AP Calculus AB – Summer Assignment

Complete all the problems on the following pages.

AP Calculus AB requires a strong foundation in polynomials, rational functions, exponential and logarithmic functions, the unit circle and trigonometric functions/equations, and graphical analysis. In addition, you must have a general idea of what the graph of each parent function looks like, including their respective end behaviors. In this course, all algebra knowledge is assumed, and we start by looking at the ideas of what makes calculus what it is. There is not much time – if any – for algebra review, so make sure you are comfortable with the problems here!

You may work together on these, but understand that working together is **NOT** dividing up the work and sharing answers – this behavior will set you behind before the class even starts, and it is also considered to be academic misconduct. Part of learning the material requires genuine practice and struggle, so give yourself the time to properly complete the questions. Use your notes from Precalculus! **You must show your work to receive credit!**

This packet will be due in the first week of class. **Do not wait until the last minute to do this!** Please email me at chris.kunz@longbeachsaints.org if you have any questions!

Linear Equations

1. Write the equation for each line in point-slope form:

(a) The line containing the point $(4,2)$ with a slope of $\frac{1}{2}$

(b) The line containing the point $(-1,3)$ with $m = 2$

(c) The line containing the points $(1,1)$ and $(4,5)$

2. Solve each system of equations by your choice of substitution or elimination:

(a) $\begin{cases} 6x - 5y = -28 \\ 7x + y = 22 \end{cases}$

(b) $\begin{cases} 5x + 3y = -7 \\ 2x + 7y = 3 \end{cases}$

Polynomial Equations

3. Factor the polynomials below completely! Check for GCF, quadratic types, difference of squares/cubes, sum of cubes, grouping, etc.:

(a) $2x^2 + 12x + 16$

(b) $x^2 - 3x + 10$

(c) $x^4 + 2x^2 - 24$

(d) $8x^3 + 27$ (e) $2x^3 - 18x$ (f) $x^3 - 4x^2 + 2x - 8$

(g) $5x^3 - x^2 - 5x + 1$ (h) $5x^2 - 3x - 2$ (i) $3x^2 + 7x + 2$

4. Solve each of the polynomial equations by factoring, or by the quadratic formula if factoring is not possible. The equation **MUST** be set equal to 0 before factoring or applying the quadratic formula! Reject complex solutions, and simplify any radicals.

(a) $12x^2 + 8x = 0$ (b) $x^4 + 5x^2 = -4$ (c) $x^3 - 3x^2 - 8x + 24 = 0$

(d) $x^2 - 4x - 9 = 0$ (e) $2x^2 + 3x = 5$ (f) $2x^3 - 3x^2 = 32x - 48$

5. Divide the polynomials using synthetic division:

(a) $(y^3 + 5y^2 - 7y + 2) \div (y - 2)$ (b) $(k^3 - 17k + 32) \div (k + 5)$

6. Divide the polynomials using ling division.

(a) $(x^4 - 5x^3 + x^2 + 20x - 10) \div (x^2 - x - 5)$ (b) $(4x^3 + 6x^2 + 2) \div (2x + 1)$

Rules of Exponents, Rational Exponents, and Radicals

7. Simplify each expression, and make sure to use positive exponents in your answer:

(a) $7x^6 \cdot 2x^4$ (b) $\frac{6x^7y^8}{2x^3y}$ (c) $-2(8xy)^0$

(d) $(4x^4y^{-2})^3$ (e) $(3ab^6)^2(-2a^2b^{-2})^3$ (f) $\sqrt[3]{\frac{x^6}{y^{12}}}$

8. Rewrite the following expressions using rational exponents. You may have negative exponents in your answer!

(a) $\sqrt[3]{x^5} + \sqrt{x}$

(b) $\sqrt[3]{x+1}$

(c) $\frac{1}{\sqrt[3]{x^5}}$

(d) $\frac{1}{x^{10}} - \frac{3}{x}$

(e) $\frac{1}{2x^6} + \frac{1}{4\sqrt{x}}$

(f) $2\sqrt{x^7+2}$

9. Rewrite the following expressions using roots/positive exponents:

(a) $x^{-\frac{5}{2}}$

(b) $x^{-8} + 6x^{-3}$

(c) $(2x+1)^{-16}$

(d) $\frac{3}{2}x^{-1}$

(e) $(3x^5+10)^{-\frac{1}{2}}$

(f) $\frac{1}{(3x)^{\frac{4}{5}}}$

10. Simplify the following expressions by using FOIL, difference of squares, or powers of binomials:

(a) $(x+2)(x-3)$

(b) $(3x-2)(2x+7)$

(c) $\left(\sqrt{3a} - \sqrt{5b}\right)\left(\sqrt{3a} + \sqrt{5b}\right)$

(d) $(x^2+y^2)(x^2-y^2)$

(e) $(2x+3y)^2$

(f) $\left(\sqrt{x} - 2\right)^2$

Rational Functions

11. Find each product or quotient by writing the result as a single, simplified fraction. Check for common factors that cancel before and after multiplying/dividing!

(a) $\frac{4x^2-4x}{2x^2+4x-6} \cdot \frac{x^2+x-6}{4x^2+8x}$

(b) $\frac{10v-5v^2}{v^2-11v+18} \cdot \frac{v^2-8v-9}{15v}$

(c) $\frac{2n-3}{n+1} \div \frac{9-4n^2}{2n^2+5n+3}$

(d) $\frac{v^2-49}{20v^3} \div \frac{v^2+v-42}{4v^2-24v}$

12. Find each sum or difference by writing the result as a single, simplified fraction. Check for factorizations in the denominator before adding/subtracting, and check for factors that cancel after adding/subtracting!

(a) $\frac{x^2-5}{x^2+5x+6} + \frac{1}{x+2}$

(b) $\frac{5}{w+1} + \frac{8}{w-7}$

(c) $\frac{2}{4p-20} - \frac{p-6}{p^2-8p+15}$

(d) $\frac{b}{b+3} - \frac{4}{b}$

13. Simplify each compound fraction by writing the result as a single, simplified fraction.

(a) $\dfrac{4-\frac{1}{x}}{\frac{8}{x}-\frac{2}{x^2}}$

(b) $\dfrac{x+\frac{1}{x}}{\frac{(x^2+1)^2}{4x}}$

14. Find the x-intercept(s), vertical asymptote(s), horizontal asymptote, and hole(s) of each function (if they exist):

(a) $f(x) = \frac{x^3+3x^2-4x}{x^2-2x-24}$

(b) $f(x) = \frac{x^2-9}{x^2+5x-14}$

(c) $f(x) = \frac{2x^3+5x^2+2x}{x^2+5x+6}$

(d) $f(x) = \frac{3x^2-6x-9}{x^3-2x^2-3x}$

Exponentials and Logarithms

15. Simplify the following expressions:

(a) $e^{\ln(5)}$

(b) $\ln(e^8)$

(c) $2e^{\ln(3)} + \frac{1}{3}\ln(e)$

(d) $-4\ln(e^{2x}) + \ln(1)$

(e) $e^0 \cdot \ln(e^6)$

(f) $e^{\ln(x^2)} + \ln(e^{x^2})$

16. Solve each equation by either finding a common base, or by taking the natural logarithm of both sides:

(a) $12^{x+5} = 144$

(b) $4^{3x-5} = 4^{x+7}$

(c) $5^{2x-1} = \sqrt{5}$

(d) $e^x + 1 = 4$

(e) $3e^x + 5 = 8$

(f) $3e^{2x} = 9$

(g) $4^x + 5 = 8$

(h) $\frac{e^x+1}{2} = 7$

(i) $2^{3x+1} = 4^x$

17. Solve each equation by exponentiating both sides:

(a) $\log_6(2x-4) = -2$

(b) $3\log_2(x+4) - 5 = 10$

(c) $2\ln(x) + 3 = 4$

(d) $3\ln(3x-2) = 3$

18. Expand each logarithmic expression:

(a) $\ln\sqrt{7x^3}$

(b) $\ln\left(\frac{\sqrt[3]{x}}{x^2+x}\right)$

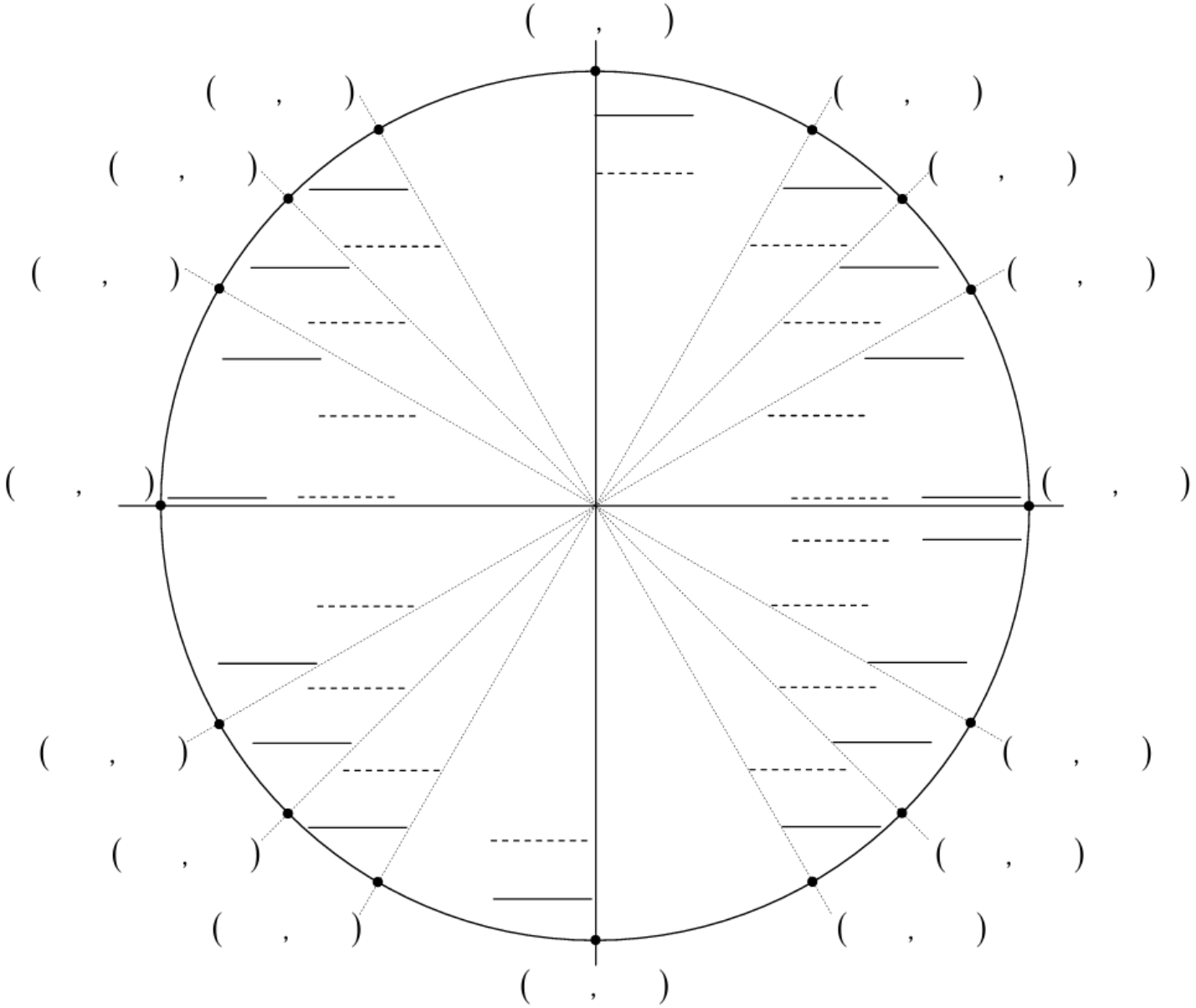
19. Condense each logarithmic expression.

(a) $3\ln(pq) + 4\ln(pq^2)$

(b) $4\ln(x) - (2\ln(y) + 3\ln(z))$

Trigonometry

20. Fill in the blanks of the unit circle. Put the corresponding degree measure on the dotted line, the corresponding radian measure on the solid line, and the corresponding ordered pair in the parentheses. This must be memorized, as it is used in every unit of the course!



21. Fill in the table of sine and cosine values at each angle of the unit circle. These too must be memorized!

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin(\theta)$																	
$\cos(\theta)$																	

22. Using the definitions for the other trigonometric functions (listed below) and the sine/cosine table above, evaluate the following expressions. Write "undefined" if you find yourself dividing by 0.

$$\sec(\theta) = \frac{1}{\cos(\theta)} \qquad \csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}$$

(a) $\tan(0)$ (b) $\tan\left(\frac{\pi}{4}\right)$ (c) $\tan\left(\frac{\pi}{3}\right)$ (d) $\tan\left(\frac{\pi}{2}\right)$ (e) $\sec\left(\frac{\pi}{4}\right)$ (f) $\sec(0)$

(g) $\sec\left(\frac{\pi}{6}\right)$ (h) $\csc\left(\frac{\pi}{4}\right)$ (i) $\csc\left(\frac{\pi}{3}\right)$ (j) $\cot\left(\frac{\pi}{2}\right)$ (k) $\cot\left(\frac{\pi}{6}\right)$ (l) $\cot(0)$

23. Each of the six trigonometric functions has an inverse function. The one we will spend the most time with is arctangent – the inverse function of tangent (NOT the reciprocal function of tangent).

$y = \sin(x) \implies x = \arcsin(y) = \sin^{-1}(y)$ $\qquad$ $y = \cos(x) \implies x = \arccos(y) = \cos^{-1}(y)$

$y = \tan(x) \implies x = \arctan(y) = \tan^{-1}(y)$ $\qquad$ $y = \csc(x) \implies x = \operatorname{arccsc}(y) = \csc^{-1}(y)$

$y = \sec(x) \implies x = \operatorname{arcsec}(y) = \sec^{-1}(y)$ $\qquad$ $y = \cot(x) \implies x = \operatorname{arccot}(y) = \cot^{-1}(y)$

(a) $\arctan(1)$ (b) $\arctan(-1)$ (c) $\arctan(0)$ (d) $\arctan\left(\sqrt{3}\right)$

(e) $\arctan\left(\frac{\sqrt{3}}{3}\right)$ (f) $\lim_{x\to\pm\infty} \arctan(x)$ (g) $\arcsin(-1)$ (h) $\arccos(-1)$

(i) $\arcsin\left(\frac{\sqrt{3}}{2}\right)$ (j) $\arccos\left(\frac{\sqrt{3}}{2}\right)$ (k) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ (l) $\arccos\left(\frac{1}{2}\right)$

24. Solve the following trigonometric equations in the interval $\mathbf{0} \leq \boldsymbol{x} \leq \mathbf{2}\boldsymbol{\pi}$ (there may be more than one solution). Isolate the trigonometric function, and then find the value(s) of x that satisfy it. You may need to factor as well! Write your solutions in radians (degrees are rarely used in calculus).

(a) $6\sin(x) - 3 = 0$

(b) $5\cos(x) + \sqrt{2} = 3\cos(x)$

(c) $3\tan(x) = \tan(x) + 2$

(d) $\sin^2(x) - \sin(x) = 0$

(e) $\cos(x)\tan(x) + \cos(x) = 0$

(f) $\cos^2(x) - 2\cos(x) - 3 = 0$

Other Trigonometric Identities Important in AP Calculus AB:

- Even/Odd Identities:

$\sin(-\theta) = -\sin(\theta)$ $\quad$ $\cos(-\theta) = \cos(\theta)$ $\quad$ $\tan(-\theta) = -\tan(\theta)$
$\csc(-\theta) = -\csc(\theta)$ $\quad$ $\sec(-\theta) = \sec(\theta)$ $\quad$ $\cot(-\theta) = -\cot(\theta)$

- Pythagorean Identities:

$\sin^2(\theta) + \cos^2(\theta) = 1$ $\quad$ $\tan^2(\theta) + 1 = \sec^2(\theta)$ $\quad$ $1 + \cot^2(\theta) = \csc^2(\theta)$

- Double Angle Formulae

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ $\quad$ $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

- Power Reduction Formulae (Come from Cosine Double Angle Formulae)

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2} \qquad \cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

25. Simplify each trigonometric expression. When in doubt, FOIL, factor, recognize the obvious double angle / Pythagorean identities, or rewrite functions in terms of sine and cosine and simplify compound fractions.

(a) $\dfrac{(1 + \sin(\theta))(1 - \sin(\theta))}{\cot^2(\theta)}$

(b) $\dfrac{\cos(\theta)\sin^2(\theta) + \cos^3(\theta)}{\sin(\theta)}$

(c) $\dfrac{\sec^2(\theta) - \tan^2(\theta)}{\sec(\theta)\tan(\theta)}$

(d) $\dfrac{1 - \cos(2\theta)}{\sin(2\theta)}$

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(e) $\dfrac{2\sin^2(\theta) + \cos(2\theta)}{1 + \tan^2(\theta)}$

(f) $\dfrac{2\cos(\theta)}{\sin(2\theta)} - \cot(\theta)\cos(\theta)$

Functions and Graphical Analysis

26. Let $f(x) = x^2 + 5$. Evaluate the following expressions, and simplify:

(a) $f(-2)$ (b) $f(x^3)$ (c) $f(x-3)$

(d) $f(2x+1)$ (e) $f(x+h)$

(f) $\frac{f(5)-f(1)}{5-1}$ (g) $\frac{f(x+h)-f(x)}{h}$

27. Let $f(x) = -3x^2 + 1$, $g(x) = x + 5$, $h(x) = \sqrt{x}$, and $k(x) = 5x - 2$. Evaluate and simplify the following expressions:

(a) $g\big(f(1)\big)$ (b) $f\big(g(x)\big)$

(c) $k\big(h(4)\big)$ (d) $h\big(k(x)\big)$

28. Find the domain of the following functions, and write your answer in interval notation. Use the rules below to help:

- **Sine, Cosine, Exponential, Polynomial, Odd Root, Arctangent, Arccotangent Functions** – Domain is "All Real Numbers", denoted $\mathbb{R}$, or $(-\infty, \infty)$.
- **Rational Functions** – Set the denominator equal to 0, and solve for x. These are the x-values that are not included in the domain.
  - Why? We cannot divide by 0!
- **Even Root Functions** – Set the radicand greater than or equal to 0, and solve the inequality for x. The solution gives the domain, but rewrite this in interval notation.
  - Why? An even root of a negative number is an imaginary number.
- **Logarithmic Functions** – Set the expression inside the logarithm strictly greater than 0, and solve the inequality for x. The solution gives the domain, but rewrite this in interval notation.
  - Why? Logarithms have a vertical asymptote at 0, and logarithms of negative numbers are actually complex numbers.
- **Composite Functions –** Combine the rules, and only accept the x-values that agree with all functions.

(a) $f(x) = \sin(3x)$

(b) $f(x) = x^5 - 3x^2 + 7$

(c) $f(x) = \sqrt[5]{x^2 + x + 4}$

(d) $f(x) = \sqrt{2x + 7}$

(e) $f(x) = \ln(2x - 10)$

(f) $f(x) = \frac{x+4}{x^2-9}$

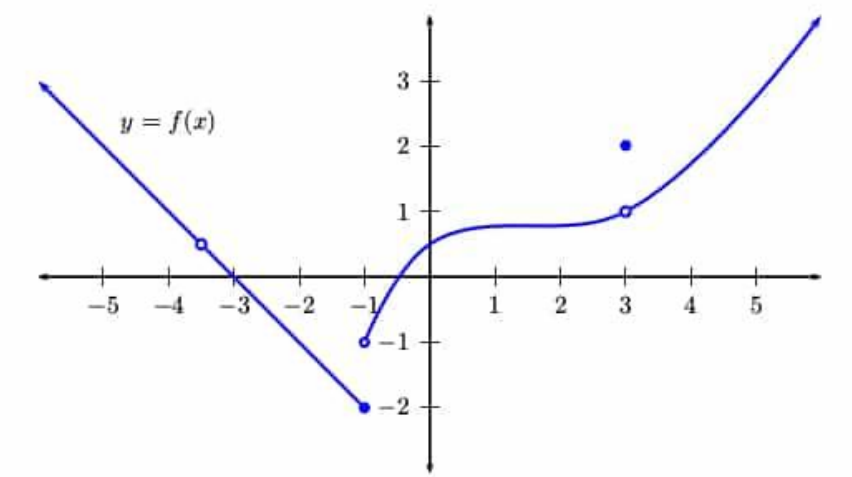
(g) $f(x) = \frac{1}{x^2+1}$

(h) $f(x) = \frac{1}{2}\sqrt[4]{\frac{1}{4}x - 2}$

(i) $f(x) = \frac{\arctan(x)}{\sqrt{x+5}}$

(j) $f(x) = \frac{2x}{x^2+5x+6}$ (k) $f(x) = \frac{x^2+e^x}{\ln(x)\cdot\sqrt{x-4}\cdot\sqrt[3]{x+6}}$ (l) $f(x) = \sin\left(\frac{1}{x-1}\right)\cdot e^{\ln(x)}$

29. Using the graph of the function f below, answer the following questions. If a value is undefined, write "undefined."



(a) $f(-4)$ (b) $f\left(-\frac{7}{2}\right)$ (c) $f(-3)$ (d) $f(-1)$ (e) $f(3)$

(f) What is the domain of f? Write in interval notation:

(g) What is the range of f? Write in interval notation:

(h) List the x-intercept(s) of f:

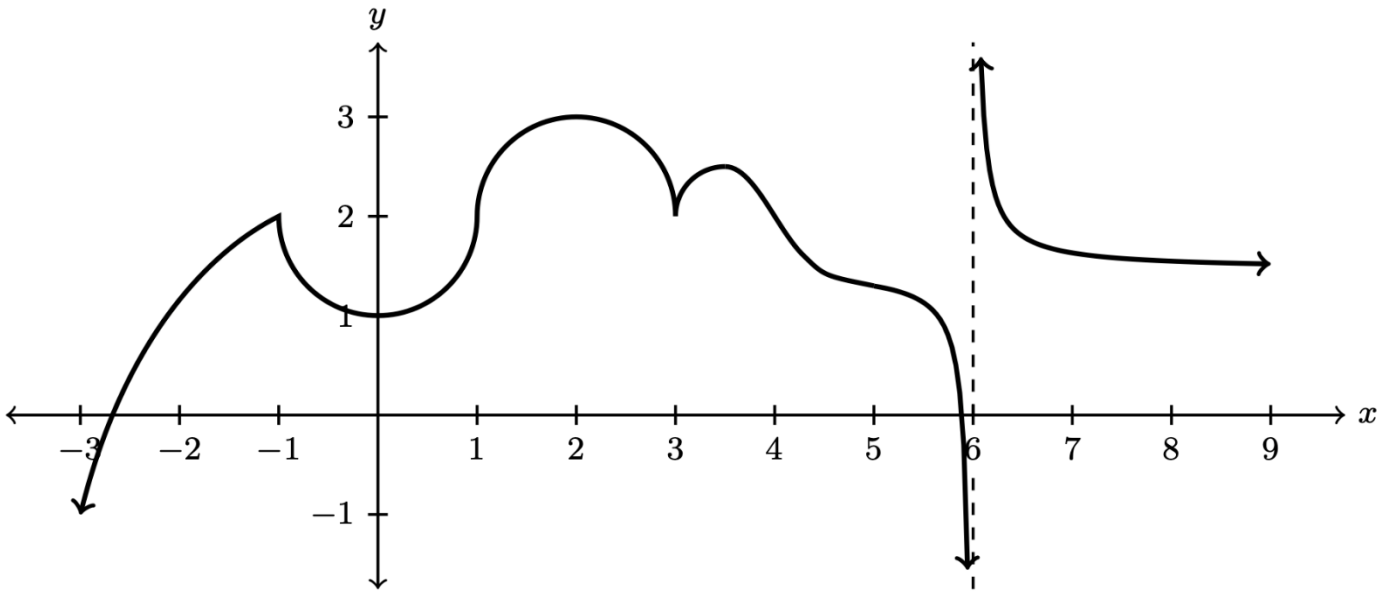
(i) List the y-intercept of f:

(j) If $f(0) = \frac{1}{2}$ and $f(5) = 3$, find the average rate of change of f over $[0,5]$.

(k) Write the end behaviors of f using limit notation.

30. Use the graph below to answer the following questions. Recall for a function f:

- f is increasing on an interval if whenever $a < b$, $f(a) < f(b)$.
- f is decreasing on an interval if whenever $a < b$, $f(a) > f(b)$.
- f has a relative/local extremum if f goes from increasing to decreasing or vice versa.
- f is concave up on an interval if the function "opens up."
- f is concave down on an interval if the function "opens down."
- f has a point of inflection / inflection point if f goes from concave up to concave down or vice versa.



(a) On what intervals is f increasing?

(b) On what intervals is f decreasing?

(c) List the x-values of the local extrema?

(d) On what intervals is f concave up?

(e) On what intervals is f concave down?

(f) List the x-values of the inflection points?