

AP CALCULUS AB SUMMER PACKET

Eastlake High School

Summer 2026

Dear Future AP Calculus AB Student,

Congrats! You have signed up for AP Calculus AB for the 2026-2027 school year at Eastlake High School. AP Calculus AB is the culmination of all the mathematics you have learned! It's truly an amazing experience that enhances *all* the mathematics you have learned through completely new concepts and applications emphasizing critical thinking, analysis, and reasoning coupled with advanced algebraic and trigonometry knowledge. If you love mathematics, get ready for a wild ride as this course will follow the College Board set curriculum. A quick pace will be followed to ensure completion of the required content and to ensure all students can have the opportunity to earn college credit with a passing score of a 3, 4 or 5 on the College Board exam.

A big piece of being successful in this class is hard work and perseverance. We expect AP Calculus AB students to possess a level of independence surrounding their learning and engaging in study habits that *enhance* the "learn how to learn" model which include:

- Engaging in daily reflections on your own learning, digging for the *WHY* behind the process
- Exhibiting care, attention, and detail in the completion of every homework problem
- Self-correction of mistakes, both in daily assignments and assessments, as mistakes will occur and are expected and the power of what can be learned from them isn't to be ignored!
- Reading the PowerPoints before, during, and after lesson presentations
- Accessing online resources such as "Extra Resources", AP Classroom videos, and Flippedmath.com to supplement your understanding
- Working in a study group and/or EHS weekly Math Lab
- Seeking teacher assistance during tutorials (before/after school) and Wolf Time
- Self-discipline and initiative to seek learning and understanding.

There are certain math skills that have been taught to you over the previous years that are necessary to be successful in *any* calculus course. If you do not have these skills, you will consistently struggle to correctly solve problems and reach solutions, even though you understand the calculus concept. The Summer Assignments enclosed are intended to help you brush up on and possibly relearn these essential skills. We have created a sampling of foundational skills in algebra and trigonometry that you can practice remotely over the summer, including an optional packet which can be found on the EHS school website under AP Calculus.

Note: For AP Calculus AB, a graphing calculator is REQUIRED! Please purchase one BEFORE the school starts from the list of approved calculators: [AP Exams Calculator Policy – AP Students | College Board](#). We will instruct the class using a TI-84 plus CE.

All the best!

Ms. Bultman (mbultman@lwsd.org), Mrs. Simon (jsimon@lwsd.org) and Mrs. Timofte (mtimofte@lwsd.org)

What is in this packet?

- Student letter (you just read this)
- AP Calculus AB Foundational Pre-Requisite Skills overview
- Intro to Limits – (3 lessons) videos links and practice with solutions - **HIGHLY RECOMMENDED**
- Summer Assignment Pre-requisite skills: (**HIGHLY RECOMMENDED Parts 1-3**)
- Part 1 – Trigonometry: foundational skills required exercises to be done with paper & pencil. The answer key can be found in this packet.
- Part 2 – Additional practice exercises and the step-by-step solutions to all these exercises
- Part 3 – Basic skills review to be completed on Khan Academy at 75% or above. Below you will see a table listing the title of each Khan Academy “practice” with a supporting video hyperlink for you to refresh your understanding if need be.
- Part 4 - The UW placement test prior to calc. (**Optional**)
- Expect 2-3 hours of work on Intro to LIMITS, 4-5 hours of work for the Summer Assignment (Part 1 and Part 2) & 3-4 hours for the Khan Academy Practice & UW placement test for Calculus.
- Note: You are expected to be able to solve ALL the problems in this packet WITHOUT any calculator!

What can you expect in the fall?

- Assessment of the Summer Assignments Part 1 & Part 2 skills will be used to determine algebraic and trigonometric readiness for AP Calculus AB during **the first month of school** and students will receive a grade in the grade book. The purpose of this assessment is to inform students if they have or not the pre-requisite skills needed for calculus, so they can take the necessary action steps, as applicable.
- Students will need to use a large majority of the summer assignment skills during the first unit of Calculus, but also in all the other units!

AP Calculus AB Foundational Pre-Requisite Skills

Algebra

- can manipulate with ease fractions, decimals and variables in a variety of settings including in equations and rational functions
- comfortable with all forms of factoring including quadratics, sum and difference of cubes, quartic and factoring by grouping
- completing the square
- synthetic and long division with polynomials
- add, subtract, multiply and divide radical expressions including rationalizing denominators
- use exponents and logarithms properties to simplify expressions
- solving equations involving logarithms and rational exponents

Graphing

- be familiar with the graphs of linear, absolute value, quadratic, cubic, quartic, logarithmic, exponential, and rational functions
- identify the domain and range of functions
- recognize end behaviors of graphs, vertical and horizontal asymptotes, and holes
- identify transformations and use them to graph

Trigonometry

- work with the basic six trig functions including manipulating them to simplify expressions and solve equations by finding all solutions
- know and apply a subset of trig identities
- Know the trig table of values - know the graphs of the six basic trig functions including their domain and range
- know how to find, without a calculator, the values of the inverse trig functions at chosen values

Intro to limits:

Video lesson	Practice	Solutions
Calculus AB/BC - 1.2 Defining Limits and Using Limit Notation - YouTube	Microsoft Word - Calc 1.2 Packet	Microsoft Word - Calc 1.2 Solutions
Calculus AB/BC – 1.3 Estimating Limit Values from Graphs	Microsoft Word - Calc 1.3 Packet	Microsoft Word - Calc 1.3 Solutions
Calculus AB/BC – 1.4 Estimating Limit Values from Tables	Microsoft Word - Calc 1.4 Packet	Microsoft Word - 1.4 Solutions

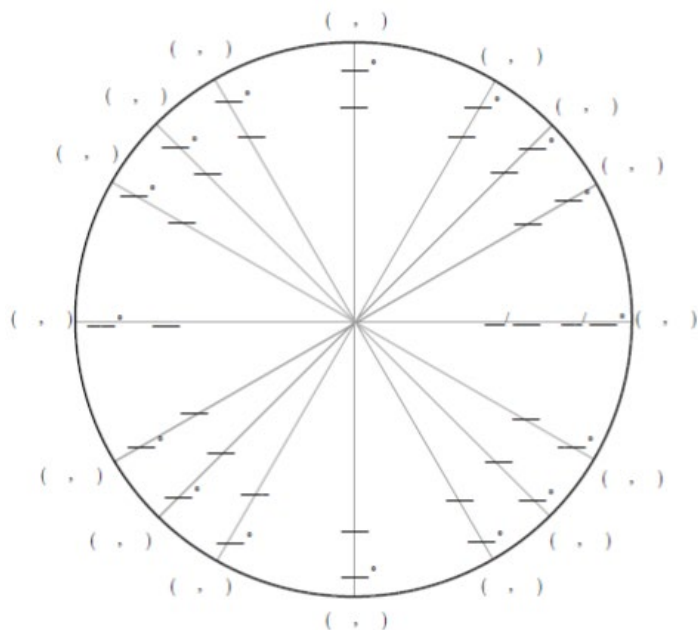
AP Calculus AB: Summer Packet pre-requisite skills

Part #1: Trigonometry – Complete all exercises below with supporting work shown. Check your work with the answer key.

UNIT CIRCLE: You MUST have Quadrant 1 memorized, and it should not take you more than a few seconds to determine the other Quadrants' values.

Fill out the Unit Circle below by finding the following:

- The measures for each angle in radian and degree (radians will be used in AP Calc AB)
- The coordinate pair for each angle



Fill in the following trig values for Quadrant 1:

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin \theta$					
$\cos \theta$					
$\tan \theta$					

- Be able to sketch **all 6** trigonometric functions and their inverse functions, label their vertical asymptotes, and state their domains and ranges.

- Know the relationship between trig functions and their inverse functions and why their domains & ranges are switched and why the restricted domains of the trig functions are required to ensure the inverse trig function exists.

Domain and Range of Trigonometry Functions

	Domain	Range
$\sin^{-1}$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}$	$\mathbb{R}$	$(-\pi/2, \pi/2)$
$\operatorname{cosec}^{-1}$	$\mathbb{R} - (-1, 1)$	$[-\pi/2, \pi/2] - \{0\}$
$\sec^{-1}$	$\mathbb{R} - (-1, 1)$	$[0, \pi] - \{\pi/2\}$
$\cot^{-1}$	$\mathbb{R}$	$(0, \pi)$

You also need to know the following **Trig Identities**:

$$\csc(x) = \frac{1}{\sin(x)}$$

$$\sin(x) = \frac{1}{\csc(x)}$$

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\cos(x) = \frac{1}{\sec(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$$

$$\tan(x) = \frac{1}{\cot(x)} = \frac{\sin(x)}{\cos(x)}$$

$$\tan^2(t) + 1 = \sec^2(t)$$

$$1 + \cot^2(t) = \csc^2(t)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

Practice Problems:

Evaluate each trigonometric function for the value or angle given.

$\sec(2\pi)$	$\tan \frac{5\pi}{3}$	$\csc \pi$	$\cot \frac{3\pi}{4}$
$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$	$\arccos \frac{-1}{2}$	$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$	$\arcsin \frac{-1}{2}$
$\csc \frac{-7\pi}{6}$	$\arctan(\sqrt{3})$	$\sec \frac{-5\pi}{6}$	$\arctan\left(\frac{\sqrt{3}}{3}\right)$

Solving equations involving trigonometric functions.

Example:

Solve the equation $2 \sin^2 x + 3 \sin x = -1$ on the interval $[0, 2\pi]$

Consider $\sin x = u$;

The equation becomes: $2u^2 + 3u + 1 = 0$ or $(2u + 1)(u + 1) = 0$.

This equation has the solutions $-1/2$ and -1 , so we need to solve on the interval $[0, 2\pi]$

$\sin x = -1/2$ and $\sin x = -1$

The solutions are: $x = 7\pi/6$, $x = 11\pi/6$, and $x = 3\pi/2$.

A. Solve the equation $2 \sin^2 x + \sin x = 1$ on the interval $[0, 2\pi]$

B. Solve the equation $2 \cos^2 x + \cos x = 1$ on the interval $[0, 2\pi]$

C. Solve the equation $\sin 2x = -1$ on the interval $[0, 3\pi]$

D. Solve the equation $2 \sin 2x = 1$ on the interval $[0, 2\pi]$

E. Solve the equation $\sin^2 x - \sin x = 0$ on the interval $[0, 2\pi]$

F. Solve the equation $\cos^2 x - \cos x = 0$ on the interval $[0, 2\pi]$

G. Verify each identity.

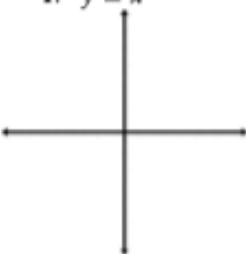
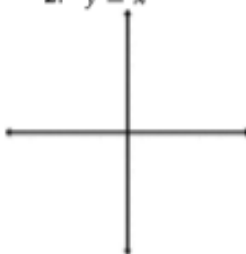
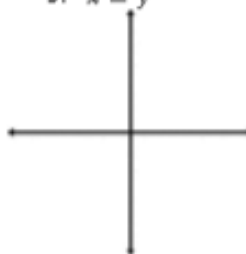
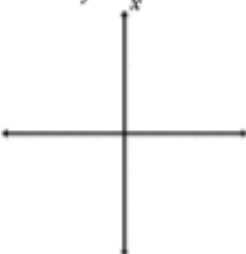
1) $\frac{1 + \csc x}{\sec x} - \cot x = \cos x$

2) $\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$

3) $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$

Part #2: Additional Practice – Complete the practice problems below. The key is at the end.

Graphing: Sketch the graph and state the Domain & Range. Use interval notation for the Domain & Range

1. $y = x$  Domain: Range:	2. $y = x^2$  Domain: Range:	3. $x = y^2$  Domain: Range:
4. $y = \frac{1}{x}$  Domain: Range:	5. $y = \sqrt{x}$  Domain: Range:	6. $y = e^x$  Domain: Range:
7. $y = \ln x$  Domain: Range:	8. $y = \sin x$  Domain: Range:	9. $y = \cos x$  Domain: Range:
10. $y = x$  Domain: Range:	11. $x^2 = 16 - y^2$  Domain: Range:	12. $y = \sqrt{4 - x^2}$  Domain: Range:

1. Evaluating Functions:

A)

$$f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$$

(a) $f(-2)$

(b) $f(1)$

(c) $f(2)$

B)

$$f(x) = \begin{cases} 3x - 1, & x < -1 \\ 4, & -1 \leq x \leq 1 \\ x^2, & x > 1 \end{cases}$$

(a) $f(-2)$

(b) $f(-\frac{1}{2})$

(c) $f(3)$

C)

$$f(s) = \frac{|s - 2|}{s - 2}$$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$					

II. Algebraic Manipulation

A. Factor each of the following completely or state that it is prime.

13. $x^2 + 5x + 6$

14. $x^2 - 4x - 12$

15. $x^2 + 5x - 24$

16. $16x^2 - 81$

17. $x^3 + 4x^2 + 3x$

18. $x^4 + 6x^2 + 9$

A'. Completing the square: Find h and k such that $x^2 + 12x + 4 = (x + h)^2 + k$

B. Simplify

19. $\frac{(x+4)^2 - 16x}{x-4}$

20. $\frac{y+3}{(y+4)^2 - (8y+25)}$

21. $\frac{\frac{25}{a} - a}{5+a}$

22. $\frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}}$

C. Solve for x without a calculator or grapher:

23. $1 + \frac{x}{3} = \frac{4}{5}$	24. $\frac{2}{3} - \frac{5}{7} = x$	25. $\frac{3}{8} = \frac{4}{1-x}$
26. $\frac{6}{x} + \frac{x}{2} = 4$	27. $\ln(x-2) = 4$	28. $-3\ln(x+1) = 2$
29. $\ln(x-2) + \ln(3) = 5$	30. $4e^{3x-2} = 8$	31. $100 = 200e^{-0.06x}$

32. $\sin x = -\frac{\sqrt{2}}{2}, \text{ for } 0 \leq x \leq 2\pi$	33. $3x^2 + 7x + 3 = 0$
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D_1 : Find the horizontal and vertical asymptotes of each function. Show work.

34. $y = \frac{1}{x-1}$

35. $y = \frac{1}{(x-1)^2}$

D_2 : Find the holes (if any). Show work.

A) $y = \frac{x-1}{x^3-1}$

B) $y = \frac{x^2-3x+2}{x^2+2x-3}$

E: Equations of lines.

36) Create the point-slope form of a line passing through the point $(0, 5)$ with a slope of $\frac{2}{3}$.

37) Create the point-slope form of a line passing through the point $(2, 8)$ that is parallel to the line $y = \frac{5}{6}x - 1$.

38) Create the point-slope form of a line perpendicular to the y -axis passing through the point $(4, 7)$.

F. Function Notation

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each.

39. $h[f(-2)] = \underline{\hspace{2cm}}$

40. $f[g(x-1)] = \underline{\hspace{2cm}}$

41. $g[h(x^3)] = \underline{\hspace{2cm}}$

G. Modeling Equations

42. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.
43. The sum of the perimeters of an equilateral triangle and a square is 10. Let s be the side length of the equilateral triangle and let x be the side length of the square. Find the equation for the area of the square in terms of s .
44. A manufacturer makes a metal can in the shape of a cylinder that holds 1000 cm^3 of oil. Write an equation for the Surface area of the can in terms of one variable.

H. Difference Quotient:

$$f(x) = x^2 - x + 1, \quad \frac{f(2+h) - f(2)}{h}, h \neq 0$$

$$f(x) = 5x - x^2, \quad \frac{f(5+h) - f(5)}{h}, h \neq 0$$

I. Division of Polynomials:

A) $(x^3 - 2x^2 - 5x + 6) \div (x - 3)$

B) $(2x^4 + 3x^3 + 5x - 1) \div (x^2 + 3x + 2)$

J. Exponents and Logarithmic properties:

a) Simplify: $\left(\frac{2x^4 3y^6}{4xy^4}\right)^{-2}$

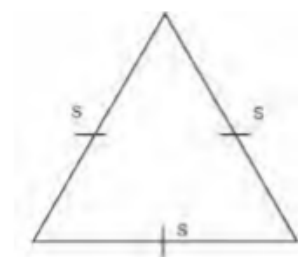
b) Expand: $\ln\left(\frac{3x^2}{2y^3}\right)$

c) rewrite in exponential form: $\sqrt[3]{2x^5}$

d) Evaluate: $e^{3\ln e^2}$

III. Area/Geometry Questions

45. What is the area of an equilateral triangle with side length s ? Hint: use a 30-60-90 triangle!



46. What is the area of a semicircle with respect to diameter d ?

47. What's the area of an isosceles right triangle with base $(4-x^2)$?

Important Reminders:

- *This Summer Packet Includes:*
 - Student Letter from the AP Calculus AB teaching team
 - Part 1 – Trigonometry
 - Part 2 – Additional Practice Exercises (AP Calculus AB Foundational Pre-Requisite Skills)
 - Part 3 – Khan Academy links to basic skills review
 - Part 4 - The UW placement test prior to calc (at the end of the packet, after the keys to Parts 1 & 2)
- Assessment of the Summer Assignments Part 1 & Part 2 skills will be used to determine algebraic and trigonometric readiness for AP Calculus AB during **the first month of school** and students will receive a grade in the grade book. The purpose of this assessment is to inform students if they have or not the pre-requisite skills needed for calculus.

Part #3: Khan Academy Skills Practice – Practice the skills listed below. If needed, watch the videos (links provided below)

Title of Assignment "practice" on Khan Academy	Supporting Video Links on Khan Academy
Polynomials	
Factor using polynomial division	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-div/x2ec2f6f830c9fb89:poly-div-by-linear/v/factor-w-poly-div?modal=1
	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-div/x2ec2f6f830c9fb89:poly-div-by-linear/v/factor-w-poly-div-missing-term?modal=1

Factor higher degree polynomials	<i>Overview of Factoring of Polynomials:</i> https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-factor/x2ec2f6f830c9fb89:mono-factor/v/factor-high-deg-poly-intro?modal=1 <i>Higher Degree Polynomials:</i> https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-factor/x2ec2f6f830c9fb89:factor-high-deg/v/factor-high-deg-poly?modal=1 <i>Additional High Degree Polynomials</i> https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-factor/x2ec2f6f830c9fb89:factor-high-deg/v/factoring-perfect-square-polynomial?modal=1
Divide polynomials with remainders	https://www.khanacademy.org/math/algebra-home/alg-polynomials/alg-practice-dividing-polynomials-with-remainders/v/dividing-polynomials-with-remainders
Zeros of polynomials (with factoring)	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-graphs/x2ec2f6f830c9fb89:poly-zeros/v/polynomial-zeros-grouping?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-graphs/x2ec2f6f830c9fb89:poly-zeros/v/polynomial-zeros-common-factor?modal=1
Composite Functions	
Evaluate composite functions: graphs & tables	https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:composing/v/evaluating-composite-functions-using-tables?modal=1 AND https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:composing/v/evaluating-composite-functions-using-graphs?modal=1
Find composite functions	https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:composing/v/new-function-from-composition?modal=1 AND https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:composing/v/evaluating-composite-functions-example-1?modal=1
Inverse Functions	
Verify inverse functions	https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:verifying-inverse/v/verifying-function-inverses-by-composition?modal=1
Rational Exponents	
Fractional exponents	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:exp/x2ec2f6f830c9fb89:rational-exp/v/rewriting-roots-as-rational-exponents?modal=1
Logarithms	

Use the properties of logarithms	<u>Product Rule:</u> https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:logs/x2ec2f6f830c9fb89:log-prop/v/sum-of-logarithms-with-same-base?modal=1 <u>Power Rule:</u> https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:logs/x2ec2f6f830c9fb89:log-prop/v/logarithm-of-a-power?modal=1 <u>Multiple Steps using Properties of Logarithm:</u> https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:logs/x2ec2f6f830c9fb89:log-prop/v/using-multiple-logarithm-properties-to-simplify?modal=1
Transformations of Functions	
Identify function transformations	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformation/s/x2ec2f6f830c9fb89:trans-all-together/v/shifting-and-reflecting-functions?modal=1
Even & Odd Functions	
Even and odd functions: Graphs and tables	<u>Graphs:</u> https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformation/s/x2ec2f6f830c9fb89:symmetry/v/recognizing-features-of-functions-2-example-2?modal=1 <u>Tables:</u> https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformation/s/x2ec2f6f830c9fb89:symmetry/v/even-and-odd-functions-tables?modal=1
Graphs of Functions	
Graphs of square and cube root functions	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformation/s/x2ec2f6f830c9fb89:radical-graphs/v/graphing-square-and-cube-root?modal=1
Graphs of exponential functions	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformation/s/x2ec2f6f830c9fb89:exp-graphs/v/transforming-exponential-graphs?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformation/s/x2ec2f6f830c9fb89:exp-graphs/v/transforming-exponential-graphs-2?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformation/s/x2ec2f6f830c9fb89:exp-graphs/v/graphing-exponential-functions-interactive?modal=1
Graphs of logarithmic functions	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformation/s/x2ec2f6f830c9fb89:log-graphs/v/graphing-logarithmic-functions-1?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformation/s/x2ec2f6f830c9fb89:log-graphs/v/graphing-logarithmic-functions-2?modal=1
Solving Equations	
Quadratic systems	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:eq/x2ec2f6f830c9fb89:quad-sys/v/line-and-parabola-system?modal=1 AND

	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:eq/x2ec2f6f830c9fb89:quad-sys/v/systems-of-nonlinear-equations-3?modal=1
Rational Functions	
End behavior of rational functions	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-end-behavior/v/end-behavior-of-rational-functions?modal=1
Rational function points of discontinuity	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:discontinuities/v/discontinuities-of-rational-functions?modal=1
Analyze vertical asymptotes of rational functions	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:discontinuities/v/analyzing-vertical-asymptotes-of-rational-functions?modal=1
Add & subtract rational expressions: unlike denominators	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-add-sub-intro/v/adding-rational-expression-w-unlike-denominators?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-add-sub-intro/v/subtracting-rational-expressions-w-unlike-denominators?modal=1
Piecewise Functions	
Evaluate piecewise functions	https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:absolute-value-piecewise-functions/x2f8bb11595b61c86:piecewise-functions/v/evaluating-piecewise-functions-example
Piecewise functions graphs	https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:absolute-value-piecewise-functions/x2f8bb11595b61c86:piecewise-functions/v/graphing-piecewise-function
Completing the Square	
Completing the square (intermediate)	https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:completing-square-quadratics/v/rewriting-quadratics-as-perfect-squares
Writing Equation of a Line: Point Slope Form	
Point-slope form	https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:forms-of-linear-equations/x2f8bb11595b61c86:point-slope-form/v/point-slope-and-slope-intercept-form-from-two-points
Rationalizing the Denominator	
Rationalizing the denominator (advanced)	Click on hint button inside the practice

Part #1: Trigonometry – KEY

Evaluate each trigonometric function for the value or angle given.

$$\sec(2\pi) = \mathbf{1 \text{ because } \sec(2\pi) = \frac{1}{\cos(2\pi)} = \frac{1}{1} = 1}$$

$$\csc \pi = \text{undefined because } \csc(\pi) = \frac{1}{\sin(\pi)} = \frac{1}{0} = \mathbf{undef.}$$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \mathbf{\pi/3}$$

$$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \mathbf{5\pi/6}$$

$$\csc \frac{-7\pi}{6} = \mathbf{2 \text{ because } \csc\left(\frac{-7\pi}{6}\right) = \frac{1}{\sin\left(\frac{-7\pi}{6}\right)} = \frac{1}{\frac{1}{2}} = 2}$$

$$\sec \frac{-5\pi}{6} = \mathbf{-2\sqrt{3}/3}$$

$$\tan \frac{5\pi}{3} = \mathbf{-\sqrt{3}}$$

$$\cot \frac{3\pi}{4} = \mathbf{-1}$$

$$\arccos \frac{-1}{2} = \mathbf{\frac{2\pi}{3} \text{ because the restricted domain of arccos x is } [0, \pi]}$$

$$\arcsin \frac{-1}{2} = \mathbf{-\pi/6 \text{ because the restricted domain of arcsin x is } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$$

$$\arctan(\sqrt{3}) = \mathbf{\pi/3 \text{ because the restricted domain of arctan x is } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and tan x is defined as opp/adj. Use a 30-60-90 triangle.}}$$

$$\arctan\left(\frac{\sqrt{3}}{3}\right) = \mathbf{\pi/6 \text{ because the restricted domain of arctan x is } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and tan x is defined as opp/adj. Use a 30-60-90 triangle.}}$$

- B. Solve the equation $2 \sin^2 x + \sin x = 1$ on the interval $[0, 2\pi]$: $\mathbf{x = \pi/6, x = 5\pi/6, x = 3\pi/2}$

$$2 \sin^2 x + \sin x = 1$$

$$2 \sin^2 x + \sin x - 1 = 0$$

$$(2 \sin x - 1)(\sin x + 1) = 0$$

$$2 \sin x - 1 = 0 \text{ OR } \sin x + 1 = 0$$

$$\sin x = \frac{1}{2} \text{ OR } \sin x = -1$$

Use your knowledge of Unit Circle on the given domain of $[0, 2\pi]$ to solve for x such that each equation is true.

- C. Solve the equation $2 \cos^2 x + \cos x = 1$ on the interval $[0, 2\pi]$: $\mathbf{x = \pi/3, x = 5\pi/3, x = \pi}$

$$2 \cos^2 x + \cos x = 1$$

$$2 \cos^2 x + \cos x - 1 = 0$$

$$(2 \cos x - 1)(\cos x + 1) = 0$$

$$2 \cos x - 1 = 0 \text{ OR } \cos x + 1 = 0$$

$$\cos x = \frac{1}{2} \text{ OR } \sin x = -1$$

Use your knowledge of Unit Circle on the given domain of $[0, 2\pi]$ to solve for x such that each equation is true.

- D. Solve the equation $\sin 2x = -1$ on the interval $[0, 3\pi]$: $\mathbf{x = 3\pi/4, x = 7\pi/4, x = 11\pi/4}$

In order for $\sin x$ to be -1, the input angle must be $\frac{3\pi}{2}$ or $\frac{7\pi}{2}$ or $\frac{11\pi}{2}$ therefore set up the equations $2x = \frac{3\pi}{2}$ or $2x = \frac{7\pi}{2}$ or $2x = \frac{11\pi}{2}$ to solve for x such that the equation is true on the given domain $[0, 3\pi]$. The function $\sin 2x$ has a period of π , so you will have a 3 complete sine curves on $[0, 3\pi]$.

E. Solve the equation $2\sin 2x = 1$ on the interval $[0, 2\pi]$: $x = \pi/12, x = 5\pi/12, x = 13\pi/12, x = 17\pi/12$

In order for $\sin x$ to be $1/2$, the input angle must be $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ therefore set up the equations $2x = \frac{\pi}{6}, 2x = \frac{5\pi}{6}, 2x = \frac{13\pi}{6}, 2x = \frac{17\pi}{6}$ to solve for x such that the equation is true on the given domain $[0, 2\pi]$. The function $\sin 2x$ has a period of π , so you will have a 2 complete sine curves on $[0, 2\pi]$.

F. Solve the equation $\sin^2 x - \sin x = 0$ on the interval $[0, 2\pi]$: $x = 0, x = \pi, x = 2\pi, x = \pi/2$

$$\sin^2 x - \sin x = 0$$

$$\sin x (\sin x - 1) = 0$$

$\sin x = 0$ OR $\sin x = 1$ use your knowledge of Unit Circle on the given domain of $[0, 2\pi]$ to solve for x such that each equation is true.

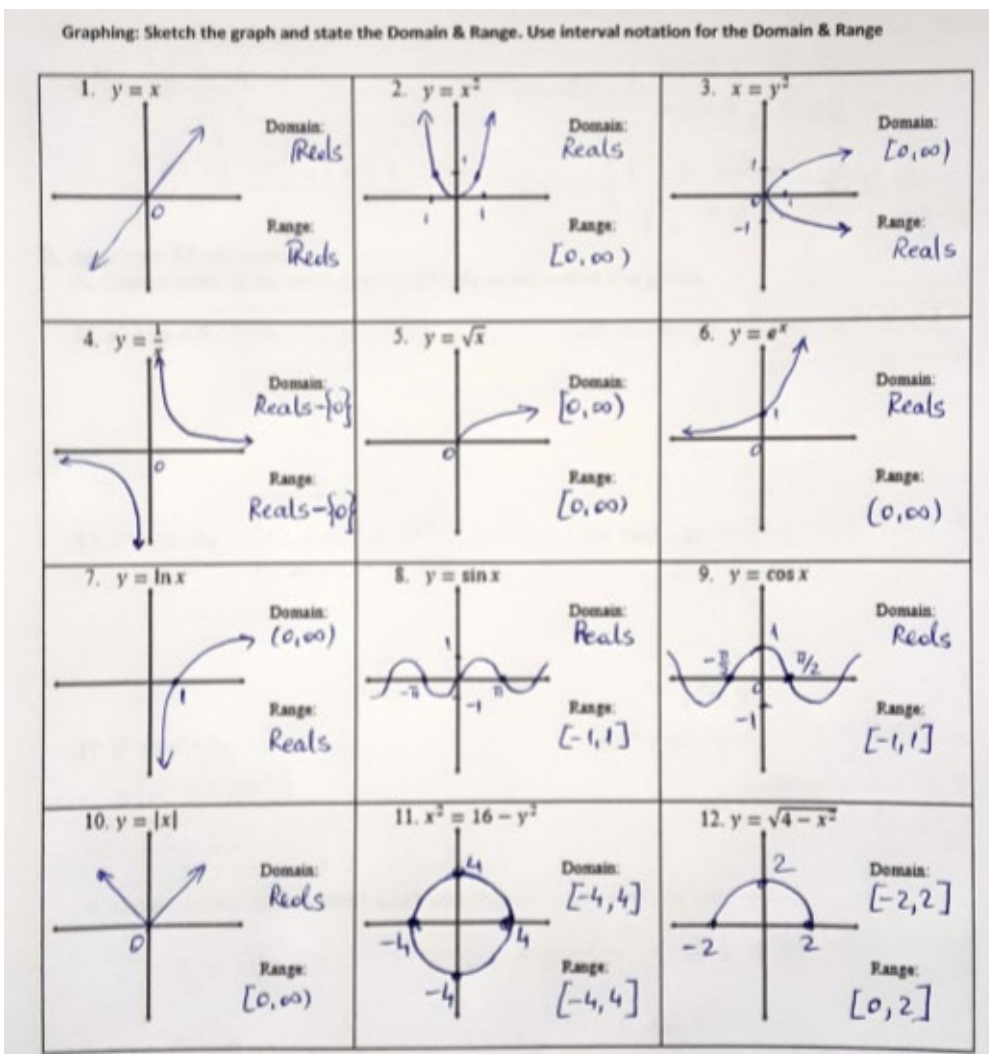
G. Solve the equation $\cos^2 x - \cos x = 0$ on the interval $[0, 2\pi]$: $x = 0, x = 2\pi, x = \pi/2, x = 3\pi/2$

$$\cos^2 x - \cos x = 0$$

$$\cos x (\cos x - 1) = 0$$

$\cos x = 0$ OR $\cos x = 1$ use your knowledge of Unit Circle on the given domain of $[0, 2\pi]$ to solve for x such that each equation is true.

Part #2: Additional Practice – KEY



1. Evaluating Functions:

A)

$$f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$$

(a) $f(-2)$

(b) $f(1)$

(c) $f(2)$

a) $f(-2) = (-2)^2 + 2 = \boxed{6}$

b) $f(1) = 1^2 + 2 = \boxed{3}$

c) $f(2) = 2 \cdot 2^2 + 2 = \boxed{10}$

B)

$$f(x) = \begin{cases} 3x - 1, & x < -1 \\ 4, & -1 \leq x \leq 1 \\ x^2, & x > 1 \end{cases}$$

(a) $f(-2)$

(b) $f(-\frac{1}{2})$

(c) $f(3)$

a) $f(-2) = 3(-2) - 1 = \boxed{-7}$

b) $f(-\frac{1}{2}) = \boxed{4}$

c) $f(3) = 3^2 = \boxed{9}$

C)

$$f(x) = \frac{|x-2|}{x-2}$$

x	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
f(x)	-1	-1	-1	1	1

$$|s-2| = \begin{cases} s-2, & s \geq 2 \\ -s+2, & s < 2 \end{cases}$$

$$f(s) = \begin{cases} 1, & s > 2 \\ -1, & s < 2 \end{cases} \quad f(2) \text{ does not exist (DNE)}$$

II. Algebraic Manipulation

A. Factor each of the following completely or state that it is prime.

13. $x^2 + 5x + 6 = \boxed{(x+2)(x+3)}$

14. $x^2 - 4x - 12 = \boxed{(x-6)(x+2)}$

15. $x^2 + 5x - 24 = \boxed{(x+8)(x-3)}$

16. $16x^2 - 81 = \boxed{(4x-9)(4x+9)}$

17. $x^3 + 4x^2 + 3x$

$= x(x^2 + 4x + 3)$

$= \boxed{x(x+3)(x+1)}$

$x^2 = u$

18. $x^4 + 6x^2 + 9 = u^2 + 6u + 9$

$= (u+3)^2 = \boxed{(x^2+3)^2}$

A'. Completing the square: Find h and k such that $x^2 + 12x + 4 = (x+h)^2 + k$

$h = \frac{12}{2} = 6$

$\rightarrow 6^2 = 36$

$x^2 + 12x + 4 = (x^2 + 12x + 36) + 4 - 36$

$= (x+6)^2 - 32$

$\boxed{h=6, k=-32}$

B. Simplify

$$19. \frac{(x+4)^2 - 16x}{x-4} = \frac{x^2 + 8x + 16 - 16x}{x-4} = \frac{x^2 - 8x + 16}{x-4} = \frac{(x-4)^2}{x-4} = \boxed{x-4}$$

$$20. \frac{y+3}{(y+4)^2 - (8y+25)} = \frac{y+3}{y^2 + 8y + 16 - 8y - 25} = \frac{y+3}{y^2 - 9} = \frac{y+3}{(y+3)(y-3)} = \boxed{\frac{1}{y-3}}$$

$$21. \frac{\frac{25}{3} - a}{3+a} = \frac{25 - a^2}{a(5+a)} = \frac{(5+a)(5-a)}{a(5+a)} = \boxed{\frac{5-a}{a}}$$

$$22. \frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}} = \frac{\frac{x^2 - x - 1}{x(x+1)}}{\frac{x^2 + x + 1}{x(x+1)}} = \boxed{\frac{x^2 - x - 1}{x^2 + x + 1}}$$

C. Solve for x without a calculator or grapher:

$$23. 1 + \frac{x}{3} = \frac{4}{5}$$

$$15 + 5x = 12$$

$$5x = -3$$

$$\boxed{x = -\frac{3}{5}}$$

$$24. \frac{2}{3} - \frac{5}{7} = x$$

$$14 - 15 = 21x$$

$$\boxed{-\frac{1}{21} = x}$$

$$25. \frac{3}{8} = \frac{4}{1-x}$$

$$3 - 3x = 32$$

$$-3x = 29$$

$$\boxed{x = -\frac{29}{3}}$$

$$26. \frac{6}{x} + \frac{x}{2} = 4 \quad x \neq 0$$

$$12 + x^2 = 8x$$

$$x^2 - 8x + 12 = 0$$

$$(x-2)(x-6) = 0$$

$$\boxed{x=2, x=6}$$

$$27. \ln(x-2) = 4$$

$$x-2 = e^4$$

$$\boxed{x = 2 + e^4}$$

$$28. -3\ln(x+1) = 2$$

$$\ln(x+1) = -\frac{2}{3}$$

$$x+1 = e^{-2/3}$$

$$\boxed{x = -1 + e^{-2/3}}$$

$$29. \ln(x-2) + \ln(3) = 5$$

$$\ln(3x-6) = 5$$

$$3x-6 = e^5$$

$$3x = 6 + e^5$$

$$\boxed{x = (6 + e^5)/3}$$

$$30. 4e^{3x-1} = 8$$

$$e^{3x-1} = 2$$

$$3x-1 = \ln 2$$

$$3x = 1 + \ln 2$$

$$\boxed{x = (1 + \ln 2)/3}$$

$$31. 100 = 200e^{-0.06x}$$

$$\frac{1}{2} = e^{-0.06x}$$

$$\ln\left(\frac{1}{2}\right) = -0.06x$$

$$-\ln 2 = -0.06x$$

$$\boxed{x = \ln 2 / 0.06}$$

$$32. \sin x = -\frac{\sqrt{3}}{2}, \text{ for } 0 \leq x \leq 2\pi$$

$\sin x < 0$ in 3rd & 4th quadrants
reference angle: $\frac{\pi}{4}$

$$x = \pi + \frac{\pi}{4} = \boxed{\frac{5\pi}{4}}; x = 2\pi - \frac{\pi}{4} = \boxed{\frac{7\pi}{4}}$$

$$33. 3x^2 + 7x + 3 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 4 \cdot 3 \cdot 3}}{2 \cdot 3}$$

$$x = \boxed{\frac{-7 \pm \sqrt{13}}{6}}$$

D. Find the asymptotes (horizontal, vertical). Show work.

$$34. y = \frac{1}{x-1}$$

$x=1$ vertical asymptote

$y=0$ horizontal asymptote

$$35. y = \frac{1}{(x-1)^2}$$

$x=1$ V.A.

$y=0$ H.A.

D'. Find the holes (if any). Show work.

$$A) y = \frac{x-1}{x^3-1}$$

$$f(x) = y = \frac{x-1}{(x-1)(x^2+x+1)} \rightarrow \frac{1}{x^2+x+1} = g(x)$$

$$g(1) = \frac{1}{3}; f(x) = g(x) \text{ except for } x=1$$

$$f(1) = \text{DNE}$$

f has a hole @ $(1, \frac{1}{3})$

$$B) y = \frac{x^2-3x+2}{x^2+2x-3}$$

$$f(x) = y = \frac{(x-1)(x-2)}{(x-1)(x+3)} \rightarrow$$

$$g(x) = \frac{x-2}{x+3} \quad g(1) = -\frac{1}{4}$$

$$f(1) = \text{DNE}, f(x) = g(x) \text{ except for } x=1$$

f has a hole @ $(1, -\frac{1}{4})$

E. Equations of Lines

36. Use point-slope form to find the equation of the line passing through the point $(0, 5)$ with a slope of $2/3$.

$$\boxed{y - 5 = \frac{2}{3}(x - 0)} \quad \text{or} \quad y = \frac{2}{3}x + 5$$

37. Using point slope form, find the equation of a line passing through the point $(2, 8)$ and parallel to the line $y = \frac{5}{6}x - 1$.

$$\boxed{y - 8 = \frac{5}{6}(x - 2)} \quad \text{or} \quad y = 8 + \frac{5}{6}(x - 2)$$

38. Using point slope form, find the equation of a line perpendicular to the y-axis passing through the point $(4, 7)$.

$$\boxed{y = 7} \quad \rightarrow \text{horizontal line}$$

F. Function Notation

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each.

$$39. h[f(-2)] = \underline{h(4)} = 16 - 1 = \boxed{15}$$

$$40. f[g(x-1)] = \underline{f(2(x-1)+5)} = f(2x+3) = \boxed{(2x+3)^2}$$

$$41. g[h(x^3)] = \underline{g((x^3)^2 - 1)} = g(x^6 - 1) = 2(x^6 - 1) + 5 = \boxed{2x^6 + 3}$$

42. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

$$\begin{cases} x \cdot y = 192 \\ x + 3y = \min. \end{cases}$$

• 192 $\times + 34 \text{ min}$

2. 86	→	104
3. 64	→	73
4. 48	→	50
5. 32	→	50
6. 24	→	48
12. 16	→	52

$$x+3y \text{ min. when } y=8, x=24$$

- x   5

$$4x + 35 = 10 \rightarrow x = (10 - 35) / 4$$

$$\text{Area (square)} = x^2 = \left(\frac{10-3d}{4}\right)^2$$

- $$V_{\text{cyl.}} = \pi r^2 h = 1000 \text{ cm}^3 \rightarrow h = \frac{1000}{\pi r^2}$$

$$\text{Surface Area}_{\text{cyl.}} = 2\pi rh = 2\pi r \cdot \frac{1000}{\pi r^2} = \boxed{\frac{2000}{r} \text{ cm}^2}$$

a) $f(x) = x^2 - x + 1, \quad \frac{f(2+h) - f(2)}{h}, h \neq 0$

b) $f(x) = 5x - x^2$, $\frac{f(5+h) - f(5)}{h}, h \neq 0$

$$a) \frac{(2+h)^2 - (2+h) + 1 - (2^2 - 2 + 1)}{h} = \frac{4 + 4h + h^2 - 2 - h + 1 - 4 + 2 + 1}{h} = \frac{h^2 + 3h}{h} = \frac{h(h+3)}{h} = \boxed{h+3}$$

$$b) \frac{5(5+h) - (5+h)^2 - (5 \cdot 5 - 5^2)}{h} = \frac{5 \cdot 5 + 5h - 25 - 10h - h^2 - 25 + 5^2}{h} = \frac{-h^2 - 5h}{h} = \boxed{-h-5}$$

A) $(x^3 - 2x^2 - 5x + 6) \div (x - 3)$

$$\begin{array}{r} 3 \mid 1 \quad -2 \quad -5 \quad 6 \\ \quad \quad 3 \quad 3 \quad -6 \\ \hline 1 \quad 1 \quad -2 \quad 10 \end{array}$$

$$\rightarrow x^2 + x - 2$$

8) $(2x^4 + 3x^3 + 5x - 1) \div (x^2 + 3x + 2)$

$$x^2 + 3x + 2 \overline{) 2x^2 - 3x + 5} = \text{Quotient}$$

$$x^2 + 3x + 2 \overline{) 2x^4 + 3x^3 + 5x - 1}$$

$$\underline{2x^4 + 6x^3 + 4x^2}$$

$$-3x^3 - 4x^2 + 5x - 1$$

$$\underline{-3x^3 - 9x^2 - 6x}$$

$$\begin{array}{r} 5x^2 + 11x - 6 \\ 5x^2 + 15x + 9 \\ \hline \end{array}$$

$$= \frac{5x^2 + 15x + 10}{3x^2}$$

b) Expand: $\ln\left(\frac{3x^2}{2y^3}\right)$

$$= \ln 3 + 2 \ln x - \ln 2 - 3 \ln y$$

a) Simplify: $\left(\frac{2x^4 3y^6}{4xy^4}\right)^{-2} = \frac{4}{9x^6 y^4}$

b) Expand: $\ln\left(\frac{3x^2}{2y^3}\right)$

c) rewrite in exponential form: $\sqrt[3]{2x^5}$

$$= \left[2^{1/3} \cdot x^{5/3} \right]$$

d) Evaluate: $e^{3 \ln e^2} = e^{\ln(e^2)^3} = e^{\ln e^6} = \boxed{e^6}$

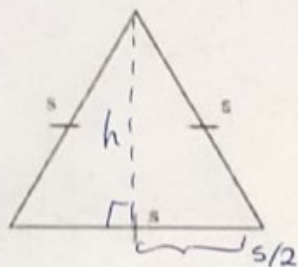
OR

$$e^{3 \cdot 2 \ln e} = e^{6 \ln e} = e^6$$

($\ln e = 1$)

III. Area/Geometry Questions

45. What is the area of an equilateral triangle with side length s ? Hint: use a 30-60-90 triangle!



$$\text{Area} = \frac{1}{2} \cdot s \cdot h$$

$$h^2 + \left(\frac{s}{2}\right)^2 = s^2 \rightarrow h^2 = s^2 - \frac{s^2}{4} = \frac{3s^2}{4} \rightarrow h = \frac{s\sqrt{3}}{2}$$

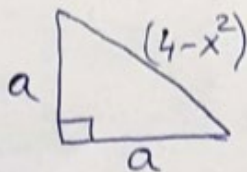
$$\text{Area} = \frac{1}{2} \cdot s \cdot \frac{s\sqrt{3}}{2} = \frac{s^2\sqrt{3}}{4}$$

46. What is the area of a semicircle with respect to diameter d ?

$$\text{Area (semicircle)} = \frac{\pi r^2}{2} = \frac{\pi}{2} \left(\frac{d}{2}\right)^2 = \frac{\pi}{2} \cdot \frac{d^2}{4} = \frac{\pi d^2}{8}$$

$$r = \frac{d}{2}$$

47. What's the area of an isosceles right triangle with base $(4-x^2)$?



$$2a^2 = (4-x^2)^2 \rightarrow a^2 = \frac{(4-x^2)^2}{2}$$

$$\text{Area}_{\Delta} = \frac{1}{2} \cdot a^2 = \frac{1}{2} \cdot \frac{(4-x^2)^2}{2}$$

$$\boxed{\text{Area} = \frac{(4-x^2)^2}{4}}$$

Part #4: UW Placement test for Calculus – Non-graphing calculators are allowed!

1. A person is walking directly away from a light on an 18-foot tall pole. At this instant, the person casts a shadow 14 feet long. If they walk 10 feet farther from the pole, they will cast a shadow 20 feet long. How tall is the person?
2. Line A passes through the points (1, 5) and (6, 3).
Line B is perpendicular to line A and passes through the point (4, 4).
At what point do Line A and Line B intersect?
3. The graph of a quadratic function f passes through the points $(-2, 4)$, $(1, 1)$ and $(5, 0)$.
What is the y -intercept of the graph of f ?
4. Let $f(x) = 2x^2 - 3x + 5$. Let $g(x) = f(6 - x)$. What is the lowest point on the graph of g ?
5. You are walking along a flat, horizontal road toward a mountain in the distance. You measure the mountain's angle of elevation and find it to be 4° . You then walk 2 kilometers closer to the mountain and measure the angle again: this time the angle is 4.5° .
How tall is the mountain?
6. The population of town A grows exponentially and doubles every 7 years. Currently, town A has a population of 4000.
The population of town B increases at the constant rate of 12% per year. Five years from now, town B will have a population that is half that of town A.
When, in years from now, will the populations of the two towns be equal?
7. Find all solutions to the equation $\sin x = -0.7$ with $-1 \leq x \leq 10$.
8. Find the equation of one of the tangent lines to the circle $x^2 + y^2 = 5^2$ that pass through the point $\left(\frac{25}{3}, 0\right)$.
9. Let $f(x) = \frac{2x + 4}{5x + 2}$. Find the equations of the horizontal and vertical asymptotes of
 $y = f(1 - 3x)$.
10. Two numbers sum to 100. How small can the sum of one number and the square of the other number be?

Answers

1. 6.75 feet tall
2. $\left(\frac{114}{29}, \frac{111}{29}\right)$
3. $y = \frac{25}{14}$
4. $\left(\frac{21}{4}, \frac{31}{8}\right)$
5. The mountain is 1254.34 meters tall.
6. 53.4458 years from now
7. There are three solutions: $x \approx -0.77539$, 3.91699 , and 5.50778 .
8. The two tangent lines are $y = -\frac{3}{4}x + \frac{25}{4}$ and $y = \frac{3}{4}x - \frac{25}{4}$.
9. The horizontal asymptote is $y = \frac{2}{5}$ and the vertical asymptote is $x = \frac{7}{15}$.
10. The sum is ≥ 99.75 with equality when one number is $\frac{1}{2}$ and the other is 99.5.