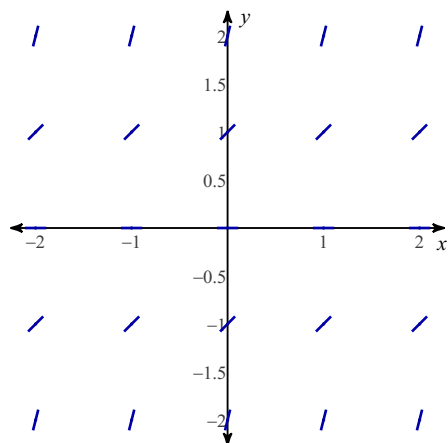


Summer Packet (No Calculator)

Block _____

For each problem, find a differential equation that could be represented with the given slope field.

1)



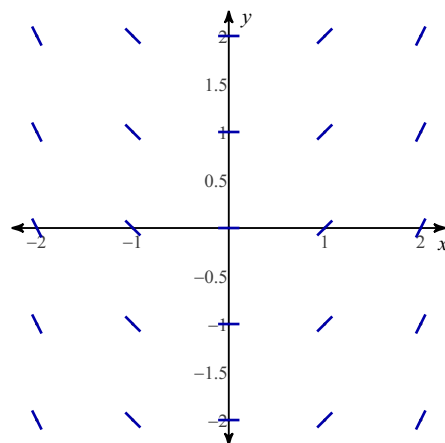
A) $\frac{dy}{dx} = -x$

B) $\frac{dy}{dx} = x$

C) $\frac{dy}{dx} = y^2$

D) $\frac{dy}{dx} = -1$

2)



A) $\frac{dy}{dx} = x$

B) $\frac{dy}{dx} = y^2$

C) $\frac{dy}{dx} = -y$

D) $\frac{dy}{dx} = \frac{1}{9}$

Evaluate each limit.

3) $\lim_{x \rightarrow \infty} \left(-\frac{x^3}{e^x} + 3 \right)$

4) $\lim_{x \rightarrow -\infty} \frac{3x^3}{4x^2 + 3}$

$$5) \lim_{x \rightarrow \infty} \sin(x)$$

$$6) \lim_{x \rightarrow -\infty} (-e^{-3x} - 2)$$

$$7) \lim_{x \rightarrow \infty} 4e^{\frac{1}{x}}$$

$$8) \lim_{x \rightarrow -\infty} (-x^5 + 4x^3 - 3x)$$

Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.

$$9) \lim_{x \rightarrow 0} \frac{3x^2}{e^x - 1 - x}$$

$$10) \lim_{x \rightarrow \infty} \frac{x}{e^{2x}}$$

$$11) \lim_{x \rightarrow \infty} \frac{2x^2}{e^x}$$

$$12) \lim_{x \rightarrow 1} \frac{5(x-1)}{\ln x}$$

Determine if each function is continuous. If the function is not continuous, find the x -axis location of and classify each discontinuity.

$$13) f(x) = \begin{cases} x - 3, & x < 2 \\ -2, & x \geq 2 \end{cases}$$

$$14) f(x) = \begin{cases} -x^2, & x \leq -1 \\ 2x, & x > -1 \end{cases}$$

Evaluate each limit.

$$15) \lim_{x \rightarrow 3^+} f(x), f(x) = \begin{cases} 2x - 3, & x \leq 3 \\ x^2 - 4x + 5, & x > 3 \end{cases}$$

$$16) \lim_{x \rightarrow -3} f(x), f(x) = \begin{cases} -\frac{x}{2} - 4, & x < -3 \\ \frac{x}{2} - 2, & x \geq -3 \end{cases}$$

Differentiate each function with respect to x .

$$17) y = 4x^{-4}$$

$$18) y = 2x^{\frac{1}{3}}$$

$$19) \ y = 2x^2 + 5x^{-1} + 3x^{-4}$$

$$20) \ y = 5x^5 + 2x^{\frac{3}{5}} + 3x^{-2}$$

$$21) \ y = (5x^4 + 2x^2 + 3)(-3x^3 - 2)$$

$$22) \ y = 3x^5(-5x^2 + 5)$$

$$23) \ y = \frac{5x^4}{5x^4 - 3}$$

$$24) \ y = \frac{5x^2}{2x^2 + 3}$$

$$25) \ y = ((3x + 4)^2 + 5)^3$$

$$26) \ y = (2x^5 + 5)^2$$

$$27) \ y = e^{x^2}$$

$$28) \ y = e^{4x^2}$$

$$29) \ y = \ln 4x^2$$

$$30) \ y = e^{5x^2}$$

$$31) \ y = \cos 4x^2$$

$$32) \ y = \tan 5x^2$$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

33) $5x = -5x^2y + 1$

34) $x = -4y^2 + 1$

35) $-2y^3 - 3x^3 + 5 = 5x^2$

36) $1 = 4x^2 + 2y^3 + 5y^2$

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

37)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	1	2	1
2	2	1	3	$-\frac{1}{2}$
3	3	1	1	-2

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(3)$

Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(1)$

Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(1)$

Part 4) Given $h_4(x) = \frac{f(x)}{g(x)}$, find $h_4'(2)$

Part 5) Given $h_5(x) = f(g(x))$, find $h_5'(2)$

38)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	2	2	-1
2	3	0	1	$\frac{1}{2}$
3	1	-2	3	2

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(1)$ Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(2)$ Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(2)$ Part 4) Given $h_4(x) = \frac{f(x)}{g(x)}$, find $h_4'(3)$ Part 5) Given $h_5(x) = f(g(x))$, find $h_5'(3)$

For each problem, find the values of c that satisfy the Mean Value Theorem.

39) $f(x) = x^2 + 4x - 2$; $[-3, 1]$

40) $f(x) = -x^2 - 4x - 4$; $[-4, 0]$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

41) $y = -x^2 + 1$ at $(2, -3)$

42) $y = (2x + 4)^{\frac{2}{3}}$ at $(2, 4)$

For each problem, find all points of absolute minima and maxima on the given interval.

43) $y = -x^2 - 4x - 3$; $[-4, -2]$

44) $y = -x^3 + 2x^2 + 3$; $[0, 2]$

For each problem, find all points of relative minima and maxima.

45) $f(x) = -\frac{x^2}{2} + x + \frac{11}{2}$

46) $f(x) = -(7x - 35)^{\frac{2}{3}}$

For each problem, find the x-coordinates of all critical points and find the open intervals where the function is increasing and decreasing.

47) $y = -2x^2 + 16x - 27$

48) $y = (5x + 20)^{\frac{1}{2}}$

For each problem, find the x-coordinates of all points of inflection and find the open intervals where the function is concave up and concave down.

49) $f(x) = -2x^2 + 12x - 18$

50) $f(x) = -\frac{x^2}{2} + 3x - \frac{1}{2}$

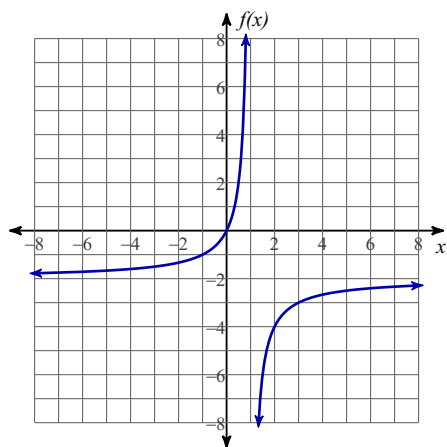
Solve each related rate problem.

- 51) A crowd gathers around a movie star, forming a circle. The radius of the crowd increases at a rate of 3 m/sec. How fast is the area taken up by the crowd increasing when the radius is 9 m?

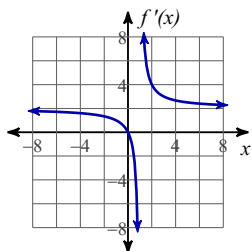
- 52) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 3 m/min. How fast is the area of the spill increasing when the radius is 3 m?

Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

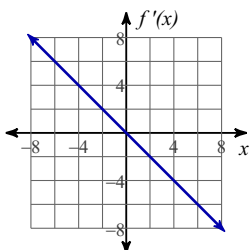
53)



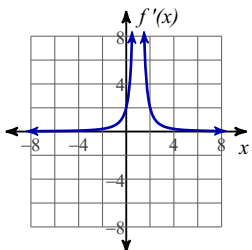
A)



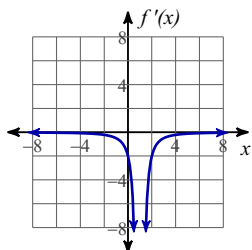
B)



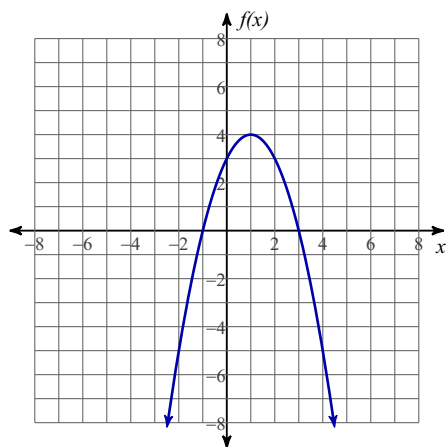
C)



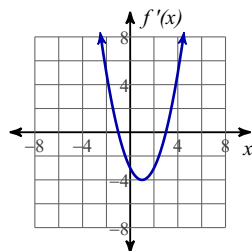
D)



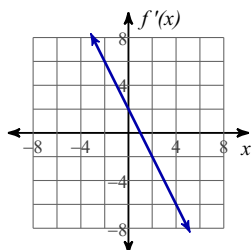
54)



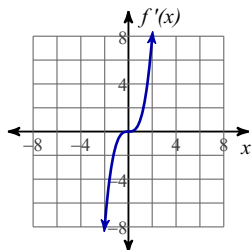
A)



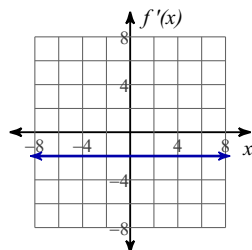
B)



C)



D)



Evaluate each indefinite integral.

$$55) \int 3 \, dx$$

$$56) \int \left(20x^3 + \frac{15\sqrt[4]{x}}{4} \right) dx$$

$$57) \int \left(\frac{25\sqrt[3]{x^2}}{3} + 6\sqrt[5]{x} \right) dx$$

$$58) \int \frac{5\sqrt[4]{x}}{4} dx$$

$$59) \int 10x^4(2x^5 - 3)^3 \, dx$$

$$60) \int 15x^2(5x^3 - 3)^4 \, dx$$

$$61) \int 5e^{5x} \cdot (e^{5x} + 5)^5 \, dx$$

$$62) \int 9x^2(3x^3 + 4)^5 \, dx$$

$$63) \int 3 \cos x \, dx$$

$$64) \int 5 \csc x \cot x \, dx$$

$$65) \int 60x^4 \cos(3x^5 + 5) \, dx$$

$$66) \int -30x \sec^2(3x^2 - 2) \, dx$$

$$67) \int 40x^3 e^{5x^4 + 1} \, dx$$

$$68) \int 48x^3 e^{3x^4 + 5} \, dx$$

$$69) \int -45x^2 e^{5x^3 + 3} \, dx$$

$$70) \int -\frac{8x}{x^2 - 2} \, dx$$

$$71) \int \frac{1}{\sqrt{16-x^2}} dx$$

$$72) \int \frac{1}{x\sqrt{x^2-4}} dx$$

For each problem, use a left-hand Riemann sum to approximate the integral based off of the values in the table.

$$73) \int_0^8 f(x) dx$$

x	0	1	4	5	8
$f(x)$	9	7	5	6	8

$$74) \int_0^8 f(x) dx$$

x	0	3	4	7	8
$f(x)$	6	7	6	4	6

For each problem, use a right-hand Riemann sum to approximate the integral based off of the values in the table.

$$75) \int_0^{10} f(x) dx$$

x	0	4	6	9	10
$f(x)$	3	5	7	5	7

$$76) \int_0^8 f(x) dx$$

x	0	2	3	4	8
$f(x)$	4	5	6	8	9

For each problem, find $F'(x)$.

$$77) F(x) = \int_{-3}^{x^3} 2t \, dt$$

$$78) F(x) = \int_{-1}^{2x} (-t^3 + 4t^2 - 5) \, dt$$

For each problem, find the area of the region enclosed by the curves.

$$79) y = -x^3 - x^2 + 4x, \quad y = -x^2$$

$$80) x = -\frac{y^3}{2} - \frac{y^2}{2} + 2y, \quad x = -\frac{y^2}{2}$$

$$81) x = -y^3 + 6y, \quad x = -y^2$$

$$82) x = -y^3 - y^2 + 5y, \quad x = -y$$

For each problem, find the area under the curve over the given interval.

$$83) y = 3\sqrt{x}; \quad [3, 5]$$

84) $y = 2\sec^2 x$; $[-\pi, -\frac{5\pi}{6}]$

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

85) $y = -x^2 - 1$, $y = -2$, $x = -1$, $x = 0$
Axis: $y = -2$

86) $x = -y^2 + 1$, $x = 0$, $y = 0$, $y = 1$
Axis: $x = 0$

87) $x = \frac{2}{y}$, $x = 0$, $y = \frac{2}{5}$, $y = 6$
Axis: $x = 0$

88) $x = y^2 + 1$, $x = 1$, $y = 1$
Axis: $x = 1$

89) $y = -4$, $y = -x^2 - 3$
Axis: $y = -1$

90) $x = y^2 + 3$, $x = 2$, $y = -2$, $y = 0$
Axis: $x = 2$