AP Calculus AB

Summer Packet

This packet is intended to prepare you for AP Calculus AB by reviewing prerequisite algebra and precalculus skill. With the amount of curriculum to be covered before the AP Exam in May, we need to cover this review chapter over the summer.

It <u>is due on the first day of school</u> and worth a summative assignment grade based on completion and accuracy. Your work should be done neatly, in pencil, in your own handwriting on the printout.

If you do not remember how to do something, you are welcome to find a video, use your texts and notes from previous math classes, or ask a friend for help. I expect you to get help if there are any problems you don't remember how to do. Please understand you are expected to know and be able to use these skills throughout AB Calculus without having to be retaught. This is essential to being able to move through the course at a reasonable pace and thus be prepared for the AP exam.

Calculus truly is a fascinating course—you will love it!
I am looking forward to teaching you!

– Mrs. Strellman

Name_____

The following formulas and identities will help you complete this packet. You are expected to know ALL of these for the course.



LINES

Slope-intercept: $y = mx + b$

Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$

Horizontal line: $y = b$ (slope $= 0$)

Vertical line: $x = a$ (slope = undefined)

Parallel → same slope

Perpendicular → opposite reciprocal slopes

QUADRATICS

Standard: $y = ax^2 + bx + c$

Vertex: $y = a(x - h)^2 + k$

Intercept: $y = a(x - p)(x - q)$

Parabola opens: up if $a > 0$
down if $a < 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXPONENTIAL PROPERTIES

$x^a \cdot x^b = x^{a+b}$ $\quad (xy)^a = x^a y^a$

$\frac{x^a}{x^b} = x^{a-b}$ $\quad \sqrt[n]{x^m} = x^{m/n}$

$x^0 = 1\ (x \neq 0)$ $\quad \left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$

$x^{-n} = \frac{1}{x^n}$

In general, it is fine to have negative exponents in your answers!

LOGARITHMS

$y = \log_a x$ is equivalent to $a^y = x$

$\log_b(mn) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \log_b m$

TRIGONOMETRIC IDENTITIES

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cot x = \frac{1}{\tan x}$ $\quad \tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1$ $\quad \tan^2 x + 1 = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

$\sin(2x) = 2 \sin x \cos x$ $\quad \cos(2x) = \cos^2 x - \sin^2 x$ **or** $1 - 2\sin^2 x$ **or** $2\cos^2 x - 1$

You are expected to know the general shape, domain, and range of each parent function in the table. Draw one example function.

"Parent" functions mean no transformations have been applied. Transformation (shifting, stretching, compressing, or reflecting) may change the domain or range.

Linear functions (x) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$	**Quadratic** functions (x^2) **D:** $(-\infty, \infty)$ **R:** $[0, \infty)$
Odd-degree polynomials (x^n for **odd** n) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$	**Even-degree** polynomials (x^n for **even** n) **D:** $(-\infty, \infty)$ **R:** $[0, \infty)$
Exponential functions $(a^x\ , e^x)$ **D:** $(-\infty, \infty)$ **R:** $(0, \infty)$ Horizontal asymptote: $y = 0$	**Logarithmic** functions $(\log_a x\ , \ln x)$ **D:** $(0, \infty)$ **R:** $(-\infty, \infty)$ Vertical asymptote: $x = 0$
Sinusoidal functions $(a \sin x\ , a \cos x)$ **D:** $(-\infty, \infty)$ **R:** $[-a, a]$	**Tangent** function $(\tan x)$ **D:** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{(2n+1)\pi}{2}, \frac{(2n+3)\pi}{2}\right)$ **R:** $(-\infty, \infty)$ Vertical asymptotes: $x = \frac{(2n+1)\pi}{2}$
Reciprocal functions $\left(\frac{1}{x}\right)$ **D:** $(-\infty, 0) \cup (0, \infty)$ **R:** $(-\infty, 0) \cup (0, \infty)$ Vertical asymptote: $x = 0$ Horizontal asymptote: $y = 0$	**Absolute value** functions $(\lvert x \rvert)$ **D:** $(-\infty, \infty)$ **R:** $[0, \infty)$
Even root functions $\left(\sqrt{x}\ , \sqrt[n]{x} \text{ for even } n\right)$ **D:** $[0, \infty)$ **R:** $[0, \infty)$	**Odd root** functions $\left(\sqrt[3]{x}\ , \sqrt[n]{x} \text{ for odd } n > 1\right)$ **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$

For #1-8, write an equation for each line in point-slope form.

1. Containing $(4, -1)$ with a slope of $\frac{1}{2}$

_____

2. Crossing the x-axis at $x = -3$ and the y-axis at $y = 6$

_____

3. Containing the points $(-6, -1)$ and $(3,2)$

_____

4. Write an equation of a line passing through $(5, -3)$ with an undefined slope.

_____

5. Write an equation of a line passing through $(-4,2)$ with a slope of 0.

_____

6. Write an equation of a line passing through $(2,8)$ that is parallel to $y = \frac{5}{6}x - 1$.

_____

7. Write an equation of a line passing through $(4,7)$ that is perpendicular to the y-axis.

_____

8. Write an equation of a line passing through $(6, -7)$ that is perpendicular to $y = -2x - 5$.

_____

For #9-16, solve each equation for x. Note that some equations with have a specific value, but most will have a solution in terms of other variables. (For example: $x = \frac{a+b}{c}$ may be a solution.)

9. $x^2 + 3x = 8x - 6$

_____

10. $\frac{2x-5}{x+y} = 3 - y$

_____

11. $3xy + 6x - xz = 12$

_____

12. $A = ax + bx$

_____

13. $cx = vx$

_____

14. $r = t - x(z - y)$

_____

15. $\frac{3+x}{5-x} = 6 + y$

_____

16. $\frac{y+2}{4-x} = 4(2 - z)$

_____

For #17-22, solve each quadratic by factoring.

17. $x^2 - 4x - 12 = 0$

_____

18. $x^2 - 6x + 9 = 0$

_____

19. $x^2 - 9x + 14 = 0$

_____

20. $x^2 - 36 = 0$

_____

21. $9x^2 - 1 = 0$

_____

22. $4x^2 + 4x + 1 = 0$

_____

For #23-27, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$.

23. $f\left(\frac{1}{2}\right) =$ _____

24. $g(-2) =$ _____

25. $f(1) + g(0) =$ _____

26. $f(0) \cdot g(0) =$ _____

27. $\frac{g(-6)}{f(-6)} =$ _____

For #28-35, use $f(x) = x^2 - 1$, $g(x) = 3x$, and $h(x) = 5 - x$ to find each composite function.

28. $f\big(g(x)\big) =$

_____

29. $g\big(f(x)\big) =$

_____

30. $f\big(f(4)\big) =$

_____

31. $g\big(h(-4)\big) =$

_____

32. $f\left(g\big(h(1)\big)\right) =$

_____

33. $f\big(g(x-1)\big) =$

_____

34. $g\big(f(x^3)\big) =$

_____

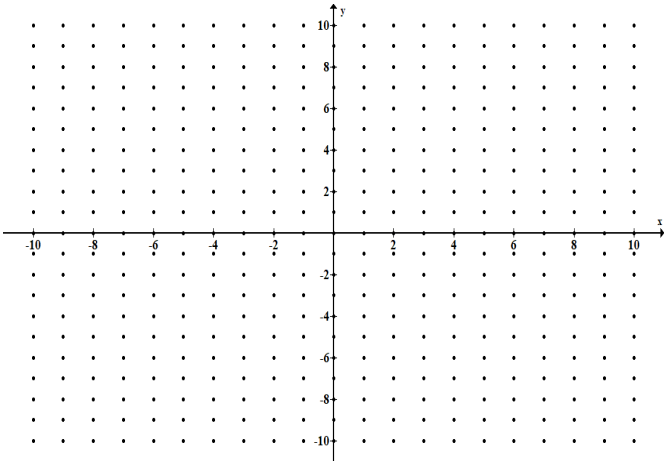
35. $\frac{f(x+h)-f(x)}{h} =$

_____

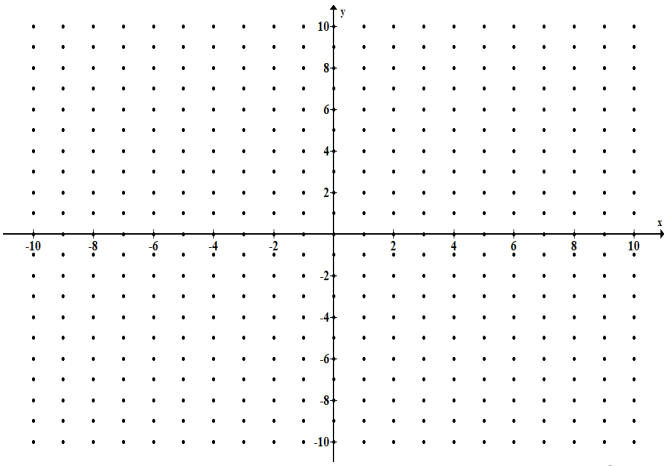
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For #36-38, graph each piecewise function.

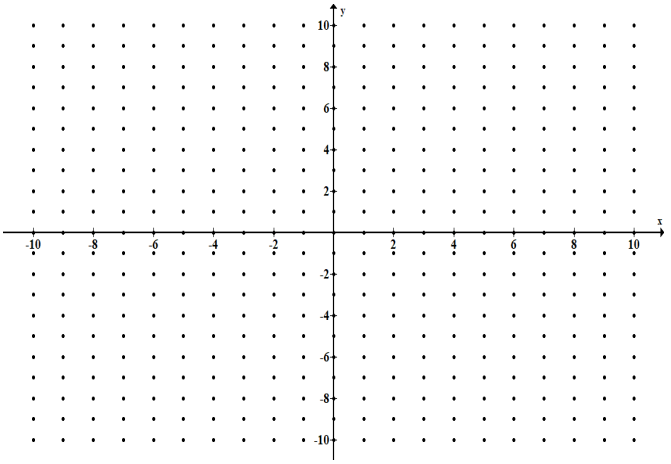
36. $f(x) = \begin{cases} x+3 & ; \ x < 0 \\ -2x+5 & ; \ x \geq 0 \end{cases}$



37. $g(x) = \begin{cases} \frac{1}{2}x & ; \ -4 \leq x \leq 2 \\ 2x-3 & ; \ x > 2 \end{cases}$



38. $h(x) = \begin{cases} |x| & ; \ x \leq 1 \\ 2 - |x - 2| & ; \ x > 1 \end{cases}$



For #39-43, solve each exponential equation and round answers to the nearest thousandth. Some equations can be solved by writing each side as the same base while others will require a logarithm.

39. $5^x = \frac{1}{5}$

_____

40. $6^x = 1296$

_____

41. $6^{2x-7} = 216$

_____

42. $5^{3x-1} = 49$

_____

43. $10^{x+5} = 125$

_____

For #44-47, simplify each expression without the use of a calculator. The exponential properties on page 2 of this packet will help.

44. $e^{\ln 4} =$ _____

45. $e^{2 \ln 3} =$ _____

46. $\ln e^9 =$ _____

47. $5 \ln e^3 =$ _____

For #48-53, solve each exponential or logarithmic equation by hand. Round answers to the nearest thousandth.

48. $e^x = 34$

_____

49. $3e^x = 120$

_____

50. $e^x - 8 = 51$

_____

51. $\ln x = 2.5$

_____

52. $\ln(3x - 2) = 2.8$

_____

53. $2\ln(e^x) = 5$

_____

For #54-66, find the exact value of the expression using the Unit Circle. To be clear, "exact" answer means no decimals!

54. $\sin 120° =$ _____

55. $\cos \frac{11\pi}{6} =$ _____

56. $\tan 225° =$ _____

57. $\sin\left(-\frac{2\pi}{3}\right) =$ _____

58. $\sin 150° =$ _____

59. $\tan \frac{7\pi}{4} =$ _____

60. $\csc\left(\frac{\pi}{4}\right) =$ _____

61. $\sec(-210°) =$ _____

62. $\cot\left(\frac{5\pi}{4}\right) =$ _____

63. $\sin\left(\frac{9\pi}{4}\right) =$ _____

64. $\sec\left(-\frac{\pi}{4}\right) =$ _____

65. $\tan\left(-\frac{4\pi}{3}\right) =$ _____

66. $\cos\left(\frac{8\pi}{3}\right) =$ _____

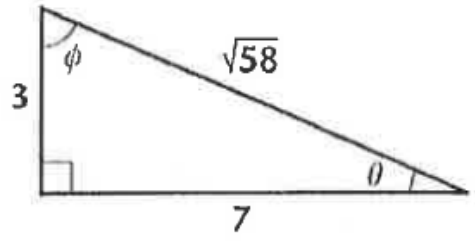
For #67-70, evaluate each trigonometric expression using the right triangle provided. You do NOT need to rationalize the denominator.

67. $\sin\theta =$ _____

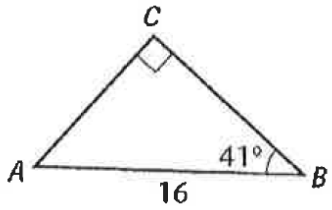
68. $\cos\theta =$ _____

69. $\tan\phi =$ _____

70. $\sec\phi =$ _____



71. Solve the triangle, rounding all angles and sides to the nearest thousandth. ("Solving a triangle" means to find all missing sides and angles.)



$m\angle A =$ _____

$AC =$ _____

$CB =$ _____

For #72-79, evaluate each inverse trigonometric function using the Unit Circle. Write all answer in radians, not degrees. Do not use a calculator.

72. $\sin^{-1}\left(\frac{1}{2}\right) =$ _____

73. $\sin^{-1}(-1) =$ _____

74. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ _____

75. $\tan^{-1}\left(\sqrt{3}\right) =$ _____

76. $\tan^{-1}(-1) =$ _____

77. $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) =$ _____

78. $\sin\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right) =$ _____

79. $\sin^{-1}(\cos(0)) =$ _____

80. Explain how the graph of $f(x)$ and its inverse, $f^{-1}(x)$, compare.

_____

_____

For #81-83, find the inverse of each function.

81. $g(x) = \frac{5}{x-2}$

$g^{-1}(x) =$ _____

82. $f(x) = \frac{x^2}{3}$

$f^{-1}(x) =$ _____

83. $y = \sqrt{4-x} + 1$

$y^{-1} =$ _____

84. If the graph of $f(x)$ has the point (2,7), then what is one point on the graph of $f^{-1}(x)$? _____

For #85-89, write each inequality in interval notation. For example, $x > 3$ becomes $(3, \infty)$.

85. $1 < x \leq 10$ _____

86. $x < 0$ **or** $x \geq 4$ _____

87. $x \geq -2$ _____

88. $x \geq 4$ **and** $x > 10$ _____

89. $x > 5$ **or** $x < 7$ _____

For #90-99, find the domain and range of each function. Write answers in interval notation. Confirm your answer by graphing the function on your calculator. The parent functions on page 3 of this packet will help.

90. $f(x) = \sqrt{x+5}$

D: _____ R: _____

91. $g(x) = x^2 - 5$

D: _____ R: _____

92. $y(t) = \frac{1}{t+7}$

D: _____ R: _____

93. $h(x) = \frac{5}{x^2+1}$

D: _____ R: _____

94. $f(x) = \sqrt{x^2+5}$

D: _____ R: _____

95. $g(t) = t^3 + 2t - 7$

D: _____ R: _____

96. $h(x) = 3\sin(\pi x) - 1$

D: _____ R: _____

97. $y(x) = \sqrt[5]{2x+3}$

D: _____ R: _____

98. $f(x) = -3e^{2x} + 5$

D: _____ R: _____

99. $g(t) = \log_4(x-2) + 1$

D: _____ R: _____

For #100-102, find the difference quotient of each function. (Refer back to #35 if needed.)

100. $g(x) = x^2 - 3x$

_____

101. $f(x) = \frac{2}{x+1}$

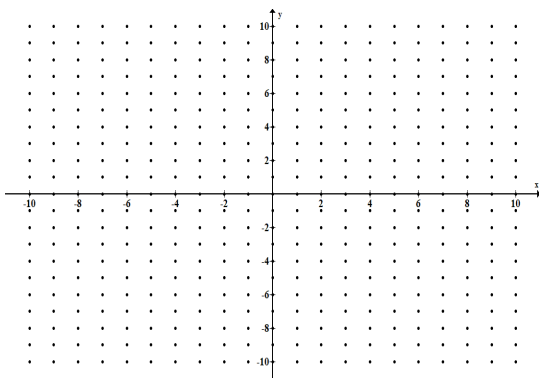
_____

102. $h(x) = \sqrt{x-3}$

_____

The remaining exercises are more challenging and specifically from concepts and skills covered in Pre-Calculus. You must show all work to earn credit.

103. State the domain and range of $f(x) = \frac{2x^2-6x-20}{x^3-2x^2-15x}$



D: _____ R: _____