AP Calculus AB



Summer Assignment

Name: _____ Period: _

The purpose of this packet is to both convey to students the foundational skills needed to be successful in this course and to provide them an opportunity to self-assess and develop these skills prior to entering the class. In order to be successful in this and all subsequent math courses at Servite, students must master and retain the content and skills from all previous math courses. As such, we ask that you please work on this assignment with integrity and diligence always striving to meet the intended purpose and goal of this assignment.

Directions: Please print this packet. You **must show all work** in this packet in the space provided. You **may not** use a calculator. For every word problem, write your answer in the form of a sentence. After you make an honest attempt at a problem, check your answer. If your answer is incorrect, try to identify where you went wrong, review the topic, and redo the problem correctly.

This packet will be **collected** on the **second day** of school. You will be given a homework grade for completing this packet. Per Servite School policy, if this packet is not turned in on the second day of school, you will receive half credit if it is turned in the following day. After that, you will receive a zero for this packet.

Have a great summer and we are looking forward to seeing you in August!

I understand that I have to show all my work.

_____ _____

(Student Signature) **(Date)**

I have checked to see that my child has shown all work and completed all problems.

_____ _____

(Parent/Guardian Signature) **(Date)**

For Algebra Topics:

http://www.purplemath.com/modules/index.htm

Beginning Algebra Topics:

- Fractional and Negative Exponents
- Equations of Lines
- Solving for Indicated Variable

Intermediate Algebra Topics:

- Domain
- Solving Inequalities
  - Absolute value
  - Quadratic
- Even and Odd Functions
- Function Transformation
- Factor theorem (*p* over *q* method/synthetic division)
- Special Factorization
- Solving Quadratic by Factoring or Quadratic Formula
- Equations of Circles

Advanced Algebra Topics:

- Asymptotes
- Complex Fractions
- Composition of Functions
- Solving Rational Equations
- Logarithms
- Inverse Functions

For Trigonometry Topics:

http://www.themathpage.com/aTrig/trigonometry-of-right-triangles.htm

- Right Triangle Trigonometry

http://www.analyzemath.com/Tutorial-Trigonometric-Equations/Tutorial-Trigonometric-Eq.html

- Solving Trigonometric Equations
- Trigonometric Identities

For Other Topics:

http://www.mathwords.com/d/difference_quotient.htm

- The Difference Quotient

http://www.regentsprep.org/Regents/math/ALGEBRA/AS1/RefARea.htm
https://www.etap.org/demo/statistics/lesson7/instructiontutor_last.html

- Area/Geometric Probability

INSTRUCTIONS: Where applicable, put your solutions in interval notation. Do not use any calculator (except on Topic 14:#5). Please do all work on separate paper and do the problems in order. SHOW ALL WORK! We've given you the answers—we care about the process that gets you there.

Topic 1: Fractional and Negative Exponents

Simplify, using positive only exponents.

1. $\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^2}\right]^{-3}$ 2. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$ 3. $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

Topic 2: Domain

Find the domain of the following functions.

1. $y=\frac{x^2-5x-6}{x^2-3x-18}$ 2. $y=\frac{\sqrt{2x-9}}{2x+9}$ 3. $y=\sqrt{x^2-5x-14}$ 4. $y=\log(2x-12)$

Topic 3: Solving Inequalities

Write the following absolute value function as a piece-wise function.

1. $y=|2x-4|$

Solve the following absolute value inequalities.

2. $|x-3|\le 4$ 3. $|3x-4|>-2$

Solve the following quadratic inequalities.

4. $x^2-3x\ge 10$ 5. $x^3+4x^2-x\ge 4$ 6. $2\sin^2 x\ge \sin x,\quad 0\le x<2\pi$

Solve the following rational inequality.

7. $\frac{2x-1}{3x-2}\le 1$

Topic 4: Even and Odd Functions

Show algebra to determine if the relation is even, odd, or neither.

1. $f(x)=2x^2-7$ 2. $f(x)=-4x^3-2x$ 3. $f(x)=4x^2-4x+4$

Topic 5: Function Transformation

If $f(x) = x^2 - 1$, *describe in words what the following would do to the graph of :*

1. $f(x) - 4$
2. $f(x-4)$
3. $-f(x+2)$
4. $5f(x) + 3$
5. $f(2x)$
6. $|f(x)|$

Topic 6: Special Factorization

Factor completely.

1. $27x^3 - 125y^3$
2. $x^4 + 11x^2 - 80$
3. $2x^2 + 50y^2 - 20xy$
4. $x^2 + 12x + 36 - 9y^2$
5. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$
6. $(3x+4)^{-3}(2x-5)^3 + (3x+4)^{-2}(2x-5)^2$
7. $\frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2}$

Topic 7: Solving by Factoring or Quadratic Formula

Solve each equation.

1. $x^2 + 6x + 4 = 0$
2. $2x^2 - (x+2)(x-3) = 12$
3. $x - 10\sqrt{x} + 9 = 0$
4. $\frac{1}{x^2} - \frac{1}{x} = 6$
5. $x^3 + x^2 - 3x - 6 = 0$

Topic 8: Asymptotes

For each function, find the equations of both the vertical and horizontal asymptote(s), if they exist.

1. $y = \frac{x+4}{x^2+1}$
2. $y = \frac{x^2-9}{x^3+3x^2-18x}$
3. $y = \frac{2x^3}{x^3-1}$

Topic 9: Complex Fractions

Simplify.

1. $\dfrac{\dfrac{3}{x}-\dfrac{4}{y}}{\dfrac{4}{x}-\dfrac{3}{y}}$

2. $\dfrac{1-\dfrac{2}{3x}}{x-\dfrac{4}{9x}}$

3. $\dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{x+y}{y}}$

4. $\dfrac{x^{-3}-x}{x^{-2}-1}$

5. $\dfrac{\dfrac{4}{x-5}+\dfrac{2}{x+2}}{\dfrac{2x}{x^2-3x-10}+3}$

Topic 10: Composition of Functions

If $f(x)=x^2$, $g(x)=2x-1$, *and* $h(x)=2^x$, *find the following:*

1. $f(g(2))$

2. $h(f(-1))$

3. $g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$

4. $g(f(x))$

5. $g(g(x))$

6. $f(h(x))$

Topic 11: Rationalizing Denominators and Numerators

For problem #1 rationalize the denominator. For problem #2 rationalize the numerator.

1. $\dfrac{3}{\sqrt[4]{(3x)^3}}$

2. $\dfrac{5\sqrt{2}+\sqrt{5}}{5}$

Topic 12: Solving Rational Equations

Solve equation for x.

1. $\dfrac{x-5}{x+1}=\dfrac{3}{5}$

2. $\dfrac{60}{x}-\dfrac{60}{x-5}=\dfrac{2}{x}$

3. $\dfrac{2}{x+5}+\dfrac{1}{x-5}=\dfrac{16}{x^2-25}$

4. $\dfrac{x}{x-2}+\dfrac{2x}{4-x^2}=\dfrac{5}{x+2}$

5. $\dfrac{2x+3}{x-1}=\dfrac{10}{x^2-1}+\dfrac{2x-3}{x+1}$

Topic 13: Right Triangle Trigonometry

1. If $\cos\theta = -\frac{5}{13}$, θ in quadrant II, find $\sin\theta$ and $\tan\theta$.

2. If $\cot\theta = 3$, θ in quadrant III, find $\sin\theta$ and $\cos\theta$.

3. A kite is 100m above the ground. If there are 200m of string out, what is the angle (in radians) between the string and the horizontal? (Assume that the string is perfectly straight.)

Topic 14: Solving Trigonometric Equations

Solve each equation on the interval [0, 2π). Please use a calculator to complete #5.

1. $\cos^2 x = \cos x$
2. $4\sin^2 x = 1$
3. $2\sin^2 x + \sin x = 1$
4. $2\sin x\cos x + \sin x = 0$
5. $8\cos^2 x - 2\cos x = 1$
6. $\sin^2 x - \cos^2 x = 0$

Topic 15: Logarithms

Simplify.

1. $\log_2 5 + \log_2\left(x^2 - 1\right) - \log_2(x - 1)$
2. $2\log_4 9 - \log_2 3$
3. $3^{2\log_3 5}$

Topic 16: Solving for Indicated Variable

Solve each equation for the indicated variable.

1. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, for a
2. $A = 2\pi r^2 + 2\pi rh$, for $r > 0$
3. $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$, for x

Topic 17: Equations of Lines

Determine the equation of each line:

1. the line through (-1, 3) and (2, -4)
2. the line through (-1, 2) and perpendicular to the line $2x - 3y + 5 = 0$
3. the line through (2, 3) and the midpoint of the line segment from (-1, 4) to (3, 2)

Topic 18: The Difference Quotient

Simplify $\frac{f(x+h)-f(x)}{h}$ *, where:*

1. $f(x) = 2x + 3$
2. $f(x) = \frac{1}{x+1}$
3. $f(x) = x^2$

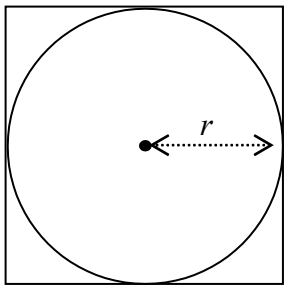
Topic 19: Inverse Functions

Find the inverse of each function:

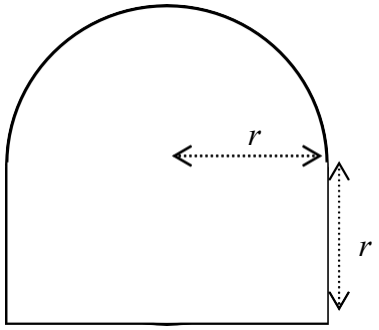
1. $f(x) = 2x + 3$
2. $f(x) = \frac{x+2}{5x-1}$

Topic 20: Area

1. Find the ratio of the area inside the square but outside the circle to the area of the square in the picture below.



2. Find a formula for the perimeter of a window of the shape in the picture below.



Calculus Summer Packet SOLUTIONS

Topic 1: **1)** $\frac{-(2-x)^5}{4}$ **2)** $\frac{2}{\sqrt[4]{(x-4)}}$ **3)** $\frac{1}{3(2x+5)^{3/2}}$ **4)** $\frac{1}{\sqrt{x^2+4xy+y^2}}$

Topic 2: **1)** $(-\infty, -3) \cup (-3, 6) \cup (6, \infty)$ **2)** $[9/2, \infty)$ **3)** $(-\infty, -2] \cup [7, \infty)$ **4)** $(6, \infty)$

Topic 3: **1)** $y = \begin{cases} 2x-4, & x \geq 2 \\ 4-2x, & x < 2 \end{cases}$ **2)** [-1, 7] **3)** $(-\infty, \infty)$ **4)** $(-\infty, -2] \cup [5, \infty)$

5) $[-4, -1] \cup [1, \infty)$ **6)** $[0] \cup [\pi/6, 5\pi/6] \cup [\pi, 2\pi)$ **7)** $\left(-\infty, \frac{2}{3}\right) \cup [1, \infty)$

Topic 4: **1)** even **2)** odd **3)** neither

Topic 5: **1)** translated 4 units down **2)** translated 4 units to the right

3) reflected over the *x*-axis and translated 2 units left

4) stretched vertically by a factor of 5 and translated up 3 units

5) stretched horizontally by a factor of ½

6) no change $(-\infty, -1) \cup (1, \infty)$, on [-1, 1] the graph would be reflected over the *x*-axis

Topic 6: **1)** $(3x-5y)(9x^2+15xy+25y^2)$ **2)** $(x^2+16)(x^2-5)$ **3)** $2(x-5y)^2$

4) $(x+6+3y)(x+6-3y)$ **5)** $(x-3)^2(2x+1)^2(3x-2)$

6) $\frac{(2x-5)^2(5x-1)}{(3x+4)^3}$ **7)** $\frac{(2x+1)^{3/2}(3x-1)}{15}$

Topic 7: **1)** $x = -3 \pm \sqrt{5}$ **2)** $x = -3$ or 2 **3)** $x = 1$ or 81 **4)** $x = -\frac{1}{2}$ or $\frac{1}{3}$

5) $x = -2, \sqrt{3}, -\sqrt{3}$

Topic 8: **1)** VA: none; HA: $y = 0$ **2)** VA: $x = 0, x = -6$; HA: $y = 0$

3) VA: $x = 1$; HA: $y = 2$

Topic 9: **1)** $\frac{3y-4x}{4y-3x}$ **2)** $\frac{3}{3x+2}, x \neq 0, \frac{2}{3}$ **3)** $\frac{x-y}{x}, x \neq -y$

4) $\frac{1+x^2}{x}, x \neq -1, 1$ **5)** $\frac{6x-2}{3x^2-7x-30}$

Topic 10: **1)** 9 **2)** 2 **3)** 3 **4)** $2x^2-1$ **5)** $4x-3$ **6)** 2^{2x}

Topic 11: **1)** $\frac{\sqrt[4]{3x}}{x}$ **2)** $\frac{9}{5\sqrt{2}-\sqrt{5}}$

Topic 12: **1)** $x=14$ **2)** $x=-145$ **3)** $x=7$ **4)** $x=\frac{5}{2}\pm\frac{i\sqrt{15}}{2}$ **5)** no solution

Topic 13: **1)** $\sin\theta=\frac{12}{13};\tan\theta=\frac{-12}{5}$ **2)** $\cos\theta=\frac{-3\sqrt{10}}{10};\sin\theta=\frac{-\sqrt{10}}{10}$ **3)** $\theta=\frac{\pi}{6}$

Topic 14: **1)** $x=0,\frac{\pi}{2},\frac{3\pi}{2}$ **2)** $x=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$ **3)** $x=\frac{\pi}{6},\frac{5\pi}{6},\frac{3\pi}{2}$

4) $x=0,\frac{2\pi}{3},\pi,\frac{4\pi}{3}$ **5)** $x=\frac{\pi}{3},\frac{5\pi}{3},\ 1.823,\ 4.460$ **6)** $x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

Topic 15: **1)** $\log_2 5(x+1), x>1$ **2)** $\log_2 3$ **3)** 25

Topic 16: **1)** $a=\frac{bcx}{bc-cy-bz}$ **2)** $r=\frac{\sqrt{h^2\pi^2+2\pi A}}{2\pi}-\frac{h}{2}$ **3)** $x=\frac{\pi}{\pi-1}$

Topic 17: **1)** $y=-\frac{7}{3}x+\frac{2}{3}$ ***2)*** $y=-\frac{3}{2}x+\frac{1}{2}$ **3)** $y=3$

Topic 18: **1)** 2 **2)** $\frac{-1}{(x+h+1)(x+1)}$ **3)** $2x+h$

Topic 19: **1)** $f^{-1}(x)=\frac{x-3}{2}$ **2)** $f^{-1}(x)=\frac{x+2}{5x-1}$

Topic 20: **1)** $\frac{4-\pi}{4}$ **2)** $\pi r+4r$ **3)** $\frac{9}{4}\pi\,\mathrm{m}^2$