

Things to Know for Calculus

TRIGONOMETRY

Trig Functions

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

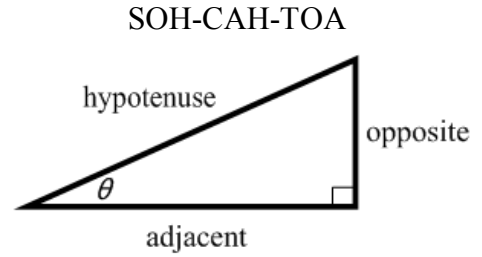
$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

Reciprocal Functions

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}}$$

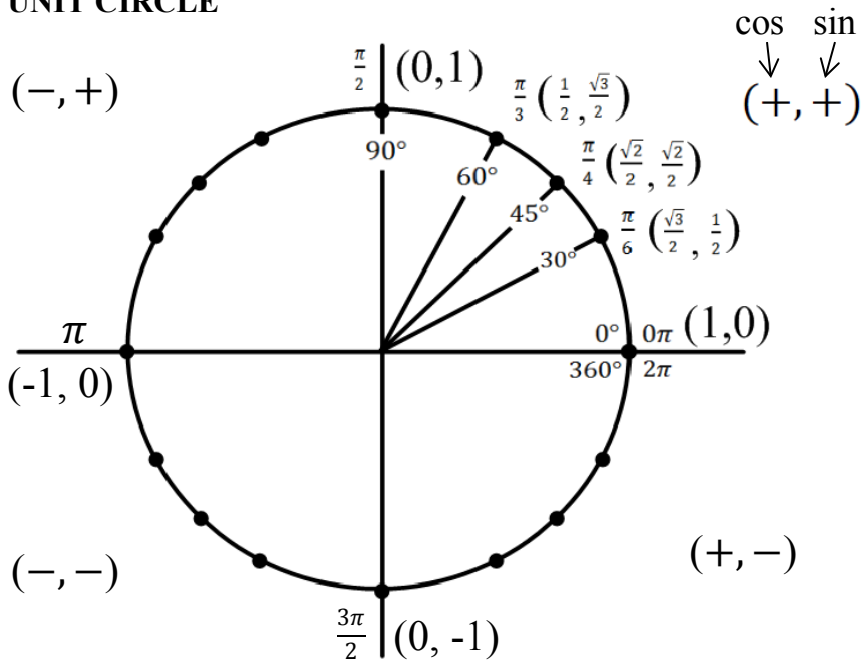
$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adj}}{\text{opp}}$$



TEST ONLY USES RADIANS!

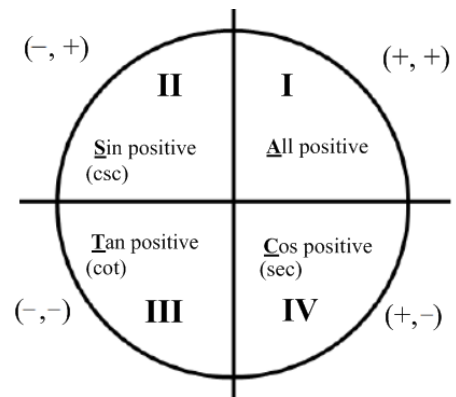
Must know trig values of special angles $0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ using Unit Circle or Special Right Triangles.

UNIT CIRCLE



To help remember the signs in each quadrant

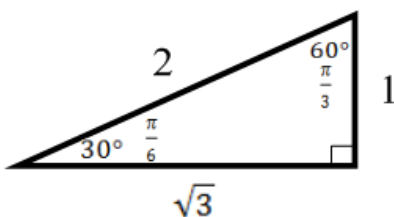
All Students Take Calculus



SPECIAL RIGHT TRIANGLES

$30^\circ - 60^\circ - 90^\circ$ Triangles

Which are $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$ Triangles

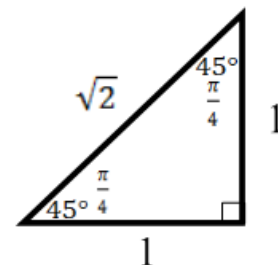


Find $\tan(\frac{\pi}{6})$

$$\tan(\frac{\pi}{6}) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} \text{ simplify to } \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$45^\circ - 45^\circ - 90^\circ$ Triangles

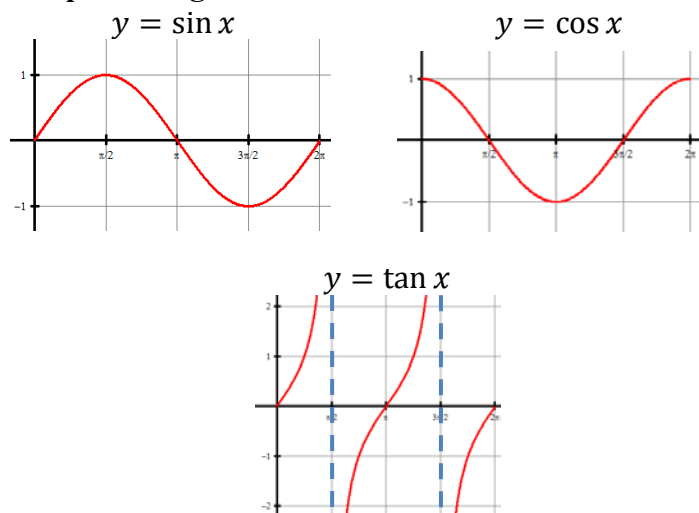
Which are $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ Triangles



Find $\sin(\frac{\pi}{4})$

$$\sin(\frac{\pi}{4}) = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} \text{ simplify to } \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Graphs of trig functions



Inverse Trig Function

$\sin^{-1}\theta$ is the same as $\arcsin \theta$

$\sin^{-1}\theta = \left(\frac{\sqrt{3}}{2}\right)$ means what angle has a sine value of $\frac{\sqrt{3}}{2}$

that means $\theta = \frac{\pi}{3} \pm 2\pi n$ or $\frac{2\pi}{3} \pm 2\pi n$

Since θ has infinite answers then it isn't a function. Bummer. To make it a function we define inverses like:

$\sin/\csc$ and $\tan/\cot$ use quadrant I and IV for inverses
 $\cos/\sec$ use quadrant I and II for inverses

So... $\theta = \frac{\pi}{3}$ because it is in the first quadrant

Trig Identities

There are a bunch, but you really only need to know Pythagorean Identity. **$\sin^2 x + \cos^2 x = 1$**

Subtract $\sin^2 x$ to get $\cos^2 x = 1 - \sin^2 x$ or subtract $\cos^2 x$ to get $\sin^2 x = 1 - \cos^2 x$

Divide by $\sin^2 x$ to get $1 + \cot^2 x = \csc^2 x$ or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

GEOMETRY

FORMULAS

AREA

Triangle = $\frac{1}{2}bh$

Circle = πr^2

Trapezoid = $\frac{1}{2}(b_1 + b_2)h$

CIRCUMFERENCE

Circle = $2\pi r$

SURFACE AREA

Sphere = $4\pi r^2$

LATERAL AREA

Cylinder = $2\pi rh$

VOLUME

Sphere = $\frac{4}{3}\pi r^3$

Cylinder = $\pi r^2 h$

Cone = $\frac{1}{3}\pi r^2 h$

Prism = Bh

Pyramid = $\frac{1}{3}Bh$

B is the area of the base

DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ALGEBRA

Linear Functions

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form

(slope-intercept Form)

$$y = mx + b$$

Point Slope Form

$$y - y_1 = m(x - x_1)$$

Parallel Lines

Have the same slope

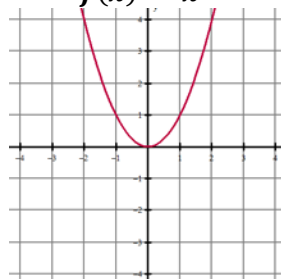
Perpendicular Lines

Have the opposite reciprocal slopes

Functions

Quadratic Function

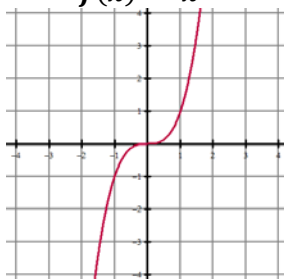
$$f(x) = x^2$$



$$y = a(x - h)^2 + k$$

Cubic Function

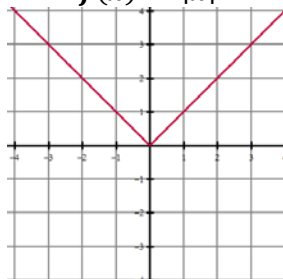
$$f(x) = x^3$$



$$y = a(x - h)^3 + k$$

Absolute Value

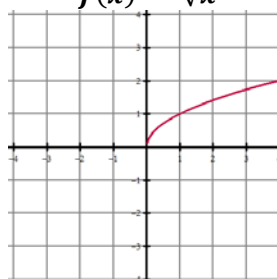
$$f(x) = |x|$$



$$y = a|x - h| + k$$

Square Root Function

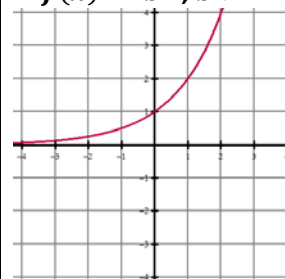
$$f(x) = \sqrt{x}$$



$$y = a\sqrt{x - h} + k$$

Exponential Function

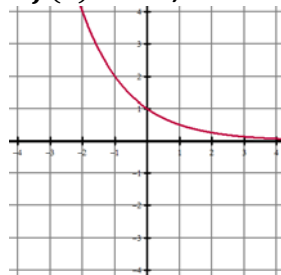
$$f(x) = b^x, b > 1$$



$$y = a \cdot b^{(x-h)} + k$$

Exponential Function

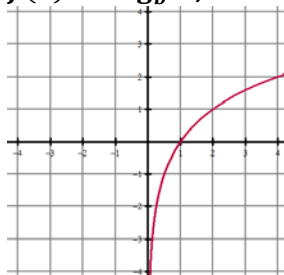
$$f(x) = b^x, b < 1$$



$$y = a \cdot b^{(x-h)} + k$$

Logarithmic Function

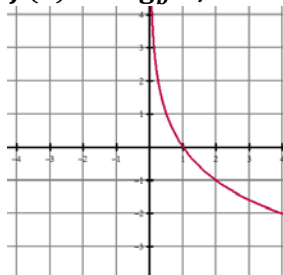
$$f(x) = \log_b x, b > 1$$



$$y = a \log_b(x - h) + k$$

Logarithmic Function

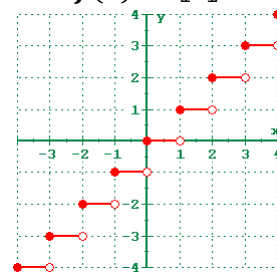
$$f(x) = \log_b x, b < 1$$



$$y = a \log_b(x - h) + k$$

Greatest Integer

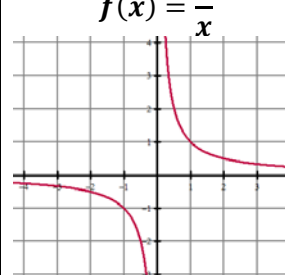
$$f(x) = \llbracket x \rrbracket$$



$$y = a \llbracket x - h \rrbracket + k$$

Rational Function

$$f(x) = \frac{1}{x}$$



$$y = \frac{a}{x - h} + k$$

Translations

All functions move the same way!

Given the parent function $y = x^2$

Move up 4

$$y = x^2 + 4$$

Move down 3

$$y = x^2 - 3$$

Move left 2

$$y = (x + 2)^2$$

Move right 1

$$y = (x - 1)^2$$

Move left 2 and down 3

$$y = (x + 2)^2 - 3$$

To flip (reflect) the function vertically $y = -x^2$

To flip (reflect) the function horizontally $y = (-x)^2$

So $f(x) = -\sqrt{x - 3} + 1$ is a square root function reflected vertically, shifted right 3 and up 1

Notation

Notice open parenthesis () versus closed []

Inequality

$$-3 < x \leq 5$$

$\longleftrightarrow$

Interval

$$(-3, 5]$$

$$-3 \leq x \leq 5$$

$\longleftrightarrow$

$$[-3, 5]$$

$$-3 < x < 5$$

$\longleftrightarrow$

$$(-3, 5)$$

$$-3 \leq x < 5$$

$\longleftrightarrow$

$$[-3, 5)$$

Infinity is always open parenthesis

Inequality

$$x < 3$$

$\longleftrightarrow$

Interval

$$(-\infty, 3)$$

$$x \leq 3 \text{ or } x > 5$$

$\longleftrightarrow$

$$(-\infty, 3] \cup (5, \infty)$$

$$x \neq 3$$

$\longleftrightarrow$

$$(-\infty, 3) \cup (3, \infty)$$

$$\text{all Real numbers}$$

$\longleftrightarrow$

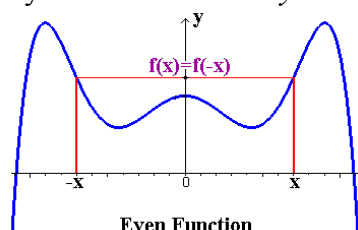
$$(-\infty, \infty)$$

Even and Odd Functions

EVEN

$$f(-x) = f(x)$$

Symmetric about the y-axis

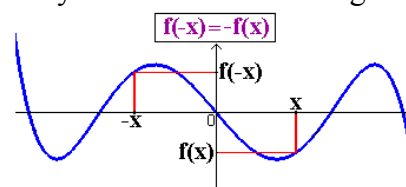


Even Function

ODD

$$f(-x) = -f(x)$$

Symmetric about the origin



Odd Function

Domain and Range

Domain = all possible x values

Range = all possible y values

Algebraically

You can't divide by zero

You can't square root a negative

$$y = \sqrt{2x + 5}$$

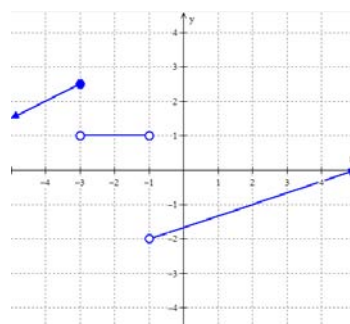
$$D: [-\frac{5}{2}, \infty)$$

$$y = \frac{x^2 - 1}{x^2 + 7x + 12}$$

$$D: (-\infty, -4)(-4, -3)(-3, \infty)$$

Graphically

Just look at it



$$D: (-\infty, -1)(-1, 5]$$

$$R: (-\infty, 2.5]$$

Finding zeros

Must be able to factor and use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

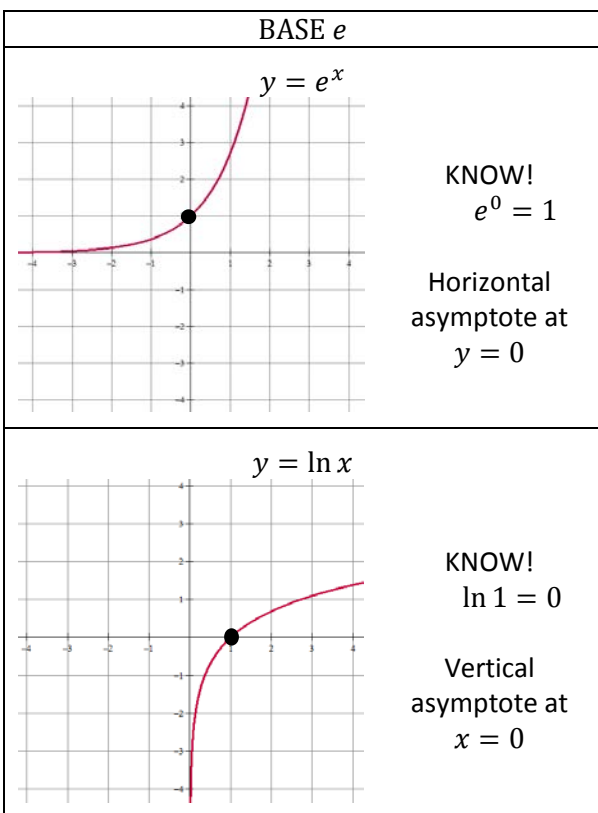
Special products

Sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Exponential and Logarithmic Properties

The exponential function b^x of base b is one-to-one which means it has an inverse which is called the logarithmic function of base b or logarithm of base b which is denoted $\log_b x$ which reads “the logarithm of base b of x ” or “log base b of x ”. So...



$$y = \log_b x \longleftrightarrow x = b^y$$

Exponential		Logarithmic
$b^x b^y = b^{x+y}$	Product Rule	$\log_b xy = \log_b x + \log_b y$
$\frac{b^x}{b^y} = b^{x-y}$	Quotient Rule	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
$(b^x)^y = b^{xy}$	Power Rule	$\log_b x^y = y \log_b x$
$b^{-x} = \frac{1}{b^x}$		$\log_b \left(\frac{1}{x}\right) = -\log_b x$
$b^0 = 1$		$\log_b 1 = 0$
$b^1 = b$		$\log_b b = 1$
	Change of Base	$\log_b x = \frac{\log_c x}{\log_c b}$
	Natural Log	$\log_e x = \ln x$
	Common Log	$\log_{10} x = \log x$

Calculus AB Summer Packet

Block _____

You are not to use a calculator for these questions. Questions involving trig. values can be solved with a unit circle!

1)

Simplify each expression.

2) $2(9n - 3) - 10(n + 7)$

3) $-8(10a - 4) - 3(10a - 3)$

4) $2(-5k + 8) - 4(1 + 9k)$

5) $9(6 + 10k) - 10(k - 3)$

Solve each equation by completing the square.

6) $k^2 - 16k + 46 = -9$

7) $r^2 - 12r + 23 = 3$

8) $n^2 + 20n + 42 = -9$

9) $r^2 + 18r - 85 = 3$

Convert each degree measure into radians and each radian measure into degrees.

10) -330°

11) $\frac{4\pi}{3}$

12) 210°

13) 350°

14) $\frac{\pi}{3}$

15) -45°

Divide.

16) $(24x^3 + 8x^2 - 2x + 56) \div (4x + 6)$

17) $(70n^3 - 102n^2 - 13n + 43) \div (7n - 6)$

18) $(8v^3 - 49v^2 + 31v + 68) \div (8v + 7)$

19) $(72b^3 - 35b^2 - 61b + 15) \div (8b - 3)$

Simplify.

$$20) \frac{\sqrt[3]{3}}{\sqrt[3]{-64}}$$

$$21) \frac{3\sqrt[3]{5}}{5\sqrt[3]{135}}$$

$$22) \frac{\sqrt{12}}{4\sqrt{16}}$$

$$23) \frac{5\sqrt{10}}{\sqrt{5}}$$

$$24) \frac{3\sqrt{10}}{\sqrt{10}-5}$$

$$25) \frac{3}{\sqrt{3}+7}$$

$$26) \frac{6}{-5-\sqrt{3}}$$

$$27) \frac{9}{9+\sqrt{6}}$$

Solve each equation.

28) $2^{2k} = 2^2$

29) $8^{-2b-1} = 64$

30) $\log_{13} (4n - 3) = \log_{13} (-3n + 4)$

31) $\log_{16} (5m - 8) = \log_{16} m$

32) $\log_{15} (v^2 + 11v) = \log_{15} (9 + 3v)$

33) $\log_{12} (-12n - 1) = \log_{12} (n^2 + 26)$

34) $\log_8 (x - 1) - \log_8 x = \log_8 24$

35) $\log_3 8 + \log_3 4x = 1$

36) $\log_2 -3x - \log_2 6 = 2$

37) $\log_3 -4x - \log_3 10 = 1$

Solve each equation by factoring.

38) $7r^2 + 47r + 30 = 0$

39) $3n^2 + 4n - 7 = 0$

40) $7n^2 + 27n - 40 = 0$

41) $7r^2 - 48r - 7 = 0$

42) $15x^2 - 8x - 12 = 0$

43) $7n^2 - 29n + 24 = 0$

Solve each equation for $0 \leq \theta < 2\pi$.

44) $\sin \theta = \frac{\sqrt{3}}{2}$

45) $0 = \tan \theta$

46) $\sin \theta = -\frac{\sqrt{3}}{2}$

47) $-1 = \cos \theta$

Factor each completely.

48) $25x^2 - 30x + 9$

49) $16n^2 + 24n + 9$

50) $9n^2 - 6n + 1$

51) $16r^2 - 24r + 9$

52) $25x^2 - 9$

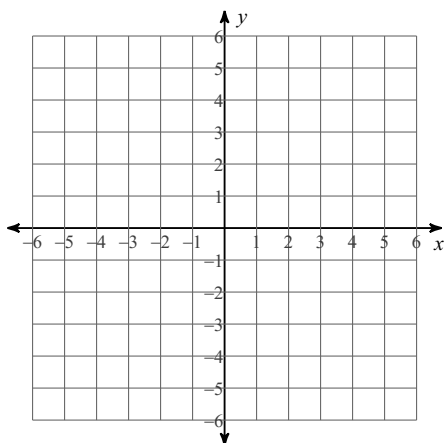
53) $4n^2 - 25$

54) $25n^2 - 1$

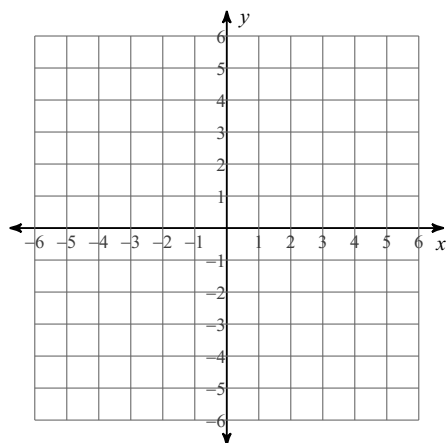
55) $9n^2 - 4$

Graph each equation.

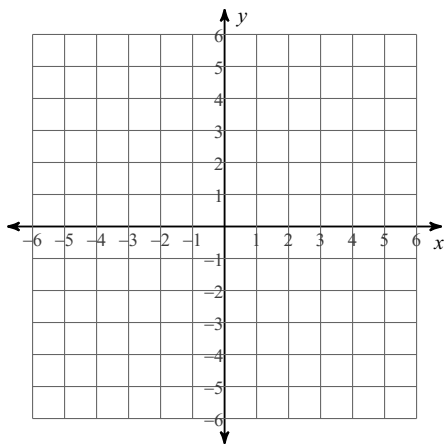
56) $y = -|x| - 4$



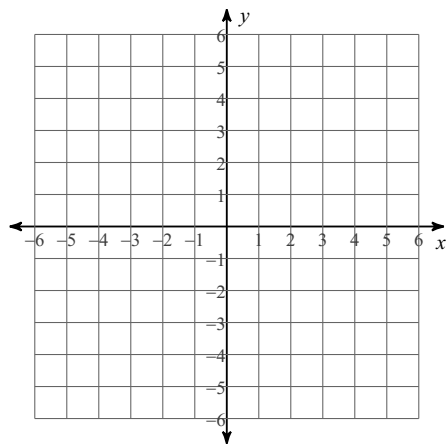
57) $y = |x - 2| - 2$



58) $y = |x + 2| - 1$



59) $y = -|x + 3| + 2$



Write the point-slope form of the equation of the line through the given points.

60) through: $(-1, 0)$ and $(0, -1)$

61) through: $(3, 5)$ and $(0, 0)$

62) through: $(5, 4)$ and $(5, 3)$

63) through: $(-5, 4)$ and $(5, -1)$

64) through: $(0, -2)$ and $(2, 2)$

65) through: $(2, -4)$ and $(1, -1)$

Condense each expression to a single logarithm.

66) $6\log_4 u + 2\log_4 v$

67) $20\log_6 3 + 5\log_6 8$

68) $5\log_4 8 + 10\log_4 3$

69) $\frac{\log_8 x}{3} + \frac{\log_8 y}{3} + \frac{\log_8 z}{3}$

Expand each logarithm.

$$70) \log_2 \left(\frac{u}{v^4} \right)^2$$

$$71) \log_9 \frac{x^6}{y^5}$$

$$72) \log_7 \left(\frac{a}{b^2} \right)^4$$

$$73) \log_9 (a^4 \cdot b)^2$$

Write each expression in exponential form.

$$74) (\sqrt[5]{3n})^7$$

$$75) \sqrt{a}$$

$$76) \frac{1}{\sqrt[3]{p}}$$

$$77) (\sqrt{7n})^3$$

78) $(\sqrt{3p})^5$

Write each expression in radical form.

79) $(2n^3)^{\frac{1}{6}}$

80) $a^{\frac{1}{3}}$

81) $(10x)^{\frac{1}{6}}$

82) $(7m)^{\frac{1}{3}}$

83) $(5x)^{\frac{1}{2}}$

Find the exact value of each trigonometric function.

84) $\sec \frac{\pi}{3}$

85) $\sec \frac{5\pi}{3}$

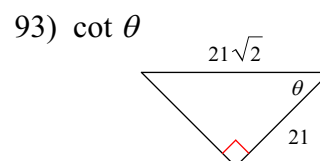
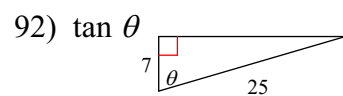
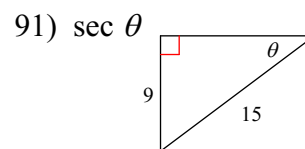
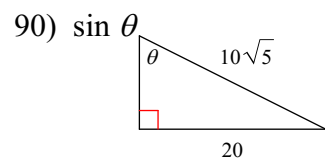
86) $\sin \frac{\pi}{6}$

87) $\sec \frac{7\pi}{6}$

88) $\sec 0$

89) $\sin \frac{11\pi}{6}$

Find the value of the trig function indicated.



Simplify each expression.

$$94) \frac{\frac{1}{8} + \frac{x^2}{4}}{16}$$

$$95) \frac{\frac{25}{x^2} - \frac{1}{x}}{\frac{5}{2}}$$

$$96) \frac{\frac{x}{x+4} + \frac{x}{25}}{\frac{5x}{x+4}}$$

$$97) \frac{4}{\frac{9}{u^2} + \frac{9}{u^2}}$$

$$98) \frac{\frac{25}{16}}{\frac{1}{5}}$$

$$99) \frac{\frac{1}{3}}{\frac{1}{8}}$$

$$100) \frac{\frac{25}{4}}{\frac{9}{5}}$$