

Course Title:	Calculus (A)
Grade(s):	11 & 12

Course Description
The course is designed for students who wish to take calculus but do not want to take the Advanced Placement course, and for students who may need to strengthen their skills from precalculus. Students in academic precalculus classes who may need to take a calculus course in college should be encouraged to take this course. Topics include: limit theory, differential calculus including the development of the derivative; derivative techniques involving polynomials, trigonometric and inverse trigonometric functions, exponential and logarithmic functions, the power, product, quotient and chain rules and implicit differentiation; integral calculus including area approximation techniques, integration of polynomials, trigonometric, exponential and logarithmic functions, integration by substitution and integration by parts. Skills are applied to solving problems involving position, velocity and acceleration, tangent lines and finding linear approximations for functions, graph analysis, optimization, related rates, areas bounded by curves, average value of a function, volumes of solids, growth and decay. Applications in business, economics and in the physical sciences will be used throughout the course. Course Units: 1. Limits and Continuity 2. Differentiation - Definition and Properties 3. Differentiation - Composite, Implicit and Inverse Functions 4. Contextual Applications of Differentiation 5. Analytical Applications of Differentiation 6. Integration and Accumulation of Change 7. Applications of Integration 8. Differential Equations
Approved Curricular Materials/Textbook(s)
<i>Calculus: Graphical, Numerical, Algebraic</i> (AP Edition)

Unit Title	Unit Description (Include Unit Objectives)	Unit Pacing
Unit 0: REVIEW OF FUNCTIONS & THEIR GRAPHS	The main prerequisite for the study of calculus is an understanding of functions. Students who have taken a precalculus course that emphasizes algebraic, numerical, and graphical representations of functions will have seen everything that is covered in this unit. The purpose of this unit of study is a brief review of the concepts covered in precalculus - Functions, their graphs, and basic properties of these functions. Since calculus is basically a tool for understanding how functions of various kinds model real-world behavior, a solid understanding of the basic functions is essential. Unit Topics Include: 1. Linear Functions 2. Quadratic Functions 3. Polynomial Functions 4. Piecewise Functions and Absolute Value Functions 5. Radical Functions (Square Root Functions) 6. Exponential & Logarithmic Functions 7. Trigonometric Functions	3 weeks - 4 weeks (August - September)
Unit 1: LIMITS & CONTINUITY	As we begin our study of calculus, we step into a branch of mathematics designed to describe change, motion, and behavior near key points. To do this effectively, we need tools that allow us to understand what a function is <i>approaching</i>, even when we cannot directly plug in a value. This is the idea behind a limit. Limits help us answer questions such as: • <i>What value does a function get close to as x approaches a certain number?</i> • <i>How can we analyze a function at points where it isn't nicely defined?</i> • <i>What does it mean for a function to behave smoothly without sudden jumps or breaks?</i> These ideas lead naturally to continuity—a formal way of describing when a function has no interruptions, holes, or jumps. Understanding limits and continuity	4 weeks (September - October)

	forms the foundation of derivatives and integrals, the central ideas of calculus. Throughout this unit, we will explore how limits work, why they matter, and how they allow us to describe functions with precision. By the end, you'll be able to examine a graph or an equation and determine not just what it <i>is</i>, but what it's <i>tending toward</i>—a powerful shift in mathematical thinking. Unit Topics Include: 1. Defining Limits and Using Limit Notation 2. Estimating Limit Values from Graphs 3. Estimating Limit Values from Tables 4. Determining Limits Using Algebraic Properties of Limits 5. Determining Limits Using Algebraic Manipulation 6. Selecting Procedures for Determining Limits 7. Determining Limits Using the Squeeze Theorem 8. Connecting Multiple Representations of Limits 9. Exploring Types of Discontinuities 10. Defining Continuity at a Point 11. Confirming Continuity over an Interval 12. Removing Discontinuities 13. Connecting Infinite Limits and Vertical Asymptotes 14. Connecting Limits at Infinity and Horizontal Asymptotes 15. Working with the Intermediate Value Theorem (IVT)	
Unit 2: DIFFERENTIATION: DEFINITION & PROPERTIES	Differentiation is one of the central ideas of calculus—a tool that allows us to understand how quantities change. While functions tell us the relationship between variables, differentiation goes a step further: it describes how <i>fast</i> that relationship is changing at any moment. At its core, differentiation is defined through a limit—the limit of average rates of change as the interval becomes infinitely small. This gives us the derivative, a new function that measures the instantaneous rate of change of the original function. From this definition comes a rich set of properties and rules, such as linearity, product and quotient rules, and the chain rule, which make it possible to	3 weeks - 4 weeks (October - November)

	differentiate even complex functions efficiently and accurately. Understanding differentiation equips students with a powerful mathematical lens. It allows them to: • Analyze motion, growth, and decay in real-world contexts • Model change in fields such as physics, biology, economics, and engineering • Describe patterns in graphs, including where functions increase, decrease, or reach maximum and minimum values • Build the foundational skills needed for optimization and related-rates problems later in the course As you work through this unit, you will see how the derivative reveals the <i>behavior</i> hidden inside a function—how it rises, falls, curves, and responds to changes. Mastering differentiation not only strengthens your mathematical reasoning but also opens the door to many of the most meaningful applications of calculus. Unit Topics Include: 1. Defining Average and Instantaneous Rates of Change at a Point 2. Defining the Derivative of a Function and Using Derivative Notation 3. Estimating Derivatives of a Function at a Point 4. Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist 5. Applying the Power Rule 6. Derivative Rules: Constant, Sum, Difference, and Constant Multiple 7. Derivatives of $\cos x$, $\sin x$, e^x, and $\ln x$ 8. The Product Rule 9. The Quotient Rule 10. Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions	
Unit 3: DIFFERENTIATION:	In this unit, students will extend their understanding of differentiation beyond basic rules and learn how to differentiate more complex relationships using composite, implicit, and inverse functions. These tools allow us to analyze situations where	3 weeks - 4 weeks (November -

COMPOSITE, IMPLICIT, & INVERSE FUNCTIONS	quantities depend on each other in layered, indirect, or hidden ways—something that happens constantly in mathematics, science, engineering, and data modeling. Composite functions help us understand change when one variable depends on another through multiple stages. In real applications—such as calculating how a medication’s concentration changes in the bloodstream or how temperature affects the pressure inside an engine—relationships are rarely simple. The chain rule, which governs composite differentiation, becomes essential for breaking down how one small change propagates through an entire system. Implicit differentiation is used when relationships are too complicated to solve for one variable explicitly. Circles, ellipses, and many physical constraints are naturally expressed implicitly. Implicit differentiation allows us to compute slopes and rates even when equations resist being rewritten neatly, making it vital in physics, optimization, and any scenario involving constraints or interconnected variables. Inverse function differentiation helps us calculate rates of change for functions that “undo” each other, such as logarithmic and exponential relationships. These arise constantly in finance, population modeling, radioactive decay, and any exponential growth or decay scenario. Understanding how to differentiate inverses gives students flexibility in modeling real-world phenomena and interpreting results. Together, these techniques expand the tools students have for describing and predicting how things change. As problems become more complex and interconnected, these forms of differentiation make it possible to analyze systems with precision and confidence—skills that are foundational for higher-level mathematics, science, economics, and engineering. Unit Topic Include: 1. The Chain Rule 2. Implicit Differentiation 3. Differentiating Inverse Functions 4. Differentiating Inverse Trigonometric Functions	December)
---	---	------------------

	5. Selecting Procedures for Calculating Derivatives 6. Calculating Higher-Order Derivatives	
Unit 4: CONTEXTUAL APPLICATIONS OF DIFFERENTIATION	In this unit, students will explore how differentiation applies to a wide range of real-world situations, focusing on motion, related rates, and local linearity. Differentiation allows us to describe how quantities change, model dynamic systems, and make accurate predictions in practical contexts. Moving beyond abstract formulas, they will see how derivatives describe rates of change in the physical world and how smooth curves can be approximated by straight lines for practical analysis. Motion is one of the most tangible applications of differentiation. By examining position, velocity, and acceleration, students learn how the derivative translates a change in one quantity over time into a deeper understanding of movement. This allows them to model cars accelerating on a highway, objects in free fall, or any situation where speed and direction change dynamically. These skills are essential in physics, engineering, robotics, and any field where predicting and controlling motion is important. Related rates extend this idea to situations where multiple quantities change simultaneously. For example, as a balloon inflates, its radius and volume change together; as a ladder slides down a wall, both the height and distance from the wall change. Understanding related rates equips students to tackle problems in engineering, economics, biology, and everyday scenarios where changes are interconnected. Local linearity emphasizes how differentiable functions can be approximated by straight lines over small intervals. This principle allows students to estimate changes, make predictions, and simplify complex problems—whether it's approximating growth trends in populations, changes in chemical concentrations, or adjustments in mechanical systems. By studying these applications, students see that differentiation is not just abstract mathematics—it is a versatile tool for analyzing change in almost any context. Mastery of these concepts prepares students to solve practical problems, make predictions, and think critically about dynamic systems in science, technology,	3 weeks - 4 weeks (January - February)

	engineering, and beyond. Unit Topics Include: 1. Interpreting the Meaning of the Derivative in Context 2. Straight-Line Motion: Connecting Position, Velocity, and Acceleration 3. Rates of Change in Applied Contexts Other Than Motion 4. Introduction to Related Rates 5. Solving Related Rates Problems 6. Approximating Values of a Function Using Local Linearity and Linearization 7. Using L'Hospital's Rule for Determining Limits of Indeterminate Forms	
Unit 5: ANALYTICAL APPLICATIONS OF DIFFERENTIATION	In this unit, students will explore how differentiation helps us analyze graphs and solve optimization problems, connecting abstract calculus concepts to practical applications. By studying critical points, increasing and decreasing intervals, and concavity, students learn to understand the shape and behavior of functions, which is essential in fields such as economics, engineering, physics, and data analysis. Graph analysis allows us to identify maxima, minima, and points of inflection, giving insight into trends, peak performance, and rates of change in real systems. Whether determining the highest profit, the fastest speed, or the steepest slope, these skills help students interpret and predict behavior accurately. Optimization builds directly on this analysis, enabling students to find the best possible solutions under given constraints. From minimizing material costs in construction to maximizing efficiency in manufacturing or revenue in business, these techniques turn calculus into a powerful decision-making tool. Through this unit, students see that differentiation is not just a set of rules—it is a method for analyzing, interpreting, and optimizing real-world problems, providing a foundation for precise reasoning and problem-solving in many professional contexts.	4 weeks (February - March)

	Unit Topics Include: 1. Using the Mean Value Theorem 2. Extreme Value Theorem, Global Versus Local Extrema, and Critical Points 3. Determining Intervals on Which a Function Is Increasing or Decreasing 4. Using the First Derivative Test to Determine Relative (Local) Extrema 5. Using the Candidates Test to Determine Absolute (Global) Extrema 6. Determining Concavity of Functions over Their Domains 7. Using the Second Derivative Test to Determine Extrema 8. Sketching Graphs of Functions and Their Derivatives 9. Connecting a Function, Its First Derivative, and Its Second Derivative 10. Introduction to Optimization Problems 11. Solving Optimization Problems 12. Exploring Behaviors of Implicit Relations	
Unit 6: INTEGRATION & ACCUMULATION OF CHANGE	In this unit, students will explore integration, a fundamental tool for understanding how quantities accumulate over time or space. While differentiation measures rates of change, integration allows us to sum small changes to find total amounts, making it essential in physics, engineering, biology, economics, and any field that models accumulation. Students will begin with Riemann sums, which approximate areas under curves and provide a foundation for understanding definite integrals. From there, they will learn techniques of integration, including substitution and other methods, to calculate exact areas, total quantities, and solve real-world problems involving distance, volume, growth, and accumulation. By mastering integration, students gain the ability to translate rates of change into total outcomes, providing insight into how systems evolve and giving them powerful tools to solve practical problems in science, engineering, and beyond. Unit Topics Include: 1. Exploring Accumulations of Change 2. Approximating Areas with Riemann Sums 3. Riemann Sums, Summation Notation, and Definite Integral Notation	4 weeks (March - April)

	4. The Fundamental Theorem of Calculus and Accumulation Functions 5. Interpreting the Behavior of Accumulation Functions Involving Area 6. Applying Properties of Definite Integrals 7. The Fundamental Theorem of Calculus and Definite Integrals 8. Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation 9. Integrating Using Substitution 10. Integrating Functions Using Long Division and Completing the Square 11. Selecting Techniques for Antidifferentiation	
Unit 7: APPLICATIONS OF INTEGRATION	In this unit, students will explore how integration can be used to solve meaningful problems involving area, volume, motion, and average values of functions. Building on the concept of accumulation, integration allows us to determine total quantities from rates of change and to measure regions and solids that cannot be easily calculated with basic geometric formulas. Students will apply integration to find the area between curves and the volumes of three-dimensional solids, tools that are widely used in engineering, architecture, and physical sciences to measure space and design efficient structures. They will also investigate the average value of a function, which provides a meaningful way to describe the typical behavior of a quantity over an interval, an idea that appears in contexts such as average temperature, average cost, or average output. In addition, students will use integration to connect position, velocity, and acceleration, reversing the process of differentiation to determine total displacement or changes in motion over time. These relationships are fundamental in physics and engineering, where understanding how motion evolves is essential. Through these applications, students will see how integration provides powerful tools for measuring, modeling, and interpreting real-world phenomena, reinforcing its role as a central concept in calculus and applied mathematics. Unit Topics Include: 1. Finding the Average Value of a Function on an Interval 2. Connecting Position, Velocity, and Acceleration Functions Using Integrals	4 weeks (April - May)

	3. Using Accumulation Functions and Definite Integrals in Applied Contexts 4. Finding the Area Between Curves Expressed as Functions of x 5. Finding the Area Between Curves Expressed as Functions of y 6. Finding the Area Between Curves That Intersect at More Than Two Points 7. Volumes with Cross Sections: Squares and Rectangles 8. Volumes with Cross Sections: Triangles and Semicircles 9. Volume with Disc Method: Revolving Around the x- or y-Axis 10. Volume with Disc Method: Revolving Around Other Axes 11. Volume with Washer Method: Revolving Around the x- or y-Axis 12. Volume with Washer Method: Revolving Around Other Axes	
Unit 8: DIFFERENTIAL EQUATIONS (Time Permitting)	In this unit, students will explore differential equations, which are equations that describe how quantities change in relation to one another. Rather than simply finding a rate of change, differential equations allow us to model entire systems whose behavior depends on changing conditions over time. These models appear throughout science, engineering, economics, and biology, where understanding how systems evolve is essential. Students will learn how to model real-world situations using differential equations, translating physical or contextual information into mathematical relationships that describe growth, decay, and dynamic change. They will also use slope fields as a graphical tool to visualize families of possible solutions and understand how solutions behave even when exact formulas are difficult to find. By studying differential equations, students gain experience with one of the most powerful applications of calculus: building mathematical models that describe and predict real-world phenomena. These tools help scientists and engineers analyze systems such as population growth, the spread of disease, cooling processes, and many other situations where change occurs continuously over time. Unit Topics Include: 1. Modeling Situations with Differential Equations 2. Verifying Solutions for Differential Equations 3. Sketching Slope Fields 4. Reasoning Using Slope Fields	4 weeks (May - June)

	5. Finding General Solutions Using Separation of Variables 6. Finding Particular Solutions Using Initial Conditions and Separation of Variables 7. Exponential Models with Differential Equations	
--	---	--