

Summer Assignment

Show ALL work clearly and circle final answers. You are to do this entire packet without a calculator.

If you do not understand something, look back at your notes from AB. This should all be review. Otherwise, email me at tjewell@canterburyfortmyers.org.

There will be a test in the first few days of class on this material.

Evaluate each limit.

1) $\lim_{x \rightarrow \frac{\pi}{3}} \cos(2x)$

2) $\lim_{s \rightarrow -3} (2s^2 + 12s + 16)$

3) $\lim_{x \rightarrow 1^-} f(x), f(x) = \begin{cases} 2x + 1, & x \leq 1 \\ x^2 - 6x + 8, & x > 1 \end{cases}$

4) $\lim_{x \rightarrow 0^+} f(x), f(x) = \begin{cases} x^2 + 4x + 3, & x < 0 \\ \frac{x}{2}, & x \geq 0 \end{cases}$

5) $\lim_{x \rightarrow 1} f(x), f(x) = \begin{cases} -2, & x < 1 \\ x - 3, & x \geq 1 \end{cases}$

6) $\lim_{x \rightarrow 1} f(x), f(x) = \begin{cases} -x^2 + 6x - 8, & x \leq 1 \\ 2x - 8, & x > 1 \end{cases}$

$$7) \lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x+6}-3}$$

$$8) \lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x+3}$$

$$9) \lim_{x \rightarrow 0} \frac{x}{\frac{1}{-2+x} + \frac{1}{2}}$$

$$10) \lim_{x \rightarrow 2} \frac{x^2 - 2x}{x-2}$$

$$11) \lim_{x \rightarrow 3^+} \frac{4x}{x-3}$$

$$12) \lim_{x \rightarrow 1^+} \frac{x-3}{x^2 - 2x + 1}$$

$$13) \lim_{x \rightarrow -\infty} \frac{2x^2}{x^2 + 4}$$

$$14) \lim_{x \rightarrow \infty} \left(-\frac{\ln x}{x^3} - 2 \right)$$

$$15) \lim_{x \rightarrow \infty} (e^x + 1)$$

$$16) \lim_{x \rightarrow -\infty} 3e^{\frac{1}{x}}$$

For each problem, find the average rate of change of the function over the given interval.

$$17) f(x) = \frac{1}{x-2}; \quad [-4, -3]$$

$$18) f(x) = -2x^2 - x - 2; \quad [-1, 1]$$

Use the definition of the derivative to find the derivative of each function with respect to x . I understand the power and chain rules can be used for this. You must use the limit definition for credit.

19) $y = \sqrt{3x - 1}$

20) $y = x^2 - 4x + 3$

Differentiate each function with respect to x .

21) $y = -\frac{4}{5}x^2 - \frac{5}{3}\sqrt[3]{x^2} + 1$

22) $y = 4x^4 - \frac{2}{3} - \frac{5}{4x^2}$

23) $y = \frac{3}{2}\sqrt[3]{x^2} + \frac{3}{2} + \frac{4}{5x^5}$

24) $y = \frac{4}{5}x^4 - \frac{3}{4}\sqrt[5]{x} + \frac{2}{3x^3}$

For each problem, find the indicated derivative with respect to x .

25) $y = \sqrt[4]{x} + 5x^{-3}$ Find $\frac{d^2y}{dx^2}$

26) $y = -3\sqrt[4]{x} + 5x^{-3}$ Find $\frac{d^2y}{dx^2}$

Differentiate each function with respect to x using the PRODUCT rule.

27) $y = (-4 + 3x^{-4}) \cdot -5x^2$

28) $y = (4x^3 + \sqrt[5]{x} + 1)(3x^3 + 2)$

Differentiate each function with respect to x using the QUOTIENT rule.

$$29) \ y = \frac{5x^3 + 4}{5x^5 + 4}$$

$$30) \ y = \frac{4x^5 - 4x^4 - 3x^3}{x^3 + 5}$$

Differentiate each function with respect to x using the CHAIN rule.

$$31) \ y = (5x^4 + 2)^3$$

$$32) \ y = (5x^3 + 3)^5$$

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

33)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	1	2	2
2	2	$\frac{3}{2}$	4	$\frac{1}{2}$
3	4	0	3	-1
4	2	-2	2	-1

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(3)$

Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(2)$

Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(3)$

Part 4) Given $h_4(x) = \frac{f(x)}{g(x)}$, find $h_4'(3)$

Part 5) Given $h_5(x) = (f(x))^2$, find $h_5'(2)$

Part 6) Given $h_6(x) = f(g(x))$, find $h_6'(4)$

Differentiate each function with respect to x .

34) $y = \csc(\cos x^3)$

35) $y = (x^2 + 4)\tan 5x^5$

36) $y = \sec x^4 \sin x^5$

37) $y = \tan (\cos 3x^2)$

Find the derivative of each transcendental function with respect to x .

38) $y = (4x^3 + 5) \cdot e^{2x^2}$

39) $y = \ln 3x^2$

40) $y = (4x^3 + 1) \ln x^5$

41) $y = \frac{x^2 + 1}{\ln 5x^4}$

42) $y = \log_5 3x^5$

43) $y = 2^{3x^4}$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

44) $2x^2 = 2y + x^2y^2$

45) $3x = 3y^3 + 4$

Find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

46) $y = 2x^2 - 16x + 34$ at $(3, 4)$

Find the equation of the line NORMAL to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intercept form.

47) $y = -x^2 - 6x - 7$ at $(0, -7)$

For each problem, find the open intervals where the function is increasing and decreasing.

48) $y = -2x^2 + 12x - 19$

49) $y = -x^3 + 3x^2 - 3$

For each problem, find the open intervals where the function is concave up and concave down.

50) $y = x^3 - 2x^2$

51) $y = x^3 - 12x^2 + 45x - 55$

For each problem, find all points of relative minima and maxima.

52) $y = \frac{x^2}{2} - x + \frac{3}{2}$

53) $y = x^3 - 3x^2 - 2$

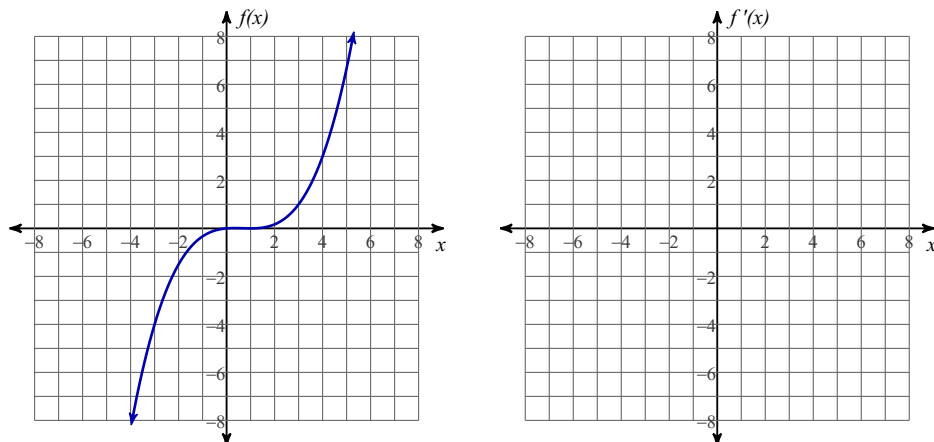
Solve each optimization problem.

- 54) A supermarket employee wants to construct an open-top box from a 10 by 16 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

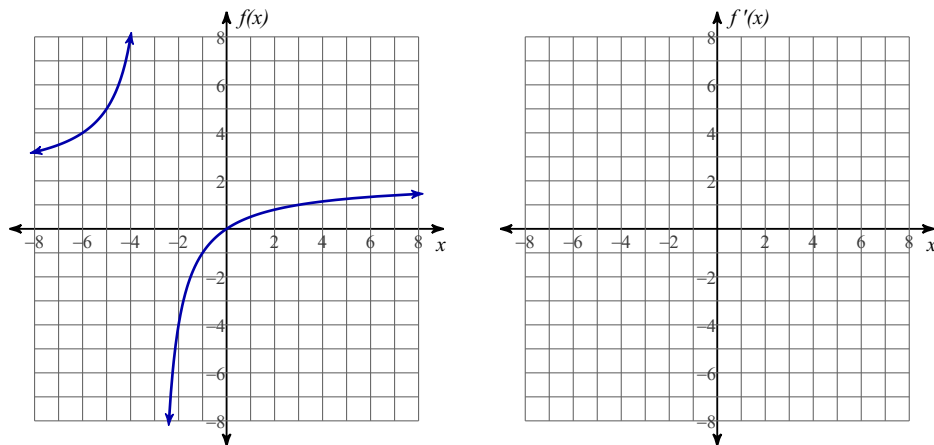
- 55) A farmer wants to construct a rectangular pigpen using 300 m of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?

Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

56)



57)



A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, and the intervals of time when the particle is slowing down and speeding up.

58) $s(t) = t^2 - 6t - 40$

59) $s(t) = t^3 - 13t^2 + 40t$

Solve each related rate problem.

60) A hypothetical square grows so that the length of its sides are increasing at a rate of 5 yd/min. How fast is the area of the square increasing when the sides are 6 yd each?

61) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 5 cm/min. How fast is the area of the pool increasing when the radius is 12 cm?

Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.

62) $\lim_{x \rightarrow 0} \frac{e^{5x} - 1}{\sin(4x)}$

63) $\lim_{x \rightarrow 1} \frac{3 \ln x}{x - 1}$

64) $\lim_{x \rightarrow 1} \frac{2 \ln x}{x}$

65) $\lim_{x \rightarrow \infty} \frac{e^{2x}}{2x}$

Evaluate each indefinite integral.

$$66) \int \frac{14x^{\frac{5}{2}}}{2} dx$$

$$67) \int -\frac{3}{x^4} dx$$

$$68) \int 18x^5 dx$$

$$69) \int -\frac{25\sqrt[3]{x^2}}{3} dx$$

$$70) \int -\frac{3}{x} dx$$

$$71) \int -5x^{-1} dx$$

$$72) \int -\frac{1}{x} dx$$

$$73) \int -e^x dx$$

$$74) \int 2 \cdot 3^x dx$$

$$75) \int 4^x dx$$

$$76) \int -3 \cdot 5^x dx$$

$$77) \int -3^x dx$$

$$78) \int -4\cos x dx$$

$$79) \int -3\sec x \tan x dx$$

$$80) \int 3\cos x dx$$

$$81) \int 4\sec^2 x dx$$

$$82) \int \frac{1}{9+x^2} dx$$

$$83) \int \frac{1}{\sqrt{9-x^2}} dx$$

$$84) \int \frac{1}{x\sqrt{x^2-4}} dx$$

$$85) \int \frac{1}{\sqrt{25-x^2}} dx$$

$$86) \int (4x^2+1)^5 \cdot 8x dx$$

$$87) \int (3x^2+4)^3 \cdot 6x dx$$

$$88) \int (5x^5-3)^3 \cdot 25x^4 dx$$

$$89) \int 3x^2(x^3-5)^3 dx$$

$$90) \int \frac{48x^2}{4x^3 + 3} dx$$

$$91) \int 18x^2 e^{3x^3 + 4} dx$$

$$92) \int \frac{5e^{\ln x}}{x} dx$$

$$93) \int \frac{6e^{3x}}{e^{3x} + 4} dx$$

$$94) \int 6x^2 \sin(2x^3 + 1) dx$$

$$95) \int 9x^2 \csc(x^3 + 5) \cot(x^3 + 5) dx$$

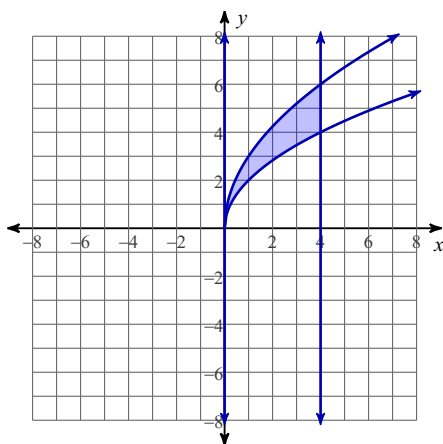
Evaluate each definite integral.

96) $\int_3^4 (-x - 1) dx$

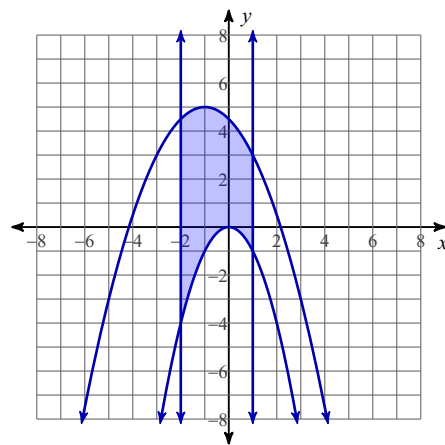
97) $\int_{-3}^0 (-x^2 - 4x - 5) dx$

For each problem, find the area of the region enclosed by the curves.

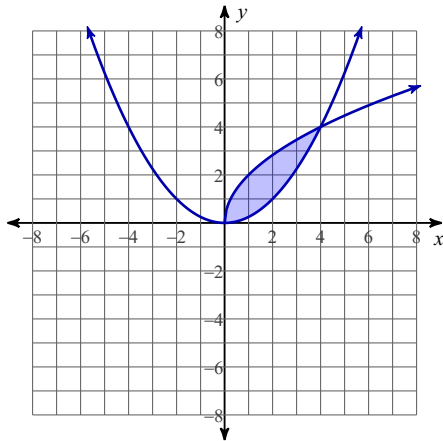
98) $y = 3\sqrt{x}$, $y = 2\sqrt{x}$,
 $x = 0$, $x = 4$



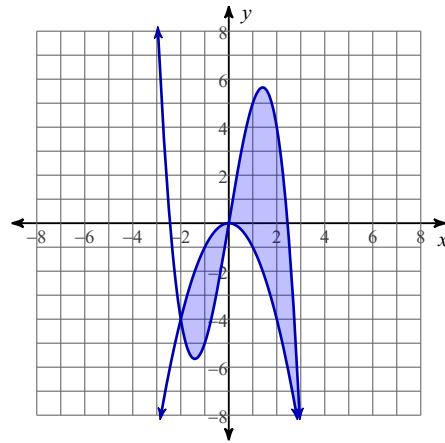
99) $y = -x^2$, $y = -\frac{x^2}{2} - x + \frac{9}{2}$,
 $x = -2$, $x = 1$



100) $y = 2\sqrt{x}$, $y = \frac{x^2}{4}$



101) $y = -x^3 + 6x$, $y = -x^2$



A particle moves along a coordinate line. Its velocity function is $v(t)$ for $t \geq 0$. Find the displacement of the particle over the given interval.

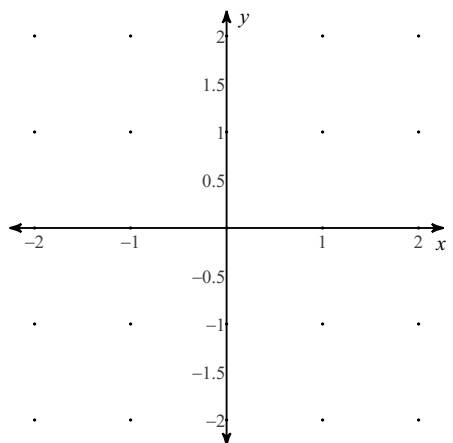
102) $v(t) = -2t + 9$; $1 \leq t \leq 9$

A particle moves along a coordinate line. Its velocity function is $v(t)$ for $t \geq 0$. Find the position function $s(t)$.

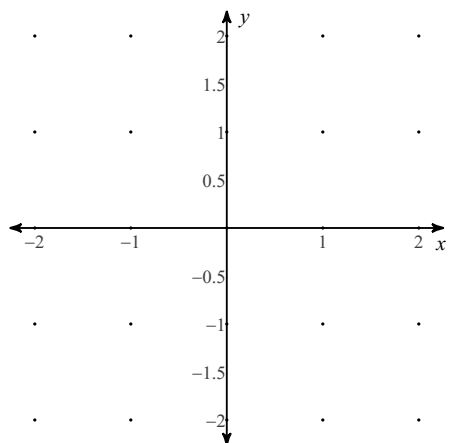
103) $v(t) = 3t^2 - 2t - 56$; $s(0) = 0$

Sketch the slope field for each differential equation. Use all integer coordinates.

104) $\frac{dy}{dx} = x - y$



105) $\frac{dy}{dx} = xy$



Find the general solution of each differential equation.

106) $\frac{dy}{dx} = \frac{1}{\sec^2 y}$

107) $\frac{dy}{dx} = 2e^{x-y}$