Regina Dominican High School

Entering Geometry

Summer Packet

Name _____

Due on the First Day of School

Following are six topics from Algebra that you should be comfortable with for success in Geometry. This packet is due when you return to school and will be followed by a brief no calculator quiz to check your understanding. Remember to ***check your answers*** with those at the end of the packet.

NO CALCULATOR

I. EXPONENTS

The exponent of a number shows you how many times the number is to be used in a multiplication.

It is written as a small number to the right and above the base number.

In this example: $8^2 = 8 \times 8 = 64$

Another name for exponent is index or power.

We have rules for working with exponents just as we have rules for working with non-exponent numbers. Here are several that you should be fluent with.

Product of Powers Property

$a^m \cdot a^n = a^{m+n}$

For example: $3^2 \cdot 3^7 = 3^{2+7} = 3^9$

Simplify – Show ALL work (See answers to all practice problems on back page):

1. $5^3 \cdot 5^6$

2. $x^2 \cdot x^3 \cdot x^4$

3. $3 \cdot 3^5$

4. $(-2)(-2)^4$

NO CALCULATOR

Power of a Power Property

$(a^m)^n = a^{m*n}$

For example: $(5^2)^4 = 5^{2*4} = 5^8$

Simplify – Show ALL work:

5. $(3^5)^2$

6. $(y^2)^4$

7. $[(-3)^3]^2$

8. $[(a + 1)^2]^5$

Power of a Product Property

$(a \cdot b)^m = a^m \cdot b^m$

For example: $(2 \cdot 3)^6 = 2^6 \cdot 3^6$

Simplify – Show ALL work:

9. $(6 \cdot 5)^2$

10. $(4yz)^3$

11. $(-2w)^2$

12. $-(2w)^2$

NO CALCULATOR

Putting it all together

Simplify:

13. $(2x^4y^2)^2 \cdot x^3$

II. RADICALS

Check out these videos for help: https://youtube.com/playlist?list=PL9mIY8V-Yex2kE1g6wnKfLC2r9EDX5ZnH

14. Write the first 10 perfect squares:

An expression with radicals in it is in SIMPLEST FORM if the following are true:

- No perfect square factors other than 1 are in the radicand
- No fractions are in the radicand
- No radicals appear in the denominator of a fraction

Product Property

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ When *a* and *b* are positive numbers

For example: $\sqrt{4 \cdot 100} = \sqrt{4} \cdot \sqrt{100} = 2 \cdot 10 = 20$

Simplify – Show ALL work:

15. $\sqrt{50}$

16. $\sqrt{44}$

17. $\sqrt{90}$

18. $\sqrt{200}$

NO CALCULATOR

Quotient Property

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ When *a* and *b* are positive numbers

For example: $\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$

Simplify – Show ALL work:

19. $\sqrt{\frac{3}{4}}$

20. $\frac{\sqrt{20}}{4}$

21. $\sqrt{\frac{32}{50}}$

22. $\sqrt{\frac{7}{9}}$

Operations with Radical Expressions

Adding and subtracting – you can only add or subtract radicals with the same radicand (number under the radical).

For example:

$2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

You may need to simplify the radicals first.

Perform the indicated operation and simplify.

23. $2\sqrt{2} + \sqrt{5} - 6\sqrt{2}$

24. $4\sqrt{3} - \sqrt{27}$ (simplify first!)

25. $\sqrt{18} + \sqrt{32}$

NO CALCULATOR

Multiplying and Dividing – use the product property from above.

Example 1: $\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$

Example 2: $\sqrt{2}(5-\sqrt{3}) = 5\sqrt{2} - \sqrt{2}\sqrt{3} = 5\sqrt{2} - \sqrt{6}$

Perform the indicated operation.

26. $\sqrt{3} \cdot \sqrt{12}$

27. $\sqrt{6}(7\sqrt{3}+6)$

28. $2\sqrt{6} \cdot 5\sqrt{10}$

III. The Distributive Property and Factoring

The Distributive Property

a(b+c) = ab + ac

a(b-c) = ab – ac

(a+b)(c+d) = a(c+d) + b(c+d) = ac + ad + bc + bd

For Example:

3(x + 2) = 3x + 6

5(y – 3) = 5y – 15

(x + 2)(x – 4) = x(x – 4) + 2(x – 4) = x^2 – 4x + 2x – 8 = x^2 – 2x – 8

Simplify:

29. 4(x – 2)

30. -2(3 + y)

31. (x – 2)(x + 5)

32. (x + b)(y + c)

NO CALCULATOR

Factoring with leading coefficient 1

If you need help, check out these videos: https://youtube.com/playlist?list=PL9mIY8V-Yex0gyNLKVIW2RSEFlh5yfjHI

To factor an expression of the form $x^2 + bx + c$ we need to put it in the form $(x + p)(x + q)$ such that if we were to use the distributive property to multiply we'd get $x^2 + bx + c$. In other words, we need to find numbers p and q such that $pq = c$ and $p+q = b$

For example:

Factor $x^2 + 3x + 2$.

I need two numbers, p and q, that when added give me 3 and when multiplied give me 2.

If $p = 1$ and $q = 2$, those numbers add to 3 and multiply to 2.

So, if I factor $x^2 + 3x + 2$ I get $(x + 1)(x + 2)$

We need to then verify this answer by applying the distributive property.

$(x + 1)(x + 2) = x(x + 2) = 1(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$

Therefore my answer is indeed $(x + 1)(x + 2)$.

Another way is to use a guess-and-check (or expansion) box:

If we want to factor $x^2 - 4x + 3$, start with x^2 in the top left and 3 in the bottom right.

	x	?
x	x^2	
?		3

We know that x should be in both spots that multiply to that top left box. Now we need to guess and check what goes in the spots with the ? so that when we multiply, we get our original expression.

We guess 3 for the top ? and 1 for the other one. When we multiply, we get:

	x	3
x	x^2	3x
1	1x	3

Since we have $x^2 + 3x + 1x + 3$, which equals $x^2 + 4x + 3$, the expression I started with, I know the correct factorization is $(x + 3)(x + 1)$

Factor:

33. $x^2 - 5x + 6$

34. $x^2 - 2x - 8$

35. $x^2 + 7x - 18$

NO CALCULATOR

Factoring with leading coefficient not one

If you need help, check out these videos: https://youtube.com/playlist?list=PL9mIY8V-Yex1bOcqQvysfNOlFbFFkjFwY

To factor an expression of the form $ax^2 + bx + c$ we need to consider more than just finding numbers that add to be and multiply to c. We need to come up with an expression of the form $(mx + p)(nx + q)$ such that $mn = a$, $pq = c$, and $mq + pn = b$.

For example:

Factor $2x^2 + 11x + 5$

m and n have to be 1 and 2, as those are the only numbers that when multiplied give us 2.

p and q have to be 1 and 5, as those are the only numbers that when multiplied give us 5.

That give us the possibilities of $(1x + 1)(2x + 5)$ or $(1x + 5)(2x + 1)$. Try them both:

$(1x + 1)(2x + 5) = 1x(2x + 5) + 1(2x + 5) = 2x^2 + 5x + 2x + 25 = 2x^2 + 7x + 25$. Not what we started with, so wrong.

$(1x + 5)(2x + 1) = 1x(2x + 1) + 5(2x + 1) = 2x^2 + 1x + 10x + 5 = 2x^2 + 11x + 5$. This one matches! We have our answer!

$(1x + 5)(2x + 1)$.

Another way is to use a guess-and-check (or expansion) box:

If we want to factor $2x^2 - 7x - 15$, start with $2x^2$ in the top left and -15 in the bottom right.

	x	?
2 x	$2x^2$	
?		-15

-

We know that the two spots that multiply to that top left box need to be x and 2x. Now we need to guess and check what goes in the spots with the ? so that when we multiply, we get our original expression.

We guess -5 for the top ? and 3 for the other one. When we multiply, we get:

	x	-5
2x	$2x^2$	-10x
3	3x	-15

Since we have $2x^2$ - 10x + 3x - 15, which equals $2x^2 – 7x$ - 15, the expression we started with, we know the correct factorization is (x -5)(2x + 3)

Factor:

36. $2x^2 + 7x + 3$

37. $3x^2 + 16x + 5$

NO CALCULATOR

IV. Solving Equations and Simplifying Expressions

Simplifying by Combining Like Terms

Example 1: $8x + 3x = 11x$

Example 2: $4x^2 + 2 - x^2 = 3x^2 + 2$

Example 3:

$3 - 2(4 + x)$

$= 3 + -2(4 + x)$

$= 3 + -8 + -2x$

$= -5 + -2x = -5 - 2x$

Simplify – Show ALL work:

38. $2y - (-4y - 3)$

39. $4x - (2x + 5)$

Solving Equations

Simplify each side of the equal sign first, then follow the order of operations in reverse to solve.

The order of operations for simplifying

Parentheses
Exponents
Multiplication/Division
Addition/Subtraction

The order to solve

Subtraction/Addition
Division/Multiplication
Exponents
Parentheses

For example:

$2x + 6 = 8$

$2x + 6 - \mathbf{6} = 8 - \mathbf{6}$

$2x = 2$

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

$5x + 3 = x + 11$

$5x + 3 - \mathbf{x} = x + 11 - \mathbf{x}$

$4x + 3 = 11$

$4x + 3 - \mathbf{3} = 11 - \mathbf{3}$

$4x = 8$

$\frac{4x}{4} = \frac{8}{4}$

$x = 2$

NO CALCULATOR

Solve – Show ALL work:

40. $3x - 1 = 8$

41. $6 = 14 - 2x$

42. $-12q + 4 = 8q - 6$

43. $12p - 7 = -3p + 8$

V. GRAPHING

Finding the slope of a line

When given two points on a line, $slope(m) = \frac{y_1 - y_2}{x_1 - x_2}$

For example:

Find the slope of the line that passes through (6,9) and (4,3).

$$m = \frac{9-3}{6-4} = \frac{6}{2} = 3$$

The slope of the line passing through (6,9) and (4,3) is 3

Find the slope through the following pairs of points:

44. (4,5) (2,3)

45. (2,4) (4,-4)

46. (2, -2) (3, -6)

NO CALCULATOR

Writing the equation of a line

One way to write the equation of a line is in slope-intercept form, or y = mx + b where m is the slope, b is the y-intercept, and (x,y) is any point on the line. We write the equation with numbers in place of the m and the b, and the x and y remaining variables.

We can write the equation of a line if we're given the slope and y-intercept, or the slope and any point on the line.

For example:

Write the equation of the line with slope (-2) and going through (6, -3).

We use our equation, y = mx + b and insert what we know, the slope for m and the point for x and y.

$-3 = -2 \cdot 6 + b$ There is only one variable that we don't know, b, so we solve for it.

First, simplify: -3 = -12 + b

Then, add 12 to both sides: 9 = b.

Now that we have both an m (given as -2) and a b (9) we can write the equation for the line: y = -2x + 9.

Write the equations for the lines:

47. Through point (1, 4) slope 3

48. Through point (-3, -5) slope -2

Graphing

Use the equation of the line (y = mx + b) to graph the line. First, plot the y-intercept (b), then use the slope (m)(rise over run) to plot additional points.
For example:

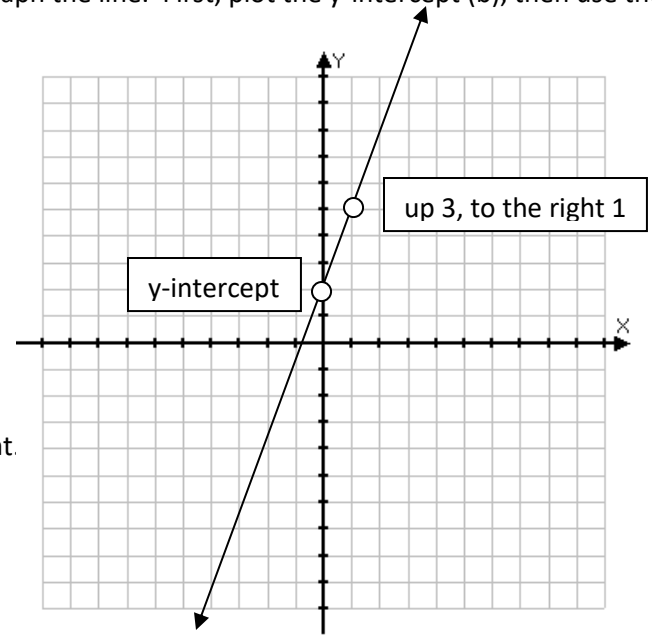
Graph y = 3x – 2

Slope: 3, y-intercept: -2

The slope is $\frac{rise}{run}$ or how far up you go over

how far to the right you go.

In this case, we go up 3 for every 1 to the right.



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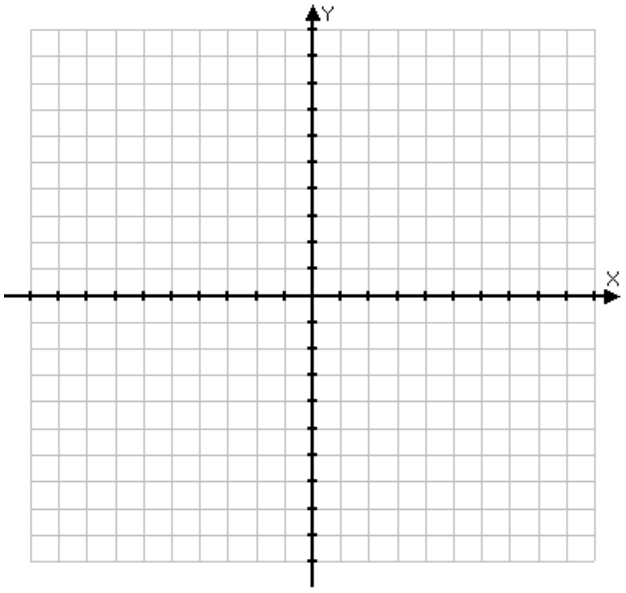
Graph the equations:

49. y = 2x – 1

Slope: _____

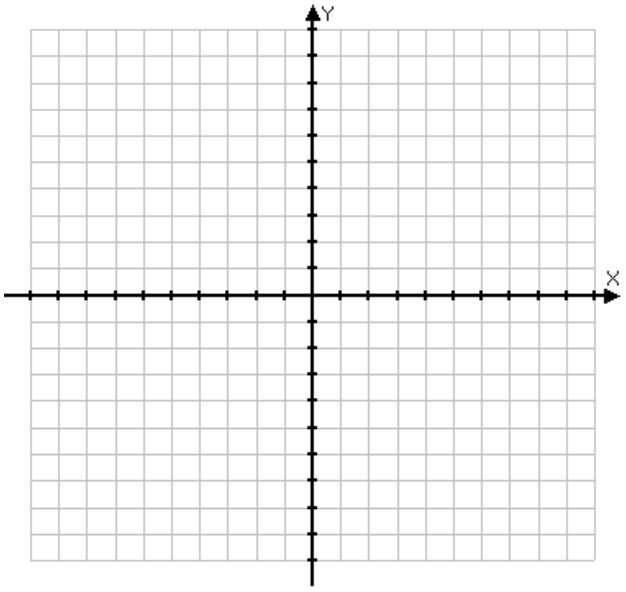
y-intercept: _____



50. y = -3x + 4

Slope: _____ (notice that it's negative!)

y-intercept _____



NO CALCULATOR

VI. SOLVING SYSTEMS OF EQUATIONS

Solving Linear Systems By Substitution

When one of the variables has 1 as a coefficient in at least one of the equations:

Step 1: Solve one of the equations for one of its variables

Step 2: Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3: Substitute the value from Step 2 into the revised equation from Step 1 and solve.

Step 4: Check the solution in each of the original equations.

For example:

Solve the linear system:

$-x + y = 1$
$2x + y = -2$

Here y has a coefficient of 1 in each of the equations.

Step 1, solve: $-x + y = 1$

$-x + y + x = 1 + x$

$y = 1 + x$

Step 2, substitute and solve: $2x + (1 + x) = -2$

$3x + 1 = -2$

$3x = -3$

$x = -1$

Step 3, substitute and solve: $2(-1) + y = -2$

$-2 + y = -2$

$y = 0$

Step 4, check: $-x + y = 1$ $-(-1) + 0 = 1$ ☑

$2(-1) + 0 = -2$ $-2 + 0 = -2$ ☑

The point that works in both equations is (-1,0)

51. Solve the system:

$2x + y = -10$

$3x – y = 0$

NO CALCULATOR

Solving Linear Systems By Elimination

Step 1: Arrange the equations with like terms in columns

Step 2: Multiply one or both of the equations by a number to obtain coefficients that are opposites for one of the variables

Step 3: Add the equations from Step 2. Combining like terms will eliminate one variable. Solve for the remaining variable.

Step 4: Substitute the value obtained in Step 3 into either of the original equations and solve for the other variable.

Step 5: Check the solution in each of the original equations.

For example:

Solve the system: $3x + 5y = 6$

$-4x + 2y = 5$

Step 1: $3x + 5y = 6$

$-4x + 2y = 5$

Step 2 (Multiply): $4(3x + 5y) = 4(6)$

$3(-4x + 2y) = 3(5)$

$12x + 20y = 24$

$-12x + 6y = 15$

Step 3 (Add and solve): $12x + 20y + \textit{-12x + 6y} = 24 + \textit{15}$

$26y = 39$

$y = 1.5$

Step 4 (Substitute): $-4x + 2(1.5) = 5$

$-4x + 3 = 5$

$-4x = 2$

$x = -0.5$

Step 5 (Check): $-4(-0.5) + 2(1.5) = 5$ $\quad 2 + 3 = 5$ ☑

$3(-0.5) + 5(1.5) = 6$ $\quad -1.5 + 7.5 = 6$ ☑

Keep going! One more problem on the last page….

NO CALCULATOR

52. Solve the system:

x + 2y = 5

5x – y = 3

Answers

Exponents

1. 5^9
2. x^9
3. 3^6
4. $(-2)^5$
5. 3^{10}
6. y^8
7. $(-3)^6$
8. $(a+1)^{10}$
9. 900
10. $64y^3z^3$
11. $4w^2$
12. $-4w^2$
13. $4x^{11}y^4$

Radicals

14. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 (because $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, and $10^2 = 100$)
15. $5\sqrt{2}$
16. $2\sqrt{11}$
17. $3\sqrt{10}$
18. $10\sqrt{2}$
19. $\frac{\sqrt{3}}{2}$
20. $\frac{\sqrt{5}}{2}$
21. $\frac{4}{5}$
22. $\frac{\sqrt{7}}{3}$
23. $\sqrt{5} - 4\sqrt{2}$
24. $\sqrt{3}$
25. $7\sqrt{2}$
26. 6
27. $21\sqrt{2} + 6\sqrt{6}$
28. $20\sqrt{15}$

The Distributive Property and Factoring

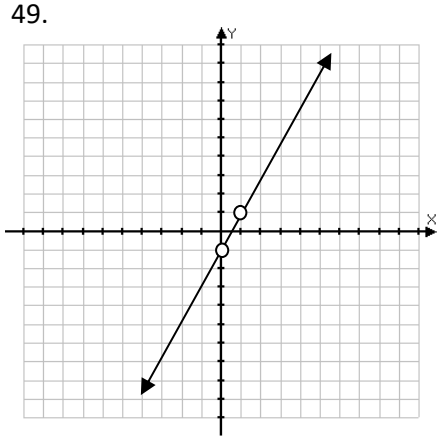
29. 4x - 8
30. -6 – 2y
31. $x^2 + 3x - 10$
32. xy + cx + by + bc
33. (x – 3)(x – 2)
34. (x – 4)(x + 2)
35. (x + 9)(x – 2)
36. (2x + 1)(x + 3)
37. (3x + 1)(x + 5)

Solving Equations and Simplifying Expressions

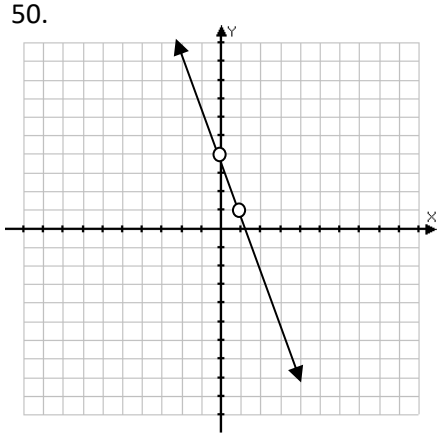
38. 6y + 3
39. 2x - 5
40. x = 3
41. x = 4
42. q = .5 = ½
43. p = 1

Graphing

44. 1
45. -4
46. -4
47. y = 3x + 1
48. y = -2x - 11
49.



50.



Systems of Equations

51. (-2, -6)
52. (1, 2)