



Pre-AP Geometry with Statistics Summer Math Packet

Instructions

The following review has several topics of basic geometry required for your next level mathematics course, **PreAP Geometry with Statistics**.

Every topic has introductory notes followed by examples. Then, there are several exercises you must complete in the space provided. If you need extra space, you can use scratch paper.

You are required to show work whenever necessary and you must set up equations that solve for variables.

Answers with no work will receive no credit.

The packet will be graded by percentage of completion. Try your best to answer all questions, even if you are not sure of your answer.

This document is due **THE FIRST DAY OF SCHOOL**.

NO LATE SUBMISSIONS WILL BE ACCEPTED.

You must print and turn in only the “practice” pages of this document with your answers.

You do not need to print the “notes” pages.

After reviewing these topics in class, you will have an assessment on all the material included within this summer math packet.

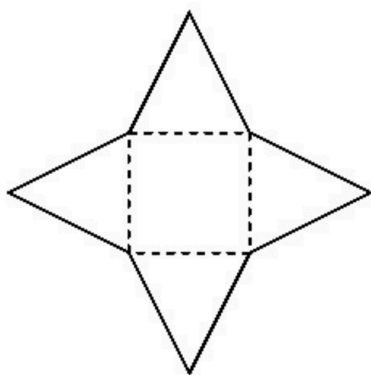
Lesson 1.1 - Nets and Drawings for Visualizing Geometry

A **net** is a two-dimensional flat diagram that represents a three-dimensional figure. It shows all of the shapes that make up the faces of a solid.

Stepping through the process of building a three-dimensional figure from a net will help you improve your ability to visualize the process. Here are the steps you would take to build a square pyramid.

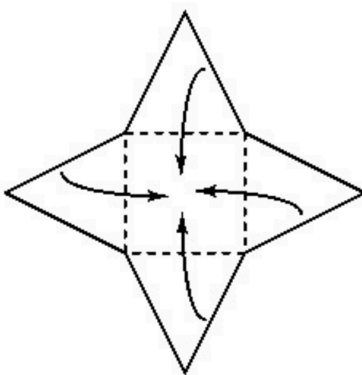
Step 1

Start with the net.



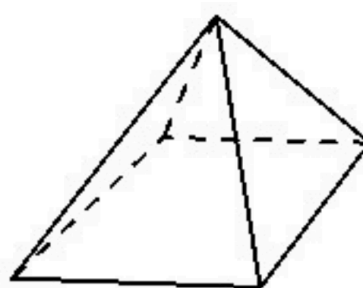
Step 2

Fold up on the dotted lines.

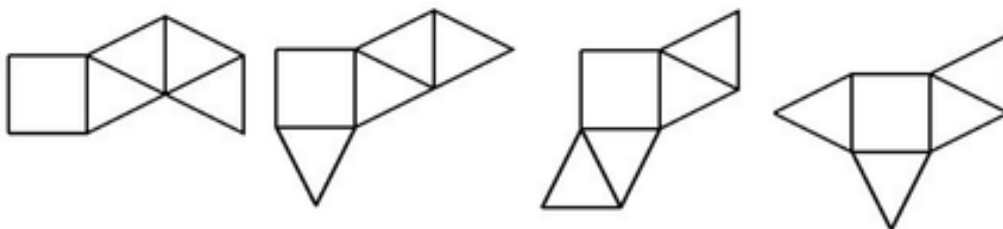


Step 3

Tape the adjacent triangle sides together.



Other examples of nets that also fold up into a square pyramid.



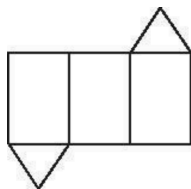
Example: How can you be sure that none of the nets shown above are the same?

Make sure you cannot rotate or flip any net and place it on top of any other net.

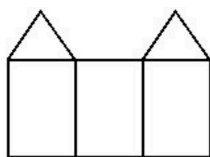
Lesson 1.1 - Nets and Drawings for Visualizing Geometry

1. What is a possible net for the figure shown to the right?

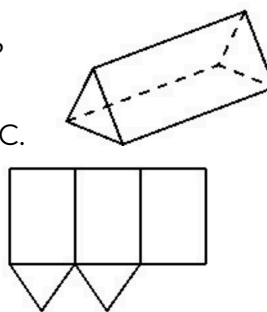
A.



B.

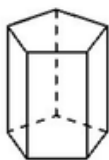


C.



2. Match each three-dimensional figure with its net.

a.



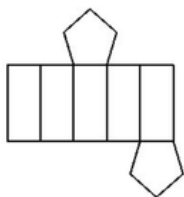
b.



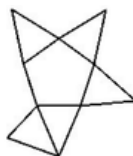
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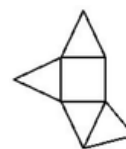
A.



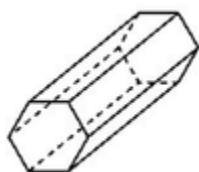
B.



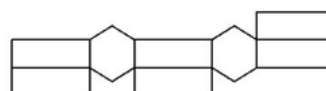
C.



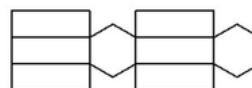
3. **Error Analysis** | Two students draw nets for the solid shown below. Who is correct, Student A or Student B? Explain.



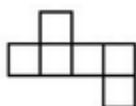
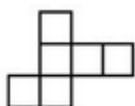
Student A:



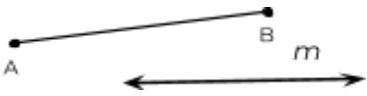
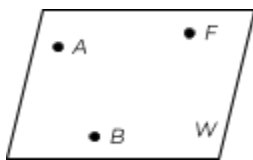
Student B



4. Choose the nets that will fold to make a cube.



Lesson 1.2 - Points, Lines and Planes

Term	Examples of Labels	Diagram
Point	Italicized capital letter: D	
Line	Two capital letters with a line drawn over them: $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ One italicized lowercase letter: m	
Line Segment	Two capital letters (called endpoints) with a segment drawn over them: $\overline{AB}$ or $\overline{BA}$	
Ray	Two capital letters with a ray symbol drawn over them: $\overrightarrow{AB}$	
Plane	Three capital letters: ABF , AFB , BAF , BFA , FAB , or FBA One italicized capital letter: W	

- * A ray starts at an endpoint and continues forever through another point.
- * To name a plane, use three points that are not all collinear.
- * An arrow means the path continues forever in one direction.
- * A plane is drawn as a parallelogram, but it is flat, has no edges, and extends forever.

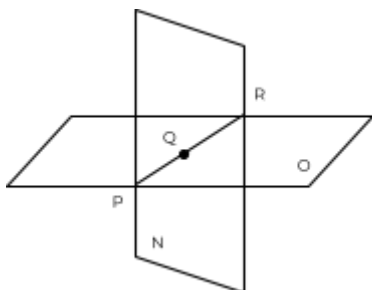
A **postulate** is a statement that is accepted as true.

Postulate 1-4: Through any three noncollinear points, there is only one plane.

Noncollinear points are points that do not all lie on the same line. In the figure to the right, points D , E , and F are noncollinear. These points all lie in one plane. Three noncollinear points lie in only one plane.

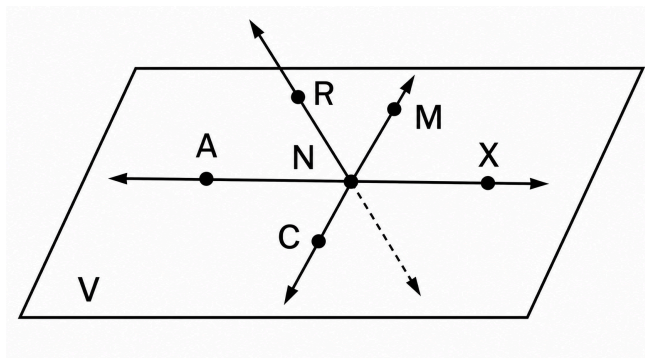


Collinear points are points that all lie on the same line. Three points that are collinear can be contained by more than one plane. In the figure to the left, points P , Q , and R are collinear, and lie in both plane O and plane N .



Lesson 1.2 - Points, Lines and Planes

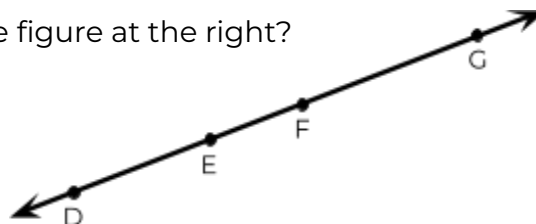
Use the figure below for exercises 1–8. Note that $\overleftrightarrow{RN}$ pierces the plane at N . It is not coplanar with V .



1. Name two segments shown in the figure.
2. What is the intersection of $\overleftrightarrow{RN}$ and $\overleftrightarrow{RN}$?
3. Name three collinear points.
4. What are two other ways to name plane V ?
5. Are points R , N , M , and X coplanar?
6. Name two rays shown in the figure.
7. Name the pair of opposite rays with endpoint N .
8. How many lines are shown in the drawing?

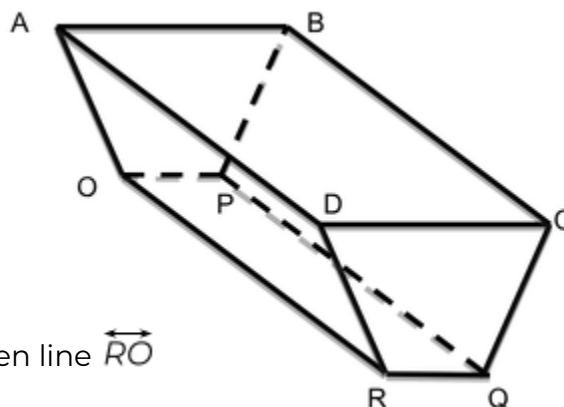
For exercises 9–14, determine whether each statement is *always*, *sometimes*, or *never true*.

9. $\overrightarrow{GH}$ and $\overrightarrow{HG}$ are the same ray.
10. $\overrightarrow{JI}$ and $\overrightarrow{JL}$ are opposite rays.
11. A plane contains only three points.
12. Three noncollinear points are contained in only one plane.
13. If $\overleftrightarrow{EG}$ lies in plane X , point G lies in plane X .
14. If three points are coplanar, they are collinear.
15. How many segments can be named from the figure at the right?



Lesson 1.2 - Points, Lines and Planes (Cont.)

Use the figure to the right for exercises 16–21. Name the intersection of each pair of planes or lines.



16. planes ABP and BCD

17. $\overleftrightarrow{RQ}$ and $\overleftrightarrow{RO}$

18. planes ADR and DCQ

19. planes BCD and BCQ

20. $\overleftrightarrow{OP}$ and $\overleftrightarrow{QP}$

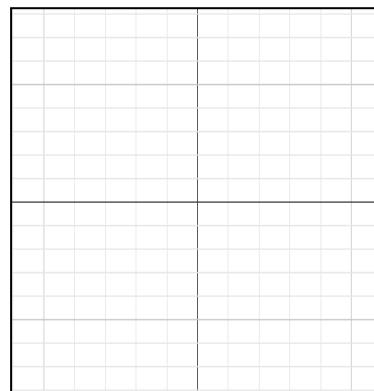
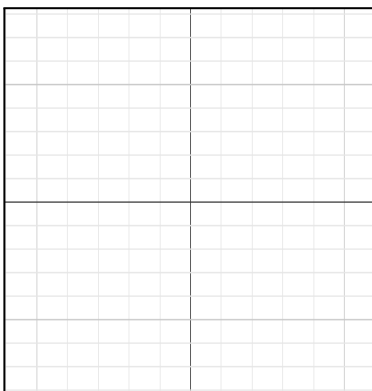
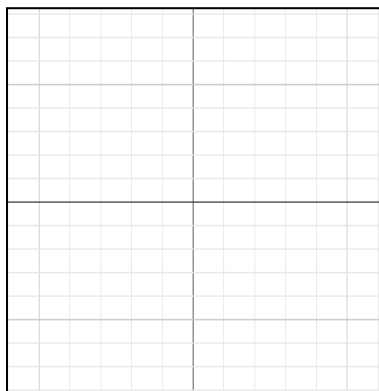
21. Name two planes that intersect in the given line $\overleftrightarrow{RO}$

Coordinate Geometry | Graph the points and state whether they are collinear.

22. $(0, 0)$, $(4, 2)$, $(6, 3)$

23. $(-2, 0)$, $(0, 4)$, $(2, 0)$

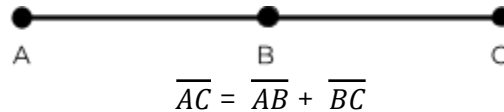
24. $(-4, -1)$, $(-1, -2)$, $(2, -3)$



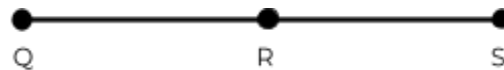
Lesson 1.3 - Measuring Segments

Segment Addition Postulate: a point lies on a line segment if and only if the sum of the distances from the point to each end point of the line segment is equal to the length of the line segment itself.

If three points, A , B , and C , are on the same line, and point B is between points A and C , then the distance $\overline{AC}$ is the sum of the distances $\overline{AB}$ and $\overline{BC}$.



Example: If $\overline{QS} = 7$ and $\overline{QR} = 3$, what is $\overline{RS}$?



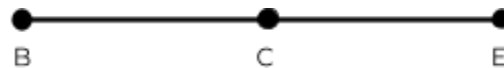
$$\begin{aligned}\overline{QS} &= \overline{QR} + \overline{RS} && \text{Segment Addition Postulate} \\ \overline{QS} - \overline{QR} &= \overline{RS} && \text{Subtract } \overline{QR} \text{ from each side} \\ 7 - 3 &= \overline{RS} && \text{Substitute} \\ 4 &= \overline{RS} && \text{Simplify}\end{aligned}$$

Midpoint: divides a line segment into two equal parts in length.



X is the midpoint of $\overline{WY}$.
 $\overline{XW} = \overline{XY}$, so $\overline{XW}$ and $\overline{XY}$ are congruent.

Example: • C is the midpoint of $\overline{BE}$. If $\overline{BC} = t + 1$, and $\overline{CE} = 15t$, what is $\overline{BE}$?



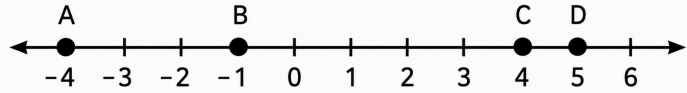
$$\begin{aligned}BC &= CE && \text{Definition of midpoint} \\ t + 1 &= 15 - t && \text{Subtract } QR \text{ from each side} \\ t + t + 1 &= 15 - t + t && \text{Substitute} \\ 2t + 1 &= 15 && \text{Simplify} \\ 2t + 1 - 1 &= 15 - 1 && \text{Subtract 1 from each side} \\ 2t &= 14 && \text{Simplify} \\ t &= 7 && \text{Divide each side by 2}\end{aligned}$$

$$\begin{aligned}BC &= t + 1 && \text{Given} \\ BC &= 7 + 1 && \text{Substitute} \\ BC &= 8 && \text{Simplify} \\ BE &= 2(BC) && \text{Definition of midpoint} \\ BE &= 2(8) && \text{Substitute} \\ BE &= 16 && \text{Simplify}\end{aligned}$$

Lesson 1.3 - Measuring Segments

Use the figure below for exercises 1-3. Find the length of each segment.

1. $\overline{AB} =$ _____
2. $\overline{BC} =$ _____
3. $\overline{AC} =$ _____

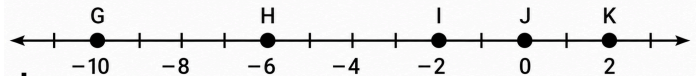


For exercises 4-6, use the figure to the right.

4. If $\overline{PQ} = 7$ and $\overline{QR} = 10$, then $\overline{PR} =$ _____
5. If $\overline{PR} = 25$ and $\overline{PQ} = 12$, then $\overline{QR} =$ _____
6. If $\overline{PR} = 10$ and $\overline{PQ} = 4$, then $\overline{QR} =$ _____



Use the number line to the right for exercises 7-9. State whether the segments are congruent.



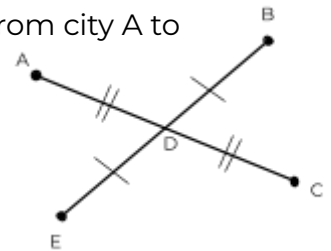
7. $\overline{GH}$ and $\overline{HI}$
8. $\overline{GH}$ and $\overline{IK}$
9. $\overline{HJ}$ and $\overline{IK}$

Algebra | Use the figure to the right for exercise 10.

10. Given: $\overline{ST} = 3x + 3$ and $\overline{TU} = 2x + 9$.
 - a. What is the value of $\overline{ST}$?
 - b. What is the value of $\overline{TU}$?



11. You plan to drive north from city A to town B and then continue north to city C. The distance between city A and town B is 39 mi, and the distance between town B and city C is 99 mi.
 - a. Assuming you follow a straight driving path, after how many miles of driving will you reach the midpoint between city A and city C?
 - b. If you drive an average of 46 mi/h, how long will it take you to drive from city A to city C?



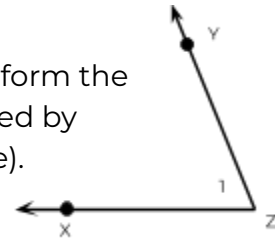
Algebra | Use the diagram to the right for exercises 12-14.

12. If $\overline{AD} = 20$ and $\overline{AC} = 3x + 4$, find the value of x . Then find $\overline{AC}$ and $\overline{DC}$.
13. If $\overline{ED} = 5y + 6$ and $\overline{DB} = y + 30$, find the value of y . Then find $\overline{ED}$, $\overline{DB}$, and $\overline{EB}$.
14. If $\overline{DC} = 6x$ and $\overline{DA} = 4x + 18$, find the value of x . Then find $\overline{AD}$, $\overline{DC}$, and $\overline{AC}$.

Lesson 1.4 - Measuring Angles

The **vertex** of an angle is the common endpoint of the rays that form the angle. An angle may be named by its vertex. It may also be named by a number or by a point on each ray and the vertex (in the middle).

This is $\angle Z$, $\angle XZY$, $\angle YZX$, or $\angle 1$.
It is **not** $\angle ZYX$, $\angle XYZ$, $\angle YXZ$, or $\angle ZXY$.

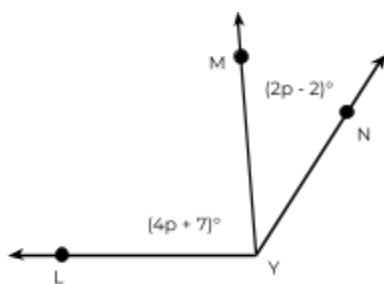
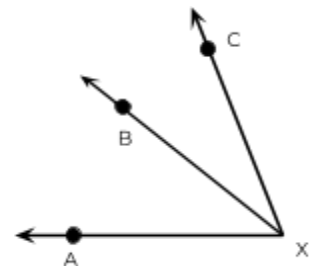


Acute	Right	Obtuse	Straight
Angle measure: Between 0° and 90°	Angle measure = 90°	Angle measure: Between 90° and 180°	Angle measure = 180°

Angle Addition Postulate: if a point or ray is inside an angle, the two smaller angles add up to the larger angle.

If $\bullet B$ is inside $\angle AXC$, then: $m\angle AXB + m\angle BXC = m\angle AXC$

Example: If $m\angle LYN = 125^\circ$, what are $m\angle LYM$ and $m\angle MYN$?



Step 1 - Solve for p

$$m\angle LYN = m\angle LYM + m\angle MYN \text{ Angle Addition Postulate}$$

$$125 = (4p + 7) + (2p - 2) \text{ Substitute}$$

$$125 = 6p + 5 \text{ Simplify}$$

$$120 = 6p \text{ Subtract 5 from each side}$$

$$20 = p \text{ Divide each side by 6}$$

Step 2 - Use the value of p to find the measures of the angles

$$m\angle LYN = 4p + 7 \text{ Given}$$

$$m\angle LYN = 4(20) + 7 \text{ Substitute}$$

$$m\angle LYN = 87^\circ \text{ Simplify}$$

$$m\angle MYN = 2p - 2 \text{ Given}$$

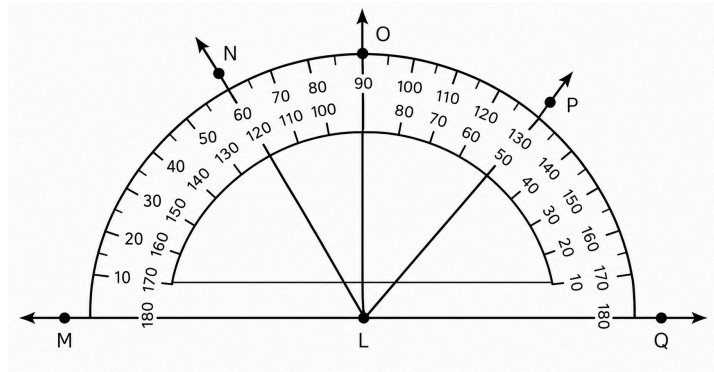
$$m\angle MYN = 2(20) - 2 \text{ Substitute}$$

$$m\angle MYN = 38^\circ \text{ Simplify}$$

Lesson 1.4 - Measuring Angles

Use the diagram below for exercises 1–11. Find the measure of each angle.

1. $\angle MLN$
2. $\angle NLP$
3. $\angle NLQ$
4. $\angle OLP$
5. $\angle MLQ$

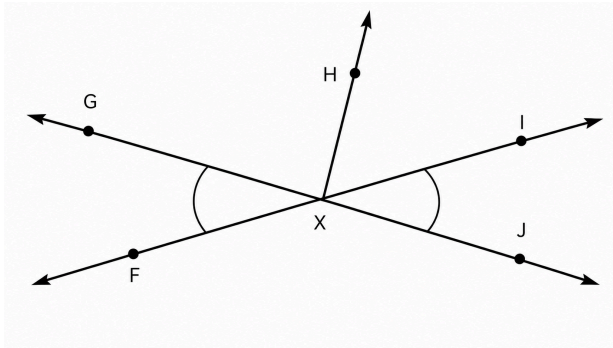


Classify each angle as *acute*, *right*, *obtuse*, or *straight*.

6. $\angle MLN$
7. $\angle NLO$
8. $\angle MLP$

Use the figure to the right for exercises 9–12. $m\angle FXH = 130^\circ$ and $m\angle FXG = 49^\circ$.

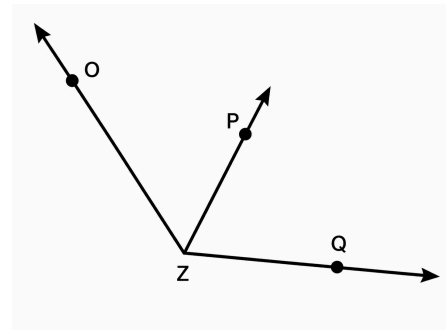
9. $\angle FXG \cong$
10. $m\angle GXH =$
11. Name a straight angle in the figure
12. $\angle IXJ =$



Algebra | Use the figure to the right for exercise 13.

13. $m\angle OZP = 4r + 2$, $m\angle PZQ = 5r - 12$, and $m\angle OZQ = 125^\circ$.

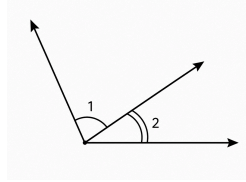
What are $m\angle OZP$ and $m\angle PZQ$?



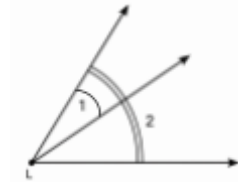
Lesson 1.5 - Exploring Angle Pairs

Adjacent Angles and Vertical Angles

Adjacent means “next to.” Angles are adjacent if they lie next to each other. In other words, the angles have the same vertex and they share a side without overlapping.

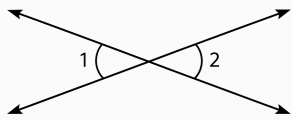


Adjacent Angles

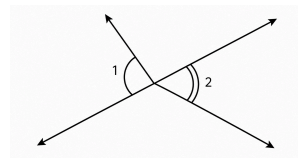


Overlapping Angles

Vertical means “related to the vertex.” Angles are vertical if they share a vertex, but not just any vertex. They share a vertex formed by the intersection of two straight lines. Vertical angles are always congruent.



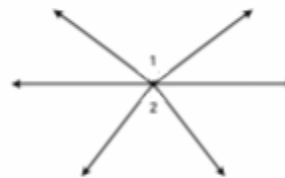
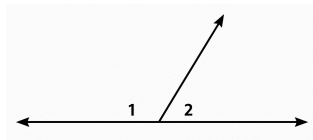
Vertical Angles



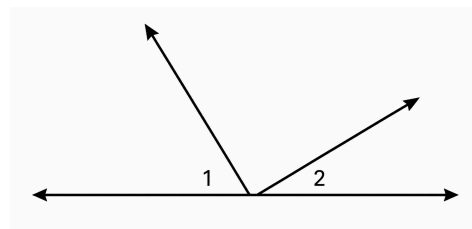
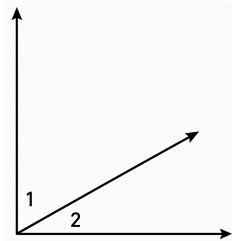
Non-Vertical Angles

Supplementary Angles and Complementary Angles

Two angles that form a line are **supplementary angles**. Another term for these angles is a **linear pair**. Any two angles with measures that sum to 180° are also considered supplementary angles. In both figures below, $m\angle 1 = 120^\circ$ and $m\angle 2 = 60^\circ$, so $\angle 1$ and $\angle 2$ are supplementary.



Two angles that form a right angle are **complementary angles**. Any two angles with measures that sum to 90° are also considered complementary angles. In both figures below, $m\angle 1 = 120^\circ$ and $m\angle 2 = 60^\circ$, so $\angle 1$ and $\angle 2$ are complementary.



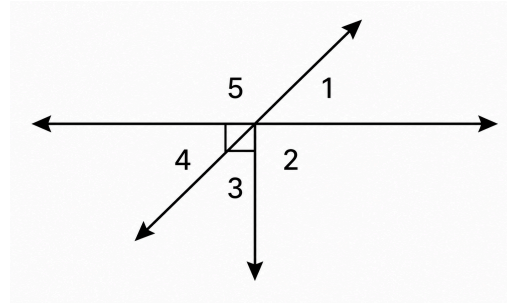
Lesson 1.5 - Exploring Angle Pairs

Use the diagram to the right for exercises 1-3. Is each statement true?

1. $\angle 2$ and $\angle 5$ are adjacent angles

2. $\angle 1$ and $\angle 4$ are vertical angles

3. $\angle 4$ and $\angle 5$ are complementary

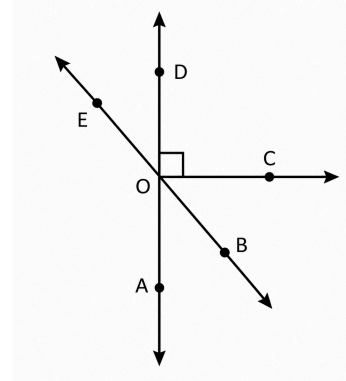


Name an angle or angles in the diagram to the right described in exercises 4-6.

4. complementary to $\angle BOC$

5. supplementary to $\angle DOB$

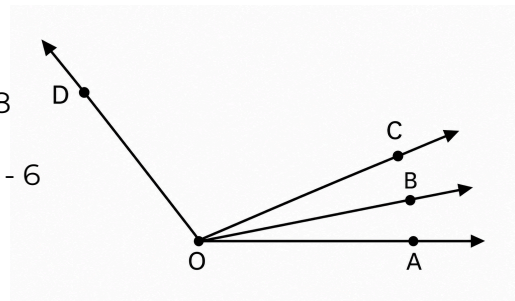
6. adjacent and supplementary to $\angle AOC$



Use the diagram to the right for exercises 7-8 to solve for x and find the angle measures.

7. $m\angle AOB = 4x - 1$; $m\angle BOC = 2x + 15$; $m\angle AOC = 8x + 8$

8. $m\angle COD = 8x + 13$; $m\angle BOC = 3x - 10$; $m\angle BOD = 12x - 6$



9. $\angle ABC$ and $\angle EBF$ are a pair of vertical angles; $m\angle ABC = 3x + 8$ and $m\angle EBF = 2x + 48$.

What are $m\angle ABC$ and $m\angle EBF$?

$\overrightarrow{GH}$ bisects $\angle PQR$. Solve for x and find $m\angle PQR$.

10. $m\angle PQS = 3x$; $m\angle SQR = 5x - 20$

11. $m\angle PQS = 2x + 1$; $m\angle RQS = 4x - 15$

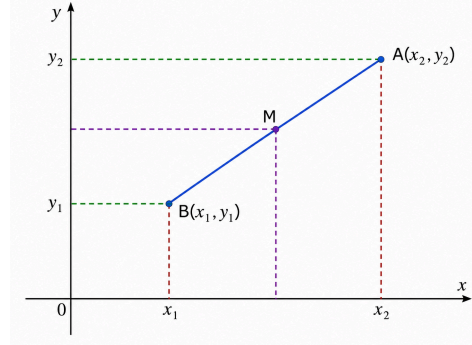
12. $m\angle PQR = 3x - 12$; $m\angle PQS = 30$

1.7 - Midpoint and Distance in the Coordinate Plane

When working with two points on a coordinate plane, we can find:

Midpoint: the point exactly halfway between two endpoints

Distance: the length of the segment between two points



Midpoint Formula: results in a point whose coordinates are the average of the x-coordinates and the y-coordinates.

If the endpoints are $A(x_1, y_1)$ and $B(x_2, y_2)$ then,

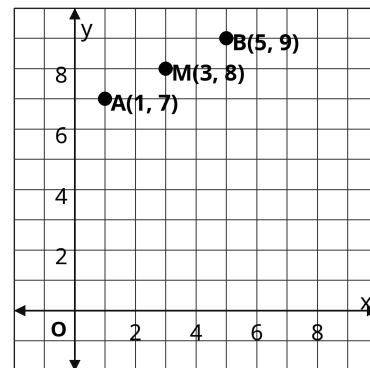
$$\text{Midpoint of } \overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example: What is the midpoint of $\overline{AB}$ if the endpoints are $A(1, 7)$ and $B(5, 9)$?

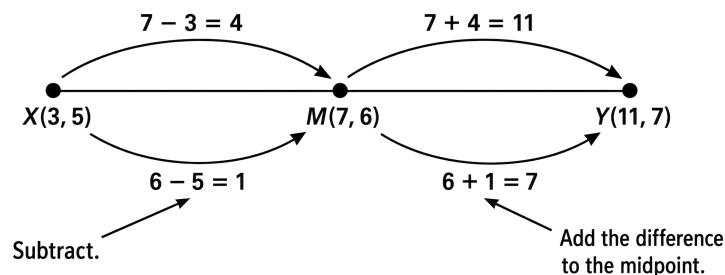
$$\text{Average of x-coordinates} \rightarrow \frac{1 + 5}{2} = 3$$

$$\text{Average of y-coordinates} \rightarrow \frac{7 + 9}{2} = 8$$

Midpoint $\overline{AB}$: $(3, 8)$



Example: The midpoint of $\overline{XY}$ is $M(7, 6)$. One endpoint is $X(3, 5)$. What are the coordinates of the other endpoint Y ?



1.7 - Midpoint and Distance in the Coordinate Plane (Cont.)

Distance Formula: distance between two points in a coordinate plane

If the endpoints are $A(x_1, y_1)$ and $B(x_2, y_2)$ then,

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: What is the distance between points A and B ?

$$\text{Distance between } A - B = D_{AB} \rightarrow \sqrt{(5 - 1)^2 + (9 - 7)^2}$$

$$\text{Distance between } A - B = D_{AB} \rightarrow \sqrt{(4)^2 + (2)^2}$$

$$\text{Distance between } A - B = D_{AB} \rightarrow \sqrt{16 + 4}$$

$$\text{Distance between } A - B = D_{AB} \rightarrow \sqrt{20} = 2\sqrt{5}$$

1.7 - Midpoint and Distance in the Coordinate Plane

Find the coordinate of the midpoint of the segment with the given endpoints.

1. 3 and 5

2. 7 and 4

Find the coordinates of the midpoint of $\overline{AB}$.

3. $A(6, 7)$, $B(4, 3)$

4. $A(-1, 5)$, $B(2, -3)$

5. $A(2.8, 1.1)$, $B(-3.4, 5.7)$

The coordinates of point Y are given. The midpoint of XY is (3, 5). Find the coordinates of point X.

6. $Y(0, 2)$

7. $Y(-10, 5)$

8. $Y(4, -8)$

9. $Y(2.5, -6.5)$

Find the distance between each pair of points. If necessary, round to the nearest tenth.

10. $A(6, 7)$, $B(-1, 7)$

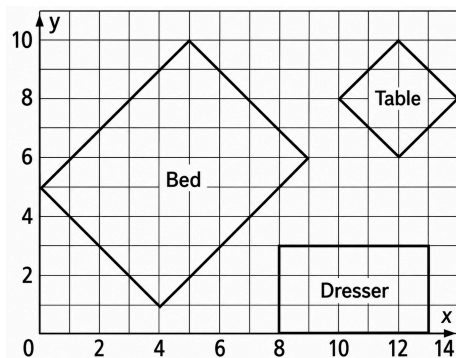
11. $H(20, -4)$, $I(-4, 3)$

The room shown below right is 14 ft by 10 ft. Find the dimensions of each piece of furniture to the nearest tenth.

12. length and width of the dresser

13. length and width of the table

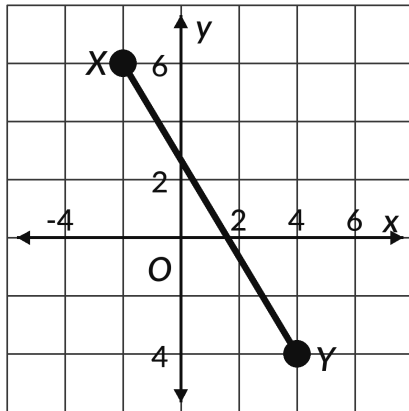
14. length and width of the bed



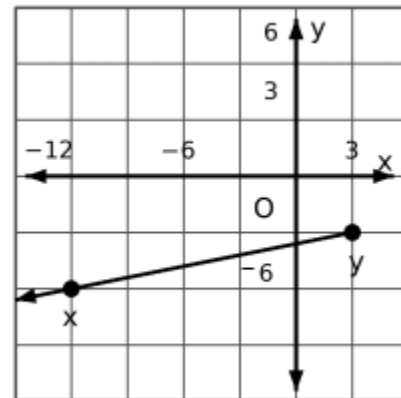
1.7 - Midpoint and Distance in the Coordinate Plane (Cont.)

For each graph, find (a) length of $\overline{XY}$ to the nearest tenth and (b) the coordinates of the midpoint of $\overline{XY}$.

15.



16.



1.8 - Perimeter, Circumference, and Area

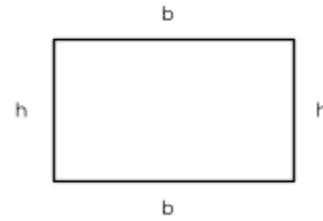
Perimeter / Circumference = the distance around a shape

Area = the amount of space inside a shape

Rectangle: has two bases (b) and two heights (h)

$$\text{Perimeter} \rightarrow P = 2(b + h) \text{ or } P = 2b + 2h$$

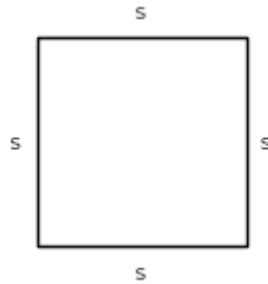
$$\text{Area} \rightarrow A = bh$$



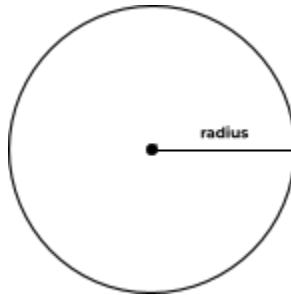
Square: rectangle with four equal sides (s) and four right angles

$$\text{Perimeter} \rightarrow P = 4s$$

$$\text{Area} \rightarrow A = s^2$$



Circle: set of points in a plane equidistant from a given point (center of circle)



Radius (r): distance from the center to a point on the circle

Diameter (d): segment that goes through the center of the circle, with both endpoints on the edge of the circle (double the radius)

Circumference: distance around the circle (perimeter of a circle)

$$C = \pi d \text{ or } C = 2\pi r$$

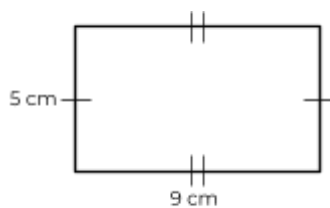
Area: amount of space inside the circle

$$A = \pi r^2$$

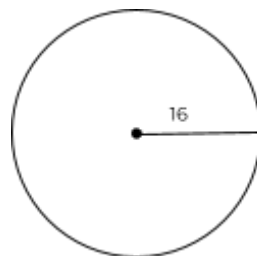
1.8 - Perimeter, Circumference, and Area

Find the perimeter of each figure.

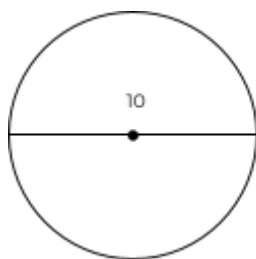
1.



2.

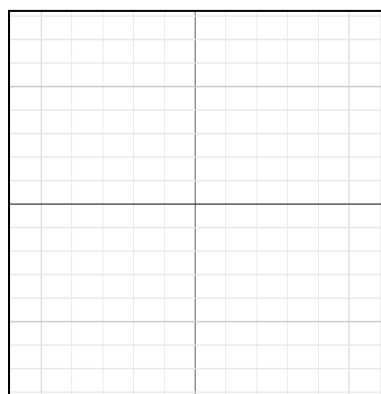


3.

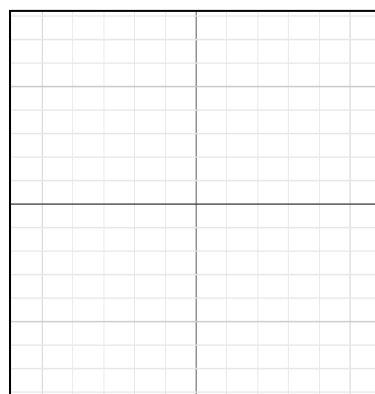


Graph each figure in the coordinate plane. Find the perimeter.

4. $X(-4, 2)$, $Y(2, 10)$, $Z(2, 2)$



5. $R(1, 2)$, $S(1, -2)$, $T(4, -2)$



Find the area of the rectangle with the given base and height.

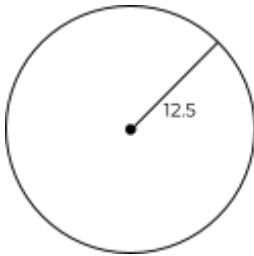
6. 4 ft, 15 in.

7. 90 in., 3 yd.

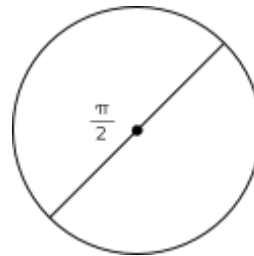
1.8 - Perimeter, Circumference, and Area (Cont.)

Find the area of each circle in terms of π .

8.

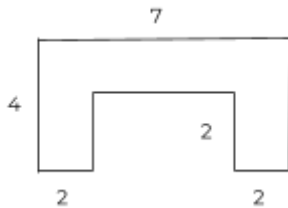


9.

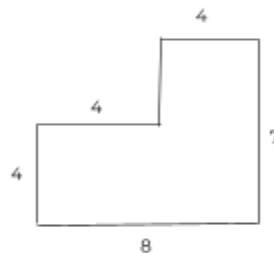


Find the area of each figure region. All angles are right angles.

10.



11.



Find the circumference and area of each circle, using $\pi = 3.14$. If necessary, round to the nearest tenth.

12. $r = 5$ m

13. $d = 2.1$ in

14. A rectangle has twice the area of a square. The rectangle is 18 in. by 4 in. What is the perimeter of the square?

15. **Coordinate Geometry** | The center of a circle is $A(-3, 3)$, and $B(1, 6)$ is on the circle. Find the area of the circle in terms of π .

16. The area of a circle is 25π in². What is its radius?

Simplify.

1) $\sqrt{36x^4}$

2) $\sqrt{72k^4}$

3) $\sqrt{32x}$

4) $\sqrt{175x^4}$

5) $-5\sqrt{128x^3}$

6) $-5\sqrt{128a}$

7) $7\sqrt{8x^4}$

8) $-5\sqrt{512x^2}$

9) $\sqrt{54x^3y}$

10) $\sqrt{32a^3b^4}$

11) $\sqrt{20a^3b^2}$

12) $\sqrt{54x^3y^3}$

$$13) -7\sqrt{144x^4y}$$

$$14) -8\sqrt{175x^2y^3}$$

$$15) -3\sqrt{27x^3y^3}$$

$$16) -3\sqrt{392x^4y^3}$$

$$17) 5\sqrt{288h^4j^3k}$$

$$18) -6\sqrt{180h^3j^4k^4}$$

$$19) 6\sqrt{105z}$$

$$20) -5\sqrt{75x^4yz^4}$$

Solve each equation.

$$21) 134 = 7(2 - 3v) + 6v$$

$$22) 6x - 8(1 - 3x) = 202$$

$$23) 8(k - 8) = -5k - 38$$

$$24) 4x + 31 = 7 - 4(x - 8)$$

$$25) -3(1 - 2x) - x = 7x - (1 + 3x)$$

$$26) -2(5x - 2) = -8x + 4(1 + 4x)$$

Solve each proportion.

$$27) \frac{3}{m-1} = \frac{2}{10}$$

$$28) \frac{9}{n-3} = \frac{2}{3}$$

$$29) \frac{9}{6} = \frac{v-1}{2}$$

$$30) \frac{7}{2} = \frac{x-7}{7}$$

$$31) \frac{n+5}{n-10} = \frac{3}{10}$$

$$32) \frac{x+5}{4} = \frac{x+5}{10}$$

$$33) \frac{n-10}{n-8} = \frac{9}{2}$$

$$34) \frac{v+5}{6} = \frac{v+5}{2}$$

Solve each equation by factoring.

$$35) b^2 + 5 = 6b$$

$$36) b^2 = 15b - 56$$

$$37) b^2 - 11b + 52 = 3b + 4$$

$$38) x^2 + 17 = 9x + 3$$

$$39) 3p^2 + 24p - 217 = -5p^2 + 7$$

$$40) -5x^2 - 5x - 57 = -1 - 6x^2 - 4x$$

$$41) 28b^2 + 7b - 99 = -1 + 7b^2$$

$$42) 30a^2 + 12a = 18$$

$$43) 6r^2 - 15r - 8 = 4r^2$$

$$44) -5b^2 + 3 = -7b^2 + 3 - 3b$$

Solve each equation by completing the square.

$$45) k^2 - 10k + 18 = 9$$

$$46) x^2 + 2x - 53 = 10$$

$$47) k^2 + 99 = 20k$$

$$48) 8p^2 - 4p = -3 + 7p^2$$

$$49) -6n^2 - 13n - 30 = -7n^2$$

$$50) n^2 - 9n - 12 = -2$$