KING SCHOOL

Summer Assignment

MAT 305: Algebra 2

Algebra 305 Summer Work

The following problems are required and must be completed on a separate sheet of paper. Work done on these pages will not be accepted. For each problem write down the page number, problem number, and the original problem. Show the important solutions steps and circle your final answer. All graphs must be done on graph paper. Please bring your completed assignment to class on the first day of school. This will be your first grade of the course.

Page 64 # 1, 3, 7, 8, 11, 13, 14, 15, 18, 21
Page 65 # 23, 28, 31, 39, 47
Page 69 # 4, 7, 14, 16
Page 76 # 7, 8, 15
Page 78 # 1, 6, 14, 21, 25
Page 79 # 31, 32, 33, 35
Page 83 # 3, 10, 13
Page 86 # 2, 4, 10, 11, 20
Page 88 # 1, 5, 9, 12, 17, 20

Example: page 76 #2

$(4x+5)(5x+3)$
$20x^2+12x+25x+15$
$\boxed{20x^2+37x+15}$

WRITING AND GRAPHING LINEAR EQUATIONS ON A FLAT SURFACE #21

SLOPE is a number that indicates the steepness (or flatness) of a line, as well as its direction (up or down) left to right.

SLOPE is determined by the ratio: $\frac{\text{vertical change}}{\text{horizontal change}}$ between any two points on a line.

For lines that go **up** (from left to right), the sign of the slope is **positive.** For lines that go **down** (left to right), the sign of the slope is **negative**.

Any linear equation written as **y = mx + b**, where m and b are any real numbers, is said to be in **SLOPE-INTERCEPT FORM**. m is the **SLOPE** of the line. b is the **Y-INTERCEPT**, that is, the point (0, b) where the line intersects (crosses) the y-axis.

If two lines have the same slope, then they are parallel. Likewise, **PARALLEL LINES** have the same slope.

Two lines are **PERPENDICULAR** if the slope of one line is the negative reciprocal of the slope of the other line, that is, m and $-\frac{1}{m}$.

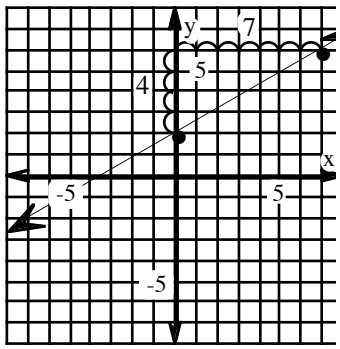
Note that $m \cdot \left(\frac{-1}{m}\right) = -1$.

Examples: 3 and $-\frac{1}{3}$, $-\frac{2}{3}$ and $\frac{3}{2}$, $\frac{5}{4}$ and $-\frac{4}{5}$

Two distinct lines on a flat surface that are not parallel intersect in a single point. See "Solving Linear Systems" to review how to find the point of intersection.

Example 1

Graph the linear equation $y = \frac{4}{7}x + 2$



Using $y = mx + b$, the slope in $y = \frac{4}{7}x + 2$ is $\frac{4}{7}$ and the y-intercept is the point (0, 2). To graph, begin at the y-intercept (0, 2). Remember that slope is $\frac{\text{vertical change}}{\text{horizontal change}}$ so go up 4 units (since 4 is positive) from (0, 2) and then move right 7 units. This gives a second point on the graph. To create the graph, draw a straight line through the two points.

Example 2

A line has a slope of $\frac{3}{4}$ and passes through (3, 2). What is the equation of the line?

Using $y = mx + b$, write $y = \frac{3}{4}x + b$. Since (3, 2) represents a point (x, y) on the line, substitute 3 for x and 2 for y, $2 = \frac{3}{4}(3) + b$, and solve for b.
$2 = \frac{9}{4} + b \Rightarrow 2 - \frac{9}{4} = b \Rightarrow -\frac{1}{4} = b$. The equation is $y = \frac{3}{4}x - \frac{1}{4}$.

Example 3

Decide whether the two lines at right are parallel, perpendicular, or neither (i.e., intersecting). 5x – 4y = -6 and -4x + 5y = 3.

First find the slope of each equation. Then compare the slopes.

$5x - 4y = -6$ $-4y = -5x - 6$ $y = \frac{-5x - 6}{-4}$ $y = \frac{5}{4}x + \frac{3}{2}$ The slope of this line is $\frac{5}{4}$.	$-4x + 5y = 3$ $5y = 4x + 3$ $y = \frac{4x + 3}{5}$ $y = \frac{4}{5}x + \frac{3}{5}$ The slope of this line is $\frac{4}{5}$.	These two slopes are not equal, so they are not parallel. The product of the two slopes is 1, not -1, so they are not perpendicular. These two lines are neither parallel nor perpendicular, but do intersect.

Example 4

Find two equations of the line through the given point, one parallel and one perpendicular to the given line: $y = -\frac{5}{2}x + 5$ and (-4, 5).

For the parallel line, use $y = mx + b$ with the same slope to write $y = -\frac{5}{2}x + b$. Substitute the point (–4, 5) for x and y and solve for b. $5 = -\frac{5}{2}(-4) + b \Rightarrow 5 = \frac{20}{2} + b \Rightarrow -5 = b$ Therefore the parallel line through (-4, 5) is $y = -\frac{5}{2}x - 5$	For the perpendicular line, use $y = mx + b$ where m is the negative reciprocal of the slope of the original equation to write $y = \frac{2}{5}x + b$. Substitute the point (–4, 5) and solve for b. $5 = \frac{2}{5}(-4) + b \Rightarrow \frac{33}{5} = b$ Therefore the perpendicular line through (-4, 5) is $y = \frac{2}{5}x + \frac{33}{5}$.

Identify the y-intercept in each equation.

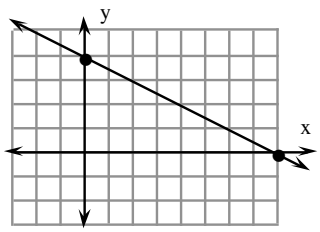
1. $y = \frac{1}{2}x - 2$
2. $y = -\frac{3}{5}x - \frac{5}{3}$
3. $3x + 2y = 12$
4. $x - y = -13$
5. $2x - 4y = 12$
6. $4y - 2x = 12$

Write the equation of the line with:

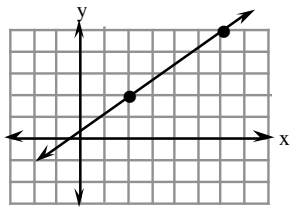
7. slope = $\frac{1}{2}$ and passing through (4, 3).
8. slope = $\frac{2}{3}$ and passing through (-3, -2).
9. slope = $-\frac{1}{3}$ and passing through (4, -1).
10. slope = -4 and passing through (-3, 5).

Determine the slope of each line using the highlighted points.

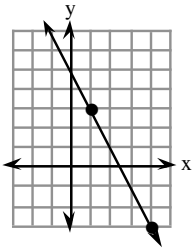
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12.

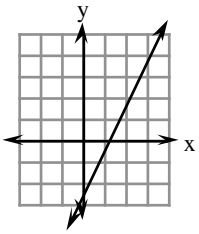


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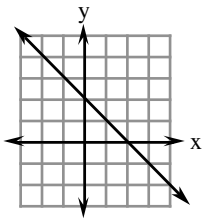


Using the slope and y-intercept, determine the equation of the line.

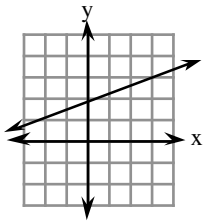
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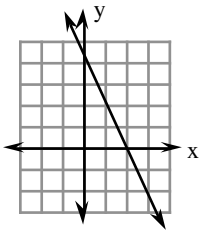
15.



16.



17.



Graph the following linear equations on graph paper.

18. $y = \frac{1}{2}x + 3$
19. $y = -\frac{3}{5}x - 1$
20. $y = 4x$
21. $y = -6x + \frac{1}{2}$
22. $3x + 2y = 12$

State whether each pair of lines is parallel, perpendicular, or intersecting.

23. $y = 2x - 2$ and $y = 2x + 4$

24. $y = \frac{1}{2}x + 3$ and $y = -2x - 4$

25. $x - y = 2$ and $x + y = 3$

26. $y - x = -1$ and $y + x = 3$

27. $x + 3y = 6$ and $y = -\frac{1}{3}x - 3$

28. $3x + 2y = 6$ and $2x + 3y = 6$

29. $4x = 5y - 3$ and $4y = 5x + 3$

30. $3x - 4y = 12$ and $4y = 3x + 7$

Find an equation of the line through the given point and parallel to the given line.

31. $y = 2x - 2$ and (-3, 5)

32. $y = \frac{1}{2}x + 3$ and (-4, 2)

33. $x - y = 2$ and (-2, 3)

34. $y - x = -1$ and (-2, 1)

35. $x + 3y = 6$ and (-1, 1)

36. $3x + 2y = 6$ and (2, -1)

37. $4x = 5y - 3$ and (1, -1)

38. $3x - 4y = 12$ and (4, -2)

Find an equation of the line through the given point and perpendicular to the given line.

39. $y = 2x - 2$ and (-3, 5)

40. $y = \frac{1}{2}x + 3$ and (-4, 2)

41. $x - y = 2$ and (-2, 3)

42. $y - x = -1$ and (-2, 1)

43. $x + 3y = 6$ and (-1, 1)

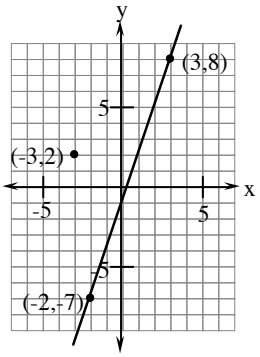
44. $3x + 2y = 6$ and (2, -1)

45. $4x = 5y - 3$ and (1, -1)

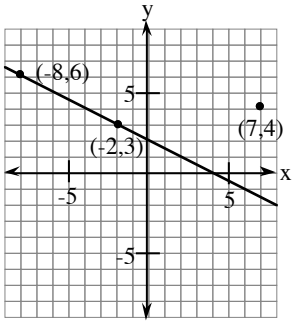
46. $3x - 4y = 12$ and (4, -2)

Write an equation of the line parallel to each line below through the given point.

47.



48.



49. Write the equation of the line through (7, -8) which is parallel to the line through (2, 5) and (8, -3).

50. Write the equation of the line through (1, -4) which is parallel to the line through (-3, -7) and (4, 3).

SOLVING LINEAR SYSTEMS #22

You can find where two lines intersect (cross) by using algebraic methods. The two most common methods are **SUBSTITUTION** and **ELIMINATION** (also known as the addition method).

Example 1

Solve the following system of equations at right by **substitution**. Check your solution.

$$y = 5x + 1$$
$$y = -3x - 15$$

When solving a system of equations, you are solving to find the x- and y-values that result in true statements when you substitute them into <u>both</u> equations. You generally find each value one at a time. Since both equations are in y-form (that is, solved for y), and we know y = y, we can substitute the right side of each equation for y in the simple equation y = y and write

$$5x + 1 = -3x - 15$$

Now solve for x. $5x + 1 = -3x - 15 \Rightarrow 8x + 1 = -15 \Rightarrow 8x = -16 \Rightarrow x = -2$

Remember you must find x <u>and</u> y. To find y, use either one of the two original equations. Substitute the value of x into the equation and find the value of y. Using the first equation,

$$y = 5x + 1 \Rightarrow y = 5(-2) + 1 \Rightarrow y = -10 + 1 = -9 .$$

The solution appears to be (-2, -9). In order for this to be a solution, it must make both equations true when you replace x with -2 and y with -9. Substitute the values in both equations to check.

$y = 5x + 1$	$y = -3x - 15$
$-9 \stackrel{?}{=} 5(-2) + 1$	$-9 \stackrel{?}{=} -3(-2) - 15$
$-9 \stackrel{?}{=} -10 + 1$	$-9 \stackrel{?}{=} 6 - 15$
$-9 = -9$ Check!	$-9 = -9$ Check!

Therefore, (-2, -9) is the solution.

Example 2

Substitution can also be used when the equations are <u>not</u> in y-form.

$$x = -3y + 1$$
$$4x - 3y = -11$$

Use substitution to rewrite the two equations as 4(-3y + 1) - 3y = -11 by replacing x with (-3y + 1), then solve for y as shown at right.

$$x = (-3y + 1)$$
$$4(\quad) - 3y = -11$$
$$4(-3y + 1) - 3y = -11$$
$$y = 1$$

Substitute y = 1 into x = -3y + 1. Solve for x, and write the answer for x and y as an ordered pair, (1, –2). Substitute y = 1 into 4x - 3y = -11 to verify that <u>either</u> original equation may be used to find the second coordinate. Check your answer as shown in example 1.

Example 3

When you have a pair of two-variable equations, sometimes it is easier to **ELIMINATE** one of the variables to obtain one single variable equation. You can do this by adding the two equations together as shown in the example below.

Solve the system at right:

$$2x + y = 11$$
$$x - y = 4$$

To eliminate the y terms, **add** the two equations together.

$$\begin{array}{r} 2x + y = 11 \\ x - y = 4 \\ \hline 3x = 15 \end{array}$$

then solve for x.

$$3x = 15$$
$$x = 5$$

Once we know the x-value we can substitute it into <u>either</u> of the original equations to find the corresponding value of y.

Using the first equation:

$$2x + y = 11$$
$$2(5) + y = 11$$
$$10 + y = 11$$
$$y = 1$$

Check the solution by substituting both the x-value and y-value into the other original equation, $x - y = 4$: $5 - 1 = 4$, checks.

Example 4

You can solve the system of equations at right by elimination, but before you can eliminate one of the variables, you must adjust the coefficients of one of the variables so that they are additive opposites.

$$3x + 2y = 11$$
$$4x + 3y = 14$$

To eliminate y, multiply the first equation by 3, then multiply the second equation by –2 to get the equations at right.

$$9x + 6y = 33$$
$$-8x - 6y = -28$$

Next eliminate the y terms by adding the two adjusted equations.

$$\begin{array}{r} 9x + 6y = 33 \\ \underline{-8x - 6y = -28} \\ x = 5 \end{array}$$

Since $x = 5$, substitute in either original equation to find that $y = -2$. The solution to the system of equations is (5, -2). You could also solve the system by first multiplying the first equation by 4 and the second equation by -3 to eliminate x, then proceeding as shown above to find y.

Solve the following systems of equations to find the point of intersection (x, y) for each pair of lines.

1. $y = x - 6$
   $y = 12 - x$

2. $y = 3x - 5$
   $y = x + 3$

3. $x = 7 + 3y$
   $x = 4y + 5$

4. $x = -3y + 10$
   $x = -6y - 2$

5. $y = x + 7$
   $y = 4x - 5$

6. $y = 7 - 3x$
   $y = 2x - 8$

7. $y = 3x - 1$
   $2x - 3y = 10$

8. $x = -\frac{1}{2}y + 4$
   $8x + 3y = 31$

9. $2y = 4x + 10$
   $6x + 2y = 10$

10. $y = \frac{3}{5}x - 2$
    $y = \frac{x}{10} + 1$

11. $y = -4x + 5$
    $y = x$

12. $4x - 3y = -10$
    $x = \frac{1}{4}y - 1$

13. $x + y = 12$
    $x - y = 4$

14. $2x - y = 6$
    $4x - y = 12$

15. $x + 2y = 7$
    $5x - 4y = 14$

16. $5x - 2y = 6$
    $4x + y = 10$

17. $x + y = 10$
    $x - 2y = 5$

18. $3y - 2x = 16$
    $y = 2x + 4$

19. $x + y = 11$
    $x = y - 3$

20. $x + 2y = 15$
    $y = x - 3$

21. $y + 5x = 10$
    $y - 3x = 14$

22. $y = 7x - 3$
    $4x + 2y = 8$

23. $y = 12 - x$
    $y = x - 4$

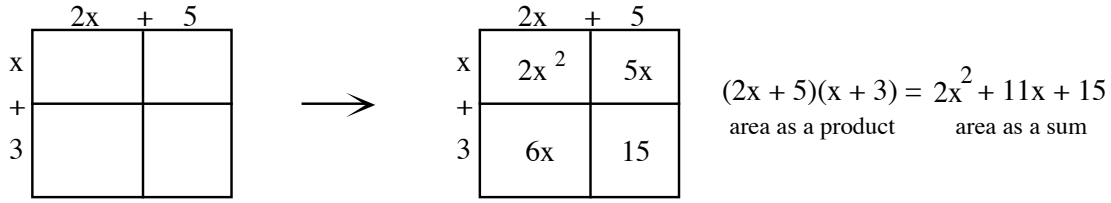
24. $y = 6 - 2x$
    $y = 4x - 12$

MULTIPLYING POLYNOMIALS #24

We can use generic rectangles as area models to find the products of polynomials. A generic rectangle helps us organize the problem. It does not have to be drawn accurately or to scale.

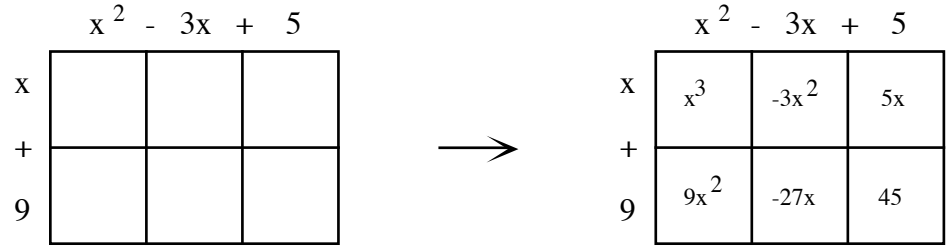
Example 1

Multiply $(2x+5)(x+3)$



Example 2

Multiply $(x+9)(x^2-3x+5)$



Therefore $(x+9)(x^2-3x+5) = x^3+9x^2-3x^2-27x+5x+45 =$

$x^3+6x^2-22x+45$

Another approach to multiplying binomials is to use the mnemonic "F.O.I.L." F.O.I.L. is an acronym for First, Outside, Inside, Last in reference to the positions of the terms in the two binomials.

Example 3

Multiply $(3x-2)(4x+5)$ using the F.O.I.L. method.

F.	multiply the FIRST terms of each binomial	$(3x)(4x) = 12x^2$
O.	multiply the OUTSIDE terms	$(3x)(5) = 15x$
I.	multiply the INSIDE terms	$(-2)(4x) = -8x$
L.	multiply the LAST terms of each binomial	$(-2)(5) = -10$

Finally, we combine like terms: $12x^2 + 15x - 8x - 10 = 12x^2 + 7x - 10$.

Find each of the following products.

1. $(3x+2)(2x+7)$
2. $(4x+5)(5x+3)$
3. $(2x-1)(3x+1)$
4. $(2a-1)(4a+7)$
5. $(m-5)(m+5)$
6. $(y-4)(y+4)$
7. $(3x-1)(x+2)$
8. $(3a-2)(a-1)$
9. $(2y-5)(y+4)$
10. $(3t-1)(3t+1)$
11. $(3y-5)^2$
12. $(4x-1)^2$
13. $(2x+3)^2$
14. $(5n+1)^2$
15. $(3x-1)(2x^2+4x+3)$
16. $(2x+7)(4x^2-3x+2)$
17. $(x+7)(3x^2-x+5)$
18. $(x-5)(x^2-7x+1)$
19. $(3x+2)(x^3-7x^2+3x)$
20. $(2x+3)(3x^2+2x-5)$

FACTORING POLYNOMIALS #25

Often we want to un-multiply or **FACTOR** a polynomial P(x). This process involves finding a constant and/or another polynomial that evenly divides the given polynomial. In formal mathematical terms, this means $P(x) = q(x) \cdot r(x)$, where q and r are also polynomials. For elementary algebra there are three general types of factoring.

1) **Common term** (finding the largest common factor):
$6x + 18 = 6(x + 3)$ where 6 is a common factor of both terms.
$2x^3 - 8x^2 - 10x = 2x(x^2 - 4x - 5)$ where $2x$ is the common factor.
$2x^2(x-1) + 7(x-1) = (x-1)(2x^2 + 7)$ where $x - 1$ is the common factor.

2) **Special products**

$a^2 - b^2 = (a+b)(a-b)$ $\quad x^2 - 25 = (x+5)(x-5)$

$9x^2 - 4y^2 = (3x+2y)(3x-2y)$

$x^2 + 2xy + y^2 = (x+y)^2$ $\quad x^2 + 8x + 16 = (x+4)^2$

$x^2 - 2xy + y^2 = (x-y)^2$ $\quad x^2 - 8x + 16 = (x-4)^2$

3a) **Trinomials** in the form $x^2 + bx + c$ where the coefficient of x^2 is 1.
Consider $x^2 + (d+e)x + d \cdot e = (x+d)(x+e)$, where the coefficient of x is the sum of two numbers d and e AND the constant is the product of the same two numbers, d and e. A quick way to determine all of the possible pairs of integers d and e is to factor the constant in the original trinomial. For example, 12 is $1 \cdot 12$, $2 \cdot 6$, and $3 \cdot 4$. The signs of the two numbers are determined by the combination you need to get the sum. The "sum and product" approach to factoring trinomials is the same as solving a "Diamond Problem" in CPM's Algebra 1 course (see below).

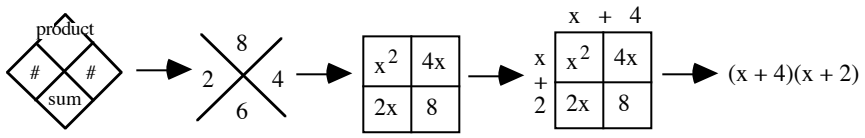
$x^2 + 8x + 15 = (x+3)(x+5); 3 + 5 = 8, 3 \cdot 5 = 15$

$x^2 - 2x - 15 = (x-5)(x+3); -5 + 3 = -2, -5 \cdot 3 = -15$

$x^2 - 7x + 12 = (x-3)(x-4); -3 + (-4) = -7, (-3)(-4) = 12$

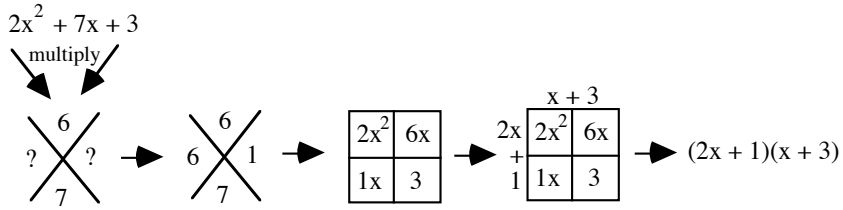
The sum and product approach can be shown visually using rectangles for an area model. The figure at far left below shows the "Diamond Problem" format for finding a sum and product. Here is how to use this method to factor $x^2 + 6x + 8$.

>> Explanation and examples continue on the next page. >>

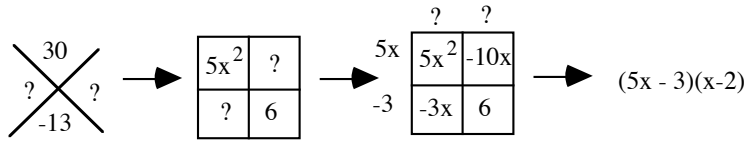


3b) **Trinomials** in the form $ax^2 + bx + c$ where $a \neq 1$.

Note that the upper value in the diamond is no longer the constant. Rather, it is the <u>product</u> of a and c, that is, the coefficient of x^2 and the constant.



Below is the process to factor $5x^2 - 13x + 6$.



Polynomials with four or more terms are generally factored by grouping the terms and using one or more of the three procedures shown above. Note that polynomials are usually factored <u>completely</u>. In the second example in part (1) above, the trinomial also needs to be factored. Thus, the complete factorization of $2x^3 - 8x^2 - 10x = 2x(x^2 - 4x - 5) = 2x(x - 5)(x + 1)$.

Factor each polynomial completely.

1. $x^2 - x - 42$
2. $4x^2 - 18$
3. $2x^2 + 9x + 9$
4. $2x^2 + 3xy + y^2$
5. $6x^2 - x - 15$
6. $4x^2 - 25$
7. $x^2 - 28x + 196$
8. $7x^2 - 847$
9. $x^2 + 18x + 81$
10. $x^2 + 4x - 21$
11. $3x^2 + 21x$
12. $3x^2 - 20x - 32$
13. $9x^2 - 16$
14. $4x^2 + 20x + 25$
15. $x^2 - 5x + 6$
16. $5x^3 + 15x^2 - 20x$
17. $4x^2 + 18$
18. $x^2 - 12x + 36$
19. $x^2 - 3x - 54$
20. $6x^2 - 21$
21. $2x^2 + 15x + 18$
22. $16x^2 - 1$
23. $x^2 - 14x + 49$
24. $x^2 + 8x + 15$
25. $3x^3 - 12x^2 - 45x$
26. $3x^2 + 24$
27. $x^2 + 16x + 64$

Factor completely.

28. $75x^3 - 27x$
29. $3x^3 - 12x^2 - 36x$
30. $4x^3 - 44x^2 + 112x$
31. $5y^2 - 125$
32. $3x^2y^2 - xy^2 - 4y^2$
33. $x^3 + 10x^2 - 24x$
34. $3x^3 - 6x^2 - 45x$
35. $3x^2 - 27$
36. $x^4 - 16$

Factor each of the following completely. Use the modified diamond approach.

37. $2x^2 + 5x - 7$
38. $3x^2 - 13x + 4$
39. $2x^2 + 9x + 10$
40. $4x^2 - 13x + 3$
41. $4x^2 + 12x + 5$
42. $6x^3 + 31x^2 + 5x$
43. $64x^2 + 16x + 1$
44. $7x^2 - 33x - 10$
45. $5x^2 + 12x - 9$

THE QUADRATIC FORMULA #27

You have used factoring and the Zero Product Property to solve quadratic equations. You can solve any quadratic equation by using the **QUADRATIC FORMULA.**

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

For example, suppose $3x^2 + 7x - 6 = 0$. Here a = 3, b = 7, and c = -6. Substituting these values into the formula results in:

$$x = \frac{-(7) \pm \sqrt{7^2 - 4(3)(-6)}}{2(3)} \quad \Rightarrow \; x = \frac{-7 \pm \sqrt{121}}{6} \quad \Rightarrow \; x = \frac{-7 \pm 11}{6}$$

Remember that non-negative numbers have both a positive and negative square root. The sign $\pm$ represents this fact for the square root in the formula and allows us to write the equation once (representing two possible solutions) until later in the solution process.

Split the numerator into the two values: $x = \dfrac{-7 + 11}{6}$ or $x = \dfrac{-7 - 11}{6}$

Thus the solution for the quadratic equation is: $x = \frac{2}{3}$ or -3.

Example 1

Solve $x^2 + 3x - 2 = 0$ using the quadratic formula.

First, identify the values for a, b, and c. In this case they are 1, 3, and –2, respectively. Next, substitute these values into the quadratic formula.

$$x = \frac{-(3) \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)} \Rightarrow x = \frac{-3 \pm \sqrt{17}}{2}$$

Then split the numerator into the two values: $x = \dfrac{-3 + \sqrt{17}}{2}$ or $x = \dfrac{-3 - \sqrt{17}}{2}$

Using a calculator, the solution for the quadratic equation is: x = 0.56 or –3.56.

Example 2

Solve $4x^2 + 4x = 3$ using the quadratic formula.

To solve any quadratic equation it must first be equal to zero. Rewrite the equation as $4x^2 + 4x - 3 = 0$. Identify the values for a, b, and c: 4, 4, and -3, respectively.

Substitute these values into the quadratic formula.

$$x = \frac{-(4) \pm \sqrt{4^2 - 4(4)(-3)}}{2(4)} \quad \Rightarrow x = \frac{-4 \pm \sqrt{64}}{8} \quad \Rightarrow \quad x = \frac{-4 \pm 8}{8}$$

Split the numerator into the two values: $x = \frac{-4+8}{8}$ or $x = \frac{-4-8}{8}$, so $x = \frac{1}{2}$ or $-\frac{3}{2}$.

Use the quadratic formula to solve each of the following equations.

1. $x^2 - x - 6 = 0$
2. $x^2 + 8x + 15 = 0$
3. $x^2 + 13x + 42 = 0$
4. $x^2 - 10x + 16 = 0$
5. $x^2 + 5x + 4 = 0$
6. $x^2 - 9x + 18 = 0$
7. $5x^2 - x - 4 = 0$
8. $4x^2 - 11x - 3 = 0$
9. $6x^2 - x - 15 = 0$
10. $6x^2 + 19x + 15 = 0$
11. $3x^2 + 5x - 28 = 0$
12. $2x^2 - x - 14 = 0$
13. $4x^2 - 9x + 4 = 0$
14. $2x^2 - 5x + 2 = 0$
15. $20x^2 + 20x = 1$
16. $13x^2 - 16x = 4$
17. $7x^2 + 28x = 0$
18. $5x^2 = -125x$
19. $8x^2 - 50 = 0$
20. $15x^2 = 3$

LAWS OF EXPONENTS #28

BASE, EXPONENT, AND VALUE

In the expression 2^5, 2 is the **base**, 5 is the **exponent**, and the **value** is 32.

2^5 means $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$ $\quad$ x^3 means $x \cdot x \cdot x$

LAWS OF EXPONENTS

Here are the basic patterns with examples:

1) $x^a \cdot x^b = x^{a+b}$ examples: $x^3 \cdot x^4 = x^{3+4} = x^7$; $\quad 2^7 \cdot 2^4 = 2^{11}$

2) $\frac{x^a}{x^b} = x^{a-b}$ examples: $x^{10} \div x^4 = x^{10-4} = x^6$; $\quad \frac{2^4}{2^7} = 2^{-3}$

3) $(x^a)^b = x^{ab}$ examples: $(x^4)^3 = x^{4 \cdot 3} = x^{12}$; $(2x^3)^4 = 2^4 x^{12} = 16x^{12}$

4) $x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^{-b}} = x^b$ examples: $3x^{-3}y^2 = \frac{3y^2}{x^3}$; $\quad \frac{2x^5}{y^{-2}} = 2x^5y^2$

Example 1

Simplify: $\left(2xy^3\right)\left(5x^2y^4\right)$

Multiply the coefficients: $2 \cdot 5 \cdot xy^3 \cdot x^2y^4 = 10xy^3 \cdot x^2y^4$

Add the exponents of x, then y: $10x^{1+2}y^{3+4} = 10x^3y^7$

Example 2

Simplify: $\frac{14x^2y^{12}}{7x^5y^7}$

Divide the coefficients: $\frac{(14 \div 7)x^2y^{12}}{x^5y^7} = \frac{2x^2y^{12}}{x^5y^7}$

Subtract the exponents: $2x^{2-5}y^{12-7} = 2x^{-3}y^5$ OR $\frac{2y^5}{x^3}$

Example 3

Simplify: $\left(3x^2y^4\right)^3$

Cube each factor: $3^3 \cdot \left(x^2\right)^3 \cdot \left(y^4\right)^3 = 27\left(x^2\right)^3\left(y^4\right)^3$

Multiply the exponents: $27x^6y^{12}$

Simplify each expression:

1. $y^5 \cdot y^7$
2. $b^4 \cdot b^3 \cdot b^2$
3. $8^6 \cdot 8^2$
4. $\left(y^5\right)^2$
5. $(3a)^4$
6. $\frac{m^8}{m^3}$
7. $\frac{12x^9}{4x^4}$
8. $\left(x^3y^2\right)^3$
9. $\frac{\left(y^4\right)^2}{\left(y^3\right)^2}$
10. $\frac{15x^2y^7}{3x^4y^5}$
11. $\left(4c^4\right)\left(ac^3\right)\left(3a^5c\right)$
12. $\left(7x^3y^5\right)^2$
13. $\left(4xy^2\right)(2y)^3$
14. $\left(\frac{4}{x^2}\right)^3$
15. $\frac{\left(2a^7\right)\left(3a^2\right)}{6a^3}$
16. $\left(\frac{5m^3n}{m^5}\right)^3$
17. $\left(3a^2x^3\right)^2\left(2ax^4\right)^3$
18. $\left(\frac{x^3y}{y^4}\right)^4$
19. $\left(\frac{6y^2x^8}{12x^3y^7}\right)^2$
20. $\frac{\left(2x^5y^3\right)^3\left(4xy^4\right)^2}{8x^7y^{12}}$

RADICALS #29

Sometimes it is convenient to leave square roots in radical form instead of using a calculator to find approximations (decimal values). Look for perfect squares (i.e., 4, 9, 16, 25, 36, 49, ...) as **factors** of the number that is inside the radical sign (**radicand**) and take the square root of any perfect square factor. Multiply the root of the perfect square times the reduced radical. When there is an existing value that multiplies the radical, multiply any root(s) times that value.

For example:

$\sqrt{9} = 3$ $\qquad 5\sqrt{9} = 5 \cdot 3 = 15$

$\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$ $\qquad 3\sqrt{98} = 3\sqrt{49 \cdot 2} = 3 \cdot 7\sqrt{2} = 21\sqrt{2}$

$\sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16} \cdot \sqrt{5} = 4\sqrt{5}$ $\qquad \sqrt{45} + 4\sqrt{20} = \sqrt{9 \cdot 5} + 4\sqrt{4 \cdot 5} = 3\sqrt{5} + 4 \cdot 2\sqrt{5} = 11\sqrt{5}$

When there are no more perfect square factors inside the radical sign, the product of the whole number (or fraction) and the remaining radical is said to be in **SIMPLE RADICAL FORM**.

Simple radical form does not allow radicals in the denominator of a fraction. If there is a radical in the denominator, **RATIONALIZE THE DENOMINATOR** by multiplying the numerator and denominator of the fraction by the radical in the original denominator. Then simplify the remaining fraction. Examples:

$$\frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2} \qquad \frac{4\sqrt{5}}{\sqrt{6}} = \frac{4\sqrt{5}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{4\sqrt{30}}{6} = \frac{2\sqrt{30}}{3}$$

In the first example, $\sqrt{2} \cdot \sqrt{2} = \sqrt{4} = 2$ and $\frac{2}{2} = 1$. In the second example, $\sqrt{6} \cdot \sqrt{6} = \sqrt{36} = 6$ and $\frac{4}{6} = \frac{2}{3}$.

Example 1

Add $\sqrt{27} + \sqrt{12} - \sqrt{48}$. Factor each radical and simplify.

$\sqrt{9 \cdot 3} + \sqrt{4 \cdot 3} - \sqrt{16 \cdot 3} = 3\sqrt{3} + 2\sqrt{3} - 4\sqrt{3} = 1\sqrt{3}$ *or* $\sqrt{3}$

Example 2

Simplify $\frac{3}{\sqrt{6}}$. Multiply by $\frac{\sqrt{6}}{\sqrt{6}}$ and simplify: $\frac{3}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$.

Write each of the following radicals in simple radical form.

1. $\sqrt{24}$
2. $\sqrt{48}$
3. $\sqrt{17}$
4. $\sqrt{31}$
5. $\sqrt{75}$
6. $\sqrt{50}$
7. $\sqrt{96}$
8. $\sqrt{243}$
9. $\sqrt{8}+\sqrt{18}$
10. $\sqrt{18}+\sqrt{32}$
11. $\sqrt{27}-\sqrt{12}$
12. $\sqrt{50}-\sqrt{32}$
13. $\sqrt{6}+\sqrt{63}$
14. $\sqrt{44}+\sqrt{99}$
15. $\sqrt{50}+\sqrt{32}-\sqrt{27}$
16. $\sqrt{75}-\sqrt{8}-\sqrt{32}$
17. $\dfrac{3}{\sqrt{3}}$
18. $\dfrac{5}{\sqrt{27}}$
19. $\dfrac{\sqrt{3}}{\sqrt{5}}$
20. $\dfrac{\sqrt{5}}{\sqrt{7}}$
21. $4\sqrt{5}-\frac{10}{\sqrt{5}}$