AP Calculus AB Summer Work **Lucy Beckham HS** **Name:**_____

*You should complete all of your work neatly on a **separate sheet of paper**. Make sure to **label each problem** and **box in your answers**.*

I. Calculator Basics

1. Find the largest and smallest values of each of the following functions on their given x intervals.
   a) $f(x) = 2^x + x^2$ on $[-4, 1]$
   b) $y = (\cos x)^x$ on $[-1.5, 4.75]$
   c) $f(x) = \frac{3x^2+x-5}{x^2+1}$ on $2 \le x \le 4$
   d) $g(x) = \frac{1}{\sqrt{4-x^2}}$ on $(-2, 2)$

2. Graph $y = \frac{x^2-9}{x-3}$ on your calculator.
   a) Explain why this graph appears to be a linear function, rather than a curve with a vertical asymptote at $x = 3$.
   b) Explain what is happening to the function at $x = 3$.
   c) IF you had to fill in a y-value when $x = 3$, what would it be?

3. An open box is to be made from cutting squares of side length s from each corner of a piece of cardboard 25" by 30".
   a) Write an expression for the volume, V, of the box in terms of s.
   b) Graph $V(s)$ using your calculator. Identify a reasonable domain and range for this graph in the context of this problem.
   c) Find the value of s that will give the maximum volume of the box.
   d) What value(s) of s will give a volume of 1225 cubic inches?

4. Find the minimum distance from point P $(1,2)$ to the line $x + 2y = 3$.

II. Cartesian Plane Basics

5. Determine the slope, length and midpoint of the line segment with endpoints $(1, -2)$ and $(3, 2)$.

6. For what value(s) of k is $5x + ky = 3$ parallel to $2x - 3y = 5$? For what value(s) of k are the two lines perpendicular?

7. Plot the line $2x - 5y = 10$ indicating your x and y-intercepts. Be sure to label your axes to indicate your scale.

8. Find the line that passes through the point $(-1, 3)$ and the point of interecetion of the lines $x + 3y = 1$ and $2x - y = -5$. Write your answer in point slope from ($y - y_1 = m(x - x_1)$).

III. Basic Functions and Transformations

9. You should be very familiar with the following functions and be able to envision them in your mind without having a graph. You should be able to identify the domain and range of each from an equation.
    a. Constant Function: $f(x) = C$, where C is a contestant
    b. Linear Functions: $f(x) = mx + b$
    c. Quadratic Function: $f(x) = ax^2 + bx + c$
    d. Polynomial Function: $f(x) = ax^n + bx^{n-1} + \cdots + mx + n$
    e. Rational Function: $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials
    f. Radical Functions: $f(x) = \sqrt{x}$ and $f(x) = \sqrt[3]{x}$
    g. Exponential Function: $f(x) = e^x$, Know the domain and range.
    h. Natural Log Function: $f(x) = \ln x$, Know the domain and range.
    i. Absolute Value Function: $f(x) = |x|$

10. Using transformations, describe how the graph of each function can be obtained from the graph of

$f(x) = \sqrt{x}$ $\quad g(x) = \frac{1}{x}$ $\quad h(x) = |x|$ $\quad k(x) = x^3$ $\quad l(x) = \ln x$ OR $m(x) = e^x$

a) $y = .5(x-4)^3 + 2$
b) $y = |x+2| - 1$
c) $y = -\sqrt{3-x} + 4$
d) $y = 2\ln(x+3) - 3$
e) $y = e^{x-1} - 3$

11. Identify the domain of the following functions. Write your answers in both inequality and interval notation.

a) $f(x) = \frac{x-1}{x^2+1}$
b) $g(x) = \sqrt{x^2 - 3x + 2}$
c) $h(x) = x^2 + x^{\frac{1}{4}}$
d) $k(x) = 2x + \frac{1}{x} + \frac{1}{x-1}$

12. Remember that for even functions, $f(-x) = f(x)$ and that for odd functions, $f(-x) = -f(x)$, show whether the following functions are even, odd or neither.

a) $y = x^4$
b) $y = x - x^4$
c) $y = \frac{1}{x^2-4}$
d) $y = -2x^3 + 4x$

IV. Trig Review

You should know the Unit Circle, basic shapes of the sine, cosine, and tangent functions, their domains and ranges and the following trig identities:

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $\sin 2x = 2\sin x \cos x$

13. **You MUST be able to evaluate the following WITHOUT a calculator or unit circle**. We always use radians, not degrees.

a) $\sin \pi$ b) $\cos \frac{3\pi}{2}$ c) $\tan \frac{5\pi}{4}$ d) $\sin \frac{4\pi}{3}$

e) $\cos \frac{7\pi}{4}$ f) $\sin \frac{2\pi}{3}$ g) $\tan \frac{5\pi}{6}$ h) $\sin \frac{11\pi}{6}$

i) $\arccos\left(\frac{1}{2}\right)$ j) $\sin^{-1}(1)$ k) $\arctan(0)$ l) $\sec^{-1}(-2)$

14. Identify which of the six trig functions are a) even, b) odd, c) neither.

15. Which of the following have the same graph?

a) $y = \sin(x)$ b) $y = -\sin(x)$ c) $y = \cos(x)$ d) $y = \sin(-x)$ e) $y = -\cos(x)$

16. Consider the function $y = \sqrt{\frac{1+\cos 2x}{2}}$

a) Can x take on any real value?

b) How large can $\cos 2x$ become? How small?

c) How large can $\frac{1+\cos 2x}{2}$ become? How small?

d) What are the domain and range of the original function?

17. Solve each of the following on the interval $[0, 2\pi)$.

a) $2\sin x - \sqrt{3} = 0$ b) $\cos^2 x = 2\sin x - 2$

18. Solve each of the following on the interval $[0, 2\pi)$.

a) $\sin^2 x < 2\cos x + 1$ b) $2\sin x \cos x + \sqrt{2}\cos x < 0$

V. Exponents, Radicals and Factoring

19. Simplify each of the following expressions.

a) $\left(\frac{2}{3}x^{-3}\right)(15x^7)$ b) $x^3(2yz^2)^3$ c) $\frac{(3x^3)(4x^5)}{(x^2)^3}$

d) $\frac{(2y^4)(3y^2)^2}{(y^3)^4}$ e) $(3a^{-2}b^3)^{-3}$

20. Simplify the following by removing all possible factors from the radical.

$\sqrt{9z^8 b}$ a) $\sqrt{24a^4b^8}$ b) $\sqrt{\frac{75}{a^6}}$

21. Factor each of the following completely.

a) $6x^3y^2 + 15x^2y^5 - 30x^7y^4z$

b) $16y^2 - 9$

c) $4x^{16} - 9y^6$

d) $6t^2 + 7t - 20$

e) $3x^2 - 5x + 2$

f) $x^3 - x^2 + 3x - 3$

g) $3a^3 + 3a^2 - 27a - 27$

h) $x^2 + 4x + 4 - 9y^2$

22. Solve the following equations.

a) $t^2 + 4t = -3$

b) $4 = \sqrt{x^2 - 8x}$

c) $0 = \frac{3y^3 - y - 2}{y^2 - 1}$

d) $\ln(y) = 2\ln(4)$

e) $0 = x^4 - 5x^2 + 4$

f) $\frac{x}{x-2} + \frac{1}{5} = \frac{2}{x-2}$

g) $2(5)^{t-1} + 4 = 20$

h) $7 = 20e^x - 5$