Summer Work for AP Calculus AB

The following practice will help you review topics from previous mathematics courses that are important for your success in AP Calculus AB.

You do not need to complete every problem, but please know there is an assumption that these algebra and trigonometry skills are sharp and that we will be ready to go on the first day of class. Highlight any questions you may have as you will have an opportunity to discuss them during our first days of school. A helpful website with video tutorials is www.khanacademy.org. Use your notes from your previous mathematics courses to assist you.

There will be an assessment on these skills within the first five days of school.

I am looking forward to working with you next year! Happy Summer!

Avon High School **Name** _____
AP Calculus AB
Summer Review Packet **Score** _____ **Period** _____

Topic A: Functions

1.) If $f(x)=4x-x^2$, find:

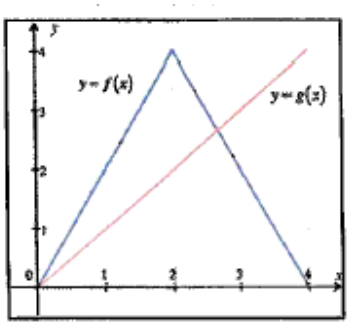
a.) $f(4)-f(-4)$ b.) $\sqrt{f\left(\frac{3}{2}\right)}$ c.) $\frac{f(x+h)-f(x)}{2h}$

2.) If $V(r)=\frac{4}{3}\pi r^3$, find:

a.) $V\left(\frac{3}{4}\right)$ b.) $V(r+1)-V(r-1)$ c.) $\frac{V(2r)}{V(r)}$

3.) If $f(x)$ and $g(x)$ are given in the graph, find:

a.) $(f-g)(3)$ b.) $f(g(3))$



4.) If $f(x)=\begin{cases}-x, & x<0\\ x^2-1, & 0\le x<2\\ \sqrt{x+2}-2, & x\ge 2\end{cases}$, find:

a.) $f(0)-f(2)$ b.) $\sqrt{5-f(-4)}$ c.) $f(f(3))$

Topic B: Domain and Range

Find the domain of the following functions using interval notation:

1.) $f(x)=3$

2.) $y=x^3-x^2+x$

3.) $y=\dfrac{x^3-x^2+x}{x}$

4.) $y=\dfrac{x-4}{x^2-16}$

5.) $f(x)=\dfrac{1}{4x^2-4x-3}$

6.) $y=\sqrt{2x-9}$

7.) $y=\log(x-10)$

8.) $y=\dfrac{\sqrt{2x+14}}{x^2-49}$

Find the range of the following functions:

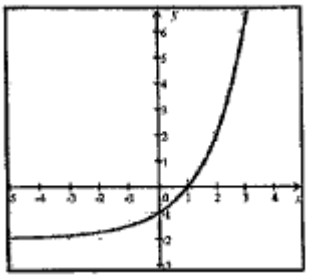
9.) $y=x^4+x^2-1$

10.) $y=100^x$

11.) $y=\sqrt{x^2+1}+1$

Find the domain and range of the following functions using interval notation.

12.)



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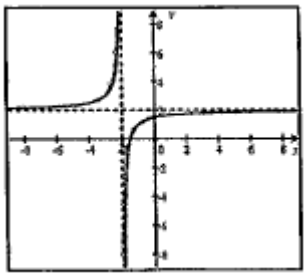

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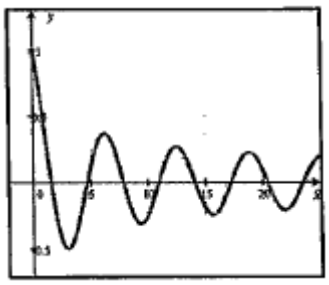
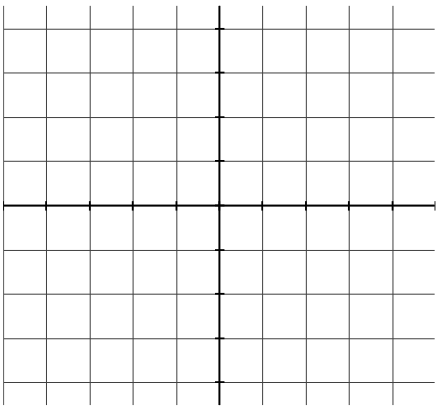


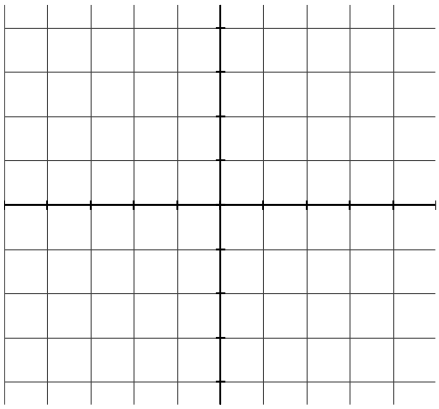
Topic C: Graphs of Common Functions

Sketch each of the following as accurately as possible. **You will need to be VERY familiar with each of these graphs throughout the year**. You may use a graphing calculator for some of them if you have access to one over the summer. If you plan on renting a TI-Nspire and thus do not have one for the summer, I strongly recommend you use try www.desmos.com. There is an app for Desmos as well that is free that you can install on your phones. Again, these are VERY important graphs to know. Be very accurate with regards to "open circles" and "closed circles" as those features may not be revealed on a graphing utility. For students who have not taken Trigonometry yet, do your best with #'s 9-14.

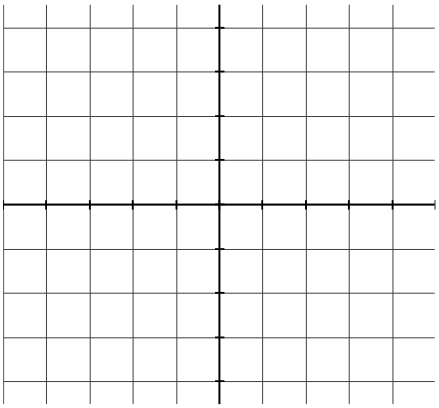
1. $y = x$



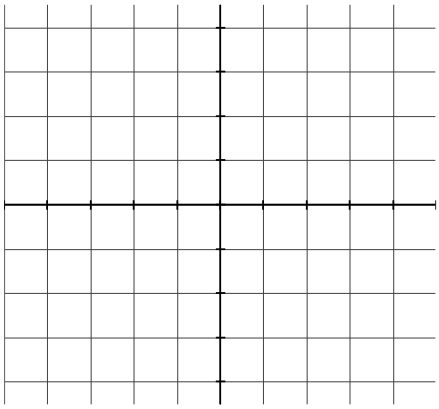
2. $y = x^2$



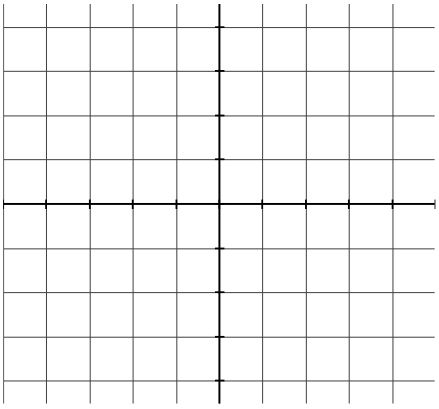
3. $y = x^3$



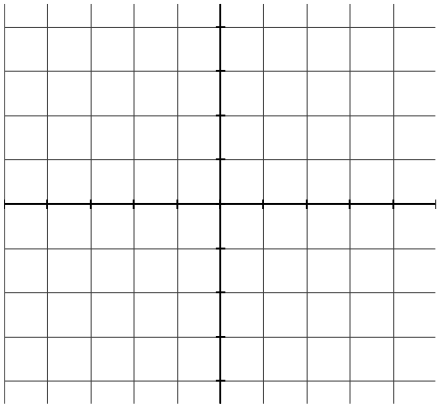
4. $y = \sqrt{x}$



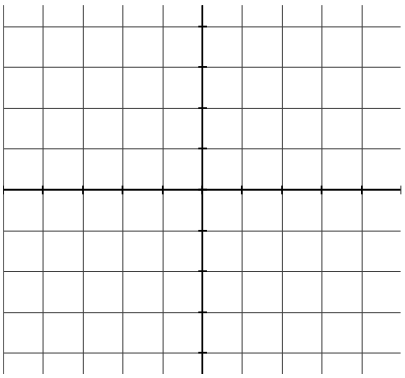
5. $y = |x|$



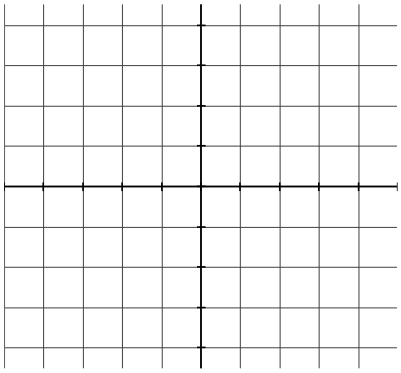
6. $y = \dfrac{|x|}{x}$



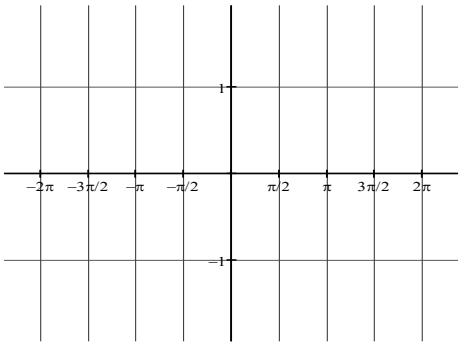
7. $y = x^{1/3}$



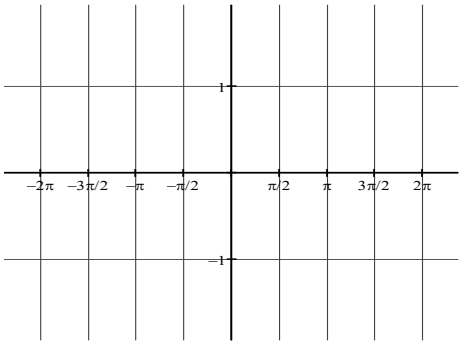
8. $y = x^{2/3}$



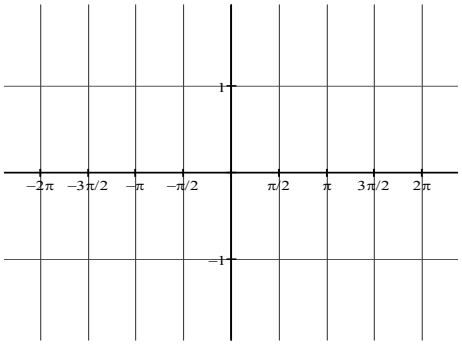
9. $y = \sin x$



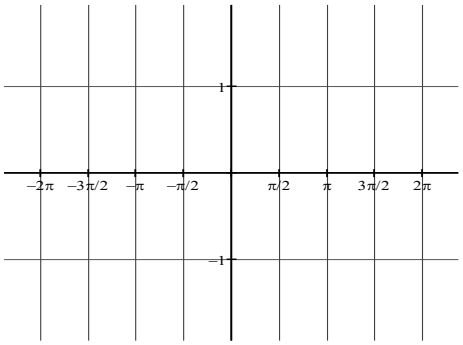
10. $y = \cos x$



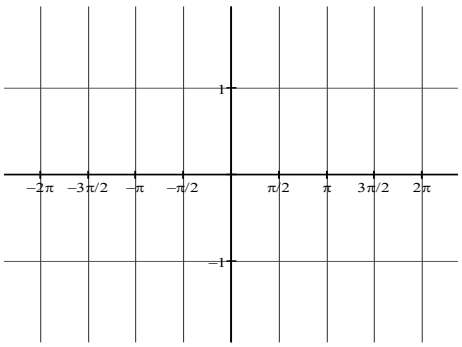
11. $y = \tan x$



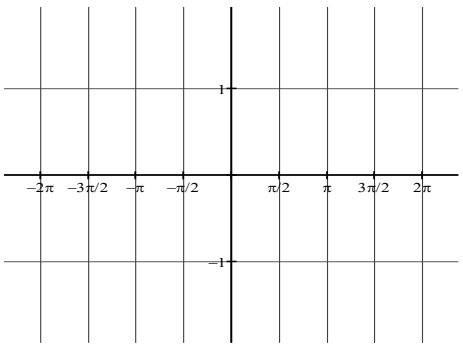
12. $y = \cot x$



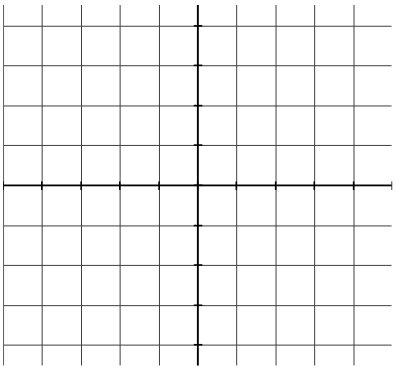
13. $y = \sec x$



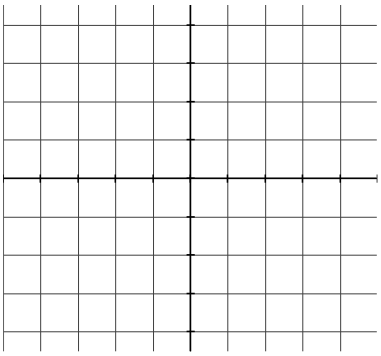
14. $y = \csc x$



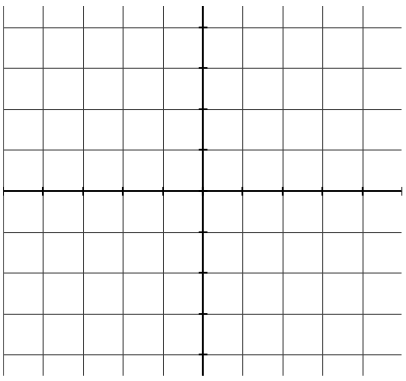
15. $y = e^x$



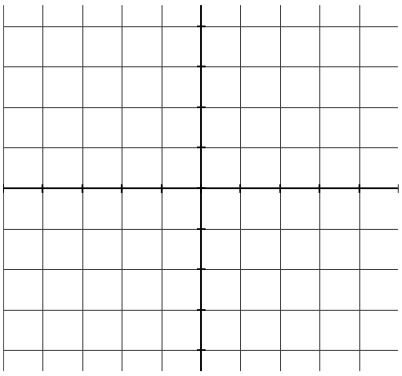
16. $y = \ln x$



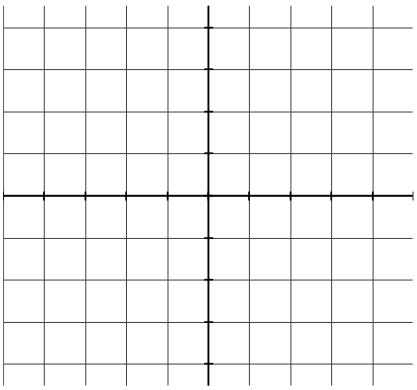
17. $y = \dfrac{1}{x}$



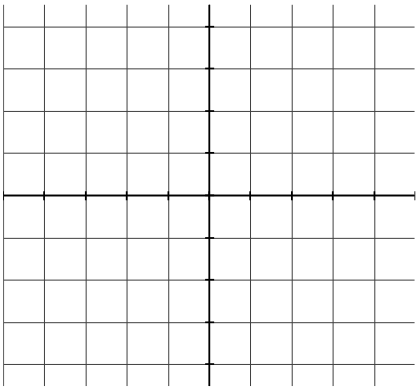
18. $y = \sqrt{x}$



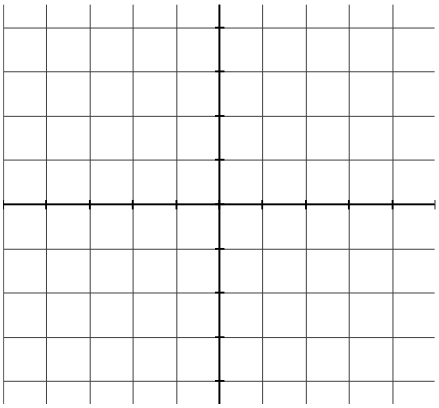
19. $y = \dfrac{1}{x^2}$



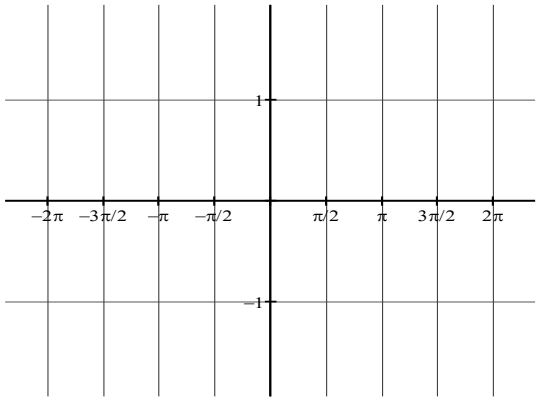
20. $y = 2^x$



21. $y = \sqrt{4 - x^2}$



22. $y = \dfrac{\sin x}{x}$



Topic D: Even/Odd Functions and Symmetry

Show work to determine if the relation is even, odd, or neither. You may want to research how to determine evenness and oddness.

1.) $f(x) = 7$

2.) $f(x) = 2x^2 - 4x$

3.) $f(x) = -3x^3 - 2x$

4.) $f(x) = \sqrt{x+1}$

5.) $f(x) = \sqrt{x^2+1}$

6.) $f(x) = |8x|$

Show work to determine if the graphs of these equations are symmetric to the *x*-axis, *y*-axis, or the origin.

7.) $4x = 1$

8.) $y^2 = 2x^4 + 6$

9.) $3x^2 = 4y^3$

10.) $x = |y|$

11.) $|x| = |y|$

12.) $|x| = y^2 + 2y + 1$

Topic E: Function Transformations

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:

1.) $f(x) - 4$

2.) $f(x-4)$

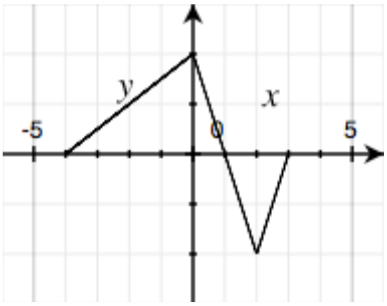
3.) $-f(x+2)$

4.) $5f(x)+3$

5.) $f(2x)$

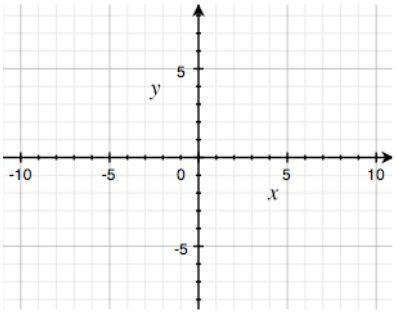
6.) $|f(x)|$

Here is a graph of $y = f(x)$:

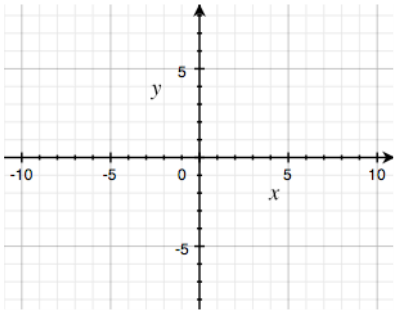


Sketch the following graphs:

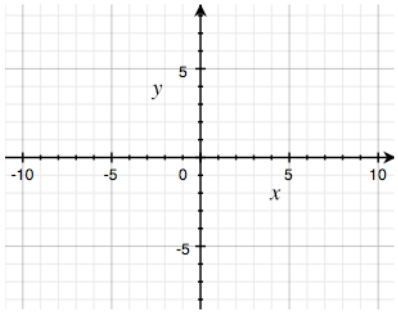
7.) $y = 2f(x)$



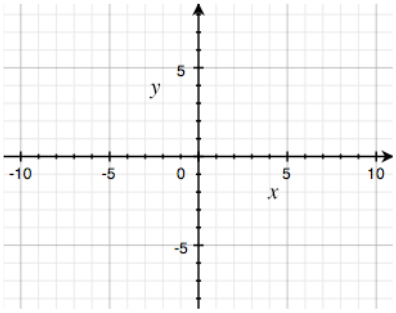
8.) $y = -f(x)$



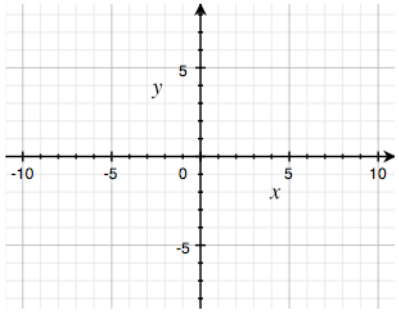
9.) $y = f(x-1)$



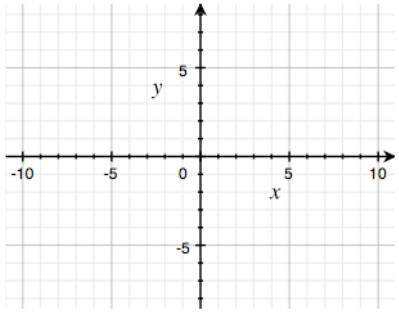
10.) $y = f(x)+2$



11.) $y = |f(x)|$



12.) $y = f(|x|)$



Topic F: Special Factorization

Factor completely.

1.) x^3+8

2.) x^3-8

3.) $27x^3-125y^3$

4.) x^4+11x^2-80

5.) $ac+cd-ab-bd$

6.) $2x^2+50y^2-20xy$

7.) $x^2+12x+36-9y^2$

8.) $x^3-xy^2+x^2y-y^3$

9.) $(x-3)^2(2x+1)^3+(x-3)^3(2x+1)^2$

Topic G: Linear Functions

1.) Find the equation of the line in point-slope form, with the given slope, passing through the given point.

a.) $m=-7,\ (-3,-7)$ b.) $m=-\frac{1}{2},\ (2,-8)$ c.) $m=\frac{2}{3},\ \left(-6,\frac{1}{3}\right)$

2.) Find the equation of the line in point-slope form, passing through the given points.

a.) $(-3,6),\ (-1,2)$ b.) $(-7,1),\ (3,-4)$ c.) $\left(-2,\frac{2}{3}\right),\ \left(\frac{1}{2},1\right)$

3.) Find the equations of the lines through the given point that are a.) parallel and b.) normal to the given line.

a.) $(5,-3),\ x+y=4$ b.) $(-6,2),\ 5x+2y=7$ c.) $(-3,-4),\ y=-2$

4.) Find the equation of the line in general form, containing the point $(4,-2)$ and parallel to the line containing the points $(-1,4)$ and $(2,3)$.

5.) Find k if the lines $3x-5y=9$ and $2x+ky=11$ are a.) parallel and b.) perpendicular.

Topic H: Solving Quadratic and Polynomial Equations

Solve each equation for x over the real number system.

1.) $x^2 + 7x - 18 = 0$

2.) $x^2 + x + \frac{1}{4} = 0$

3.) $2x^2 - 72 = 0$

4.) $12x^2 - 5x = 2$

5.) $20x^2 - 56x + 15 = 0$

6.) $81x^2 + 72x + 16 = 0$

7.) $x + \frac{1}{x} = \frac{17}{4}$

8.) $x^3 - 5x^2 + 5x - 25 = 0$

9.) $2x^4 - 15x^3 + 18x^2 = 0$

10.) If $y = x^2 + kx - k$, for what values of k will the quadratic have two real solutions?

Topic I: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptote (if it exists) and the location of any holes.

1.) $y = \frac{x-1}{x+5}$

2.) $y = \frac{8}{x^2}$

3.) $y = \frac{2x+16}{x+8}$

4.) $y = \frac{2x^2+6x}{x^2+5x+6}$

5.) $y = \frac{x}{x^2-25}$

6.) $y = \frac{x^2-5}{2x^2-12}$

7.) $y = \frac{x^3}{x^2+4}$

8.) $y = \frac{x^3+4x}{x^3-2x^2+4x-8}$

9.) $y = \frac{10x+20}{x^3-2x^2-4x+8}$

10.) $y = \frac{1}{x} - \frac{x}{x+2}$ (Hint: Express with a common denominator)

Topic J: Negative and Fractional Exponents

Simplify and write with positive exponents.

1.) $-12^2 x^{-5}$

2.) $\left(-12x^5\right)^{-2}$

3.) $\left(4x^{-1}\right)^{-1}$

4.) $\left(\frac{-4}{x^4}\right)^{-3}$

5.) $\left(\frac{5x^3}{y^2}\right)^{-3}$

6.) $\left(x^3-1\right)^{-2}$

7.) $\left(121x^8\right)^{1/2}$

8.) $\left(8x^2\right)^{-4/3}$

9.) $\left(-32x^{-5}\right)^{-3/5}$

10.) $\frac{1}{4}\left(16x^2\right)^{-3/4}\left(32x\right)$

11.) $\frac{\left(x^2-1\right)^{-1/2}}{\left(x^2+1\right)^{1/2}}$

12.) $\left(x^{-2}+2^{-2}\right)^{-1}$

Topic K: Complex Fractions

Eliminate the complex fractions:

1.) $\dfrac{\frac{5}{8}}{-\frac{2}{3}}$

2.) $\dfrac{4-\frac{2}{9}}{3+\frac{4}{3}}$

3.) $\dfrac{2+\frac{7}{2}+\frac{3}{5}}{5-\frac{3}{4}}$

4.) $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

5.) $\dfrac{1+x^{-1}}{1-x^{-2}}$

6.) $\dfrac{x^{-1}+y^{-1}}{x+y}$

7.) $\dfrac{x^{-2}+x^{-1}+1}{x^{-2}-x}$

8.) $\dfrac{\frac{1}{3}(3x-4)^{-3/4}}{-\frac{3}{4}}$

9.) $\dfrac{2x(2x-1)^{1/2}-2x^2(2x-1)^{-1/2}}{(2x-1)}$

Topic L: Inverses

Find the inverse of each of the following functions and use a graphing utility (TI-Nspire or Desmos) to show graphically that its inverse is a function.

1.) $2x-6y=1$

2.) $y=ax+b$

3.) $y=9-x^2,\ x\geq 0$

4.) $y=\sqrt{1-x^3}$

5.) $y=\dfrac{9}{x}$

6.) $y=\dfrac{2x+1}{3-2x}$

Find the inverse of each of the following functions and show that $f\left(f^{-1}(x)\right)=x$

7.) $f(x)=\dfrac{1}{2}x-\dfrac{4}{5}$

8.) $f(x)=x^2-4$

9.) $f(x)=\dfrac{x^2}{x^2+1}$

10.) Without finding the inverse, find the domain and range of the inverse to $f(x)=\dfrac{\sqrt{x+1}}{x^2}$

Topic M: Adding Fractions and Solving Rational Equations

1.) Combine the following fractions:

a.) $\frac{2}{3}-\frac{1}{x}$

b.) $\frac{1}{x-3}+\frac{1}{x+3}$

c.) $\frac{5}{2x}-\frac{5}{3x+15}$

d.) $\frac{2x-1}{x-1}-\frac{3x}{2x+1}$

2.) Solve the equation for *x*.

a.) $\frac{2}{3}-\frac{1}{x}=\frac{5}{6}$

b.) $\frac{1}{x-3}+\frac{1}{x+3}=\frac{10}{x^2-9}$

c.) $\frac{5}{2x}-\frac{5}{3(x+5)}=\frac{5}{x}$

d.) $\frac{2x-1}{x-1}-\frac{3x}{2x+1}=\frac{x^2+11}{2x^2-x-1}$

Topic N: Absolute Value Equations

Solve the following equations:

1.) $4|x+8| = 20$

2.) $|1-7x| = 13$

3.) $|8+2x| + 2x = 40$

4.) $|4x-5| + 5x + 2 = 0$

5.) $|x^2 - 2x - 1| = 7$

6.) $|12 - x| = x^2 - 12x$

Topic O: Solving Inequalities

Solve the following inequalities:

1.) $5(x-3) \le 8(x+5)$

2.) $4 - \frac{5x}{3} > -\left(2x + \frac{1}{2}\right)$

3.) $\frac{3}{4} > x + 1 > \frac{1}{2}$

4.) $x + 7 \ge |5 - 3x|$

5.) $(x+2)^2 < 25$

6.) $x^3 < 4x^2$

7.) $\frac{5}{x-6} \ge \frac{1}{x+2}$

8.) Find the domain of: $\sqrt{\frac{x^2 - x - 6}{x - 4}}$

Topic P: Exponential Functions and Logarithms

Simplify the following:

1.) $\log_2 \frac{1}{4}$

2.) $\log_8 4$

3.) $\ln \frac{1}{\sqrt[3]{e^2}}$

4.) $5^{\log_5 40}$

5.) $e^{\ln 12}$

6.) $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

7.) $\log_2 \frac{2}{3} + \log_2 \frac{3}{32}$

8.) $\log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$

9.) $\log_3 \left(\sqrt{3}\right)^5$

Solve the following:

10.) $\log_5 (3x-8) = 2$

11.) $\log_9 \left(x^2 - x + 3\right) = \frac{1}{2}$

12.) $\log(x-3) + \log 5 = 2$

13.) $\log_2 (x-1) + \log_2 (x+3) = 5$

14.) $\log_5 (x+3) - \log_5 x = 2$

15.) $\ln x^3 - \ln x^2 = \frac{1}{2}$

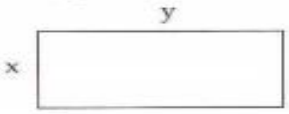
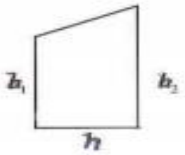
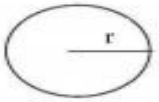
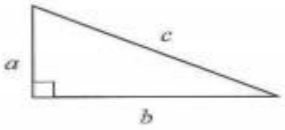
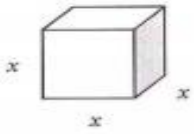
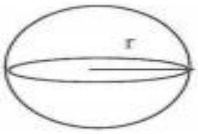
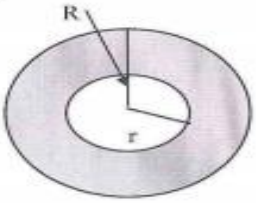
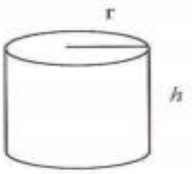
16.) $3^{x-2} = 18$

17.) $e^{3x+1} = 10$

18.) $8^x = 5^{2x-1}$

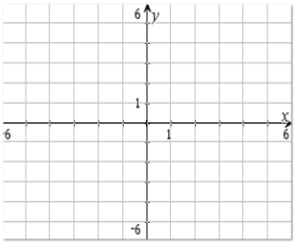
Topic Q: Geometry

1.) You will use each of the following forumlas in AP Calculus AB. Complete each of the following.

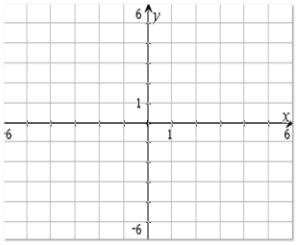
Square  Perimeter = _____ Area = _____	**Rectangle**  Perimeter = _____ Area = _____	**Trapezoid**  Area = _____
Circle  Circumference = _____ Area = _____	**Triangle**  Pythagorean Theorem (only good for right triangles) = _____ Area (of any triangle) = _____	**Cube**  Volume = _____ Surface Area = _____
Sphere  Volume = _____	**"Washer"**  Area of the shaded region = _____	**Cylinder**  Volume = _____

Find the area between the x-axis and $f(x)$ from $x = 0$ to $x = 5$. Sketch the region to verify.

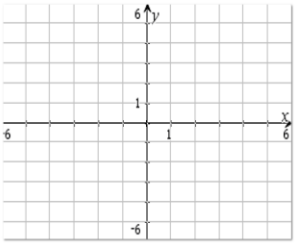
2.) $f(x) = 4$



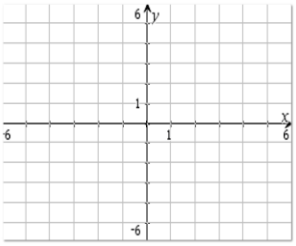
3.) $f(x) = x$



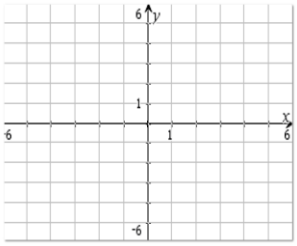
4.) $f(x) = x + 3$



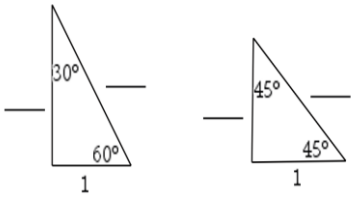
5.) $f(x) = \sqrt{9 - x^2}$



6.) $f(x) = \begin{cases} x+1, & x \le 2 \\ 5-x, & x > 2 \end{cases}$



7.) Fill in the four blanks.



Topic R: Basic Right Angle Trigonometry

(Some ACE AP Calculus students will have to complete this portion of the packet after the school year begins.)

Solve the following:

If point *P* is on the terminal side of θ, find all 6 trigonometric functions of θ. (Answers need not be rationalized.)

1.) $P(-2,4)$

2.) $P\left(\sqrt{5},-2\right)$

3.) If $\cos\theta = -\frac{5}{13}$, in quadrant II, find $\sin\theta$ and $\tan\theta$.

4.) If $\cot\theta = \frac{2\sqrt{10}}{3}$, in quadrant III, find $\sin\theta$ and $\cos\theta$.

5.) State the quadrant in which each of the following is true.

a.) $\sin\theta > 0$ and $\cos\theta < 0$

b.) $\csc\theta < 0$ and $\cot\theta > 0$

c.) $\tan\theta > 0$ and $\sec\theta < 0$