10^{52}
Milky Way galaxy	7×10^{41}
Sun	2×10^{30}
Earth	6×10^{24}
Moon	7×10^{22}
Shark	1×10^{2}
Human	7×10^{1}
Frog	1×10^{-1}
Mosquito	1×10^{-5}
Bacterium	1×10^{-15}
Hydrogen atom	2×10^{-27}
Electron	9×10^{-31}

team). Appendix A reviews the notation for powers of 10, such as the expression of the number 50 000 in the form 5×10^4.

Systems of units commonly used in physics are the Système International, in which the units of length, mass, and time are the meter (m), kilogram (kg), and second (s); the cgs, or Gaussian, system, in which the units of length, mass, and time are the centimeter (cm), gram (g), and second; and the U.S. customary system, in which the units of length, mass, and time are the foot (ft), slug, and second. SI units are almost universally accepted in science and industry, and will be used throughout the book. Limited use will be made of Gaussian and U.S. customary units.

Some of the most frequently used "metric" (SI and cgs) prefixes representing powers of 10 and their abbreviations are listed in Table 1.4. For example, 10^{-3} m is

TABLE 1.3

Approximate Values of Some Time Intervals

	Time Interval (s)
Age of Universe	5×10^{17}
Age of Earth	1×10^{17}
Average age of college student	6×10^{8}
One year	3×10^{7}
One day	9×10^{4}
Time between normal heartbeats	8×10^{-1}
Period[a] of audible sound waves	1×10^{-3}
Period[a] of typical radio waves	1×10^{-6}
Period[a] of vibration of atom in solid	1×10^{-13}
Period[a] of visible light waves	2×10^{-15}
Duration of nuclear collision	1×10^{-22}
Time required for light to travel across a proton	3×10^{-24}

[a]A *period* is defined as the time required for one complete vibration.

TABLE 1.4

Some Prefixes for Powers of Ten Used with "Metric" (SI and cgs) Units

Power	Prefix	Abbreviation
10^{-18}	atto-	a
10^{-15}	femto-	f
10^{-12}	pico-	p
10^{-9}	nano-	n
10^{-6}	micro-	μ
10^{-3}	milli-	m
10^{-2}	centi-	c
10^{-1}	deci-	d
10^{1}	deka-	da
10^{3}	kilo-	k
10^{6}	mega-	M
10^{9}	giga-	G
10^{12}	tera-	T
10^{15}	peta-	P
10^{18}	exa-	E

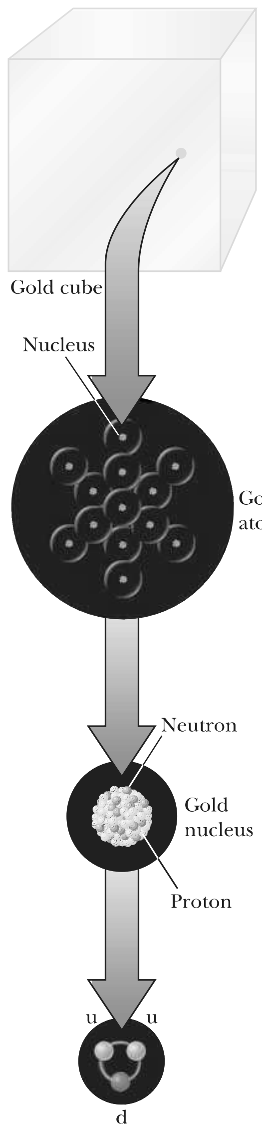
equivalent to 1 millimeter (mm), and 10^3 m is 1 kilometer (km). Likewise, 1 kg is equal to 10^3 g, and 1 megavolt (MV) is 10^6 volts (V).

1.2 THE BUILDING BLOCKS OF MATTER

A 1-kg ($\approx$ 2-lb) cube of solid gold has a length of about 3.73 cm ($\approx$ 1.5 in.) on a side. If the cube is cut in half, the two resulting pieces retain their chemical identity as solid gold. But what happens if the pieces of the cube are cut again and again, indefinitely? The Greek philosophers Leucippus and Democritus couldn't accept the idea that such cutting could go on forever. They speculated that the process ultimately would end when it produced a particle that could no longer be cut. In Greek, *atomos* means "not sliceable." From this term comes our English word *atom*, once believed to be the smallest particle of matter but since found to be a composite of more elementary particles.

The atom can be naively visualized as a miniature Solar System, with a dense, positively charged nucleus occupying the position of the Sun and negatively charged electrons orbiting like planets. This model of the atom, first developed by the great Danish physicist Niels Bohr nearly a century ago, led to the understanding of certain properties of the simpler atoms such as hydrogen but failed to explain many fine details of atomic structure.

Notice the size of a hydrogen atom, listed in Table 1.1, and the size of a proton—the nucleus of a hydrogen atom—one hundred thousand times smaller. If the proton were the size of a Ping Pong ball, the electron would be a tiny speck about the size of a bacterium, orbiting the proton a kilometer away! Other atoms are similarly constructed. So there is a surprising amount of empty space in ordinary matter.



Quark composition of a proton

FIGURE 1.2 Levels of organization in matter. Ordinary matter consists of atoms, and at the center of each atom is a compact nucleus consisting of protons and neutrons. Protons and neutrons are composed of quarks. The quark composition of a proton is shown.

After the discovery of the nucleus in the early 1900s, questions arose concerning its structure. The exact composition of the nucleus hasn't been defined completely even today, but by the early 1930s scientists determined that two basic entities—protons and neutrons—occupy the nucleus. The *proton* is nature's fundamental carrier of positive charge, equal in magnitude but opposite in sign to the charge on the electron. The number of protons in a nucleus determines what the element is. For instance, a nucleus containing only one proton is the nucleus of an atom of hydrogen, regardless of how many neutrons may be present. Extra neutrons correspond to different isotopes of hydrogen—deuterium and tritium—which react chemically in exactly the same way as hydrogen, but are more massive. An atom having two protons in its nucleus, similarly, is always helium, although again, differing numbers of neutrons are possible.

The existence of *neutrons* was verified conclusively in 1932. A neutron has no charge and has a mass about equal to that of a proton. One of its primary purposes is to act as a "glue" to hold the nucleus together. If neutrons were not present, the repulsive electrical force between the positively charged protons would cause the nucleus to fly apart.

The division doesn't stop here; it turns out that protons, neutrons, and a zoo of other exotic particles are now thought to be composed of six particles called **quarks** (rhymes with "forks," though some rhyme it with "sharks"). These particles have been given the names *up, down, strange, charm, bottom,* and *top*. The up, charm, and top quarks each carry a charge equal to $+\frac{2}{3}$ that of the proton, whereas the down, strange, and bottom quarks each carry a charge equal to $-\frac{1}{3}$ the proton charge. The proton consists of two up quarks and one down quark (see Fig. 1.2), giving the correct charge for the proton, +1. The neutron is composed of two down quarks and one up quark and has a net charge of zero.

The up and down quarks are sufficient to describe all normal matter, so the existence of the other four quarks, indirectly observed in high-energy experiments, is something of a mystery. It's also possible that quarks themselves have internal structure. Many physicists believe that the most fundamental particles may be tiny loops of vibrating string.

1.3 DIMENSIONAL ANALYSIS

In physics the word *dimension* denotes the physical nature of a quantity. The distance between two points, for example, can be measured in feet, meters, or furlongs, which are different ways of expressing the dimension of *length*.

The symbols used in this section to specify the dimensions of length, mass, and time are L, M, and T, respectively. Brackets [ ] will often be used to denote the dimensions of a physical quantity. In this notation, for example, the dimensions of velocity v are written $[v] = \mathrm{L/T}$, and the dimensions of area A are $[A] = \mathrm{L}^2$. The dimensions of area, volume, velocity, and acceleration are listed in Table 1.5, along with their units in the three common systems. The dimensions of other quantities, such as force and energy, will be described later as they are introduced.

In physics it's often necessary either to derive a mathematical expression or equation or to check its correctness. A useful procedure for doing this is called **dimensional analysis**, which makes use of the fact that **dimensions can be treated as algebraic quantities**. Such quantities can be added or subtracted only if they have the same dimensions. It follows that the terms on the opposite sides of an equation must have the same dimensions. If they don't, the equation is wrong. If they do, the equation is probably correct, except for a possible constant factor.

To illustrate this procedure, suppose we wish to derive a formula for the distance x traveled by a car in a time t if the car starts from rest and moves with constant acceleration a. The quantity x has the dimension length: $[x] = \mathrm{L}$. Time t, of course, has dimension $[t] = \mathrm{T}$. Acceleration is the change in velocity v with time. Because v has dimensions of length per unit time, or $[v] = \mathrm{L/T}$, acceleration must have dimensions $[a] = \mathrm{L/T}^2$. We organize this information in the form of an equation:

$$[a] = \frac{[v]}{[t]} = \frac{\mathrm{L/T}}{\mathrm{T}} = \frac{\mathrm{L}}{\mathrm{T}^2} = \frac{[x]}{[t]^2}$$

Looking at the left- and right-hand sides of this equation, we might now guess that

$$a = \frac{x}{t^2} \quad \rightarrow \quad x = at^2$$

This expression is not quite correct, however, because there's a constant of proportionality—a simple numerical factor—that can't be determined solely through dimensional analysis. As will be seen in Chapter 2, it turns out that the correct expression is $x = \frac{1}{2}at^2$.

When we work algebraically with physical quantities, dimensional analysis allows us to check for errors in calculation, which often show up as discrepancies in units. If, for example, the left-hand side of an equation is in meters and the right-hand side is in meters per second, we know immediately that we've made an error.

TABLE 1.5

Dimensions and Some Units of Area, Volume, Velocity, and Acceleration

System	Area (L^2)	Volume (L^3)	Velocity (L/T)	Acceleration ($\mathrm{L/T}^2$)
SI	m^2	m^3	m/s	$\mathrm{m/s}^2$
cgs	cm^2	cm^3	cm/s	$\mathrm{cm/s}^2$
U.S. customary	ft^2	ft^3	ft/s	$\mathrm{ft/s}^2$

EXAMPLE 1.1 Analysis of an Equation

Goal Check an equation using dimensional analysis.

Problem Show that the expression $v = v_0 + at$ is dimensionally correct, where v and v_0 represent velocities, a is acceleration, and t is a time interval.

Strategy Analyze each term, finding its dimensions, and then check to see if all the terms agree with each other.

Solution

Find dimensions for v and v_0.

$$[v] = [v_0] = \frac{\text{L}}{\text{T}}$$

Find the dimensions of at.

$$[at] = \frac{\text{L}}{\text{T}^2}(\text{T}) = \frac{\text{L}}{\text{T}}$$

Remarks All the terms agree, so the equation is dimensionally correct.

QUESTION 1.1

True or False. An equation that is dimensionally correct is always physically correct, up to a constant of proportionality.

EXERCISE 1.1

Determine whether the equation $x = vt^2$ is dimensionally correct. If not, provide a correct expression, up to an overall constant of proportionality.

Answer Incorrect. The expression $x = vt$ is dimensionally correct.

EXAMPLE 1.2 Find an Equation

Goal Derive an equation by using dimensional analysis.

Problem Find a relationship between a constant acceleration a, speed v, and distance r from the origin for a particle traveling in a circle.

Strategy Start with the term having the most dimensionality, a. Find its dimensions, and then rewrite those dimensions in terms of the dimensions of v and r. The dimensions of time will have to be eliminated with v, because that's the only quantity in which the dimension of time appears.

Solution

Write down the dimensions of a:

$$[a] = \frac{\text{L}}{\text{T}^2}$$

Solve the dimensions of speed for T:

$$[v] = \frac{\text{L}}{\text{T}} \quad \rightarrow \quad \text{T} = \frac{\text{L}}{[v]}$$

Substitute the expression for T into the equation for $[a]$:

$$[a] = \frac{\text{L}}{\text{T}^2} = \frac{\text{L}}{(\text{L}/[v])^2} = \frac{[v]^2}{\text{L}}$$

Substitute $\text{L} = [r]$, and guess at the equation:

$$[a] = \frac{[v]^2}{[r]} \quad \rightarrow \quad a = \frac{v^2}{r}$$

Remarks This is the correct equation for centripetal acceleration—acceleration towards the center of motion—to be discussed in Chapter 7. In this case it isn't necessary to introduce a numerical factor. Such a factor is often displayed explicitly as a constant k in front of the right-hand side—for example, $a = kv^2/r$. As it turns out, $k = 1$ gives the correct expression.

QUESTION 1.2

True or False: Replacing v by r/t in the final answer also gives a dimensionally correct equation.

EXERCISE 1.2

In physics, energy E carries dimensions of mass times length squared divided by time squared. Use dimensional analysis to derive a relationship for energy in terms of mass m and speed v, up to a constant of proportionality. Set the speed equal to c, the speed of light, and the constant of proportionality equal to 1 to get the most famous equation in physics.

Answer $E = kmv^2 \quad \rightarrow \quad E = mc^2$ when $k = 1$ and $v = c$.

1.4 UNCERTAINTY IN MEASUREMENT AND SIGNIFICANT FIGURES

Physics is a science in which mathematical laws are tested by experiment. No physical quantity can be determined with complete accuracy because our senses are physically limited, even when extended with microscopes, cyclotrons, and other gadgets.

Knowing the experimental uncertainties in any measurement is very important. Without this information, little can be said about the final measurement. Using a crude scale, for example, we might find that a gold nugget has a mass of 3 kilograms. A prospective client interested in purchasing the nugget would naturally want to know about the accuracy of the measurement, to ensure paying a fair price. He wouldn't be happy to find that the measurement was good only to within a kilogram, because he might pay for three kilograms and get only two. Of course, he might get four kilograms for the price of three, but most people would be hesitant to gamble that an error would turn out in their favor.

Accuracy of measurement depends on the sensitivity of the apparatus, the skill of the person carrying out the measurement, and the number of times the measurement is repeated. There are many ways of handling uncertainties, and here we'll develop a basic and reliable method of keeping track of them in the measurement itself and in subsequent calculations.

Suppose that in a laboratory experiment we measure the area of a rectangular plate with a meter stick. Let's assume that the accuracy to which we can measure a particular dimension of the plate is ± 0.1 cm. If the length of the plate is measured to be 16.3 cm, we can claim only that it lies somewhere between 16.2 cm and 16.4 cm. In this case, we say that the measured value has three significant figures. Likewise, if the plate's width is measured to be 4.5 cm, the actual value lies between 4.4 cm and 4.6 cm. This measured value has only two significant figures. We could write the measured values as 16.3 ± 0.1 cm and 4.5 ± 0.1 cm. In general, **a significant figure is a reliably known digit** (other than a zero used to locate a decimal point).

Suppose we would like to find the area of the plate by multiplying the two measured values together. The final value can range between $(16.3 - 0.1 \text{ cm})(4.5 - 0.1 \text{ cm}) = (16.2 \text{ cm})(4.4 \text{ cm}) = 71.28 \text{ cm}^2$ and $(16.3 + 0.1 \text{ cm})(4.5 + 0.1 \text{ cm}) = (16.4 \text{ cm})(4.6 \text{ cm}) = 75.44 \text{ cm}^2$. Claiming to know anything about the hundredths place, or even the tenths place, doesn't make any sense, because it's clear we can't even be certain of the units place, whether it's the 1 in 71, the 5 in 75, or somewhere in between. The tenths and the hundredths places are clearly not significant. We have some information about the units place, so that number is significant. Multiplying the numbers at the middle of the uncertainty ranges gives $(16.3 \text{ cm})(4.5 \text{ cm}) = 73.35 \text{ cm}^2$, which is also in the middle of the area's uncertainty range. Because the hundredths and tenths are not significant, we drop them and take the answer to be 73 cm^2, with an uncertainty of $\pm 2 \text{ cm}^2$. Note that the answer has two significant figures, the same number of figures as the least accurately known quantity being multiplied, the 4.5-cm width.

There are two useful rules of thumb for determining the number of significant figures. The first, concerning multiplication and division, is as follows: **In multiplying (dividing) two or more quantities, the number of significant figures in the final product (quotient) is the same as the number of significant figures in the *least accurate* of the factors being combined, where *least accurate* means *having the lowest number of significant figures.***

To get the final number of significant figures, it's usually necessary to do some rounding. If the last digit dropped is less than 5, simply drop the digit. If the last digit dropped is greater than or equal to 5, raise the last retained digit by one.

EXAMPLE 1.3 Installing a Carpet

Goal Apply the multiplication rule for significant figures.

Problem A carpet is to be installed in a room of length 12.71 m and width 3.46 m. Find the area of the room, retaining the proper number of significant figures.

Strategy Count the significant figures in each number. The smaller result is the number of significant figures in the answer.

Solution

Count significant figures:

$12.71\text{ m} \rightarrow$ 4 significant figures

$3.46\text{ m} \rightarrow$ 3 significant figures

Multiply the numbers, keeping only three digits:

$12.71\text{ m} \times 3.46\text{ m} = 43.976\,6\text{ m}^2 \rightarrow 44.0\text{ m}^2$

Remarks In reducing 43.976 6 to three significant figures, we used our rounding rule, adding 1 to the 9, which made 10 and resulted in carrying 1 to the unit's place.

QUESTION 1.3

What would the answer have been if the width were given as 3.460 m?

EXERCISE 1.3

Repeat this problem, but with a room measuring 9.72 m long by 5.3 m wide.

Answer 52 m^2

TIP 1.2 Using Calculators

Calculators were designed by engineers to yield as many digits as the memory of the calculator chip permitted, so be sure to round the final answer down to the correct number of significant figures.

Zeros may or may not be significant figures. Zeros used to position the decimal point in such numbers as 0.03 and 0.007 5 are not significant (but are useful in avoiding errors). Hence, 0.03 has one significant figure, and 0.007 5 has two.

When zeros are placed after other digits in a whole number, there is a possibility of misinterpretation. For example, suppose the mass of an object is given as 1 500 g. This value is ambiguous, because we don't know whether the last two zeros are being used to locate the decimal point or whether they represent significant figures in the measurement.

Using scientific notation to indicate the number of significant figures removes this ambiguity. In this case, we express the mass as 1.5×10^3 g if there are two significant figures in the measured value, 1.50×10^3 g if there are three significant figures, and 1.500×10^3 g if there are four. Likewise, 0.000 15 is expressed in scientific notation as 1.5×10^{-4} if it has two significant figures or as 1.50×10^{-4} if it has three significant figures. The three zeros between the decimal point and the digit 1 in the number 0.000 15 are not counted as significant figures because they only locate the decimal point. In this book, **most of the numerical examples and end-of-chapter problems will yield answers having two or three significant figures.**

For addition and subtraction, it's best to focus on the number of decimal places in the quantities involved rather than on the number of significant figures. **When numbers are added (subtracted), the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum (difference)**. For example, if we wish to compute 123 (zero decimal places) + 5.35 (two decimal places), the answer is 128 (zero decimal places) and not 128.35. If we compute the sum 1.000 1 (four decimal places) + 0.000 3 (four decimal places) = 1.000 4, the result has the correct number of decimal places, namely four. Observe that the rules for multiplying significant figures don't work here because the answer has five significant figures even though one of the terms in the sum, 0.000 3, has only one significant figure. Likewise, if we perform the subtraction 1.002 − 0.998 = 0.004, the result has three decimal places because each term in the subtraction has three decimal places.

To show why this rule should hold, we return to the first example in which we added 123 and 5.35, and rewrite these numbers as 123.*xxx* and 5.35*x*. Digits written with an *x* are completely unknown and can be any digit from 0 to 9. Now we line up 123.*xxx* and 5.35*x* relative to the decimal point and perform the addition, using the rule that an unknown digit added to a known or unknown digit yields an unknown:

$$\begin{array}{r} 123.xxx \\ +\quad 5.35x \\ \hline 128.xxx \end{array}$$

The answer of 128.xxx means that we are justified only in keeping the number 128 because everything after the decimal point in the sum is actually unknown. The example shows that the controlling uncertainty is introduced into an addition or subtraction by the term with the smallest number of decimal places.

In performing any calculation, especially one involving a number of steps, there will always be slight discrepancies introduced by both the rounding process and the algebraic order in which steps are carried out. For example, consider $2.35 \times 5.89/1.57$. This computation can be performed in three different orders. First, we have $2.35 \times 5.89 = 13.842$, which rounds to 13.8, followed by $13.8/1.57 = 8.789\,8$, rounding to 8.79. Second, $5.89/1.57 = 3.751\,6$, which rounds to 3.75, resulting in $2.35 \times 3.75 = 8.812\,5$, rounding to 8.81. Finally, $2.35/1.57 = 1.496\,8$ rounds to 1.50, and $1.50 \times 5.89 = 8.835$ rounds to 8.84. So three different algebraic orders, following the rules of rounding, lead to answers of 8.79, 8.81, and 8.84, respectively. Such minor discrepancies are to be expected, because the last significant digit is only one representative from a range of possible values, depending on experimental uncertainty. The discrepancies can be reduced by carrying one or more extra digits during the calculation. In our examples, however, intermediate results will be rounded off to the proper number of significant figures, and only those digits will be carried forward. In experimental work, more sophisticated techniques are used to determine the accuracy of an experimental result.

1.5 CONVERSION OF UNITS

Sometimes it's necessary to convert units from one system to another. Conversion factors between the SI and U.S. customary systems for units of length are as follows:

$$1 \text{ mile} = 1\,609 \text{ m} = 1.609 \text{ km} \qquad 1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$

$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \qquad 1 \text{ in.} = 0.025\,4 \text{ m} = 2.54 \text{ cm}$$

A more extensive list of conversion factors can be found on the inside front cover of this book.

Units can be treated as algebraic quantities that can "cancel" each other. We can make a fraction with the conversion that will cancel the units we don't want,

Billy E. Barnes/Stock Boston

This road sign near Raleigh, North Carolina, shows distances in miles and kilometers. How accurate are the conversions?

and multiply that fraction by the quantity in question. For example, suppose we want to convert 15.0 in. to centimeters. Because 1 in. = 2.54 cm, we find that

$$15.0 \text{ in.} = 15.0 \cancel{\text{in.}} \times \left(\frac{2.54 \text{ cm}}{1.00 \cancel{\text{in.}}}\right) = 38.1 \text{ cm}$$

The next two examples show how to deal with problems involving more than one conversion and with powers.

EXAMPLE 1.4 Pull Over, Buddy!

Goal Convert units using several conversion factors.

Problem If a car is traveling at a speed of 28.0 m/s, is the driver exceeding the speed limit of 55.0 mi/h?

Strategy Meters must be converted to miles and seconds to hours, using the conversion factors listed on the inside front cover of the book. Here, three factors will be used.

Solution

Convert meters to miles:

$$28.0 \text{ m/s} = \left(28.0 \frac{\cancel{\text{m}}}{\text{s}}\right)\left(\frac{1.00 \text{ mi}}{1\,609 \cancel{\text{m}}}\right) = 1.74 \times 10^{-2} \text{ mi/s}$$

Convert seconds to hours:

$$1.74 \times 10^{-2} \text{ mi/s} = \left(1.74 \times 10^{-2} \frac{\text{mi}}{\cancel{\text{s}}}\right)\left(60.0 \frac{\cancel{\text{s}}}{\cancel{\text{min}}}\right)\left(60.0 \frac{\cancel{\text{min}}}{\text{h}}\right)$$

$$= \boxed{62.6 \text{ mi/h}}$$

Remarks The driver should slow down because he's exceeding the speed limit.

QUESTION 1.4

Repeat the conversion, using the relationship 1.00 m/s = 2.24 mi/h. Why is the answer slightly different?

EXERCISE 1.4

Convert 152 mi/h to m/s.

Answer 68.0 m/s

EXAMPLE 1.5 Press the Pedal to the Metal

Goal Convert a quantity featuring powers of a unit.

Problem The traffic light turns green, and the driver of a high-performance car slams the accelerator to the floor. The accelerometer registers 22.0 m/s^2. Convert this reading to km/min^2.

Strategy Here we need one factor to convert meters to kilometers and another two factors to convert seconds squared to minutes squared.

Solution

Multiply by the three factors:

$$\frac{22.0 \cancel{\text{m}}}{1.00 \cancel{\text{s}}^2}\left(\frac{1.00 \text{ km}}{1.00 \times 10^3 \cancel{\text{m}}}\right)\left(\frac{60.0 \cancel{\text{s}}}{1.00 \text{ min}}\right)^2 = \boxed{79.2 \frac{\text{km}}{\text{min}^2}}$$

Remarks Notice that in each conversion factor the numerator equals the denominator when units are taken into account. A common error in dealing with squares is to square the units inside the parentheses while forgetting to square the numbers!

QUESTION 1.5
What time conversion factor would be used to further convert the answer to km/h^2?

EXERCISE 1.5
Convert $4.50 \times 10^3\ kg/m^3$ to g/cm^3.

Answer $4.50\ g/cm^3$

1.6 ESTIMATES AND ORDER-OF-MAGNITUDE CALCULATIONS

Getting an exact answer to a calculation may often be difficult or impossible, either for mathematical reasons or because limited information is available. In these cases, estimates can yield useful approximate answers that can determine whether a more precise calculation is necessary. Estimates also serve as a partial check if the exact calculations are actually carried out. If a large answer is expected but a small exact answer is obtained, there's an error somewhere.

For many problems, knowing the approximate value of a quantity—within a factor of 10 or so—is sufficient. This approximate value is called an **order-of-magnitude** estimate, and requires finding the power of 10 that is closest to the actual value of the quantity. For example, $75\ kg \sim 10^2\ kg$, where the symbol $\sim$ means "is on the order of" or "is approximately." Increasing a quantity by three orders of magnitude means that its value increases by a factor of $10^3 = 1\ 000$.

Occasionally the process of making such estimates results in fairly crude answers, but answers ten times or more too large or small are still useful. For example, suppose you're interested in how many people have contracted a certain disease. Any estimates under ten thousand are small compared with Earth's total population, but a million or more would be alarming. So even relatively imprecise information can provide valuable guidance.

In developing these estimates, you can take considerable liberties with the numbers. For example, $\pi \sim 1$, $27 \sim 10$, and $65 \sim 100$. To get a less crude estimate, it's permissible to use slightly more accurate numbers (e.g., $\pi \sim 3$, $27 \sim 30$, $65 \sim 70$). Better accuracy can also be obtained by systematically underestimating as many numbers as you overestimate. Some quantities may be completely unknown, but it's standard to make reasonable guesses, as the examples show.

EXAMPLE 1.6 Brain Cells Estimate

Goal Develop a simple estimate.

Problem Estimate the number of cells in the human brain.

Strategy Estimate the volume of a human brain and divide by the estimated volume of one cell. The brain is located in the upper portion of the head, with a volume that could be approximated by a cube $\ell = 20$ cm on a side. Brain cells, consisting of about 10% neurons and 90% glia, vary greatly in size, with dimensions ranging from a few microns to a meter or so. As a guess, take $d = 10$ microns as a typical dimension and consider a cell to be a cube with each side having that length.

Solution

Estimate of the volume of a human brain:

$$V_{brain} = \ell^3 \approx (0.2\ m)^3 = 8 \times 10^{-3}\ m^3 \approx 1 \times 10^{-2}\ m^3$$

Estimate the volume of a cell:

$$V_{cell} = d^3 \approx (10 \times 10^{-6}\ m)^3 = 1 \times 10^{-15}\ m^3$$

Divide the volume of a brain by the volume of a cell:

$$\text{number of cells} = \frac{V_{brain}}{V_{cell}} = \frac{0.01\ m^3}{1 \times 10^{-15}\ m^3} = \boxed{1 \times 10^{13}\ \text{cells}}$$

Remarks Notice how little attention was paid to obtaining precise values. That's the nature of an estimate.

QUESTION 1.6
Would 10^{12} cells also be a reasonable estimate? What about 10^9 cells? Explain.

EXERCISE 1.6
Estimate the total number of cells in the human body.

Answer 10^{14} (Answers may vary.)

EXAMPLE 1.7 Stack One-Dollar Bills to the Moon

Goal Estimate the number of stacked objects required to reach a given height.

Problem How many one-dollar bills, stacked one on top of the other, would reach the Moon?

Strategy The distance to the Moon is about 400 000 km. Guess at the number of dollar bills in a millimeter, and multiply the distance by this number, after converting to consistent units.

Solution

We estimate that ten stacked bills form a layer of 1 mm. Convert mm to km:

$$\frac{10 \text{ bills}}{1 \text{ mm}}\left(\frac{10^3 \text{ mm}}{1 \text{ m}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right) = \frac{10^7 \text{ bills}}{1 \text{ km}}$$

Multiply this value by the approximate lunar distance:

$$\# \text{ of dollar bills } \sim (4 \times 10^5 \text{ km})\left(\frac{10^7 \text{ bills}}{1 \text{ km}}\right) = \boxed{4 \times 10^{12} \text{ bills}}$$

Remarks That's the same order of magnitude as the U.S. national debt!

QUESTION 1.7
Based on the answer, about how many stacked pennies would reach the Moon?

EXERCISE 1.7
How many pieces of cardboard, typically found at the back of a bound pad of paper, would you have to stack up to match the height of the Washington monument, about 170 m tall?

Answer $\sim 10^5$ (Answers may vary.)

EXAMPLE 1.8 Number of Galaxies in the Universe

Goal Estimate a volume and a number density, and combine.

Problem Given that astronomers can see about 10 billion light years into space and that there are 14 galaxies in our local group, 2 million light years from the next local group, estimate the number of galaxies in the observable universe. (Note: One light year is the distance traveled by light in one year, about 9.5×10^{15} m.) (See Fig. 1.3.)

Strategy From the known information, we can estimate the number of galaxies per unit volume. The local group of 14 galaxies is contained in a sphere a million light years in radius, with the Andromeda group in a similar sphere, so there are about 10 galaxies within a volume of radius 1million light years. Multiply that number density by the volume of the observable universe.

R. Williams (STScI), the HDF-S team, and NASA

FIGURE 1.3 In this deep-space photograph, there are few stars—just galaxies without end.

Solution

Compute the approximate volume V_{lg} of the local group of galaxies:

$$V_{lg} = \frac{4}{3}\pi r^3 \sim (10^6 \text{ ly})^3 = 10^{18} \text{ ly}^3$$

Estimate the density of galaxies:

$$\text{density of galaxies} = \frac{\#\text{ of galaxies}}{V_{lg}}$$
$$\sim \frac{10\text{ galaxies}}{10^{18}\text{ ly}^3} = 10^{-17}\,\frac{\text{galaxies}}{\text{ly}^3}$$

Compute the approximate volume of the observable universe:

$$V_u = \tfrac{4}{3}\pi r^3 \sim (10^{10}\text{ ly})^3 = 10^{30}\text{ ly}^3$$

Multiply the density of galaxies by V_u:

$$\#\text{ of galaxies} \sim (\text{density of galaxies})V_u$$
$$= \left(10^{-17}\,\frac{\text{galaxies}}{\text{ly}^3}\right)(10^{30}\text{ ly}^3)$$
$$= 10^{13}\text{ galaxies}$$

Remarks Notice the approximate nature of the computation, which uses $4\pi/3 \sim 1$ on two occasions and $14 \sim 10$ for the number of galaxies in the local group. This is completely justified: Using the actual numbers would be pointless, because the other assumptions in the problem—the size of the observable universe and the idea that the local galaxy density is representative of the density everywhere—are also very rough approximations. Further, there was nothing in the problem that required using volumes of spheres rather than volumes of cubes. Despite all these arbitrary choices, the answer still gives useful information, because it rules out a lot of reasonable possible answers. Before doing the calculation, a guess of a billion galaxies might have seemed plausible.

QUESTION 1.8

Of the fourteen galaxies in the local group, only one, the Milky Way, is not a dwarf galaxy. Estimate the number of galaxies in the universe that are not dwarfs.

EXERCISE 1.8

Given that the nearest star is about 4 light years away and that the galaxy is roughly a disk 100 000 light years across and a thousand light years thick, estimate the number of stars in the Milky Way galaxy.

Answer $\sim 10^{12}$ stars (Estimates will vary. The actual answer is probably close to 4×10^{11} stars.)

1.7 COORDINATE SYSTEMS

Many aspects of physics deal with locations in space, which require the definition of a coordinate system. A point on a line can be located with one coordinate, a point in a plane with two coordinates, and a point in space with three.

A coordinate system used to specify locations in space consists of the following:

- A fixed reference point O, called the *origin*
- A set of specified axes, or directions, with an appropriate scale and labels on the axes
- Instructions on labeling a point in space relative to the origin and axes

One convenient and commonly used coordinate system is the **Cartesian coordinate system**, sometimes called the **rectangular coordinate system**. Such a system in two dimensions is illustrated in Figure 1.4. An arbitrary point in this system is labeled with the coordinates (x, y). For example, the point P in the figure has coordinates (5, 3). If we start at the origin O, we can reach P by moving 5 meters horizontally to the right and then 3 meters vertically upwards. In the same way, the point Q has coordinates (−3, 4), which corresponds to going 3 meters horizontally to the left of the origin and 4 meters vertically upwards from there.

Positive x is usually selected as right of the origin and positive y upward from the origin, but in two dimensions this choice is largely a matter of taste. (In three dimensions, however, there are "right-handed" and "left-handed" coordinates, which lead to minus sign differences in certain operations. These will be addressed as needed.)

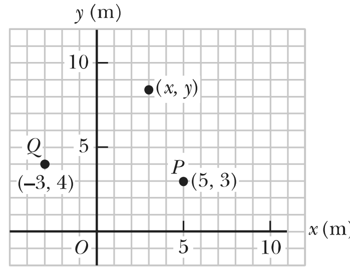


FIGURE 1.4 Designation of points in a two-dimensional Cartesian coordinate system. Every point is labeled with coordinates (x, y).

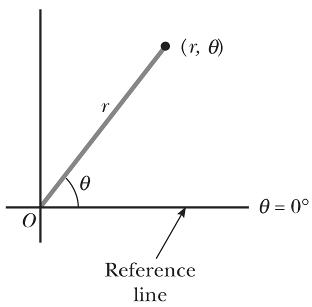
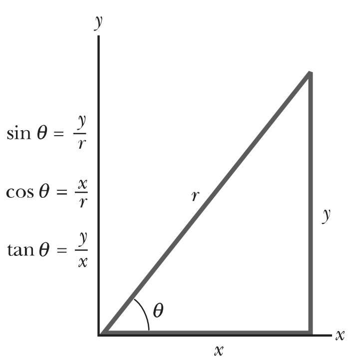


FIGURE 1.5 A polar coordinate system.

Sometimes it's more convenient to locate a point in space by its **plane polar coordinates** (r, θ), as in Figure 1.5. In this coordinate system, an origin O and a reference line are selected as shown. A point is then specified by the distance r from the origin to the point and by the angle θ between the reference line and a line drawn from the origin to the point. The standard reference line is usually selected to be the positive x-axis of a Cartesian coordinate system. The angle θ is considered positive when measured counterclockwise from the reference line and negative when measured clockwise. For example, if a point is specified by the polar coordinates 3 m and 60°, we locate this point by moving out 3 m from the origin at an angle of 60° above (counterclockwise from) the reference line. A point specified by polar coordinates 3 m and −60° is located 3 m out from the origin and 60° below (clockwise from) the reference line.

1.8 TRIGONOMETRY



ACTIVE FIGURE 1.6
Certain trigonometric functions of a right triangle.

Consider the right triangle shown in Active Figure 1.6, where side y is opposite the angle θ, side x is adjacent to the angle θ, and side r is the hypotenuse of the triangle. The basic trigonometric functions defined by such a triangle are the ratios of the lengths of the sides of the triangle. These relationships are called the sine (sin), cosine (cos), and tangent (tan) functions. In terms of θ, the basic trigonometric functions are as follows:[1]

$$\sin\theta = \frac{\text{side opposite }\theta}{\text{hypotenuse}} = \frac{y}{r}$$

$$\cos\theta = \frac{\text{side adjacent to }\theta}{\text{hypotenuse}} = \frac{x}{r} \qquad \textbf{[1.1]}$$

$$\tan\theta = \frac{\text{side opposite }\theta}{\text{side adjacent to }\theta} = \frac{y}{x}$$

For example, if the angle θ is equal to 30°, then the ratio of y to r is always 0.50; that is, $\sin 30° = 0.50$. Note that the sine, cosine, and tangent functions are quantities without units because each represents the ratio of two lengths.

Another important relationship, called the **Pythagorean theorem**, exists between the lengths of the sides of a right triangle:

$$r^2 = x^2 + y^2 \qquad \textbf{[1.2]}$$

Finally, it will often be necessary to find the values of inverse relationships. For example, suppose you know that the sine of an angle is 0.866, but you need to know the value of the angle itself. The inverse sine function may be expressed as $\sin^{-1}(0.866)$, which is a shorthand way of asking the question "What angle has a sine of 0.866?" Punching a couple of buttons on your calculator reveals that this angle is 60.0°. Try it for yourself and show that $\tan^{-1}(0.400) = 21.8°$. Be sure that your calculator is set for degrees and not radians. In addition, the inverse tangent function can return only values between −90° and +90°, so when an angle is in the second or third quadrant, it's necessary to add 180° to the answer in the calculator window.

The definitions of the trigonometric functions and the inverse trigonometric functions, as well as the Pythagorean theorem, can be applied to *any* right triangle, regardless of whether its sides correspond to x- and y-coordinates.

These results from trigonometry are useful in converting from rectangular coordinates to polar coordinates, or vice versa, as the next example shows.

TIP 1.3 Degrees vs. Radians
When calculating trigonometric functions, make sure your calculator setting—degrees or radians—is consistent with the degree measure you're using in a given problem.

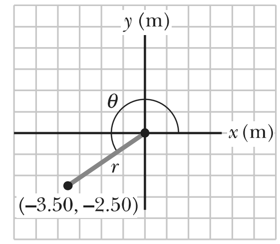
[1]Many people use the mnemonic *SOHCAHTOA* to remember the basic trigonometric formulas: *S*ine = *O*pposite/*H*ypotenuse, *C*osine = *A*djacent/*H*ypotenuse, and *T*angent = *O*pposite/*A*djacent. (Thanks go to Professor Don Chodrow for pointing this out.)

EXAMPLE 1.9 Cartesian and Polar Coordinates

Goal Understand how to convert from plane rectangular coordinates to plane polar coordinates and vice versa.

Problem **(a)** The Cartesian coordinates of a point in the xy-plane are $(x, y) = (-3.50\text{ m}, -2.50\text{ m})$, as shown in Active Figure 1.7. Find the polar coordinates of this point. **(b)** Convert $(r, \theta) = (5.00\text{ m}, 37.0°)$ to rectangular coordinates.

Strategy Apply the trigonometric functions and their inverses, together with the Pythagorean theorem.



ACTIVE FIGURE 1.7 (Example 1.9) Converting from Cartesian coordinates to polar coordinates.

Solution

(a) Cartesian to Polar

Take the square root of both sides of Equation 1.2 to find the radial coordinate:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = \boxed{4.30\text{ m}}$$

Use Equation 1.1 for the tangent function to find the angle with the inverse tangent, adding 180° because the angle is actually in third quadrant:

$$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

$$\theta = \tan^{-1}(0.714) = 35.5° + 180° = \boxed{216°}$$

(b) Polar to Cartesian

Use the trigonometric definitions, Equation 1.1.

$$x = r\cos\theta = (5.00\text{ m})\cos 37.0° = 3.99\text{ m}$$

$$y = r\sin\theta = (5.00\text{ m})\sin 37.0° = \boxed{3.01\text{ m}}$$

Remarks When we take up vectors in two dimensions in Chapter 3, we will routinely use a similar process to find the direction and magnitude of a given vector from its components, or, conversely, to find the components from the vector's magnitude and direction.

QUESTION 1.9

Starting with the answers to part (b), work backwards to recover the given radius and angle. Why are there slight differences from the original quantities?

EXERCISE 1.9

(a) Find the polar coordinates corresponding to $(x, y) = (-3.25\text{ m}, 1.50\text{ m})$. **(b)** Find the Cartesian coordinates corresponding to $(r, \theta) = (4.00\text{ m}, 53.0°)$

Answers **(a)** $(r, \theta) = (3.58\text{ m}, 155°)$ **(b)** $(x, y) = (2.41\text{ m}, 3.19\text{ m})$

EXAMPLE 1.10 How High Is the Building?

Goal Apply basic results of trigonometry.

Problem A person measures the height of a building by walking out a distance of 46.0 m from its base and shining a flashlight beam toward the top. When the beam is elevated at an angle of 39.0° with respect to the horizontal, as shown in Figure 1.8, the beam just strikes the top of the building. Find the height of the building and the distance the flashlight beam has to travel before it strikes the top of the building.

Strategy Refer to the right triangle shown in the figure. We know the angle, 39.0°, and the length of the side adjacent to it. Because the height of the building is the side opposite the angle, we can use the tangent function. With the adjacent and opposite sides known, we can then find the hypotenuse with the Pythagorean theorem.

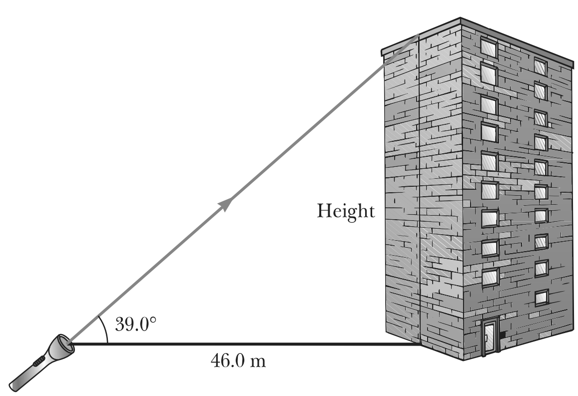


FIGURE 1.8 (Example 1.10)

Solution

Use the tangent of the given angle: $\tan 39.0° = \dfrac{\text{height}}{46.0 \text{ m}}$

Solve for the height: $\text{Height} = (\tan 39.0°)(46.0 \text{ m}) = (0.810)(46.0 \text{ m})$

$= \boxed{37.3 \text{ m}}$

Find the hypotenuse of the triangle: $r = \sqrt{x^2 + y^2} = \sqrt{(37.3 \text{ m})^2 + (46.0 \text{ m})^2} = \boxed{59.2 \text{ m}}$

Remarks In a later chapter, right-triangle trigonometry is often used when working with vectors.

QUESTION 1.10

Could the distance traveled by the light beam be found without using the Pythagorean Theorem? How?

EXERCISE 1.10

While standing atop a building 50.0 m tall, you spot a friend standing on a street corner. Using a protractor and dangling a plumb bob, you find that the angle between the horizontal and the direction to the spot on the sidewalk where your friend is standing is 25.0°. Your eyes are located 1.75 m above the top of the building. How far away from the foot of the building is your friend?

Answer 111 m

1.9 PROBLEM-SOLVING STRATEGY

Most courses in general physics require the student to learn the skills used in solving problems, and examinations usually include problems that test such skills. This brief section presents some useful suggestions that will help increase your success in solving problems. An organized approach to problem solving will also enhance your understanding of physical concepts and reduce exam stress. Throughout the book, there will be a number of sections labeled "Problem-Solving Strategy," many of them just a specializing of the list given below (and illustrated in Fig. 1.9).

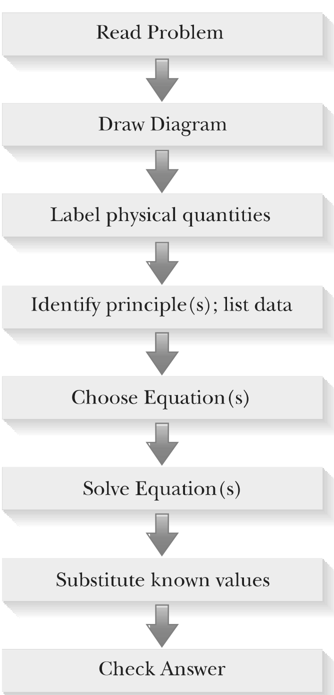


FIGURE 1.9 A guide to problem solving.

General Problem-Solving Strategy

1. **Read** the problem carefully at least twice. Be sure you understand the nature of the problem before proceeding further.
2. **Draw** a diagram while rereading the problem.
3. **Label** all physical quantities in the diagram, using letters that remind you what the quantity is (e.g., m for mass). Choose a coordinate system and label it.
4. **Identify** physical principles, the knowns and unknowns, and list them. Put circles around the unknowns.
5. **Equations**, the relationships between the labeled physical quantities, should be written down next. Naturally, the selected equations should be consistent with the physical principles identified in the previous step.
6. **Solve** the set of equations for the unknown quantities in terms of the known. Do this algebraically, without substituting values until the next step, except where terms are zero.
7. **Substitute** the known values, together with their units. Obtain a numerical value with units for each unknown.
8. **Check** your answer. Do the units match? Is the answer reasonable? Does the plus or minus sign make sense? Is your answer consistent with an order of magnitude estimate?

This same procedure, with minor variations, should be followed throughout the course. The first three steps are extremely important, because they get you mentally oriented. Identifying the proper concepts and physical principles assists you in choosing the correct equations. The equations themselves are essential, because when you understand them, you also understand the relationships between the physical quantities. This understanding comes through a lot of daily practice.

Equations are the tools of physics: To solve problems, you have to have them at hand, like a plumber and his wrenches. Know the equations, and understand what they mean and how to use them. Just as you can't have a conversation without knowing the local language, you can't solve physics problems without knowing and understanding the equations. This understanding grows as you study and apply the concepts and the equations relating them.

Carrying through the algebra for as long as possible, substituting numbers only at the end, is also important, because it helps you think in terms of the physical quantities involved, not merely the numbers that represent them. Many beginning physics students are eager to substitute, but once numbers are substituted it's harder to understand relationships and easier to make mistakes.

The physical layout and organization of your work will make the final product more understandable and easier to follow. Although physics is a challenging discipline, your chances of success are excellent if you maintain a positive attitude and keep trying.

EXAMPLE 1.11 A Round Trip by Air

Goal Illustrate the Problem-Solving Strategy.

Problem An airplane travels 4.50×10^2 km due east and then travels an unknown distance due north. Finally, it returns to its starting point by traveling a distance of 525 km. How far did the airplane travel in the northerly direction?

Strategy We've finished reading the problem (step 1), and have drawn a diagram (step 2) in Figure 1.10 and labeled it (step 3). From the diagram, we recognize a right triangle and identify (step 4) the principle involved: the Pythagorean theorem. Side y is the unknown quantity, and the other sides are known.

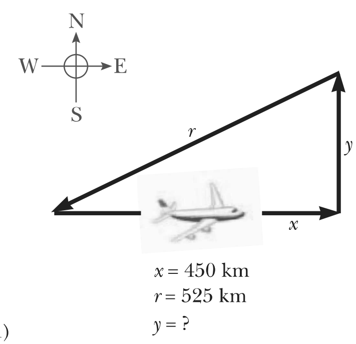


FIGURE 1.10 (Example 1.11)

Solution

Write the Pythagorean theorem (step 5): $r^2 = x^2 + y^2$

Solve symbolically for y (step 6): $y^2 = r^2 - x^2 \quad \rightarrow \quad y = +\sqrt{r^2 - x^2}$

Substitute the numbers, with units (step 7): $y = \sqrt{(525 \text{ km})^2 - (4.50 \times 10^2 \text{ km})^2} = 270 \text{ km}$

Remarks Note that the negative solution has been disregarded, because it's not physically meaningful. In checking (step 8), note that the units are correct and that an approximate answer can be obtained by using the easier quantities, 500 km and 400 km. Doing so gives an answer of 300 km, which is approximately the same as our calculated answer of 270 km.

QUESTION 1.11

What is the answer if both the distance traveled due east and the direct return distance are both doubled?

EXERCISE 1.11

A plane flies 345 km due south, then turns and flies 615 km at a heading 45.0° north of east, until it's due east of its starting point. If the plane now turns and heads for home, how far will it have to go?

Answer 509 km

SUMMARY

1.1 Standards of Length, Mass, and Time

The physical quantities in the study of mechanics can be expressed in terms of three fundamental quantities: length, mass, and time, which have the SI units meters (m), kilograms (kg), and seconds (s), respectively.

1.2 The Building Blocks of Matter

Matter is made of atoms, which in turn are made up of a relatively small nucleus of protons and neutrons within a cloud of electrons. Protons and neutrons are composed of still smaller particles, called quarks.

1.3 Dimensional Analysis

Dimensional analysis can be used to check equations and to assist in deriving them. When the dimensions on both sides of the equation agree, the equation is often correct up to a numerical factor. When the dimensions don't agree, the equation must be wrong.

1.4 Uncertainty in Measurement and Significant Figures

No physical quantity can be determined with complete accuracy. The concept of significant figures affords a basic method of handling these uncertainties. A significant figure is a reliably known digit, other than a zero, used to locate the decimal point. The two rules of significant figures are as follows:

1. When multiplying or dividing using two or more quantities, the result should have the same number of significant figures as the quantity having the fewest significant figures.
2. When quantities are added or subtracted, the number of decimal places in the result should be the same as in the quantity with the fewest decimal places.

Use of scientific notation can avoid ambiguity in significant figures. In rounding, if the last digit dropped is less than 5, simply drop the digit, otherwise raise the last retained digit by one.

1.5 Conversion of Units

Units in physics equations must always be consistent. In solving a physics problem, it's best to start with consistent units, using the table of conversion factors on the inside front cover as necessary.

Converting units is a matter of multiplying the given quantity by a fraction, with one unit in the numerator and its equivalent in the other units in the denominator, arranged so the unwanted units in the given quantity are cancelled out in favor of the desired units.

1.6 Estimates and Order-of-Magnitude Calculations

Sometimes it's useful to find an approximate answer to a question, either because the math is difficult or because information is incomplete. A quick estimate can also be used to check a more detailed calculation. In an order-of-magnitude calculation, each value is replaced by the closest power of ten, which sometimes must be guessed or estimated when the value is unknown. The computation is then carried out. For quick estimates involving known values, each value can first be rounded to one significant figure.

1.7 Coordinate Systems

The Cartesian coordinate system consists of two perpendicular axes, usually called the x-axis and y-axis, with each axis labeled with all numbers from negative infinity to positive infinity. Points are located by specifying the x- and y-values. Polar coordinates consist of a radial coordinate r which is the distance from the origin, and an angular coordinate θ which is the angular displacement from the positive x-axis.

1.8 Trigonometry

The three most basic trigonometric functions of a right triangle are the sine, cosine, and tangent, defined as follows:

$$\sin\theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{y}{r}$$

$$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{x}{r} \qquad \textbf{[1.1]}$$

$$\tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{y}{x}$$

The **Pythagorean theorem** is an important relationship between the lengths of the sides of a right triangle:

$$r^2 = x^2 + y^2 \qquad \textbf{[1.2]}$$

where r is the hypotenuse of the triangle and x and y are the other two sides.

FOR ADDITIONAL STUDENT RESOURCES, GO TO WWW.SERWAYPHYSICS.COM

MULTIPLE-CHOICE QUESTIONS

1. Newton's second law of motion (Chapter 4) says that the mass of an object times its acceleration is equal to the net force on the object. Which of the following gives the correct units for force? (a) $\text{kg}\cdot\text{m/s}^2$ (b) $\text{kg}\cdot\text{m}^2/\text{s}^2$ (c) $\text{kg/m}\cdot\text{s}^2$ (d) $\text{kg}\cdot\text{m}^2/\text{s}$ (e) none of these
2. Suppose two quantities, A and B, have different dimensions. Determine which of the following arithmetic operations *could* be physically meaningful. (a) $A + B$ (b) $B - A$ (c) $A - B$ (d) A/B (e) AB

3. A rectangular airstrip measures 32.30 m by 210 m, with the width measured more accurately than the length. Find the area, taking into account significant figures. (a) $6.783\ 0 \times 10^3\ m^2$ (b) $6.783 \times 10^3\ m^2$ (c) $6.78 \times 10^3\ m^2$ (d) $6.8 \times 10^3\ m^2$ (e) $7 \times 10^3\ m^2$

4. Use the rules for significant figures to find the answer to the addition problem 21.4 + 15 + 17.17 + 4.003. (a) 57.573 (b) 57.57 (c) 57.6 (d) 58 (e) 60

5. The Roman cubitus is an ancient unit of measure equivalent to about 445 mm. Convert the 2.00-m-height of a basketball forward to cubiti. (a) 2.52 cubiti (b) 3.12 cubiti (c) 4.49 cubiti (d) 5.33 cubiti (e) none of these

6. A house is advertised as having 1 420 square feet under roof. What is the area of this house in square meters? (a) $115\ m^2$ (b) $132\ m^2$ (c) $176\ m^2$ (d) $222\ m^2$ (e) none of these

7. Which of the following is the best estimate for the mass of all the people living on Earth? (a) 2×10^8 kg (b) 1×10^9 kg (c) 2×10^{10} kg (d) 3×10^{11} kg (e) 4×10^{12} kg

8. Find the polar coordinates corresponding to a point located at (−5.00, 12.00) in Cartesian coordinates. (a) (13.0, −67.4°) (b) (13.0, 113°) (c) (14.2, −67.4°) (d) (14.2, 113°) (e) (19, −72.5°)

9. At a horizontal distance of 45 m from a tree, the angle of elevation to the top of the tree is 26°. How tall is the tree? (a) 22 m (b) 31 m (c) 45 m (d) 16 m (e) 11 m

10. What is the approximate number of breaths a person takes over a period of 70 years? (a) 3×10^6 breaths (b) 3×10^7 breaths (c) 3×10^8 breaths (d) 3×10^9 breaths (e) 3×10^{10} breaths

11. Which of the following relationships is dimensionally consistent with an expression yielding a value for acceleration? Acceleration has the units of distance divided by time squared. In these equations, x is a distance, t is time, and v is velocity with units of distance divided by time. (a) v/t^2 (b) v/x^2 (c) v^2/t (d) v^2/x (e) none of these

CONCEPTUAL QUESTIONS

1. Estimate the order of magnitude of the length, in meters, of each of the following: (a) a mouse, (b) a pool cue, (c) a basketball court, (d) an elephant, (e) a city block.

2. What types of natural phenomena could serve as time standards?

3. Find the order of magnitude of your age in seconds.

4. An object with a mass of 1 kg weighs approximately 2 lb. Use this information to estimate the mass of the following objects: (a) a baseball; (b) your physics textbook; (c) a pickup truck.

5. (a) Estimate the number of times your heart beats in a month. (b) Estimate the number of human heartbeats in an average lifetime.

6. Estimate the number of atoms in 1 cm^3 of a solid. (Note that the diameter of an atom is about 10^{-10} m.)

7. The height of a horse is sometimes given in units of "hands." Why is this a poor standard of length?

8. How many of the lengths or time intervals given in Tables 1.2 and 1.3 could you verify, using only equipment found in a typical dormitory room?

9. If an equation is dimensionally correct, does this mean that the equation must be true? If an equation is not dimensionally correct, does this mean that the equation can't be true?

10. Why is the metric system of units considered superior to most other systems of units?

11. How can an estimate be of value even when it is off by an order of magnitude? Explain and give an example.

PROBLEMS

ENHANCED WebAssign The Problems for this chapter may be assigned online at WebAssign.

1, 2, 3 = straightforward, intermediate, challenging

GP = denotes guided problem

ecp = denotes enhanced content problem

= biomedical application

☐ = denotes full solution available in *Student Solutions Manual/Study Guide*

SECTION 1.3 DIMENSIONAL ANALYSIS

1. The period of a simple pendulum, defined as the time necessary for one complete oscillation, is measured in time units and is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where ℓ is the length of the pendulum and g is the acceleration due to gravity, in units of length divided by time squared. Show that this equation is dimensionally consistent. (You might want to check the formula using your keys at the end of a string and a stopwatch.)

2. (a) Suppose that the displacement of an object is related to time according to the expression $x = Bt^2$. What are the dimensions of B? (b) A displacement is related to time as $x = A \sin(2\pi ft)$, where A and f are constants. Find the dimensions of A. (*Hint:* A trigonometric function appearing in an equation must be dimensionless.)

3. A shape that covers an area A and has a uniform height h has a volume $V = Ah$. (a) Show that $V = Ah$ is dimensionally correct. (b) Show that the volumes of a cylinder and of a rectangular box can be written in the form $V = Ah$, identifying A in each case. (Note that A, sometimes called the "footprint" of the object, can have any shape and that the height can, in general, be replaced by the average thickness of the object.)

4. Each of the following equations was given by a student during an examination:

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_0{}^2 + \sqrt{mgh} \qquad v = v_0 + at^2 \qquad ma = v^2$$

Do a dimensional analysis of each equation and explain why the equation can't be correct.

5. Newton's law of universal gravitation is represented by

$$F = G\frac{Mm}{r^2}$$

where F is the gravitational force, M and m are masses, and r is a length. Force has the SI units $\text{kg}\cdot\text{m/s}^2$. What are the SI units of the proportionality constant G?

6. ecp Kinetic energy KE (Chapter 5) has dimensions $\text{kg}\cdot\text{m}^2/\text{s}^2$. It can be written in terms of the momentum p (Chapter 6) and mass m as

$$KE = \frac{p^2}{2m}$$

(a) Determine the proper units for momentum using dimensional analysis. (b) Refer to Problem 5. Given the units of force, write a simple equation relating a constant force F exerted on an object, an interval of time t during which the force is applied, and the resulting momentum of the object, p.

SECTION 1.4 UNCERTAINTY IN MEASUREMENT AND SIGNIFICANT FIGURES

7. A fisherman catches two striped bass. The smaller of the two has a measured length of 93.46 cm (two decimal places, four significant figures), and the larger fish has a measured length of 135.3 cm (one decimal place, four significant figures). What is the total length of fish caught for the day?

8. A rectangular plate has a length of (21.3 ± 0.2) cm and a width of (9.8 ± 0.1) cm. Calculate the area of the plate, including its uncertainty.

9. How many significant figures are there in (a) 78.9 ± 0.2, (b) 3.788×10^9, (c) 2.46×10^{-6}, (d) 0.003 2

10. The speed of light is now defined to be $2.997\,924\,58 \times 10^8$ m/s. Express the speed of light to (a) three significant figures, (b) five significant figures, and (c) seven significant figures.

11. ecp A block of gold has length 5.62 cm, width 6.35 cm, and height 2.78 cm. (a) Calculate the length times the width and round the answer to the appropriate number of significant figures. (b) Now multiply the rounded result of part (a) by the height and again round, obtaining the volume. (c) Repeat the process, first finding the width times the height, rounding it, and then obtaining the volume by multiplying by the length. (d) Explain why the answers don't agree in the third significant figure.

12. The radius of a circle is measured to be (10.5 ± 0.2) m. Calculate (a) the area and (b) the circumference of the circle, and give the uncertainty in each value.

13. Carry out the following arithmetic operations: (a) the sum of the measured values 756, 37.2, 0.83, and 2.5; (b) the product $0.003\,2 \times 356.3$; (c) the product $5.620 \times \pi$.

14. (a) Using your calculator, find, in scientific notation with appropriate rounding, (a) the value of $(2.437 \times 10^4)(6.521\,1 \times 10^9)/(5.37 \times 10^4)$ and (b) the value of $(3.141\,59 \times 10^2)(27.01 \times 10^4)/(1\,234 \times 10^6)$.

SECTION 1.5 CONVERSION OF UNITS

15. A fathom is a unit of length, usually reserved for measuring the depth of water. A fathom is approximately 6 ft in length. Take the distance from Earth to the Moon to be 250 000 miles, and use the given approximation to find the distance in fathoms.

16. A furlong is an old British unit of length equal to 0.125 mi, derived from the length of a furrow in an acre of ploughed land. A fortnight is a unit of time corresponding to two weeks, or 14 days and nights. Find the speed of light in megafurlongs per fortnight. (One megafurlong equals a million furlongs.)

17. A firkin is an old British unit of volume equal to 9 gallons. How many cubic meters are there in 6.00 firkins?

18. Find the height or length of these natural wonders in kilometers, meters, and centimeters: (a) The longest cave system in the world is the Mammoth Cave system in Central Kentucky, with a mapped length of 348 miles. (b) In the United States, the waterfall with the greatest single drop is Ribbon Falls in California, which drops 1 612 ft. (c) At 20 320 feet, Mount McKinley in Alaska is America's highest mountain. (d) The deepest canyon in the United States is King's Canyon in California, with a depth of 8 200 ft.

19. A rectangular building lot measures 1.00×10^2 ft by 1.50×10^2 ft. Determine the area of this lot in square meters (m^2).

20. Using the data in Table 1.3 and the appropriate conversion factors, find the age of Earth in years.

21. Using the data in Table 1.1 and the appropriate conversion factors, find the distance to the nearest star in feet.

22. Suppose your hair grows at the rate of 1/32 inch per day. Find the rate at which it grows in nanometers per second. Because the distance between atoms in a molecule is on the order of 0.1 nm, your answer suggests how rapidly atoms are assembled in this protein synthesis.

23. The speed of light is about 3.00×10^8 m/s. Convert this figure to miles per hour.

24. A house is 50.0 ft long and 26 ft wide and has 8.0-ft-high ceilings. What is the volume of the interior of the house in cubic meters and in cubic centimeters?

25. The amount of water in reservoirs is often measured in acre-ft. One acre-ft is a volume that covers an area of one acre to a depth of one foot. An acre is 43 560 ft^2. Find the volume in SI units of a reservoir containing 25.0 acre-ft of water.

26. The base of a pyramid covers an area of 13.0 acres (1 acre = 43 560 ft^2) and has a height of 481 ft (Fig. P1.26). If the volume of a pyramid is given by the expression $V = bh/3$, where b is the area of the base and h is the height, find the volume of this pyramid in cubic meters.

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FIGURE P1.26

27. A quart container of ice cream is to be made in the form of a cube. What should be the length of a side, in centimeters? (Use the conversion 1 gallon = 3.786 liter.)

SECTION 1.6 ESTIMATES AND ORDER-OF-MAGNITUDE CALCULATIONS

Note: In developing answers to the problems in this section, you should state your important assumptions, including the numerical values assigned to parameters used in the solution.

28. A hamburger chain advertises that it has sold more than 50 billion hamburgers. Estimate how many pounds of hamburger meat must have been used by the chain and how many head of cattle were required to furnish the meat.

29. Estimate the number of Ping-Pong balls that would fit into a typical-size room (without being crushed). In your solution, state the quantities you measure or estimate and the values you take for them.

30. Estimate the number of people in the world who are suffering from the common cold on any given day. (Answers may vary. Remember that a person suffers from a cold for about a week.)

31. ecp (a) About how many microorganisms are found in the human intestinal tract? (A typical bacterial length scale is 10^{-6} m. Estimate the intestinal volume and assume one hundredth of it is occupied by bacteria.) (b) Discuss your answer to part (a). Are these bacteria beneficial, dangerous, or neutral? What functions could they serve?

32. ecp Grass grows densely everywhere on a quarter-acre plot of land. What is the order of magnitude of the number of blades of grass? Explain your reasoning. Note that 1 acre = 43 560 ft^2.

33. An automobile tire is rated to last for 50 000 miles. Estimate the number of revolutions the tire will make in its lifetime.

34. ecp Bacteria and other prokaryotes are found deep underground, in water, and in the air. One micron (10^{-6} m) is a typical length scale associated with these microbes. (a) Estimate the total number of bacteria and other prokaryotes in the biosphere of the Earth. (b) Estimate the total mass of all such microbes. (c) Discuss the relative importance of humans and microbes to the ecology of planet Earth. Can *Homo sapiens* survive without them?

SECTION 1.7 COORDINATE SYSTEMS

35. A point is located in a polar coordinate system by the coordinates $r = 2.5$ m and $\theta = 35°$. Find the x- and y-coordinates of this point, assuming that the two coordinate systems have the same origin.

36. A certain corner of a room is selected as the origin of a rectangular coordinate system. If a fly is crawling on an adjacent wall at a point having coordinates (2.0, 1.0), where the units are meters, what is the distance of the fly from the corner of the room?

37. Express the location of the fly in Problem 36 in polar coordinates.

38. Two points in a rectangular coordinate system have the coordinates (5.0, 3.0) and (−3.0, 4.0), where the units are centimeters. Determine the distance between these points.

39. Two points are given in polar coordinates by $(r, \theta) =$ (2.00 m, 50.0°) and $(r, \theta =$ (5.00 m, −50.0°), respectively. What is the distance between them?

40. ecp Given points (r_1, θ_1) and (r_2, θ_2) in polar coordinates, obtain a general formula for the distance between them. Simplify it as much as possible using the identity $\cos^2\theta + \sin^2\theta = 1$. *Hint:* Write the expressions for the two points in Cartesian coordinates and substitute into the usual distance formula.

SECTION 1.8 TRIGONOMETRY

41. For the triangle shown in Figure P1.41, what are (a) the length of the unknown side, (b) the tangent of θ, and (c) the sine of ϕ?

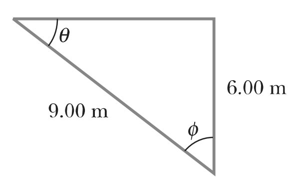

FIGURE P1.41

42. A ladder 9.00m long leans against the side of a building. If the ladder is inclined at an angle of 75.0° to the horizontal, what is the horizontal distance from the bottom of the ladder to the building?

43. A high fountain of water is located at the center of a circular pool as shown in Figure P1.43. Not wishing to get his feet wet, a student walks around the pool and measures its circumference to be 15.0 m. Next, the student stands at the edge of the pool and uses a protractor to gauge the angle of elevation at the bottom of the fountain to be 55.0°. How high is the fountain?

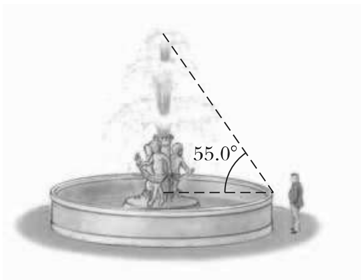

FIGURE P1.43

44. A right triangle has a hypotenuse of length 3.00 m, and one of its angles is 30.0°. What are the lengths of (a) the side opposite the 30.0° angle and (b) the side adjacent to the 30.0° angle?

45. In Figure P1.45, find (a) the side opposite θ, (b) the side adjacent to ϕ, (c) $\cos\theta$, (d) $\sin\phi$, and (e) $\tan\phi$.

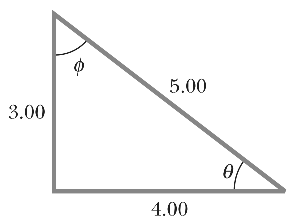

FIGURE P1.45

46. In a certain right triangle, the two sides that are perpendicular to each other are 5.00 m and 7.00 m long. What is the length of the third side of the triangle?

47. In Problem 46, what is the tangent of the angle for which 5.00 m is the opposite side?

48. GP A woman measures the angle of elevation of a mountaintop as 12.0°. After walking 1.00 km closer to the mountain on level ground, she finds the angle to be 14.0°. (a) Draw a picture of the problem, neglecting the height of the woman's eyes above the ground. *Hint:* Use two triangles. (b) Select variable names for the mountain height (suggestion: y) and the woman's original distance from the mountain (suggestion: x) and label the picture. (c) Using the labeled picture and the tangent function, write two trigonometric equations relating the two selected variables. (d) Find the height y of the mountain by first solving one equation for x and substituting the result into the other equation.

49. A surveyor measures the distance across a straight river by the following method: Starting directly across from a tree on the opposite bank, he walks 100 m along the riverbank to establish a baseline. Then he sights across to the tree. The angle from his baseline to the tree is 35.0°. How wide is the river?

50. ecp Refer to Problem 48. Suppose the mountain height is y, the woman's original distance from the mountain is x, and the angle of elevation she measures from the horizontal to the top of the mountain is θ. If she moves a distance d closer to the mountain and measures an angle of elevation ϕ, find a general equation for the height of the mountain y in terms of d, ϕ, and θ, neglecting the height of her eyes above the ground.

ADDITIONAL PROBLEMS

51. (a) One of the fundamental laws of motion states that the acceleration of an object is directly proportional to the resultant force on it and inversely proportional to its mass. If the proportionality constant is defined to have no dimensions, determine the dimensions of force. (b) The newton is the SI unit of force. According to the results for (a), how can you express a force having units of newtons by using the fundamental units of mass, length, and time?

52. (a) Find a conversion factor to convert from miles per hour to kilometers per hour. (b) For a while, federal law mandated that the maximum highway speed would be 55 mi/h. Use the conversion factor from part (a) to find the speed in kilometers per hour. (c) The maximum highway speed has been raised to 65 mi/h in some places. In kilometers per hour, how much of an increase is this over the 55-mi/h limit?

53. One cubic centimeter ($1.0\ \text{cm}^3$) of water has a mass of 1.0×10^{-3} kg. (a) Determine the mass of $1.0\ \text{m}^3$ of water. (b) Assuming that biological substances are 98% water, estimate the masses of a cell with a diameter of 1.0 μm, a human kidney, and a fly. Take a kidney to be roughly a sphere with a radius of 4.0 cm and a fly to be roughly a cylinder 4.0 mm long and 2.0 mm in diameter.

54. Soft drinks are commonly sold in aluminum containers. To an order of magnitude, how many such containers are thrown away or recycled each year by U.S. consumers? How many tons of aluminum does this represent? In your solution, state the quantities you measure or estimate and the values you take for them.

55. The displacement of an object moving under uniform acceleration is some function of time and the acceleration. Suppose we write this displacement as $s = ka^m t^n$, where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can the analysis give the value of k?

56. Compute the order of magnitude of the mass of (a) a bathtub filled with water and (b) a bathtub filled with pennies. In your solution, list the quantities you estimate and the value you estimate for each.

57. You can obtain a rough estimate of the size of a molecule by the following simple experiment: Let a droplet of oil spread out on a smooth surface of water. The resulting oil slick will be approximately one molecule thick. Given an oil droplet of mass 9.00×10^{-7} kg and density $918\ \text{kg/m}^3$ that spreads out into a circle of radius 41.8 cm on the water surface, what is the order of magnitude of the diameter of an oil molecule?

58. ecp Sphere 1 has surface area A_1 and volume V_1, and sphere 2 has surface area A_2 and volume V_2. If the radius of sphere 2 is double the radius of sphere 1, what is the ratio of (a) the areas, A_2/A_1 and (b) the volumes, V_2/V_1?

59. Estimate the number of piano tuners living in New York City. This question was raised by the physicist Enrico Fermi, who was well known for making order-of-magnitude calculations.

60. In 2007, the U.S. national debt was about \$9 trillion. (a) If payments were made at the rate of \$1 000 per second, how many years would it take to pay off the debt, assuming that no interest were charged? (b) A dollar bill is about 15.5 cm long. If nine trillion dollar bills were laid end to end around the Earth's equator, how many times would they encircle the planet? Take the radius of the Earth at the equator to be 6 378 km. (*Note:* Before doing any of these calculations, try to guess at the answers. You may be very surprised.)

61. (a) How many seconds are there in a year? (b) If one micrometeorite (a sphere with a diameter on the order of 10^{-6} m) struck each square meter of the Moon each second, estimate the number of years it would take to cover the Moon with micrometeorites to a depth of one meter. (*Hint:* Consider a cubic box, 1 m on a side, on the Moon, and find how long it would take to fill the box.)

62. Imagine that you are the equipment manager of a professional baseball team. One of your jobs is to keep baseballs on hand for games. Balls are sometimes lost when players hit them into the stands as either home runs or foul balls. Estimate how many baseballs you have to buy per season in order to make up for such losses. Assume that your team plays an 81-game home schedule in a season.

2

Craig Breedlove, five times world land speed record holder, accelerates across the Black Rock Desert in Gerlach, Nevada, in his jet-powered car, Spirit of America, on its first test run on September 6, 1997. Subsequent jet-powered cars have broken the sound barrier on land.

AP Photo/Eric Risberg

MOTION IN ONE DIMENSION

Life is motion. Our muscles coordinate motion microscopically to enable us to walk and jog. Our hearts pump tirelessly for decades, moving blood through our bodies. Cell wall mechanisms move select atoms and molecules in and out of cells. From the prehistoric chase of antelopes across the savanna to the pursuit of satellites in space, mastery of motion has been critical to our survival and success as a species.

The study of motion and of physical concepts such as force and mass is called **dynamics**. The part of dynamics that describes motion without regard to its causes is called **kinematics**. In this chapter the focus is on kinematics in one dimension: motion along a straight line. This kind of motion—and, indeed, *any* motion—involves the concepts of displacement, velocity, and acceleration. Here, we use these concepts to study the motion of objects undergoing constant acceleration. In Chapter 3 we will repeat this discussion for objects moving in two dimensions.

The first recorded evidence of the study of mechanics can be traced to the people of ancient Sumeria and Egypt, who were interested primarily in understanding the motions of heavenly bodies. The most systematic and detailed early studies of the heavens were conducted by the Greeks from about 300 B.C. to A.D. 300. Ancient scientists and laypeople regarded the Earth as the center of the Universe. This **geocentric model** was accepted by such notables as Aristotle (384–322 B.C.) and Claudius Ptolemy (about A.D. 140). Largely because of the authority of Aristotle, the geocentric model became the accepted theory of the Universe until the 17th century.

About 250 B.C., the Greek philosopher Aristarchus worked out the details of a model of the Solar System based on a spherical Earth that rotated on its axis and revolved around the Sun. He proposed that the sky appeared to turn westward because the Earth was turning eastward. This model wasn't given much consideration because it was believed that a turning Earth would generate powerful winds as it moved through the air. We now know that the Earth carries the air and everything else with it as it rotates.

The Polish astronomer Nicolaus Copernicus (1473–1543) is credited with initiating the revolution that finally replaced the geocentric model. In his system, called the **heliocentric model**, Earth and the other planets revolve in circular orbits around the Sun.

This early knowledge formed the foundation for the work of Galileo Galilei (1564–1642), who stands out as the dominant facilitator of the entrance of physics into the modern era. In 1609 he became one of the first to make astronomical observations with a telescope. He observed mountains on the Moon, the larger satellites of Jupiter, spots on the Sun, and the phases of Venus. Galileo's observations convinced him of the correctness of the Copernican theory. His quantitative study of motion formed the foundation of Newton's revolutionary work in the next century.

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2.1 DISPLACEMENT

Motion involves the displacement of an object from one place in space and time to another. Describing motion requires some convenient coordinate system and a specified origin. A **frame of reference** is a choice of coordinate axes that defines the starting point for measuring any quantity, an essential first step in solving virtually any problem in mechanics (Fig. 2.1). In Active Figure 2.2a, for example, a car moves along the x-axis. The coordinates of the car at any time describe its position in space and, more importantly, its *displacement* at some given time of interest.

← Definition of displacement

The **displacement** Δx of an object is defined as its *change in position*, and is given by

$$\Delta x \equiv x_f - x_i \qquad [2.1]$$

where the initial position of the car is labeled x_i and the final position is x_f. (The indices i and f stand for initial and final, respectively.)

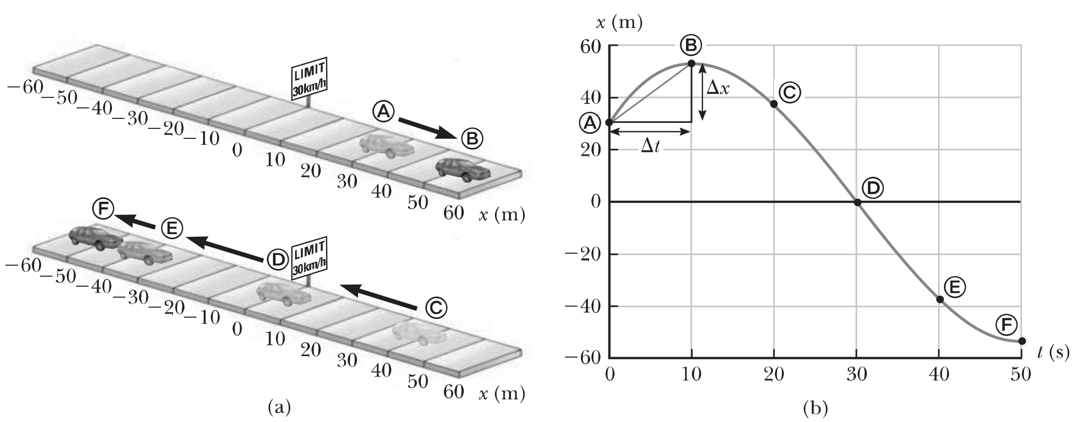
SI unit: meter (m)

We will use the Greek letter delta, Δ, to denote a change in any physical quantity. From the definition of displacement, we see that Δx (read "delta ex") is positive if x_f is greater than x_i and negative if x_f is less than x_i. For example, if the car moves from point Ⓐ to point Ⓑ so that the initial position is $x_i = 30$ m and the final position is $x_f = 52$ m, the displacement is $\Delta x = x_f - x_i = 52\text{ m} - 30\text{ m} = +22$ m. However, if the car moves from point Ⓒ to point Ⓕ, then the initial position is $x_i = 38$ m and the final position is $x_f = -53$ m, and the displacement is $\Delta x = x_f - x_i = -53\text{ m} - 38\text{ m} = -91$ m. A positive answer indicates a displacement in the positive x-direction, whereas a negative answer indicates a displacement in the negative x-direction. Active Figure 2.2b displays the graph of the car's position as a function of time.

Tip 2.1 A Displacement Isn't a Distance!

The displacement of an object is *not* the same as the distance it travels. Toss a tennis ball up and catch it. The ball travels a *distance* equal to twice the maximum height reached, but its *displacement* is zero.

Because displacement has both a magnitude (size) and a direction, it's a vector quantity, as are velocity and acceleration. In general, **a vector quantity is characterized by having both a magnitude and a direction**. By contrast, **a scalar quantity**



ACTIVE FIGURE 2.2
(a) A car moves back and forth along a straight line taken to be the x-axis. Because we are interested only in the car's translational motion, we can model it as a particle. (b) Graph of position vs. time for the motion of the "particle."

Tip 2.2 Vectors Have Both a Magnitude and a Direction.

Scalars have size. Vectors, too, have size, but they also indicate a direction.

has magnitude, but no direction. Scalar quantities such as mass and temperature are completely specified by a numeric value with appropriate units; no direction is involved.

Vector quantities will be usually denoted in boldface type with an arrow over the top of the letter. For example, $\vec{\mathbf{v}}$ represents velocity and $\vec{\mathbf{a}}$ denotes an acceleration, both vector quantities. In this chapter, however, it won't be necessary to use that notation because in one-dimensional motion an object can only move in one of two directions, and these directions are easily specified by plus and minus signs.

2.2 VELOCITY

In everyday usage the terms *speed* and *velocity* are interchangeable. In physics, however, there's a clear distinction between them: Speed is a scalar quantity, having only magnitude, whereas velocity is a vector, having both magnitude and direction.

Why must velocity be a vector? If you want to get to a town 70 km away in an hour's time, it's not enough to drive at a speed of 70 km/h; you must travel in the correct direction as well. This is obvious, but shows that velocity gives considerably more information than speed, as will be made more precise in the formal definitions.

Definition of average speed →

The **average speed** of an object over a given time interval is the total distance traveled divided by the total time elapsed:

$$\text{Average speed} \equiv \frac{\text{total distance}}{\text{total time}}$$

SI unit: meter per second (m/s)

In symbols, this equation might be written $v = d/t$, with the letter v understood in context to be the average speed, not a velocity. Because total distance and total time are always positive, the average speed will be positive, also. The definition of average speed completely ignores what may happen between the beginning and the end of the motion. For example, you might drive from Atlanta, Georgia, to St. Petersburg, Florida, a distance of about 500 miles, in 10 hours. Your average speed is 500 mi/10 h = 50 mi/h. It doesn't matter if you spent two hours in a traffic jam traveling only 5 mi/h and another hour at a rest stop. For average speed, only the total distance traveled and total elapsed time are important.

EXAMPLE 2.1 The Tortoise and the Hare

Goal Apply the concept of average speed.

Problem A turtle and a rabbit engage in a footrace over a distance of 4.00 km. The rabbit runs 0.500 km and then stops for a 90.0-min nap. Upon awakening, he remembers the race and runs twice as fast. Finishing the course in a total time of 1.75 h, the rabbit wins the race. **(a)** Calculate the average speed of the rabbit. **(b)** What was his average speed before he stopped for a nap?

Strategy Finding the overall average speed in part (a) is just a matter of dividing the total distance by the total time. Part (b) requires two equations and two unknowns, the latter turning out to be the two different average speeds: v_1 before the nap and v_2 after the nap. One equation is given in the statement of the problem ($v_2 = 2v_1$), whereas the other comes from the fact the rabbit ran for only 15 minutes because he napped for 90 minutes.

Solution

(a) Find the rabbit's overall average speed.

Apply the equation for average speed:

$$\text{Average speed} \equiv \frac{\text{total distance}}{\text{total time}} = \frac{4.00\ \text{km}}{1.75\ \text{h}}$$

$$= 2.29\ \text{km/h}$$

(b) Find the rabbit's average speed before his nap.

Sum the running times, and set the sum equal to 0.25 h: $t_1 + t_2 = 0.250\text{ h}$

Substitute $t_1 = d_1/v_1$ and $t_2 = d_2/v_2$: (1) $\dfrac{d_1}{v_1} + \dfrac{d_2}{v_2} = 0.250\text{ h}$

Substitute $v_2 = 2v_1$ and the values of d_1 and d_2 into Equation (1): (2) $\dfrac{0.500\text{ km}}{v_1} + \dfrac{3.50\text{ km}}{2v_1} = 0.250\text{ h}$

Solve Equation (2) for v_1: $v_1 = 9.00\text{ km/h}$

Remark As seen in this example, average speed can be calculated regardless of any variation in speed over the given time interval.

QUESTION 2.1

Does a doubling of an object's average speed always double the magnitude of its displacement in a given amount of time? Explain.

EXERCISE 2.1

Estimate the average speed of the Apollo spacecraft in meters per second, given that the craft took five days to reach the Moon from Earth. (The Moon is 3.8×10^8 m from Earth.)

Answer ~ 900 m/s

Unlike average speed, **average velocity** is a vector quantity, having both a magnitude and a direction. Consider again the car of Figure 2.2, moving along the road (the x-axis). Let the car's position be x_i at some time t_i and x_f at a later time t_f. In the time interval $\Delta t = t_f - t_i$, the displacement of the car is $\Delta x = x_f - x_i$.

← Definition of average velocity

The average velocity $\bar{v}$ during a time interval Δt is the displacement Δx divided by Δt:

$$\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad [2.2]$$

SI unit: meter per second (m/s)

Unlike the average speed, which is always positive, the average velocity of an object in one dimension can be either positive or negative, depending on the sign of the displacement. (The time interval Δt is always positive.) In Figure 2.2a, for example, the average velocity of the car is positive in the upper illustration, a positive sign indicating motion to the right along the x-axis. Similarly, a negative average velocity for the car in the lower illustration of the figure indicates that it moves to the left along the x-axis.

As an example, we can use the data in Table 2.1 to find the average velocity in the time interval from point Ⓐ to point Ⓑ (assume two digits are significant):

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{52\text{ m} - 30\text{ m}}{10\text{ s} - 0\text{ s}} = 2.2\text{ m/s}$$

TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Aside from meters per second, other common units for average velocity are feet per second (ft/s) in the U.S. customary system and centimeters per second (cm/s) in the cgs system.

To further illustrate the distinction between speed and velocity, suppose we're watching a drag race from the Goodyear blimp. In one run we see a car follow the straight-line path from Ⓟ to Ⓠ shown in Figure 2.3 during the time interval Δt,

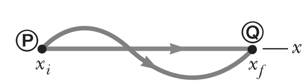


FIGURE 2.3 A drag race viewed from a blimp. One car follows the red straight-line path from Ⓟ to Ⓠ, and a second car follows the blue curved path.

and in a second run a car follows the curved path during the same interval. From the definition in Equation 2.2, the two cars had the same average velocity because they had the same displacement $\Delta x = x_f - x_i$ during the same time interval Δt. The car taking the curved route, however, traveled a greater distance and had the higher average speed.

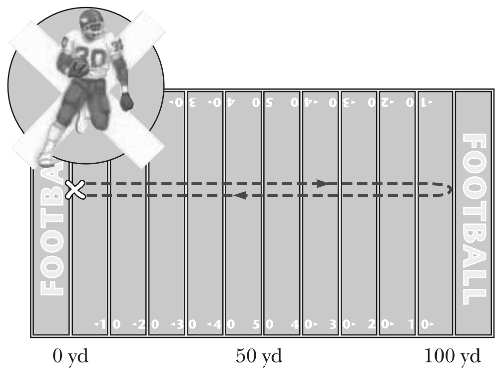


FIGURE 2.4 (Quick Quiz 2.1) The path followed by a confused football player.

QUICK QUIZ 2.1 Figure 2.4 shows the unusual path of a confused football player. After receiving a kickoff at his own goal, he runs downfield to within inches of a touchdown, then reverses direction and races back until he's tackled at the exact location where he first caught the ball. During this run, which took 25 s, what is (a) the total distance he travels, (b) his displacement, and (c) his average velocity in the *x*-direction? (d) What is his average speed?

Graphical Interpretation of Velocity

TIP 2.3 Slopes of Graphs

The word *slope* is often used in reference to the graphs of physical data. Regardless of the type of data, the *slope* is given by

$$\text{Slope} = \frac{\text{change in vertical axis}}{\text{change in horizontal axis}}$$

Slope carries units.

If a car moves along the *x*-axis from Ⓐ to Ⓑ to Ⓒ, and so forth, we can plot the positions of these points as a function of the time elapsed since the start of the motion. The result is a **position vs. time graph** like those of Figure 2.5. In Figure 2.5a, the graph is a straight line because the car is moving at constant velocity. The same displacement Δx occurs in each time interval Δt. In this case, the average velocity is always the same and is equal to $\Delta x/\Delta t$. Figure 2.5b is a graph of the data in Table 2.1. Here, the position vs. time graph is not a straight line because the velocity of the car is changing. Between any two points, however, we can draw a straight line just as in Figure 2.5a, and the slope of that line is the average velocity $\Delta x/\Delta t$ in that time interval. In general, **the average velocity of an object during the time interval Δt is equal to the slope of the straight line joining the initial and final points on a graph of the object's position versus time.**

TIP 2.4 Average Velocity vs. Average Speed

Average velocity is *not* the same as average speed. If you run from $x = 0$ m to $x = 25$ m and back to your starting point in a time interval of 5 s, the average velocity is zero, whereas the average speed is 10 m/s.

From the data in Table 2.1 and the graph in Figure 2.5b, we see that the car first moves in the positive *x*-direction as it travels from Ⓐ to Ⓑ, reaches a position of 52 m at time $t = 10$ s, then reverses direction and heads backwards. In the first 10 s of its motion, as the car travels from Ⓐ to Ⓑ, its average velocity is 2.2 m/s, as previously calculated. In the first 40 seconds, as the car goes from Ⓐ to Ⓔ, its displacement is $\Delta x = -37 \text{ m} - (30 \text{ m}) = -67$ m. So the average velocity in this interval, which equals the slope of the blue line in Figure 2.5b from Ⓐ to Ⓔ, is $\overline{v} = \Delta x/\Delta t = (-67 \text{ m})/(40 \text{ s}) = -1.7$ m/s. In general, there will be a different average velocity between any distinct pair of points.

Instantaneous Velocity

Average velocity doesn't take into account the details of what happens during an interval of time. On a car trip, for example, you may speed up or slow down a number of times in response to the traffic and the condition of the road, and on rare occasions even pull over to chat with a police officer about your speed. What is most important to the police (and to your own safety) is the speed of your car and the direction it was going at a particular instant in time, which together determine the car's **instantaneous velocity.**

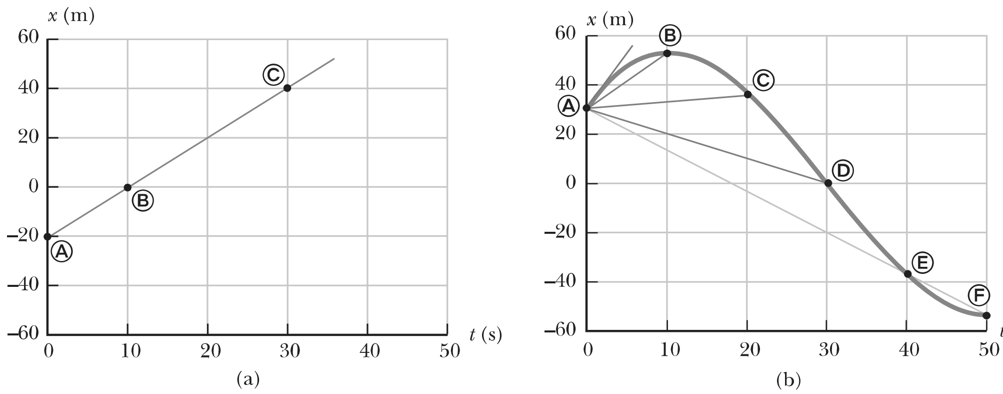


FIGURE 2.5 (a) Position vs. time graph for the motion of a car moving along the x-axis at constant velocity. (b) Position vs. time graph for the motion of a car with changing velocity, using the data in Table 2.1. The average velocity in the time interval from $t = 0$ s to $t = 30$ s is the slope of the blue straight line connecting Ⓐ and Ⓓ.

So in driving a car between two points, the average velocity must be computed over an interval of time, but the magnitude of instantaneous velocity can be read on the car's speedometer.

← Definition of instantaneous velocity

The instantaneous velocity v is the limit of the average velocity as the time interval Δt becomes infinitesimally small:

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad [2.3]$$

SI unit: meter per second (m/s)

The notation $\lim_{\Delta t \to 0}$ means that the ratio $\Delta x/\Delta t$ is repeatedly evaluated for smaller and smaller time intervals Δt. As Δt gets extremely close to zero, the ratio $\Delta x/\Delta t$ gets closer and closer to a fixed number, which is defined as the instantaneous velocity.

To better understand the formal definition, consider data obtained on our vehicle via radar (Table 2.2). At $t = 1.00$ s, the car is at $x = 5.00$ m, and at $t = 3.00$ s, it's at $x = 52.5$ m. The average velocity computed for this interval $\Delta x/\Delta t = (52.5 \text{ m} - 5.00 \text{ m})/(3.00 \text{ s} - 1.00 \text{ s}) = 23.8$ m/s. This result could be used as an estimate for the velocity at $t = 1.00$ s, but it wouldn't be very accurate because the speed changes considerably in the two-second time interval. Using the rest of the data, we can construct Table 2.3. As the time interval gets smaller, the average velocity more closely approaches the instantaneous velocity. Using the final interval of only 0.010 0 s, we find that the average velocity is $\bar{v} = \Delta x/\Delta t = 0.470 \text{ m}/0.010\,0 \text{ s} = 47.0$ m/s. Because 0.010 0 s is a very short time interval, the actual instantaneous velocity is probably very close to this latter average velocity, given the limits on the car's ability to accelerate. Finally using the conversion factor on the inside front cover of the book, we see that this is 105 mi/h, a likely violation of the speed limit.

TABLE 2.2

Positions of a Car at Specific Instants of Time

t (s)	x (m)
1.00	5.00
1.01	5.47
1.10	9.67
1.20	14.3
1.50	26.3
2.00	34.7
3.00	52.5

TABLE 2.3

Calculated Values of the Time Intervals, Displacements, and Average Velocities for the Car of Table 2.2

Time Interval (s)	Δt (s)	Δx (m)	$\bar{v}$ (m/s)
1.00 to 3.00	2.00	47.5	23.8
1.00 to 2.00	1.00	29.7	29.7
1.00 to 1.50	0.50	21.3	42.6
1.00 to 1.20	0.20	9.30	46.5
1.00 to 1.10	0.10	4.67	46.7
1.00 to 1.01	0.01	0.470	47.0

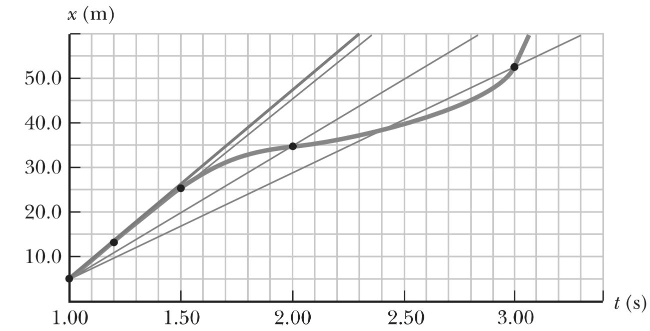


FIGURE 2.6 Graph representing the motion of the car from the data in Table 2.2. The slope of the blue line represents the average velocity for smaller and smaller time intervals and approaches the slope of the green tangent line.

As can be seen in Figure 2.6, the chords formed by the blue lines gradually approach a tangent line as the time interval becomes smaller. **The slope of the line tangent to the position vs. time curve at "a given time" is defined to be the instantaneous velocity at that time.**

The instantaneous speed of an object, which is a scalar quantity, is defined as the magnitude of the instantaneous velocity. Like average speed, instantaneous speed (which we will usually call, simply, "speed") has no direction associated with it and hence carries no algebraic sign. For example, if one object has an instantaneous velocity of +15 m/s along a given line and another object has an instantaneous velocity of −15 m/s along the same line, both have an instantaneous speed of 15 m/s.

EXAMPLE 2.2 Slowly Moving Train

Goal Obtain average and instantaneous velocities from a graph.

Problem A train moves slowly along a straight portion of track according to the graph of position versus time in Figure 2.7a. Find **(a)** the average velocity for the total trip, **(b)** the average velocity during the first 4.00 s of motion, **(c)** the average velocity during the next 4.00 s of motion, **(d)** the instantaneous velocity at $t = 2.00$ s, and **(e)** the instantaneous velocity at $t = 9.00$ s.

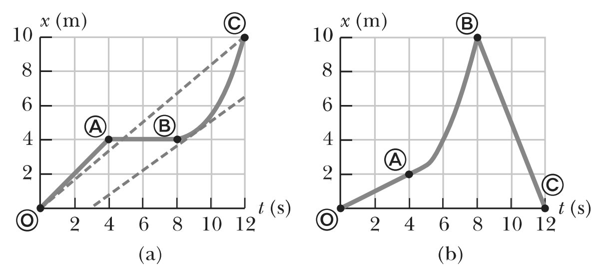


FIGURE 2.7 (a) (Example 2.2) (b) (Exercise 2.2)

Strategy The average velocities can be obtained by substituting the data into the definition. The instantaneous velocity at $t = 2.00$ s is the same as the average velocity at that point because the position vs. time graph is a straight line, indicating constant velocity. Finding the instantaneous velocity when $t = 9.00$ s requires sketching a line tangent to the curve at that point and finding its slope.

Solution

(a) Find the average velocity from Ⓞ to Ⓒ.

Calculate the slope of the dashed blue line:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{10.0\text{ m}}{12.0\text{ s}} = \boxed{+0.833\text{ m/s}}$$

(b) Find the average velocity during the first 4 seconds of the train's motion.

Again, find the slope:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{4.00\text{ m}}{4.00\text{ s}} = \boxed{+1.00\text{ m/s}}$$

(c) Find the average velocity during the next 4 seconds.

Here, there is no change in position, so the displacement Δx is zero:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0\text{ m}}{4.00\text{ s}} = \boxed{0\text{ m/s}}$$

(d) Find the instantaneous velocity at $t = 2.00$ s.

This is the same as the average velocity found in (b), because the graph is a straight line:

$$v = \boxed{1.00 \text{ m/s}}$$

(e) Find the instantaneous velocity at $t = 9.00$ s.

The tangent line appears to intercept the x-axis at (3.0 s, 0 m) and graze the curve at (9.0 s, 4.5 m). The instantaneous velocity at $t = 9.00$ s equals the slope of the tangent line through these points:

$$v = \frac{\Delta x}{\Delta t} = \frac{4.5 \text{ m} - 0 \text{ m}}{9.0 \text{ s} - 3.0 \text{ s}} = \boxed{0.75 \text{ m/s}}$$

Remarks From the origin to Ⓐ, the train moves at constant speed in the positive x-direction for the first 4.00 s, because the position vs. time curve is rising steadily toward positive values. From Ⓐ to Ⓑ, the train stops at $x = 4.00$ m for 4.00 s. From Ⓑ to Ⓒ, the train travels at increasing speed in the positive x-direction.

QUESTION 2.2

Would a vertical line in a graph of position versus time make sense? Explain.

EXERCISE 2.2

Figure 2.7b graphs another run of the train. Find (a) the average velocity from Ⓞ to Ⓒ; (b) the average and instantaneous velocities from Ⓞ to Ⓐ; (c) the approximate instantaneous velocity at $t = 6.0$ s; and (d) the average and instantaneous velocity at $t = 9.0$ s.

Answers (a) 0 m/s (b) both are +0.5 m/s (c) 2 m/s (d) both are −2.5 m/s

2.3 ACCELERATION

Going from place to place in your car, you rarely travel long distances at constant velocity. The velocity of the car increases when you step harder on the gas pedal and decreases when you apply the brakes. The velocity also changes when you round a curve, altering your direction of motion. The changing of an object's velocity with time is called **acceleration.**

Average Acceleration

A car moves along a straight highway as in Figure 2.8. At time t_i it has a velocity of v_i, and at time t_f its velocity is v_f, with $\Delta v = v_f - v_i$ and $\Delta t = t_f - t_i$.

← Definition of average acceleration

The average acceleration $\bar{a}$ during the time interval Δt is the change in velocity Δv divided by Δt:

$$\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad [2.4]$$

SI unit: meter per second per second (m/s²)

For example, suppose the car shown in Figure 2.8 accelerates from an initial velocity of $v_i = +10$ m/s to a final velocity of $v_f = +20$ m/s in a time interval of 2 s. (Both velocities are toward the right, selected as the positive direction.) These values can be inserted into Equation 2.4 to find the average acceleration:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{20 \text{ m/s} - 10 \text{ m/s}}{2 \text{ s}} = +5 \text{ m/s}^2$$

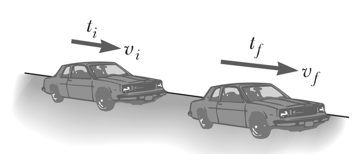


FIGURE 2.8 A car moving to the right accelerates from a velocity of v_i to a velocity of v_f in the time interval $\Delta t = t_f - t_i$.

Acceleration is a vector quantity having dimensions of length divided by the time squared. Common units of acceleration are meters per second per second ((m/s)/s, which is usually written m/s²) and feet per second per second (ft/s²). An average acceleration of $+5\ \text{m/s}^2$ means that, on average, the car increases its velocity by 5 m/s every second in the positive x-direction.

> **TIP 2.5 Negative Acceleration**
> Negative acceleration doesn't necessarily mean an object is slowing down. If the acceleration is negative and the velocity is also negative, the object is speeding up!

For the case of motion in a straight line, the direction of the velocity of an object and the direction of its acceleration are related as follows: **When the object's velocity and acceleration are in the same direction, the speed of the object increases with time. When the object's velocity and acceleration are in opposite directions, the speed of the object decreases with time.**

To clarify this point, suppose the velocity of a car changes from -10 m/s to -20 m/s in a time interval of 2 s. The minus signs indicate that the velocities of the car are in the negative x-direction; they do *not* mean that the car is slowing down! The average acceleration of the car in this time interval is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-20\ \text{m/s} - (-10\ \text{m/s})}{2\ \text{s}} = -5\ \text{m/s}^2$$

> **TIP 2.6 Deceleration**
> The word *deceleration* means a reduction in speed, a slowing down. Some confuse it with a negative acceleration, which can speed something up. (See Tip 2.5.)

The minus sign indicates that the acceleration vector is also in the negative x-direction. Because the velocity and acceleration vectors are in the same direction, the speed of the car must increase as the car moves to the left. Positive and negative accelerations specify directions relative to chosen axes, not "speeding up" or "slowing down." The terms "speeding up" or "slowing down" refer to an increase and a decrease in speed, respectively.

QUICK QUIZ 2.2 **True or False? (a)** A car must always have an acceleration in the same direction as its velocity. **(b)** It's possible for a slowing car to have a positive acceleration. **(c)** An object with constant nonzero acceleration can never stop and remain at rest.

Instantaneous Acceleration

The value of the average acceleration often differs in different time intervals, so it's useful to define the **instantaneous acceleration**, which is analogous to the instantaneous velocity discussed in Section 2.2.

Definition of instantaneous acceleration →

The instantaneous acceleration a is the limit of the average acceleration as the time interval Δt goes to zero:

$$a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \qquad \textbf{[2.5]}$$

SI unit: meter per second per second (m/s²)

Here again, the notation $\lim_{\Delta t \to 0}$ means that the ratio $\Delta v/\Delta t$ is evaluated for smaller and smaller values of Δt. The closer Δt gets to zero, the closer the ratio gets to a fixed number, which is the instantaneous acceleration.

Figure 2.9, a **velocity vs. time graph**, plots the velocity of an object against time. The graph could represent, for example, the motion of a car along a busy street. The average acceleration of the car between times t_i and t_f can be found by determining the slope of the line joining points Ⓟ and Ⓠ. If we imagine that point Ⓠ is brought closer and closer to point Ⓟ, the line comes closer and closer to becoming tangent at Ⓟ. The **instantaneous acceleration of an object at a given time equals the slope of the tangent to the velocity vs. time graph at that time.** From now on, we will use the term *acceleration* to mean "instantaneous acceleration."

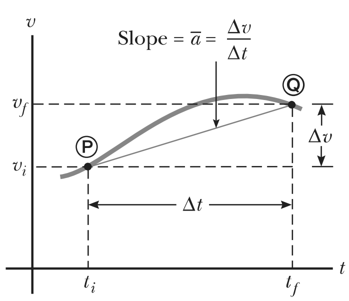


FIGURE 2.9 Velocity vs. time graph for an object moving in a straight line. The slope of the blue line connecting points Ⓟ and Ⓠ is defined as the average acceleration in the time interval $\Delta t = t_f = t_i$.

In the special case where the velocity vs. time graph of an object's motion is a straight line, the instantaneous acceleration of the object at any point is equal to its average acceleration. This also means that the tangent line to the graph overlaps the graph itself. In that case, the object's acceleration is said to be *uniform*, which means that it has a constant value. Constant acceleration problems are important in kinematics and will be studied extensively in this and the next chapter.

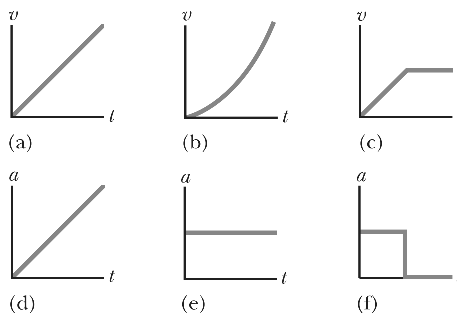


FIGURE 2.10 (Quick Quiz 2.3) Match each velocity vs. time graph to its corresponding acceleration vs. time graph.

QUICK QUIZ 2.3 Parts (a), (b), and (c) of Figure 2.10 represent three graphs of the velocities of different objects moving in straight-line paths as functions of time. The possible accelerations of each object as functions of time are shown in parts (d), (e), and (f). Match each velocity vs. time graph with the acceleration vs. time graph that best describes the motion.

EXAMPLE 2.3 Catching a Fly Ball

Goal Apply the definition of instantaneous acceleration.

Problem A baseball player moves in a straight-line path in order to catch a fly ball hit to the outfield. His velocity as a function of time is shown in Figure 2.11a. Find his instantaneous acceleration at points Ⓐ, Ⓑ, and Ⓒ.

Strategy At each point, the velocity vs. time graph is a straight line segment, so the instantaneous acceleration will be the slope of that segment. Select two points on each segment and use them to calculate the slope.

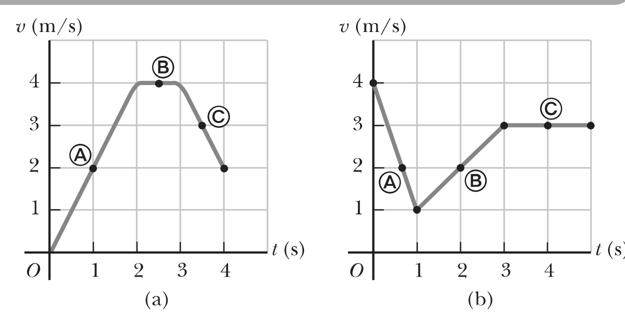


FIGURE 2.11 (a) (Example 2.3) (b) (Exercise 2.3)

Solution

Acceleration at Ⓐ.

The acceleration at Ⓐ equals the slope of the line connecting the points (0 s, 0 m/s) and (2.0 s, 4.0 m/s):

$$a = \frac{\Delta v}{\Delta t} = \frac{4.0\text{ m/s} - 0}{2.0\text{ s} - 0} = \boxed{+2.0\text{ m/s}^2}$$

Acceleration at Ⓑ.

$\Delta v = 0$, because the segment is horizontal:

$$a = \frac{\Delta v}{\Delta t} = \frac{4.0\text{ m/s} - 4.0\text{ m/s}}{3.0\text{ s} - 2.0\text{ s}} = \boxed{0\text{ m/s}^2}$$

Acceleration at Ⓒ.

The acceleration at Ⓒ equals the slope of the line connecting the points (3.0 s, 4.0 m/s) and (4.0 s, 2.0 m/s):

$$a = \frac{\Delta v}{\Delta t} = \frac{2.0\text{ m/s} - 4.0\text{ m/s}}{4.0\text{ s} - 3.0\text{ s}} = \boxed{-2.0\text{ m/s}^2}$$

Remarks Assume the player is initially moving in the positive x-direction. For the first 2.0 s, the ballplayer moves in the positive x-direction (the velocity is positive) and steadily accelerates (the curve is steadily rising) to a maximum speed of 4.0 m/s. He moves for 1.0 s at a steady speed of 4.0 m/s and then slows down in the last second (the v vs. t curve is falling), still moving in the positive x-direction (v is always positive).

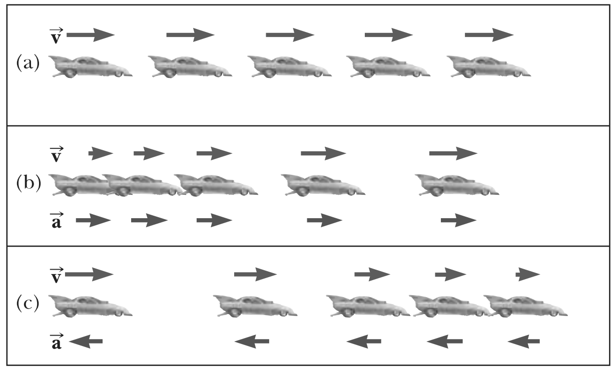
QUESTION 2.3

Can the tangent line to a velocity vs. time graph ever be vertical? Explain.

EXERCISE 2.3

Repeat the problem, using Figure 2.11b.

Answer The accelerations at Ⓐ, Ⓑ, and Ⓒ are -3.0 m/s^2, 1.0 m/s^2, and 0 m/s^2, respectively.



ACTIVE FIGURE 2.12
(a) Motion diagram for a car moving at constant velocity (zero acceleration). (b) Motion diagram for a car undergoing constant acceleration in the direction of its velocity. The velocity vector at each instant is indicated by a red arrow and the constant acceleration vector by a violet arrow. (c) Motion diagram for a car undergoing constant acceleration in the direction *opposite* the velocity at each instant.

2.4 MOTION DIAGRAMS

Velocity and acceleration are sometimes confused with each other, but they're very different concepts, as can be illustrated with the help of motion diagrams. A **motion diagram** is a representation of a moving object at successive time intervals, with velocity and acceleration vectors sketched at each position, red for velocity vectors and violet for acceleration vectors, as in Active Figure 2.12. The time intervals between adjacent positions in the motion diagram are assumed equal.

A motion diagram is analogous to images resulting from a stroboscopic photograph of a moving object. Each image is made as the strobe light flashes. Active Figure 2.12 represents three sets of strobe photographs of cars moving along a straight roadway from left to right. The time intervals between flashes of the stroboscope are equal in each diagram.

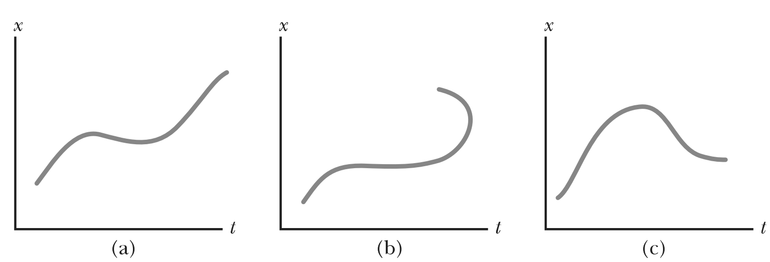
In Active Figure 2.12a, the images of the car are equally spaced: The car moves the same distance in each time interval. This means that the car moves with *constant positive velocity* and has *zero acceleration.* The red arrows are all the same length (constant velocity) and there are no violet arrows (zero acceleration).

In Active Figure 2.12b, the images of the car become farther apart as time progresses and the velocity vector increases with time, because the car's displacement between adjacent positions increases as time progresses. The car is moving with a *positive velocity* and a constant *positive acceleration.* The red arrows are successively longer in each image, and the violet arrows point to the right.

In Active Figure 2.12c, the car slows as it moves to the right because its displacement between adjacent positions decreases with time. In this case, the car moves initially to the right with a constant negative acceleration. The velocity vector decreases in time (the red arrows get shorter) and eventually reaches zero, as would happen when the brakes are applied. Note that the acceleration and velocity vectors are *not* in the same direction. The car is moving with a *positive velocity,* but with a *negative acceleration.*

Try constructing your own diagrams for various problems involving kinematics.

QUICK QUIZ 2.4 The three graphs in Active Figure 2.13 represent the position vs. time for objects moving along the x-axis. Which, if any, of these graphs is not physically possible?



ACTIVE FIGURE 2.13
(Quick Quiz 2.4) Which position vs. time curve is impossible?

QUICK QUIZ 2.5 Figure 2.14a is a diagram of a multiflash image of an air puck moving to the right on a horizontal surface. The images sketched are separated by equal time intervals, and the first and last images show the puck at rest. (a) In Figure 2.14b, which color graph best shows the puck's position as a function of time? (b) In Figure 2.14c, which color graph best shows the puck's velocity as a function of time? (c) In Figure 2.14d, which color graph best shows the puck's acceleration as a function of time?

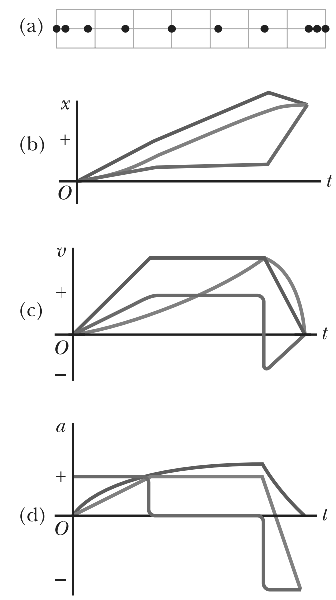


FIGURE 2.14 (Quick Quiz 2.5) Choose the correct graphs.

2.5 ONE-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

Many applications of mechanics involve objects moving with *constant acceleration*. This type of motion is important because it applies to numerous objects in nature, such as an object in free fall near Earth's surface (assuming air resistance can be neglected). A graph of acceleration versus time for motion with constant acceleration is shown in Active Figure 2.15a. **When an object moves with constant acceleration, the instantaneous acceleration at any point in a time interval is equal to the value of the average acceleration over the entire time interval.** Consequently, the velocity increases or decreases at the same rate throughout the motion, and a plot of v versus t gives a straight line with either positive, zero, or negative slope.

Because the average acceleration equals the instantaneous acceleration when a is constant, we can eliminate the bar used to denote average values from our defining equation for acceleration, writing $\bar{a} = a$, so that Equation 2.4 becomes

$$a = \frac{v_f - v_i}{t_f - t_i}$$

The observer timing the motion is always at liberty to choose the initial time, so for convenience, let $t_i = 0$ and t_f be any arbitrary time t. Also, let $v_i = v_0$ (the initial velocity at $t = 0$) and $v_f = v$ (the velocity at any arbitrary time t). With this notation, we can express the acceleration as

$$a = \frac{v - v_0}{t}$$

or

$$v = v_0 + at \qquad \text{(for constant } a\text{)} \qquad [2.6]$$

Equation 2.6 states that the acceleration a steadily changes the initial velocity v_0 by an amount at. For example, if a car starts with a velocity of +2.0 m/s to the right and accelerates to the right with $a = +6.0\ \text{m/s}^2$, it will have a velocity of +14 m/s after 2.0 s have elapsed:

$$v = v_0 + at = +2.0\ \text{m/s} + (6.0\ \text{m/s}^2)(2.0\ \text{s}) = +14\ \text{m/s}$$

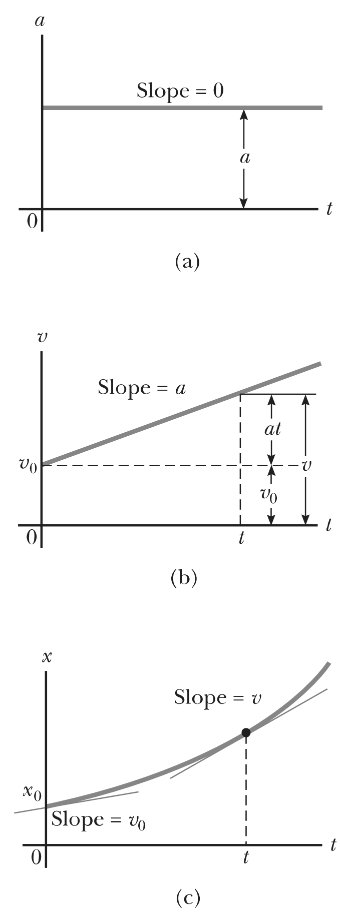
The graphical interpretation of v is shown in Active Figure 2.15b. The velocity varies linearly with time according to Equation 2.6, as it should for constant acceleration.

Because the velocity is increasing or decreasing *uniformly* with time, we can express the average velocity in any time interval as the arithmetic average of the initial velocity v_0 and the final velocity v:

$$\bar{v} = \frac{v_0 + v}{2} \qquad \text{(for constant } a\text{)} \qquad [2.7]$$

Remember that this expression is valid only when the acceleration is constant, in which case the velocity increases uniformly.

We can now use this result along with the defining equation for average velocity, Equation 2.2, to obtain an expression for the displacement of an object as a



ACTIVE FIGURE 2.15
A particle moving along the x-axis with constant acceleration a.
(a) the acceleration vs. time graph,
(b) the velocity vs. time graph, and
(c) the position vs. time graph.

TABLE 2.4

Equations for Motion in a Straight Line Under Constant Acceleration

Equation	Information Given by Equation
$v = v_0 + at$	Velocity as a function of time
$\Delta x = v_0 t + \frac{1}{2}at^2$	Displacement as a function of time
$v^2 = v_0^2 + 2a\Delta x$	Velocity as a function of displacement

Note: Motion is along the x-axis. At $t = 0$, the velocity of the particle is v_0.

function of time. Again, we choose $t_i = 0$ and $t_f = t$, and for convenience, we write $\Delta x = x_f - x_i = x - x_0$. This results in

$$\Delta x = \bar{v}t = \left(\frac{v_0 + v}{2}\right)t$$

$$\Delta x = \tfrac{1}{2}(v_0 + v)t \qquad \text{(for constant } a\text{)} \qquad \textbf{[2.8]}$$

We can obtain another useful expression for displacement by substituting the equation for v (Eq. 2.6) into Equation 2.8:

$$\Delta x = \tfrac{1}{2}(v_0 + v_0 + at)t$$

$$\Delta x = v_0 t + \tfrac{1}{2}at^2 \qquad \text{(for constant } a\text{)} \qquad \textbf{[2.9]}$$

This equation can also be written in terms of the position x, since $\Delta x = x - x_0$. Active Figure 2.15c shows a plot of x versus t for Equation 2.9, which is related to the graph of velocity vs. time: The area under the curve in Active Figure 2.15b is equal to $v_0 t + \frac{1}{2}at^2$, which is equal to the displacement Δx. In fact, **the area under the graph of *v* versus *t* for any object is equal to the displacement Δx of the object.**

Finally, we can obtain an expression that doesn't contain time by solving Equation 2.6 for t and substituting into Equation 2.8, resulting in

$$\Delta x = \tfrac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right) = \frac{v^2 - v_0^2}{2a}$$

$$v^2 = v_0^2 + 2a\Delta x \qquad \text{(for constant } a\text{)} \qquad \textbf{[2.10]}$$

Equations 2.6 and 2.9 together can solve any problem in one-dimensional motion with constant acceleration, but Equations 2.7, 2.8, and, especially, 2.10 are sometimes convenient. The three most useful equations—Equations 2.6, 2.9, and 2.10—are listed in Table 2.4.

The best way to gain confidence in the use of these equations is to work a number of problems. There is usually more than one way to solve a given problem, depending on which equations are selected and what quantities are given. The difference lies mainly in the algebra.

PROBLEM-SOLVING STRATEGY

ACCELERATED MOTION

The following procedure is recommended for solving problems involving accelerated motion.

1. **Read** the problem.
2. **Draw** a diagram, choosing a coordinate system, labeling initial and final points, and indicating directions of velocities and accelerations with arrows.
3. **Label** all quantities, circling the unknowns. Convert units as needed.

4. **Equations** from Table 2.4 should be selected next. All kinematics problems in this chapter can be solved with the first two equations, and the third is often convenient.
5. **Solve** for the unknowns. Doing so often involves solving two equations for two unknowns. It's usually more convenient to substitute all known values before solving.
6. **Check** your answer, using common sense and estimates.

TIP 2.7 Pigs Don't Fly
After solving a problem, you should think about your answer and decide whether it seems reasonable. If it isn't, look for your mistake!

Most of these problems reduce to writing the kinematic equations from Table 2.4 and then substituting the correct values into the constants a, v_0, and x_0 from the given information. Doing this produces two equations—one linear and one quadratic—for two unknown quantities.

EXAMPLE 2.4 The Daytona 500

Goal Apply the basic kinematic equations.

Problem **(a)** A race car starting from rest accelerates at a constant rate of 5.00 m/s². What is the velocity of the car after it has traveled 1.00×10^2 ft? **(b)** How much time has elapsed?

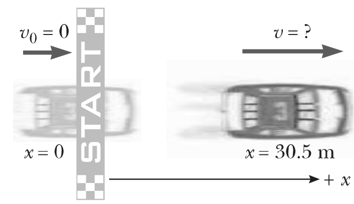


FIGURE 2.16 (Example 2.4)

Strategy **(a)** We've read the problem, drawn the diagram in Figure 2.16, and chosen a coordinate system (steps 1 and 2). We'd like to find the velocity v after a certain known displacement Δx. The acceleration a is also known, as is the initial velocity v_0 (step 3, labeling, is complete), so the third equation in Table 2.4 looks most useful for solving part (a). Given the velocity, the first equation in Table 2.4 can then be used to find the time in part (b).

Solution

(a) Convert units of Δx to SI, using the information in the inside front cover.

$$1.00 \times 10^2 \text{ ft} = (1.00 \times 10^2 \text{ ft})\left(\frac{1 \text{ m}}{3.28 \text{ ft}}\right) = 30.5 \text{ m}$$

Write the kinematics equation for v^2 (step 4):

$$v^2 = v_0{}^2 + 2a\,\Delta x$$

Solve for v, taking the positive square root because the car moves to the right (step 5):

$$v = \sqrt{v_0^2 + 2a\,\Delta x}$$

Substitute $v_0 = 0$, $a = 5.00 \text{ m/s}^2$, and $\Delta x = 30.5$ m:

$$v = \sqrt{v_0^2 + 2a\,\Delta x} = \sqrt{(0)^2 + 2(5.00 \text{ m/s}^2)(30.5 \text{ m})}$$

$$= \boxed{17.5 \text{ m/s}}$$

(b) How much time has elapsed?

Apply the first equation of Table 2.4:

$$v = at + v_0$$

Substitute values and solve for time t:

$$17.5 \text{ m/s} = (5.00 \text{ m/s}^2)t$$

$$t = \frac{17.5 \text{ m/s}}{5.0 \text{ m/s}^2} = \boxed{3.50 \text{ s}}$$

Remarks The answers are easy to check. An alternate technique is to use $\Delta x = v_0 t + \frac{1}{2}at^2$ to find t and then use the equation $v = v_0 + at$ to find v.

QUESTION 2.4
What is the final speed if the displacement is increased by a factor of 4?

EXERCISE 2.4

Suppose the driver in this example now slams on the brakes, stopping the car in 4.00 s. Find (a) the acceleration and (b) the distance the car travels while braking, assuming the acceleration is constant.

Answers (a) $a = -4.38 \text{ m/s}^2$ (b) $d = 35.0$ m

EXAMPLE 2.5 Car Chase

Goal Solve a problem involving two objects, one moving at constant acceleration and the other at constant velocity.

Problem A car traveling at a constant speed of 24.0 m/s passes a trooper hidden behind a billboard, as in Figure 2.17. One second after the speeding car passes the billboard, the trooper sets off in chase with a constant acceleration of 3.00 m/s². **(a)** How long does it take the trooper to overtake the speeding car? **(b)** How fast is the trooper going at that time?

Strategy Solving this problem involves two simultaneous kinematics equations of position, one for the trooper and the other for the car. Choose $t = 0$ to correspond to the time the trooper takes up the chase, when the car is at $x_{car} = 24.0$ m because of its head start (24.0 m/s × 1.00 s). The trooper catches up with the car when their positions are the same, which suggests setting $x_{trooper} = x_{car}$ and solving for time, which can then be used to find the trooper's speed in part (b).

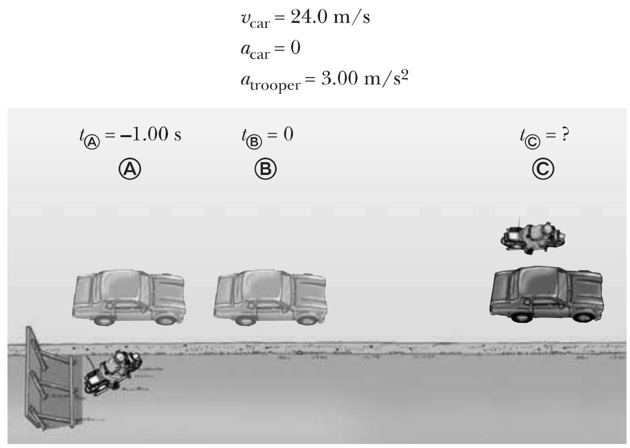


FIGURE 2.17 (Example 2.5) A speeding car passes a hidden trooper. When does the trooper catch up to the car?

Solution

(a) How long does it take the trooper to overtake the car?

Write the equation for the car's displacement:

$$\Delta x_{car} = x_{car} - x_0 = v_0 t + \tfrac{1}{2} a_{car} t^2$$

Take $x_0 = 24.0$ m, $v_0 = 24.0$ m/s and $a_{car} = 0$. Solve for x_{car}:

$$x_{car} = x_0 + vt = 24.0 \text{ m} + (24.0 \text{ m/s})t$$

Write the equation for the trooper's position, taking $x_0 = 0$, $v_0 = 0$, and $a_{trooper} = 3.00 \text{ m/s}^2$:

$$x_{trooper} = \tfrac{1}{2} a_{trooper} t^2 = \tfrac{1}{2}(3.00 \text{ m/s}^2)t^2 = (1.50 \text{ m/s}^2)t^2$$

Set $x_{trooper} = x_{car}$, and solve the quadratic equation. (The quadratic formula appears in Appendix A, Equation A.8.) Only the positive root is meaningful.

$$(1.50 \text{ m/s}^2)t^2 = 24.0 \text{ m} + (24.0 \text{ m/s})t$$

$$(1.50 \text{ m/s}^2)t^2 - (24.0 \text{ m/s})t - 24.0 \text{ m} = 0$$

$$t = \boxed{16.9 \text{ s}}$$

(b) Find the trooper's speed at this time.

Substitute the time into the trooper's velocity equation:

$$v_{trooper} = v_0 + a_{trooper}\, t = 0 + (3.00 \text{ m/s}^2)(16.9 \text{ s})$$

$$= \boxed{50.7 \text{ m/s}}$$

Remarks The trooper, traveling about twice as fast as the car, must swerve or apply his brakes strongly to avoid a collision! This problem can also be solved graphically by plotting position versus time for each vehicle on the same graph. The intersection of the two graphs corresponds to the time and position at which the trooper overtakes the car.

QUESTION 2.5

The graphical solution corresponds to finding the intersection of what two types of curves in the *xt*-plane?

EXERCISE 2.5

A motorist with an expired license tag is traveling at 10.0 m/s down a street, and a policeman on a motorcycle, taking another 5.00 s to finish his donut, gives chase at an acceleration of 2.00 m/s^2. Find (a) the time required to catch the car and (b) the distance the trooper travels while overtaking the motorist.

Answers (a) 13.7 s (b) 188 m

EXAMPLE 2.6 Runway Length

Goal Apply kinematics to horizontal motion with two phases.

Problem A typical jetliner lands at a speed of 160 mi/h and decelerates at the rate of (10 mi/h)/s. If the plane travels at a constant speed of 160 mi/h for 1.0 s after landing before applying the brakes, what is the total displacement of the aircraft between touchdown on the runway and coming to rest?

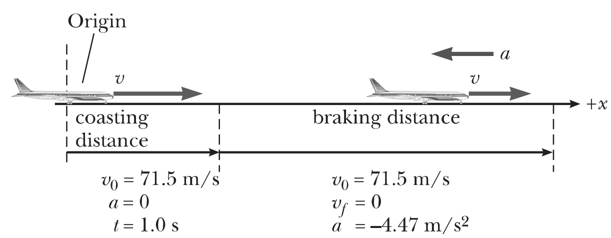


FIGURE 2.18 (Example 2.6) Coasting and braking distances for a landing jetliner.

Strategy See Figure 2.18. First, convert all quantities to SI units. The problem must be solved in two parts, or phases, corresponding to the initial coast after touchdown, followed by braking. Using the kinematic equations, find the displacement during each part and add the two displacements.

Solution

Convert units of speed and acceleration to SI:

$$v_0 = (160\ \text{mi/h})\left(\frac{0.447\ \text{m/s}}{1.00\ \text{mi/h}}\right) = 71.5\ \text{m/s}$$

$$a = (-10.0\ (\text{mi/h})/\text{s})\left(\frac{0.447\ \text{m/s}}{1.00\ \text{mi/h}}\right) = -4.47\ \text{m/s}^2$$

Taking $a = 0$, $v_0 = 71.5$ m/s, and $t = 1.00$ s, find the displacement while the plane is coasting:

$$\Delta x_{\text{coasting}} = v_0 t + \tfrac{1}{2}at^2 = (71.5\ \text{m/s})(1.00\ \text{s}) + 0 = 71.5\ \text{m}$$

Use the time-independent kinematic equation to find the displacement while the plane is braking.

$$v^2 = {v_0}^2 + 2a\Delta x_{\text{braking}}$$

Take $a = -4.47$ m/s^2 and $v_0 = 71.5$ m/s. The negative sign on a means that the plane is slowing down.

$$\Delta x_{\text{braking}} = \frac{v^2 - {v_0}^2}{2a} = \frac{0 - (71.5\ \text{m/s})^2}{2.00(-4.47\ \text{m/s}^2)} = 572\ \text{m}$$

Sum the two results to find the total displacement:

$$\Delta x_{\text{coasting}} + \Delta x_{\text{braking}} = 72\ \text{m} + 572\ \text{m} = 644\ \text{m}$$

Remarks To find the displacement while braking, we could have used the two kinematics equations involving time, namely, $\Delta x = v_0 t + \frac{1}{2}at^2$ and $v = v_0 + at$, but because we weren't interested in time, the time-independent equation was easier to use.

QUESTION 2.6

How would the answer change if the plane coasted for 2.0 s before the pilot applied the brakes?

EXERCISE 2.6

A jet lands at 80.0 m/s, the pilot applying the brakes 2.00 s after landing. Find the acceleration needed to stop the jet within 5.00×10^2 m.

Answer $a = -9.41$ m/s^2

EXAMPLE 2.7 The Acela: The Porsche of American Trains

Goal Find accelerations and displacements from a velocity vs. time graph.

Problem The sleek high-speed electric train known as the Acela (pronounced ahh-sell-ah) is currently in service on the Washington-New York-Boston run. The Acela consists of two power cars and six coaches and can carry 304 passengers at speeds up to 170 mi/h. In order to negotiate curves comfortably at high speeds, the train carriages tilt as much as 6° from the vertical, preventing passengers from being pushed to the side. A velocity vs. time graph for the Acela is shown in Figure 2.19a. **(a)** Describe the motion of the Acela. **(b)** Find the peak acceleration of the Acela in miles per hour per second ((mi/h)/s) as the train speeds up from 45 mi/h to 170 mi/h. **(c)** Find the train's displacement in miles between $t = 0$ and $t = 200$ s. **(d)** Find the average acceleration of the Acela and its displacement in miles in the interval from 200 s to 300 s. (The train has regenerative braking, which means that it feeds energy back into the utility lines each time it stops!) **(e)** Find the total displacement in the interval from 0 to 400 s.

Strategy For part (a), remember that the slope of the tangent line at any point of the velocity vs. time graph gives the acceleration at that time. To find the peak acceleration in part (b), study the graph and locate the point at which the slope is steepest. In parts (c) through (e), estimating the area under the curve gives the displacement during a given period, with areas below the time axis, as in part (e), subtracted from the total. The average acceleration in part (d) can be obtained by substituting numbers taken from the graph into the definition of average acceleration, $\overline{a} = \Delta v/\Delta t$.

Solution

(a) Describe the motion.

From about −50 s to 50 s, the Acela cruises at a constant velocity in the $+x$-direction. Then the train accelerates in the $+x$-direction from 50 s to 200 s, reaching a top speed of about 170 mi/h, whereupon it brakes to rest at 350 s and reverses, steadily gaining speed in the $-x$-direction.

(b) Find the peak acceleration.

Calculate the slope of the steepest tangent line, which connects the points (50 s, 50 mi/h) and (100 s, 150 mi/h) (the light blue line in Figure 2.19b):

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(1.5 \times 10^2 - 5.0 \times 10^1)\text{ mi/h}}{(1.0 \times 10^2 - 5.0 \times 10^1)\text{s}}$$

$$= 2.0\text{ (mi/h)/s}$$

(c) Find the displacement between 0 s and 200 s.

Using triangles and rectangles, approximate the area in Figure 2.19c:

$$\begin{aligned}\Delta x_{0 \to 200\text{ s}} &= \text{area}_1 + \text{area}_2 + \text{area}_3 + \text{area}_4 + \text{area}_5 \\ &\approx (5.0 \times 10^1\text{ mi/h})(5.0 \times 10^1\text{ s}) \\ &\quad + (5.0 \times 10^1\text{ mi/h})(5.0 \times 10^1\text{ s}) \\ &\quad + (1.6 \times 10^2\text{ mi/h})(1.0 \times 10^2\text{ s}) \\ &\quad + \tfrac{1}{2}(5.0 \times 10^1\text{ s})(1.0 \times 10^2\text{ mi/h}) \\ &\quad + \tfrac{1}{2}(1.0 \times 10^2\text{ s})(1.7 \times 10^2\text{ mi/h} - 1.6 \times 10^2\text{ mi/h}) \\ &= 2.4 \times 10^4\text{ (mi/h)s}\end{aligned}$$

Convert units to miles by converting hours to seconds:

$$\Delta x_{0 \to 200\text{ s}} \approx 2.4 \times 10^4 \frac{\text{mi}\cdot\text{s}}{\text{h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 6.7\text{ mi}$$

(d) Find the average acceleration from 200 s to 300 s, and find the displacement.

The slope of the green line is the average acceleration from 200 s to 300 s (Fig. 2.19b):

$$\overline{a} = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(1.0 \times 10^1 - 1.7 \times 10^2)\text{ mi/h}}{1.0 \times 10^2\text{ s}}$$

$$= -1.6\text{ (mi/h)/s}$$

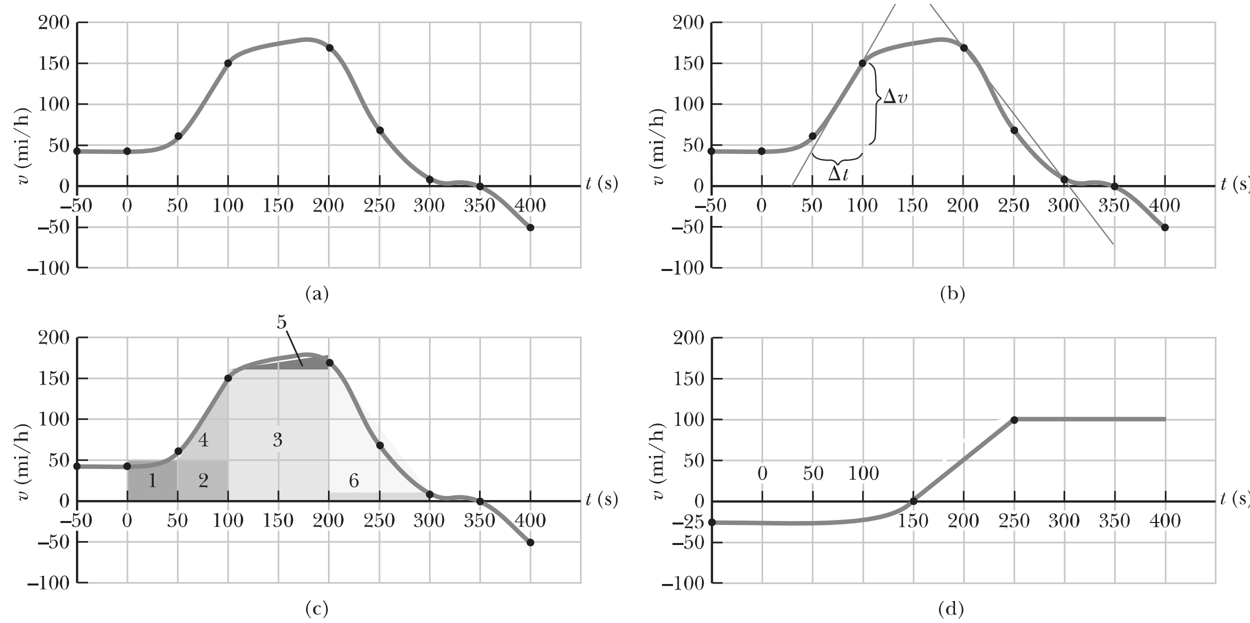


FIGURE 2.19 (Example 2.7) (a) Velocity vs. time graph for the Acela. (b) The slope of the steepest tangent blue line gives the peak acceleration, and the slope of the green line is the average acceleration between 200 s and 300 s. (c) The area under the velocity vs. time graph in some time interval gives the displacement of the Acela in that time interval. (d) (Exercise 2.7).

The displacement from 200 s to 300 s is equal to area$_6$, which is the area of a triangle plus the area of a very narrow rectangle beneath the triangle:

$$\Delta x_{200 \to 300\text{ s}} \approx \tfrac{1}{2}(1.0 \times 10^2 \text{ s})(1.7 \times 10^2 - 1.0 \times 10^1) \text{ mi/h} + (1.0 \times 10^1 \text{ mi/h})(1.0 \times 10^2 \text{ s})$$
$$= 9.0 \times 10^3 (\text{mi/h})(\text{s}) = \boxed{2.5 \text{ mi}}$$

(e) Find the total displacement from 0 s to 400 s.

The total displacement is the sum of all the individual displacements. We still need to calculate the displacements for the time intervals from 300 s to 350 s and from 350 s to 400 s. The latter is negative because it's below the time axis.

$$\Delta x_{300 \to 350\text{ s}} \approx \tfrac{1}{2}(5.0 \times 10^1 \text{ s})(1.0 \times 10^1 \text{ mi/h})$$
$$= 2.5 \times 10^2 (\text{mi/h})(\text{s})$$
$$\Delta x_{350 \to 400\text{ s}} \approx \tfrac{1}{2}(5.0 \times 10^1 \text{ s})(-5.0 \times 10^1 \text{ mi/h})$$
$$= -1.3 \times 10^3 (\text{mi/h})(\text{s})$$

Find the total displacement by summing the parts:

$$\Delta x_{0 \to 400\text{ s}} \approx (2.4 \times 10^4 + 9.0 \times 10^3 + 2.5 \times 10^2 - 1.3 \times 10^3)(\text{mi/h})(\text{s}) = \boxed{8.9 \text{ mi}}$$

Remarks There are a number of ways to find the approximate area under a graph. Choice of technique is a personal preference.

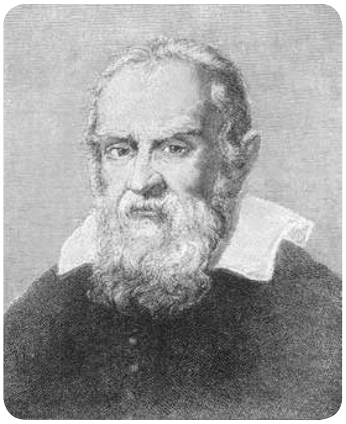
QUESTION 2.7

According to the graph in Figure 2.19a, at what different times is the acceleration zero?

EXERCISE 2.7

Suppose the velocity vs. time graph of another train is given in Figure 2.19d. Find (a) the maximum instantaneous acceleration and (b) the total displacement in the interval from 0 s to 4.00×10^2 s.

Answers (a) 1.0 (mi/h)/s (b) 4.7 mi



North Wind Archive

GALILEO GALILEI
Italian Physicist and Astronomer (1564–1642)

Galileo formulated the laws that govern the motion of objects in free fall. He also investigated the motion of an object on an inclined plane, established the concept of relative motion, invented the thermometer, and discovered that the motion of a swinging pendulum could be used to measure time intervals. After designing and constructing his own telescope, he discovered four of Jupiter's moons, found that our own Moon's surface is rough, discovered sunspots and the phases of Venus, and showed that the Milky Way consists of an enormous number of stars. Galileo publicly defended Nicolaus Copernicus's assertion that the Sun is at the center of the Universe (the heliocentric system). He published *Dialogue Concerning Two New World Systems* to support the Copernican model, a view the Church declared to be heretical. After being taken to Rome in 1633 on a charge of heresy, he was sentenced to life imprisonment and later was confined to his villa at Arcetri, near Florence, where he died in 1642.

2.6 FREELY FALLING OBJECTS

When air resistance is negligible, all objects dropped under the influence of gravity near Earth's surface fall toward Earth with the same constant acceleration. This idea may seem obvious today, but it wasn't until about 1600 that it was accepted. Prior to that time, the teachings of the great philosopher Aristotle (384–322 B.C.) had held that heavier objects fell faster than lighter ones.

According to legend, Galileo discovered the law of falling objects by observing that two different weights dropped simultaneously from the Leaning Tower of Pisa hit the ground at approximately the same time. Although it's unlikely that this particular experiment was carried out, we know that Galileo performed many systematic experiments with objects moving on inclined planes. In his experiments he rolled balls down a slight incline and measured the distances they covered in successive time intervals. The purpose of the incline was to reduce the acceleration and enable Galileo to make accurate measurements of the intervals. (Some people refer to this experiment as "diluting gravity.") By gradually increasing the slope of the incline he was finally able to draw mathematical conclusions about freely falling objects, because a falling ball is equivalent to a ball going down a vertical incline. Galileo's achievements in the science of mechanics paved the way for Newton in his development of the laws of motion, which we will study in Chapter 4.

Try the following experiment: Drop a hammer and a feather simultaneously from the same height. The hammer hits the floor first because air drag has a greater effect on the much lighter feather. On August 2, 1971, this same experiment was conducted on the Moon by astronaut David Scott, and the hammer and feather fell with exactly the same acceleration, as expected, hitting the lunar surface at the same time. In the idealized case where air resistance is negligible, such motion is called *free fall.*

The expression *freely falling object* doesn't necessarily refer to an object dropped from rest. **A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion.** Objects thrown upward or downward and those released from rest are all considered freely falling.

We denote the magnitude of the **free-fall acceleration** by the symbol g. The value of g decreases with increasing altitude, and varies slightly with latitude as well. At Earth's surface, the value of g is approximately 9.80 m/s^2. Unless stated otherwise, we will use this value for g in doing calculations. For quick estimates, use $g \approx 10$ m/s^2.

If we neglect air resistance and assume that the free-fall acceleration doesn't vary with altitude over short vertical distances, then the motion of a freely falling object is the same as motion in one dimension under constant acceleration. This means that the kinematics equations developed in Section 2.6 can be applied. It's conventional to define "up" as the $+\,y$-direction and to use y as the position variable. In that case the acceleration is $a = -g = -9.80$ m/s^2. In Chapter 7, we study the variation in g with altitude.

QUICK QUIZ 2.6 A tennis player on serve tosses a ball straight up. While the ball is in free fall, does its acceleration (a) increase, (b) decrease, (c) increase and then decrease, (d) decrease and then increase, or (e) remain constant?

QUICK QUIZ 2.7 As the tennis ball of Quick Quiz 2.6 travels through the air, does its speed (a) increase, (b) decrease, (c) decrease and then increase, (d) increase and then decrease, or (e) remain the same?

QUICK QUIZ 2.8 A skydiver jumps out of a hovering helicopter. A few seconds later, another skydiver jumps out, so they both fall along the same vertical line relative to the helicopter. Both skydivers fall with the same acceleration. Does the vertical distance between them (a) increase, (b) decrease, or (c) stay the same? Does the difference in their velocities (d) increase, (e) decrease, or (f) stay the same? (Assume g is constant.)

EXAMPLE 2.8 Not a Bad Throw for a Rookie!

Goal Apply the kinematic equations to a freely falling object with a nonzero initial velocity.

Problem A stone is thrown from the top of a building with an initial velocity of 20.0 m/s straight upward, at an initial height of 50.0 m above the ground. The stone just misses the edge of the roof on its way down, as shown in Figure 2.20. Determine **(a)** the time needed for the stone to reach its maximum height, **(b)** the maximum height, **(c)** the time needed for the stone to return to the height from which it was thrown and the velocity of the stone at that instant, **(d)** the time needed for the stone to reach the ground, and **(e)** the velocity and position of the stone at $t = 5.00$ s.

Strategy The diagram in Figure 2.20 establishes a coordinate system with $y_0 = 0$ at the level at which the stone is released from the thrower's hand, with y positive upward. Write the velocity and position kinematic equations for the stone, and substitute the given information. All the answers come from these two equations by using simple algebra or by just substituting the time. In part (a), for example, the stone comes to rest for an instant at its maximum height, so set $v = 0$ at this point and solve for time. Then substitute the time into the displacement equation, obtaining the maximum height.

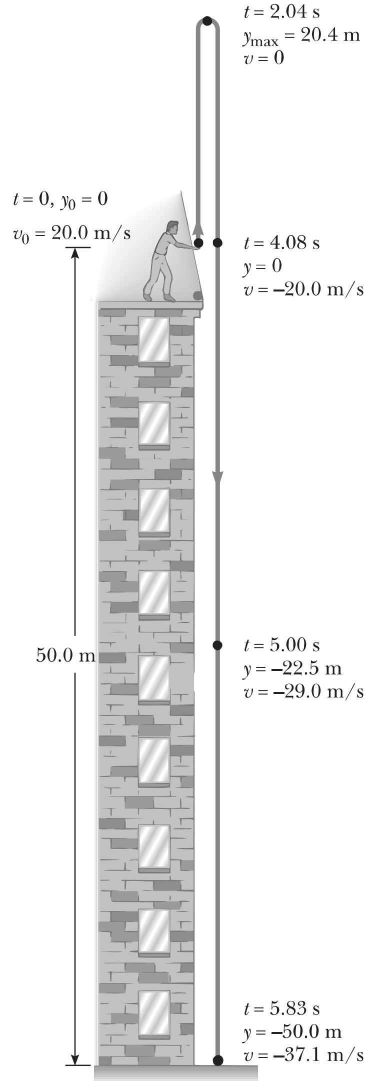


FIGURE 2.20 (Example 2.8) A freely falling object is thrown upward with an initial velocity of $v_0 = +20.0$ m/s. Positions and velocities are given for several times.

Solution

(a) Find the time when the stone reaches its maximum height.

Write the velocity and position kinematic equations:

$$v = at + v_0$$
$$\Delta y = y - y_0 = v_0 t + \tfrac{1}{2}at^2$$

Substitute $a = -9.80\ \text{m/s}^2$, $v_0 = 20.0$ m/s, and $y_0 = 0$ into the preceding two equations:

$$\textbf{(1)}\quad v = (-9.80\ \text{m/s}^2)t + 20.0\ \text{m/s}$$
$$\textbf{(2)}\quad y = (20.0\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$$

Substitute $v = 0$, the velocity at maximum height, into Equation (1) and solve for time:

$$0 = (-9.80\ \text{m/s}^2)t + 20.0\ \text{m/s}$$
$$t = \frac{-20.0\ \text{m/s}}{-9.80\ \text{m/s}^2} = \boxed{2.04\ \text{s}}$$

(b) Determine the stone's maximum height.

Substitute the time $t = 2.04$ s into Equation (2):

$$y_{max} = (20.0\ \text{m/s})(2.04\ \text{s}) - (4.90\ \text{m/s}^2)(2.04\ \text{s})^2 = \boxed{20.4\ \text{m}}$$

(c) Find the time the stone takes to return to its initial position, and find the velocity of the stone at that time.

Set y = 0 in Equation (2) and solve t:

$$0 = (20.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2$$
$$= t(20.0\text{ m/s} - 4.90\text{ m/s}^2 t)$$
$$t = \boxed{4.08\text{ s}}$$

Substitute the time into Equation (1) to get the velocity:

$$v = 20.0\text{ m/s} + (-9.80\text{ m/s}^2)(4.08\text{ s}) = \boxed{-20.0\text{ m/s}}$$

(d) Find the time required for the stone to reach the ground.

In Equation (2), set $y = -50.0$ m:

$$-50.0\text{ m} = (20.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2$$

Apply the quadratic formula and take the positive root:

$$\boxed{t = 5.83\text{ s}}$$

(e) Find the velocity and position of the stone at $t = 5.00$ s.

Substitute values into Equations (1) and (2):

$$v = (-9.80\text{ m/s}^2)(5.00\text{ s}) + 20.0\text{ m/s} = \boxed{-29.0\text{ m/s}}$$
$$y = (20.0\text{ m/s})(5.00\text{ s}) - (4.90\text{ m/s}^2)(5.00\text{ s})^2 = \boxed{-22.5\text{ m}}$$

Remarks Notice how everything follows from the two kinematic equations. Once they are written down and the constants correctly identified as in Equations (1) and (2), the rest is relatively easy. If the stone were thrown downward, the initial velocity would have been negative.

QUESTION 2.8

How would the answer to part (b), the maximum height, change if the person throwing the ball jumped upward at the instant he released the ball?

EXERCISE 2.8

A projectile is launched straight up at 60.0 m/s from a height of 80.0 m, at the edge of a sheer cliff. The projectile falls, just missing the cliff and hitting the ground below. Find (a) the maximum height of the projectile above the point of firing, (b) the time it takes to hit the ground at the base of the cliff, and (c) its velocity at impact.

Answers (a) 184 m (b) 13.5 s (c) −72.3 m/s

EXAMPLE 2.9 Maximum Height Derived

Goal Find the maximum height of a thrown projectile using symbols.

Problem Refer to Example 2.8. Use symbolic manipulation to find **(a)** the time t_{max} it takes the ball to reach its maximum height and **(b)** an expression for the maximum height that doesn't depend on time. Answers should be expressed in terms of the quantities v_0, g, and y_0 only.

Strategy When the ball reaches its maximum height, its velocity is zero, so for part (a) solve the kinematics velocity equation for time t and set $v = 0$. For part (b), substitute the expression for time found in part (a) into the displacement equation, solving it for the maximum height.

Solution

(a) Find the time it takes the ball to reach its maximum height.

Write the velocity kinematics equation:

$$v = at + v_0$$

Move v_0 to the left side of the equation:

$$v - v_0 = at$$

Divide both sides by a:

$$\frac{v - v_0}{a} = \frac{\cancel{a}t}{\cancel{a}} = t$$

Turn the equation around so that t is on the left and substitute $v = 0$, corresponding to the velocity at maximum height:

$$\textbf{(1)}\quad t = \frac{-v_0}{a}$$

Replace t by t_{max} and substitute $a = -g$:

$$\textbf{(2)}\quad t_{max} = \frac{v_0}{g}$$

(b) Find the maximum height.

Write the equation for the position y at any time:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

Substitute $t = -v_0/a$, which corresponds to the time it takes to reach y_{max}, the maximum height:

$$y_{max} = y_0 + v_0\left(\frac{-v_0}{a}\right) + \tfrac{1}{2}a\left(\frac{-v_0}{a}\right)^2$$

$$= y_0 - \frac{v_0^2}{a} + \tfrac{1}{2}\frac{v_0^2}{a}$$

Combine the last two terms and substitute $a = -g$:

$$\textbf{(3)}\quad y_{max} = y_0 + \frac{v_0^2}{2g}$$

Remarks Notice that $g = +9.8\ \text{m/s}^2$, so the second term is positive overall. Equations (1)–(3) are much more useful than a numerical answer because the effect of changing one value can be seen immediately. For example, doubling the initial velocity v_0 quadruples the displacement above the point of release. Notice also that y_{max} could be obtained more readily from the time-independent equation, $v^2 - v_0^2 = 2a\,\Delta y$.

QUESTION 2.9

By what factor would the maximum displacement above the rooftop be increased if the building were transported to the Moon, where $a = -\frac{1}{6}g$?

EXERCISE 2.9

(a) Using symbols, find the time t_E it takes for a ball to reach the ground on Earth if released from rest at height y_0. (b) In terms of t_E, how much time t_M would be required if the building were on Mars, where $a = -0.385g$?

Answers (a) $t_E = \sqrt{\dfrac{2y_0}{g}}$ (b) $t_M = 1.61t_E$

EXAMPLE 2.10 A Rocket Goes Ballistic

Goal Solve a problem involving a powered ascent followed by free-fall motion.

Problem A rocket moves straight upward, starting from rest with an acceleration of $+29.4\ \text{m/s}^2$. It runs out of fuel at the end of 4.00 s and continues to coast upward, reaching a maximum height before falling back to Earth. **(a)** Find the rocket's velocity and position at the end of 4.00 s. **(b)** Find the maximum height the rocket reaches. **(c)** Find the velocity the instant before the rocket crashes on the ground.

Strategy Take $y = 0$ at the launch point and y positive upward, as in Figure 2.21. The problem consists of two phases. In phase 1 the rocket has a net *upward* acceleration of $29.4\ \text{m/s}^2$, and we can use the kinematic equations with constant acceleration a to find the height and velocity of the rocket at the end of phase 1, when the fuel is burned up. In phase 2 the rocket is in free fall and has an acceleration of $-9.80\ \text{m/s}^2$, with initial velocity and position given by the results of phase 1. Apply the kinematic equations for free fall.

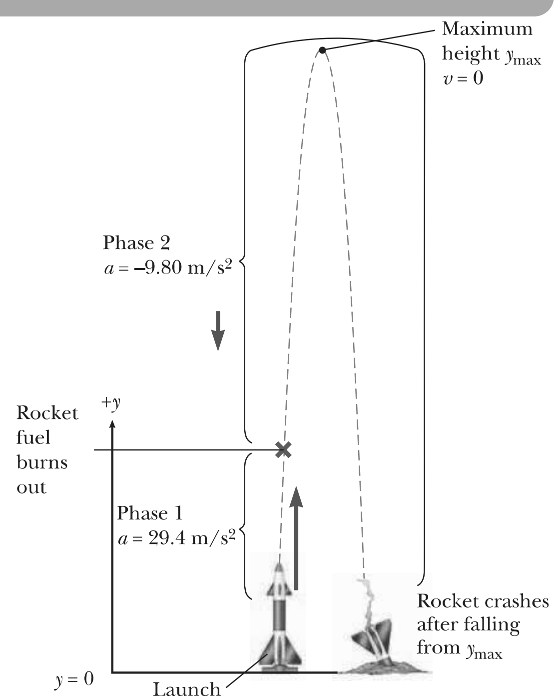


FIGURE 2.21 (Example 2.10) Two linked phases of motion for a rocket that is launched, uses up its fuel, and crashes.

Solution

(a) Phase 1: Find the rocket's velocity and position after 4.00 s.

Write the velocity and position kinematic equations:

(1) $v = v_0 + at$

(2) $\Delta y = y - y_0 = v_0 t + \frac{1}{2} a t^2$

Adapt these equations to phase 1, substituting $a = 29.4\ \text{m/s}^2$, $v_0 = 0$, and $y_0 = 0$:

(3) $v = (29.4\ \text{m/s}^2)t$

(4) $y = \frac{1}{2}(29.4\ \text{m/s}^2)t^2 = (14.7\ \text{m/s}^2)t^2$

Substitute $t = 4.00$ s into Equations (3) and (4) to find the rocket's velocity v and position y at the time of burnout. These will be called v_b and y_b, respectively.

$v_b = 118\ \text{m/s}$ and $y_b = 235\ \text{m}$

(b) Phase 2: Find the maximum height the rocket attains.

Adapt Equations (1) and (2) to phase 2, substituting $a = -9.8\ \text{m/s}^2$, $v_0 = v_b = 118\ \text{m/s}$, and $y_0 = y_b = 235$ m:

(5) $v = (-9.8\ \text{m/s}^2)t + 118\ \text{m/s}$

(6) $y = 235\ \text{m} + (118\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$

Substitute $v = 0$ (the rocket's velocity at maximum height) in Equation (5) to get the time it takes the rocket to reach its maximum height:

$$0 = (-9.8\ \text{m/s}^2)t + 118\ \text{m/s} \quad \rightarrow \quad t = \frac{118\ \text{m/s}}{9.80\ \text{m/s}^2} = 12.0\ \text{s}$$

Substitute $t = 12.0$ s into Equation (6) to find the rocket's maximum height:

$$y_{\text{max}} = 235\ \text{m} + (118\ \text{m/s})(12.0\ \text{s}) - (4.90\ \text{m/s}^2)(12.0\ \text{s})^2$$
$$= 945\ \text{m}$$

(c) Phase 2: Find the velocity of the rocket just prior to impact.

Find the time to impact by setting $y = 0$ in Equation (6) and using the quadratic formula:

$$0 = 235\ \text{m} + (118\ \text{m/s})t - (4.90\ \text{m/s})t^2$$
$$t = 25.9\ \text{s}$$

Substitute this value of t into Equation (5):

$$v = (-9.80\ \text{m/s}^2)(25.9\ \text{s}) + 118\ \text{m/s} = -136\ \text{m/s}$$

Remarks You may think that it is more natural to break this problem into three phases, with the second phase ending at the maximum height and the third phase a free fall from maximum height to the ground. Although this approach gives the correct answer, it's an unnecessary complication. Two phases are sufficient, one for each different acceleration.

QUESTION 2.10

If, instead, some fuel remains, at what height should the engines be fired again to brake the rocket's fall and allow a perfectly soft landing? (Assume the same acceleration as during the initial ascent.)

EXERCISE 2.10

An experimental rocket designed to land upright falls freely from a height of 2.00×10^2 m, starting at rest. At a height of 80.0 m, the rocket's engines start and provide constant upward acceleration until the rocket lands. What acceleration is required if the speed on touchdown is to be zero? (Neglect air resistance.)

Answer $14.7\ \text{m/s}^2$

SUMMARY

2.1 Displacement

The **displacement** of an object moving along the x-axis is defined as the change in position of the object,

$$\Delta x \equiv x_f - x_i \quad [2.1]$$

where x_i is the initial position of the object and x_f is its final position.

A **vector** quantity is characterized by both a magnitude and a direction. A **scalar** quantity has a magnitude only.

2.2 Velocity

The **average speed** of an object is given by

$$\text{Average speed} \equiv \frac{\text{total distance}}{\text{total time}}$$

The **average velocity** $\overline{v}$ during a time interval Δt is the displacement Δx divided by Δt.

$$\overline{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \quad [2.2]$$

The average velocity is equal to the slope of the straight line joining the initial and final points on a graph of the position of the object versus time.

The slope of the line tangent to the position vs. time curve at some point is equal to the **instantaneous velocity** at that time. The **instantaneous speed** of an object is defined as the magnitude of the instantaneous velocity.

2.3 Acceleration

The **average acceleration** $\overline{a}$ of an object undergoing a change in velocity Δv during a time interval Δt is

$$\overline{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \quad [2.4]$$

The **instantaneous acceleration** of an object at a certain time equals the slope of a velocity vs. time graph at that instant.

2.5 One-Dimensional Motion with Constant Acceleration

The most useful equations that describe the motion of an object moving with constant acceleration along the x-axis are as follows:

$$v = v_0 + at \quad [2.6]$$

$$\Delta x = v_0 t + \tfrac{1}{2}at^2 \quad [2.9]$$

$$v^2 = v_0^{\,2} + 2a\Delta x \quad [2.10]$$

All problems can be solved with the first two equations alone, the last being convenient when time doesn't explicitly enter the problem. After the constants are properly identified, most problems reduce to one or two equations in as many unknowns.

2.6 Freely Falling Objects

An object falling in the presence of Earth's gravity exhibits a free-fall acceleration directed toward Earth's center. If air friction is neglected and if the altitude of the falling object is small compared with Earth's radius, then we can assume that the free-fall acceleration $g = 9.8 \text{ m/s}^2$ is constant over the range of motion. Equations 2.6, 2.9, and 2.10 apply, with $a = -g$.

FOR ADDITIONAL STUDENT RESOURCES, GO TO WWW.SERWAYPHYSICS.COM

MULTIPLE-CHOICE QUESTIONS

1. An arrow is shot straight up in the air at an initial speed of 15.0 m/s. After how much time is the arrow heading downward at a speed of 8.00 m/s? (a) 0.714 s (b) 1.24 s (c) 1.87 s (d) 2.35 s (e) 3.22 s

2. A cannon shell is fired straight up in the air at an initial speed of 225 m/s. After how much time is the shell at a height of 6.20×10^2 m and heading down? (a) 2.96 s (b) 17.3 s (c) 25.4 s (d) 33.6 s (e) 43.0 s

3. When applying the equations of kinematics for an object moving in one dimension, which of the following statements *must* be true? (a) The velocity of the object must remain constant. (b) The acceleration of the object must remain constant. (c) The velocity of the object must increase with time. (d) The position of the object must increase with time. (e) The velocity of the object must always be in the same direction as its acceleration.

4. A juggler throws a bowling pin straight up in the air. After the pin leaves his hand and while it is in the air, which statement is true? (a) The velocity of the pin is always in the same direction as its acceleration. (b) The velocity of the pin is never in the same direction as its acceleration. (c) The acceleration of the pin is zero. (d) The velocity of the pin is opposite its acceleration on the way up. (e) The velocity of the pin is in the same direction as its acceleration on the way up.

5. A racing car starts from rest and reaches a final speed v in a time t. If the acceleration of the car is constant during this time, which of the following statements must be true? (a) The car travels a distance vt. (b) The average speed of the car is $v/2$. (c) The acceleration of the car is v/t. (d) The velocity of the car remains constant. (e) None of these

6. A pebble is dropped from rest from the top of a tall cliff and falls 4.9 m after 1.0 s has elapsed. How much farther does it drop in the next 2.0 seconds? (a) 9.8 m (b) 19.6 m (c) 39 m (d) 44 m (e) 27 m

7. An object moves along the x-axis, its position measured at each instant of time. The data are organized into an accurate graph of x vs. t. Which of the following quantities *cannot* be obtained from this graph? (a) the velocity at any instant (b) the acceleration at any instant (c) the displacement during some time interval (d) the average velocity during some time interval (e) the speed of the particle at any instant

8. People become uncomfortable in an elevator if it accelerates from rest at a rate such that it attains a speed of about 6 m/s after descending ten stories (about 30 m). What is the approximate magnitude of its acceleration? (Choose the closest answer.) (a) 10 m/s^2 (b) 0.3 m/s^2 (c) 0.6 m/s^2 (d) 1 m/s^2 (e) 0.8 m/s^2

9. Races are timed to an accuracy of 1/1 000 of a second. What distance could a person rollerblading at a speed of 8.5 m/s travel in that period of time? (a) 85 mm (b) 85 cm (c) 8.5 m (d) 8.5 mm (e) 8.5 km

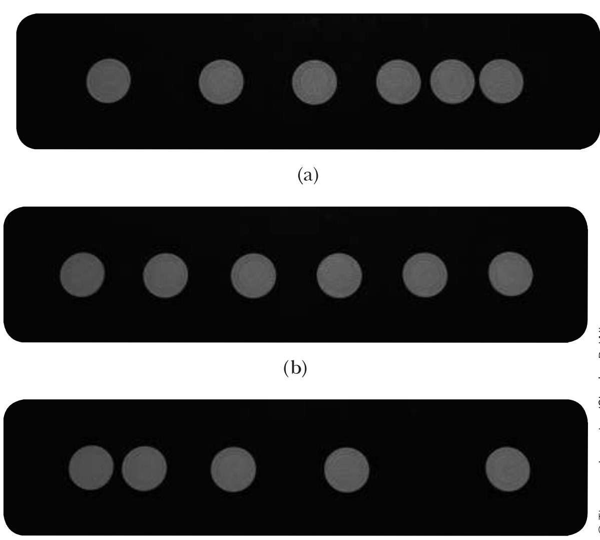
10. A student at the top of a building throws a red ball upward with speed v_0 and then throws a blue ball downward with the same initial speed v_0. Immediately before the two balls reach the ground, which of the following statements are true? (Choose all correct statements; neglect air friction.) (a) The speed of the red ball is less than that of the blue ball. (b) The speed of the red ball is greater than that of the blue ball. (c) Their velocities are equal. (d) The speed of each ball is greater than v_0. (e) The acceleration of the blue ball is greater than that of the red ball.

11. A rock is thrown downward from the top of a 40.0 m tower with an initial speed of 12 m/s. Assuming negligible air resistance, what is the speed of the rock just before hitting the ground? (a) 28 m/s (b) 30 m/s (c) 56 m/s (d) 784 m/s (e) More information is needed.

12. A ball is thrown straight up in the air. For which situation are both the instantaneous velocity and the acceleration zero? (a) on the way up (b) at the top of the flight path (c) on the way down (d) halfway up and halfway down (e) none of these

CONCEPTUAL QUESTIONS

1. If the velocity of a particle is nonzero, can the particle's acceleration be zero? Explain.
2. If the velocity of a particle is zero, can the particle's acceleration be zero? Explain.
3. If a car is traveling eastward, can its acceleration be westward? Explain.
4. Can the equations of kinematics be used in a situation where the acceleration varies with time? Can they be used when the acceleration is zero?
5. If the average velocity of an object is zero in some time interval, what can you say about the displacement of the object during that interval?
6. Figure CQ2.6 shows strobe photographs taken of a disk moving from left to right under different conditions. The time interval between images is constant. Taking the direction to the right to be positive, describe the motion of the disk in each case. For which case is



(c)

FIGURE CQ2.6

(a) the acceleration positive? (b) the acceleration negative? (c) the velocity constant?

7. Can the instantaneous velocity of an object at an instant of time ever be greater in magnitude than the average velocity over a time interval containing that instant? Can it ever be less?
8. A ball is thrown vertically upward. (a) What are its velocity and acceleration when it reaches its maximum altitude? (b) What is the acceleration of the ball just before it hits the ground?
9. Consider the following combinations of signs and values for the velocity and acceleration of a particle with respect to a one-dimensional x-axis:

	Velocity	Acceleration
a.	Positive	Positive
b.	Positive	Negative
c.	Positive	Zero
d.	Negative	Positive
e.	Negative	Negative
f.	Negative	Zero
g.	Zero	Positive
h.	Zero	Negative

Describe what the particle is doing in each case and give a real-life example for an automobile on an east–west one-dimensional axis, with east considered the positive direction.

10. A ball rolls in a straight line along the horizontal direction. Using motion diagrams (or multiflash photographs), describe the velocity and acceleration of the ball for each of the following situations: (a) The ball moves to the right at a constant speed. (b) The ball moves from right to left and continually slows down. (c) The ball moves from right to left and continually speeds up. (d) The ball moves to the right, first speeding up at a constant rate and then slowing down at a constant rate.

PROBLEMS

ENHANCED WebAssign The Problems for this chapter may be assigned online at WebAssign.

1, 2, 3 = straightforward, intermediate, challenging

GP = denotes guided problem

ecp = denotes enhanced content problem

= biomedical application

☐ = denotes full solution available in *Student Solutions Manual/Study Guide*

SECTION 2.1 DISPLACEMENT

SECTION 2.2 VELOCITY

1. The speed of a nerve impulse in the human body is about 100 m/s. If you accidentally stub your toe in the dark, estimate the time it takes the nerve impulse to travel to your brain.

2. Light travels at a speed of about 3×10^8 m/s. How many miles does a pulse of light travel in a time interval of 0.1 s, which is about the blink of an eye? Compare this distance to the diameter of Earth.

3. A person travels by car from one city to another with different constant speeds between pairs of cities. She drives for 30.0 min at 80.0 km/h, 12.0 min at 100 km/h, and 45.0 min at 40.0 km/h and spends 15.0 min eating lunch and buying gas. (a) Determine the average speed for the trip. (b) Determine the distance between the initial and final cities along the route.

4. (a) Sand dunes on a desert island move as sand is swept up the windward side to settle in the leeward side. Such "walking" dunes have been known to travel 20 feet in a year and can travel as much as 100 feet per year in particularly windy times. Calculate the average speed in each case in meters per second. (b) Fingernails grow at the rate of drifting continents, about 10 mm/yr. Approximately how long did it take for North America to separate from Europe, a distance of about 3 000 mi?

5. Two boats start together and race across a 60-km-wide lake and back. Boat A goes across at 60 km/h and returns at 60 km/h. Boat B goes across at 30 km/h, and its crew, realizing how far behind it is getting, returns at 90 km/h. Turnaround times are negligible, and the boat that completes the round trip first wins. (a) Which boat wins and by how much? (Or is it a tie?) (b) What is the average velocity of the winning boat?

6. A graph of position versus time for a certain particle moving along the x-axis is shown in Figure P2.6. Find the average velocity in the time intervals from (a) 0 to 2.00 s, (b) 0 to 4.00 s, (c) 2.00 s to 4.00 s, (d) 4.00 s to 7.00 s, and (e) 0 to 8.00 s.

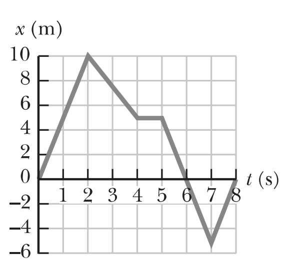


FIGURE P2.6 (Problems 6 and 17)

7. A motorist drives north for 35.0 minutes at 85.0 km/h and then stops for 15.0 minutes. He then continues north, traveling 130 km in 2.00 h. (a) What is his total displacement? (b) What is his average velocity?

8. A tennis player moves in a straight-line path as shown in Figure P2.8. Find her average velocity in the time intervals from (a) 0 to 1.0 s, (b) 0 to 4.0 s, (c) 1.0 s to 5.0 s, and (d) 0 to 5.0 s.

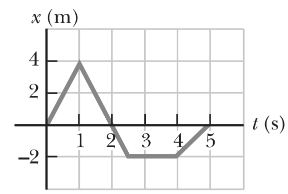


FIGURE P2.8 (Problems 8 and 9)

9. Find the instantaneous velocities of the tennis player of Figure P2.8 at (a) 0.50 s, (b) 2.0 s, (c) 3.0 s, and (d) 4.5 s.

10. Two cars travel in the same direction along a straight highway, one at a constant speed of 55 mi/h and the other at 70 mi/h. (a) Assuming they start at the same point, how much sooner does the faster car arrive at a destination 10 mi away? (b) How far must the faster car travel before it has a 15-min lead on the slower car?

11. If the average speed of an orbiting space shuttle is 19 800 mi/h, determine the time required for it to circle Earth. Make sure you consider that the shuttle is orbiting about 2.00×10^2 mi above Earth's surface and assume that Earth's radius is 3 963 miles.

12. ecp An athlete swims the length L of a pool in a time t_1 and makes the return trip to the starting position in a time t_2. If she is swimming initially in the positive x-direction, determine her average velocities symbolically in (a) the first half of the swim, (b) the second half of the swim, and (c) the round trip. (d) What is her average speed for the round trip?

13. A person takes a trip, driving with a constant speed of 89.5 km/h, except for a 22.0-min rest stop. If the person's average speed is 77.8 km/h, how much time is spent on the trip and how far does the person travel?

14. A tortoise can run with a speed of 0.10 m/s, and a hare can run 20 times as fast. In a race, they both start at the same time, but the hare stops to rest for 2.0 minutes. The tortoise wins by a shell (20 cm). (a) How long does the race take? (b) What is the length of the race?

15. To qualify for the finals in a racing event, a race car must achieve an average speed of 250 km/h on a track with a total length of 1 600 m. If a particular car covers the first half of the track at an average speed of 230 km/h, what minimum average speed must it have in the second half of the event in order to qualify?

16. **ecp** One athlete in a race running on a long, straight track with a constant speed v_1 is a distance d behind a second athlete running with a constant speed v_2. (a) Under what circumstances is the first athlete able to overtake the second athlete? (b) Find the time t it takes the first athlete to overtake the second athlete, in terms of d, v_1, and v_2. (c) At what minimum distance d_2 from the leading athlete must the finish line be located so that the trailing athlete can at least tie for first place? Express d_2 in terms of d, v_1, and v_2 by using the result of part (b).

17. A graph of position versus time for a certain particle moving along the x-axis is shown in Figure P2.6. Find the instantaneous velocity at the instants (a) $t = 1.00$ s, (b) $t = 3.00$ s, (c) $t = 4.50$ s, and (d) $t = 7.50$ s.

18. A race car moves such that its position fits the relationship

$$x = (5.0\text{ m/s})t + (0.75\text{ m/s}^3)t^3$$

where x is measured in meters and t in seconds. (a) Plot a graph of the car's position versus time. (b) Determine the instantaneous velocity of the car at $t = 4.0$ s, using time intervals of 0.40 s, 0.20 s, and 0.10 s. (c) Compare the average velocity during the first 4.0 s with the results of part (b).

19. Runner A is initially 4.0 mi west of a flagpole and is running with a constant velocity of 6.0 mi/h due east. Runner B is initially 3.0 mi east of the flagpole and is running with a constant velocity of 5.0 mi/h due west. How far are the runners from the flagpole when they meet?

SECTION 2.3 ACCELERATION

20. **ecp** Assume a canister in a straight tube moves with a constant acceleration of -4.00 m/s^2 and has a velocity of 13.0 m/s at $t = 0$. (a) What is its velocity at $t = 1.00$ s? (b) At $t = 2.00$ s? (c) At $t = 2.50$ s? (d) At $t = 4.00$ s? (e) Describe the shape of the canister's velocity versus time graph. (f) What two things must be known at a given time to predict the canister's velocity at any later time?

21. Secretariat ran the Kentucky Derby with times of 25.2 s, 24.0 s, 23.8 s, 23.2 s, and 23.0 s for the quarter mile. (a) Find his average speed during each quarter-mile segment in ft/s. (b) Assuming that Secretariat's instantaneous speed at the finish line was the same as his average speed during the final quarter mile, find his average acceleration for the entire race in ft/s^2. (*Hint:* Recall that horses in the Derby start from rest.)

22. The average person passes out at an acceleration of 7 g (that is, seven times the gravitational acceleration on Earth). Suppose a car is designed to accelerate at this rate. How much time would be required for the car to accelerate from rest to 60.0 miles per hour? (The car would need rocket boosters!)

23. A certain car is capable of accelerating at a rate of 10.60 m/s^2. How long does it take for this car to go from a speed of 55 mi/h to a speed of 60 mi/h?

24. The velocity vs. time graph for an object moving along a straight path is shown in Figure P2.24. (a) Find the average acceleration of the object during the time intervals 0 to 5.0 s, 5.0 s to 15 s, and 0 to 20 s. (b) Find the instantaneous acceleration at 2.0 s, 10 s, and 18 s.

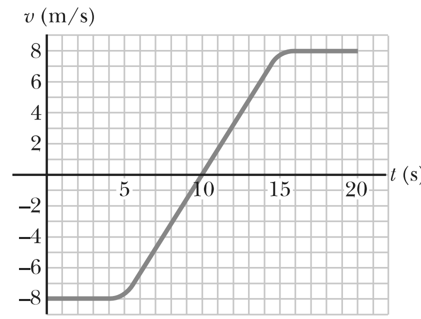


FIGURE P2.24

25. A steam catapult launches a jet aircraft from the aircraft carrier *John C. Stennis*, giving it a speed of 175 mi/h in 2.50 s. (a) Find the average acceleration of the plane. (b) Assuming the acceleration is constant, find the distance the plane moves.

SECTION 2.5 ONE-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

26. A car is traveling due east at 25.0 m/s at some instant. (a) If its constant acceleration is 0.750 m/s^2 due east, find its velocity after 8.50 s have elapsed. (b) If its constant acceleration is 0.750 m/s^2 due west, find its velocity after 8.50 s have elapsed.

27. A car traveling east at 40.0 m/s passes a trooper hiding at the roadside. The driver uniformly reduces his speed to 25.0 m/s in 3.50 s. (a) What is the magnitude and direction of the car's acceleration as it slows down? (b) How far does the car travel in the 3.5-s time period?

28. In 1865 Jules Verne proposed sending men to the Moon by firing a space capsule from a 220-m-long cannon with final speed of 10.97 km/s. What would have been the unrealistically large acceleration experienced by the space travelers during their launch? (A human can stand an acceleration of 15g for a short time.) Compare your answer with the free-fall acceleration, 9.80 m/s^2.

29. A truck covers 40.0 m in 8.50 s while smoothly slowing down to a final velocity of 2.80 m/s. (a) Find the truck's original speed. (b) Find its acceleration.

30. **GP** A speedboat increases its speed uniformly from v_i = 20.0 m/s to v_f = 30.0 m/s in a distance of 2.00×10^2 m. (a) Draw a coordinate system for this situation and label the relevant quantities, including vectors. (b) For the given information, what single equation is most appropriate for finding the acceleration? (c) Solve the equation selected in part (b) symbolically for the boat's acceleration in terms of v_f, v_a, and Δx. (d) Substitute given values, obtaining that acceleration. (e) Find the time it takes the boat to travel the given distance.

31. A Cessna aircraft has a liftoff speed of 120 km/h. (a) What minimum constant acceleration does the aircraft require if it is to be airborne after a takeoff run of 240 m? (b) How long does it take the aircraft to become airborne?

32. A truck on a straight road starts from rest and accelerates at 2.0 m/s^2 until it reaches a speed of 20 m/s. Then the

truck travels for 20 s at constant speed until the brakes are applied, stopping the truck in a uniform manner in an additional 5.0 s. (a) How long is the truck in motion? (b) What is the average velocity of the truck during the motion described?

33. ecp In a test run, a certain car accelerates uniformly from zero to 24.0 m/s in 2.95 s. (a) What is the magnitude of the car's acceleration? (b) How long does it take the car to change its speed from 10.0 m/s to 20.0 m/s? (c) Will doubling the time always double the change in speed? Why?

34. ecp A jet plane lands with a speed of 100 m/s and can accelerate at a maximum rate of -5.00 m/s^2 as it comes to rest. (a) From the instant the plane touches the runway, what is the minimum time needed before it can come to rest? (b) Can this plane land on a small tropical island airport where the runway is 0.800 km long?

35. ecp Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can accelerate only at -2.00 m/s^2 because the road is wet. Will there be a collision? State how you decide. If yes, determine how far into the tunnel and at what time the collision occurs. If no, determine the distance of closest approach between Sue's car and the van.

36. A record of travel along a straight path is as follows:

1. Start from rest with a constant acceleration of 2.77 m/s^2 for 15.0 s.
2. Maintain a constant velocity for the next 2.05 min.
3. Apply a constant negative acceleration of -9.47 m/s^2 for 4.39 s.

(a) What was the total displacement for the trip? (b) What were the average speeds for legs 1, 2, and 3 of the trip, as well as for the complete trip?

37. A train is traveling down a straight track at 20 m/s when the engineer applies the brakes, resulting in an acceleration of -1.0 m/s^2 as long as the train is in motion. How far does the train move during a 40-s time interval starting at the instant the brakes are applied?

38. A car accelerates uniformly from rest to a speed of 40.0 mi/h in 12.0 s. Find (a) the distance the car travels during this time and (b) the constant acceleration of the car.

39. A car starts from rest and travels for 5.0 s with a uniform acceleration of $+1.5\text{ m/s}^2$. The driver then applies the brakes, causing a uniform acceleration of -2.0 m/s^2. If the brakes are applied for 3.0 s, (a) how fast is the car going at the end of the braking period, and (b) how far has the car gone?

40. ecp A car starts from rest and travels for t_1 seconds with a uniform acceleration a_1. The driver then applies the brakes, causing a uniform acceleration a_2. If the brakes are applied for t_2 seconds, (a) how fast is the car going just before the beginning of the braking period? (b) How far does the car go before the driver begins to brake? (c) Using the answers to parts (a) and (b) as the initial velocity and position for the motion of the car during braking, what total distance does the car travel? Answers are in terms of the variables a_1, a_2, t_1, and t_2.

41. In the Daytona 500 auto race, a Ford Thunderbird and a Mercedes Benz are moving side by side down a straightaway at 71.5 m/s. The driver of the Thunderbird realizes that she must make a pit stop, and she smoothly slows to a stop over a distance of 250 m. She spends 5.00 s in the pit and then accelerates out, reaching her previous speed of 71.5 m/s after a distance of 350 m. At this point, how far has the Thunderbird fallen behind the Mercedes Benz, which has continued at a constant speed?

42. A certain cable car in San Francisco can stop in 10 s when traveling at maximum speed. On one occasion, the driver sees a dog a distance d m in front of the car and slams on the brakes instantly. The car reaches the dog 8.0 s later, and the dog jumps off the track just in time. If the car travels 4.0 m beyond the position of the dog before coming to a stop, how far was the car from the dog? (Hint: You will need three equations.)

43. A hockey player is standing on his skates on a frozen pond when an opposing player, moving with a uniform speed of 12 m/s, skates by with the puck. After 3.0 s, the first player makes up his mind to chase his opponent. If he accelerates uniformly at 4.0 m/s^2, (a) how long does it take him to catch his opponent, and (b) how far has he traveled in that time? (Assume the player with the puck remains in motion at constant speed.)

44. A train 400 m long is moving on a straight track with a speed of 82.4 km/h. The engineer applies the brakes at a crossing, and later the last car passes the crossing with a speed of 16.4 km/h. Assuming constant acceleration, determine how long the train blocked the crossing. Disregard the width of the crossing.

SECTION 2.6 FREELY FALLING OBJECTS

45. A ball is thrown vertically upward with a speed of 25.0 m/s. (a) How high does it rise? (b) How long does it take to reach its highest point? (c) How long does the ball take to hit the ground after it reaches its highest point? (d) What is its velocity when it returns to the level from which it started?

46. It is possible to shoot an arrow at a speed as high as 100 m/s. (a) If friction is neglected, how high would an arrow launched at this speed rise if shot straight up? (b) How long would the arrow be in the air?

47. A certain freely falling object requires 1.50 s to travel the last 30.0 m before it hits the ground. From what height above the ground did it fall?

48. ecp An attacker at the base of a castle wall 3.65 m high throws a rock straight up with speed 7.40 m/s at a height of 1.55 m above the ground. (a) Will the rock reach the top of the wall? (b) If so, what is the rock's speed at the top? If not, what initial speed must the rock have to reach the top? (c) Find the change in the speed of a rock thrown straight down from the top of the wall at an initial speed of 7.40 m/s and moving between the same two points. (d) Does the change in speed of the downward-moving rock agree with the magnitude of the speed change of the rock moving upward between the same elevations? Explain physically why or why not.

49. Traumatic brain injury such as concussion results when the head undergoes a very large acceleration. Generally, an acceleration less than 800 m/s^2 lasting for any length of time will not cause injury, whereas an acceleration greater than 1 000 m/s^2 lasting for at least 1 ms will cause injury. Suppose a small child rolls off a bed that is 0.40 m above the floor. If the floor is hardwood, the child's head is brought to rest in approximately 2.0 mm. If the floor is carpeted, this stopping distance is increased to about 1.0 cm. Calculate the magnitude and duration of the deceleration in both cases, to determine the risk of injury. Assume the child remains horizontal during the fall to the floor. Note that a more complicated fall could result in a head velocity greater or less than the speed you calculate.

50. A small mailbag is released from a helicopter that is descending steadily at 1.50 m/s. After 2.00 s, (a) what is the speed of the mailbag, and (b) how far is it below the helicopter? (c) What are your answers to parts (a) and (b) if the helicopter is rising steadily at 1.50 m/s?

51. A tennis player tosses a tennis ball straight up and then catches it after 2.00 s at the same height as the point of release. (a) What is the acceleration of the ball while it is in flight? (b) What is the velocity of the ball when it reaches its maximum height? Find (c) the initial velocity of the ball and (d) the maximum height it reaches.

52. ecp A package is dropped from a helicopter that is descending steadily at a speed v_0. After t seconds have elapsed, (a) what is the speed of the package in terms of v_0, g, and t? (b) What distance d is it from the helicopter in terms of g and t? (c) What are the answers to parts (a) and (b) if the helicopter is rising steadily at the same speed?

53. ecp A model rocket is launched straight upward with an initial speed of 50.0 m/s. It accelerates with a constant upward acceleration of 2.00 m/s^2 until its engines stop at an altitude of 150 m. (a) What can you say about the motion of the rocket after its engines stop? (b) What is the maximum height reached by the rocket? (c) How long after liftoff does the rocket reach its maximum height? (d) How long is the rocket in the air?

54. A parachutist with a camera descends in free fall at a speed of 10 m/s. The parachutist releases the camera at an altitude of 50 m. (a) How long does it take the camera to reach the ground? (b) What is the velocity of the camera just before it hits the ground?

ADDITIONAL PROBLEMS

55. A truck tractor pulls two trailers, one behind the other, at a constant speed of 100 km/h. It takes 0.600 s for the big rig to completely pass onto a bridge 400 m long. For what duration of time is all or part of the truck–trailer combination on the bridge?

56. A speedboat moving at 30.0 m/s approaches a no-wake buoy marker 100 m ahead. The pilot slows the boat with a constant acceleration of -3.50 m/s^2 by reducing the throttle. (a) How long does it take the boat to reach the buoy? (b) What is the velocity of the boat when it reaches the buoy?

57. A bullet is fired through a board 10.0 cm thick in such a way that the bullet's line of motion is perpendicular to the face of the board. If the initial speed of the bullet is 400 m/s and it emerges from the other side of the board with a speed of 300 m/s, find (a) the acceleration of the bullet as it passes through the board and (b) the total time the bullet is in contact with the board.

58. An indestructible bullet 2.00 cm long is fired straight through a board that is 10.0 cm thick. The bullet strikes the board with a speed of 420 m/s and emerges with a speed of 280 m/s. (a) What is the average acceleration of the bullet through the board? (b) What is the total time that the bullet is in contact with the board? (c) What thickness of board (calculated to 0.1 cm) would it take to stop the bullet, assuming the acceleration through all boards is the same?

59. A student throws a set of keys vertically upward to his fraternity brother, who is in a window 4.00 m above. The brother's outstretched hand catches the keys 1.50 s later. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

60. ecp A student throws a set of keys vertically upward to his fraternity brother, who is in a window a distance h above. The brother's outstretched hand catches the keys on their way up a time t later. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught? (Answers should be in terms of h, g, and t.)

61. It has been claimed that an insect called the froghopper (*Philaenus spumarius*) is the best jumper in the animal kingdom. This insect can accelerate at 4 000 m/s^2 over a distance of 2.0 mm as it straightens its specially designed "jumping legs." (a) Assuming a uniform acceleration, what is the velocity of the insect after it has accelerated through this short distance, and how long did it take to reach that velocity? (b) How high would the insect jump if air resistance could be ignored? Note that the actual height obtained is about 0.7 m, so air resistance is important here.

62. A ranger in a national park is driving at 35.0 mi/h when a deer jumps into the road 200 ft ahead of the vehicle. After a reaction time t, the ranger applies the brakes to produce an acceleration $a = -9.00$ ft/s^2. What is the maximum reaction time allowed if she is to avoid hitting the deer?

63. A ball is thrown upward from the ground with an initial speed of 25 m/s; at the same instant, another ball is dropped from a building 15 m high. After how long will the balls be at the same height?

64. To pass a physical education class at a university, a student must run 1.0 mi in 12 min. After running for 10 min, she still has 500 yd to go. If her maximum acceleration is 0.15 m/s^2, can she make it? If the answer is no, determine what acceleration she would need to be successful.

65. Two students are on a balcony 19.6 m above the street. One student throws a ball vertically downward at 14.7 m/s; at the same instant, the other student throws a ball vertically upward at the same speed. The second ball just misses the balcony on the way down. (a) What is the difference in the two balls' time in the air? (b) What is the velocity of each ball as it strikes the ground? (c) How far apart are the balls 0.800 s after they are thrown?

66. ecp Two students are on a balcony a distance h above the street. One student throws a ball vertically downward at a speed v_0; at the same time, the other student throws a ball vertically upward at the same speed. Answer the following symbolically in terms of v_0, g, h, and t. (a) Write the kinematic equation for the y-coordinate of each ball. (b) Set the equations found in part (a) equal to height 0 and solve each for t symbolically using the quadratic formula. What is the difference in the two balls' time in the air? (c) Use the time-independent kinematics equation to find the velocity of each ball as it strikes the ground. (d) How far apart are the balls at a time t after they are released and before they strike the ground?

67. You drop a ball from a window on an upper floor of a building and it is caught by a friend on the ground when the ball is moving with speed v_f. You now repeat the drop, but you have a friend on the street below throw another ball upward at speed v_f exactly at the same time that you drop your ball from the window. The two balls are initially separated by 28.7 m. (a) At what time do they pass each other? (b) At what location do they pass each other relative the window?

68. The driver of a truck slams on the brakes when he sees a tree blocking the road. The truck slows down uniformly with an acceleration of $-5.60\ \text{m/s}^2$ for 4.20 s, making skid marks 62.4 m long that end at the tree. With what speed does the truck then strike the tree?

69. ecp Emily challenges her husband, David, to catch a \$1 bill as follows. She holds the bill vertically as in Figure P2.69, with the center of the bill between David's index finger and thumb. David must catch the bill after Emily releases it without moving his hand downward. If his reaction time is 0.2 s, will he succeed? Explain your reasoning. (This challenge is a good trick you might want to try with your friends.)

FIGURE P2.69

70. A mountain climber stands at the top of a 50.0-m cliff that overhangs a calm pool of water. She throws two stones vertically downward 1.00 s apart and observes that they cause a single splash. The first stone had an initial velocity of -2.00 m/s. (a) How long after release of the first stone did the two stones hit the water? (b) What initial velocity must the second stone have had, given that they hit the water simultaneously? (c) What was the velocity of each stone at the instant it hit the water?

71. An ice sled powered by a rocket engine starts from rest on a large frozen lake and accelerates at $+40\ \text{ft/s}^2$. After some time t_1, the rocket engine is shut down and the sled moves with constant velocity v for a time t_2. If the total distance traveled by the sled is 17 500 ft and the total time is 90 s, find (a) the times t_1 and t_2 and (b) the velocity v. At the 17 500-ft mark, the sled begins to accelerate at $-20\ \text{ft/s}^2$. (c) What is the final position of the sled when it comes to rest? (d) How long does it take to come to rest?

72. In Bosnia, the ultimate test of a young man's courage used to be to jump off a 400-year-old bridge (now destroyed) into the River Neretva, 23 m below the bridge. (a) How long did the jump last? (b) How fast was the jumper traveling upon impact with the river? (c) If the speed of sound in air is 340 m/s, how long after the jumper took off did a spectator on the bridge hear the splash?

73. A person sees a lightning bolt pass close to an airplane that is flying in the distance. The person hears thunder 5.0 s after seeing the bolt and sees the airplane overhead 10 s after hearing the thunder. The speed of sound in air is 1 100 ft/s. (a) Find the distance of the airplane from the person at the instant of the bolt. (Neglect the time it takes the light to travel from the bolt to the eye.) (b) Assuming the plane travels with a constant speed toward the person, find the velocity of the airplane. (c) Look up the speed of light in air and defend the approximation used in part (a).

74. ecp A glider on an air track carries a flag of length ℓ through a stationary photogate, which measures the time interval Δt_d during which the flag blocks a beam of infrared light passing across the photogate. The ratio $v_d = \ell/\Delta t_d$ is the average velocity of the glider over this part of its motion. Suppose the glider moves with constant acceleration. (a) Is v_d necessarily equal to the instantaneous velocity of the glider when it is halfway through the photogate in space? Explain. (b) Is v_d equal to the instantaneous velocity of the glider when it is halfway through the photogate in time? Explain.

75. A stuntman sitting on a tree limb wishes to drop vertically onto a horse galloping under the tree. The constant speed of the horse is 10.0 m/s, and the man is initially 3.00 m above the level of the saddle. (a) What must be the horizontal distance between the saddle and the limb when the man makes his move? (b) How long is he in the air?