AB Calculus – summer work

NO calculators *are permitted in doing this worksheet. It is best if the work is done individually as its intent is to review concepts and skills that are necessary for calculus. Should you need assistance in solving a problem, videos from the Khan Academy or notes from last year's class can be used as source material.*

All work should be done on the worksheet. Clearly show the methods used as they structure and clarity of your solution is as important in calculus as the accuracy of your answer. All trig solutions should be expressed in radians. All lines should be left in point-slope form.

1. Evaluate the following:

(a) $\sin\frac{\pi}{2}$ (b) $\cos\frac{\pi}{3}$ (c) $\sin\frac{\pi}{6}$ (d) $\tan\frac{\pi}{4}$

(e) $\sin^{-1}\frac{1}{2}$ (f) $\cos^{-1}1$ (g) $\tan^{-1}\frac{\sqrt{3}}{3}$ (h) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(i) $\ln e^2$ (j) $e^{2\ln 3}$ (k) $\sin(\ln e^{\pi})$ (l) $\ln\sqrt{e}$

2. Write the equation of the line……

(a) with a slope of 2 that contains the point (-1 2)

(b) with a slope of e that passes through the point $(1, e)$

(c) parallel to the line $2x+3y=5$ that contains the point $(3, -1)$

(d) perpendicular to the line $3x-2y=7$ that contains the point $(\ln 2, e^2)$

3. Solve the following for x in terms of y.

(a) $y = \ln(x+1)$

(b) $y = \sin x$

(c) $y = e^x - 2$

(d) $y = \dfrac{1}{x^2+2}$

4. Evaluate the following limits:

(a) $\lim\limits_{x\to 3}(2x+1)$

(b) $\lim\limits_{x\to 2}\dfrac{x-2}{x^2-4}$

(c) $\lim\limits_{x\to \pi}\dfrac{\sin x}{x}$

(d) $\lim\limits_{x\to 0}\dfrac{\sin 6x}{x}$

(e) $\lim\limits_{x\to \infty}\dfrac{\sin 6x}{x}$

(f) $\lim\limits_{x\to \infty}\cos(\dfrac{3}{x^2})$

(g) $\lim\limits_{x\to 3^+}\dfrac{x-5}{x-3}$

(h) $\lim\limits_{x\to 7}\dfrac{\sqrt{x+2}-3}{x-7}$

(i) $\lim\limits_{x\to \infty}\dfrac{x^4+3}{2x^2-5}$

(j) $\lim\limits_{x\to -\infty}\dfrac{|2x-1|}{6x+7}$

(k) $\lim\limits_{x\to 2^-}[x]$

(l) $\lim\limits_{x\to \infty}\dfrac{\sqrt{5x}-1}{\sqrt{x^2+3}}$

5. Consider the piecewise function defined below.

$$f(x)=\begin{cases} x^2+1 & x<2 \\ k & x=2 \\ 3x-1 & x>2 \end{cases}$$

(a) Evaluate : $f(0)=$ $f(1)=$ $f(3)=$

(b) Determine $\lim_{x\to 2} f(x)$. Show the analysis that leads to your answer.

(c) Determine the value of *k* so that $f(x)$ is defined and continuous at $x=2$.

6. Simplify the following:

(a) $\dfrac{\sqrt{x+3}-\dfrac{x}{\sqrt{x+3}}}{(x+3)}$

(b) $\dfrac{(3x-1)(2)-(2x+3)(3)}{(3x-1)^2}$

7. If $g(x)=f^{-1}(x)$ and $f(2)=3;\quad f(3)=7;\quad f(-1)=\dfrac{1}{3}$, Determine $g(3)$.

8. State the domain and range of each of the following functions:

(a) $f(x) = \sqrt{x-3}$

(b) $g(x) = \sin x$

(c) $k(x) = \ln(x+2)$

(d) $h(x) = e^x - 3$

(e) $j(x) = x^3 + 4$

(f) $p(x) = x^2 - 4$

(g) $r(x) = \ln(x^2 + 3)$

(h) $w(x) = \sqrt{9 - x^2}$

9. $\frac{1}{2}\ln\frac{1}{3} =$ (A) $\ln\sqrt{3}$ (B) $-\ln\frac{3}{2}$ (C) $-\ln\sqrt{3}$

10. $-\frac{1}{2}\ln 4 =$ (A) $\ln\frac{1}{2}$ (B) $\ln 2$ (C) $\ln(-2)$

11. Solve the following inequalities using an interval line analysis

(a) $(x-3)^2(x+2)<0$

(b) $\dfrac{(2x+1)(x-1)}{(x-2)(x+5)^2}\geq 0$

12. Sketch a rudimentary graph of each of the following. State the domain, range, zeros and *y*-intercept of each.

(a) $y=x^2-4$

(b) $y=\ln(x)$

(c) $y=e^x$

(d) $y=\sqrt{x+4}$

(e) $y=\sin x$ over the interval $[0,\,2\pi]$

(f) $y=\cos x$ over the interval $[0,\,2\pi]$

13. Solve for *y*:

(a) $\frac{y^2}{2} = \frac{x^3}{3} - x^2 + 3x - 5$

(b) $-\frac{1}{y} = x + \frac{1}{2}$

14. $\sin^2 x + \cos^2 x =$ (A) 1 (B) $\cos 2x$ (C) $\sin 2x$

15. $\sin x \cos x =$ (A) $\sin 2x$ (B) $\frac{1}{2}\cos 2x$ (C) $\frac{1}{2}\sin 2x$

16. Determine the area of the region enclosed by the *y*-axis, the *x*-axis, the line $x = 3$ and the line $y = x + 2$. You might want to consider what the graph looks like.