AP Calculus Summer Review Packet

The problems included in this packet are intended to be a review of some of the skills and concepts that you learned in Algebra I, Geometry, Algebra II, and PreCalculus and will need to know to be successful in AP Calculus. **This assignment should not be a group effort, as students are expected to complete this assignment independently.**

Please do not wait until a day or two before school begins to begin working on this packet. If you plan ahead and work a few pages at a time throughout the summer, this packet should act as a gentle reminder of previously learned material.

There is not enough room to complete problems on the pages of the packet itself. **Please rewrite each problem and do your work on separate sheets of paper.** You can write on the backs of pages, but please write neatly and clearly. **You must show all of your work.**

This packet must be completed and ready to turn in on the FIRST day of your AP Calculus class. The work will be counted as three homework grades.

AP Calculus Summer Packet

1. Using the attached unit circle, find the six trig functions of $\frac{5\pi}{4}$.

2. Using the attached unit circle, find the six trig functions of $\frac{-\pi}{3}$.

3. Using the attached unit circle, find the value of the trig functions.

$$\sin 5\pi =$$

$$\cos 5\pi =$$

$$\cos\frac{8\pi}{3} =$$

$$\sin\frac{9\pi}{4} =$$

$$\cos\frac{-13\pi}{6} =$$

$$\sin\frac{-9\pi}{4} =$$

$$\sin\frac{-19\pi}{6} =$$

$$\cos\frac{-8\pi}{3} =$$

4. Create a triangle in the appropriate quadrant. Solve for missing sides. Find the remaining 5 trig functions of θ.

$$tan\,\theta = \frac{-15}{8} \qquad\qquad sin\,\theta < 0$$

5. Find the reference angle of the given angle.

$$300° \qquad \theta' =$$

$$-60° \qquad \theta' =$$

$$\frac{4\pi}{5} \qquad \theta' =$$

6. Rewrite the logarithmic equation in exponential form.

$$\log_3 81 = 4$$

7. Rewrite the exponential equation in log form.

$$5^3 = 125$$

8. Solve for x.

$$\log_4 4^2 = x$$

9. Solve for x.

$$\log_8 x = \log_8 10^{-1}$$

10. Simplify.

$$\log_2 2^{3x}$$

11. Simplify algebraically. This is *not* a calculator-based problem.

$$4 \log_2 \frac{1}{2}$$

12. Simplify. This is *not* a calculator-based problem.

$$\ln \sqrt{e}$$

13. Expand.

$$\log_2 3x^2y^5$$

14. Condense.

$$\frac{1}{3} \log_8 x + 4 \log_8(x-1)$$

15. Expand.

$$\ln \frac{\sqrt{2x-3}}{6}$$

16. Condense.

$$4 \ln(x+8) - \ln x$$

17. Find the exact solution for x. (No decimals)

$$e^x = 12$$

18. Solve for x. This is *not* a calculator-based problem.

$$2^{x+3} = 256$$

19. Solve for x.

$$\ln x - \ln 4 = 0$$

20. Solve for the exact value of x. No decimal solutions.

$$\ln x = -2$$

21. Solve for x.

$$\log_{10} x = -1$$

22. Solve for the exact value of x. No decimal solutions.

$$\ln(2x - 1) = 5$$

23. Simplify.

$$5 + e^{\ln(x^2+1)}$$

24. Find the exact value of x. No decimal approximations.

$$7 - 2e^x = 5$$

25. Find the exact value of x. No decimal approximations.

$$5\,(2^{3-x}) - 13 = 100$$

26. Find the exact value of x.

$$e^{x^2-3x} = e^{x-2}$$

27. Use your TI-84 calculator to model the following data.

x	-4	-3	-2	0	2	3	4
Y	21	14	9	5	9	14	21

- Load data into L1, L2
- Create a scatter plot
- Calculate regression

28. Find the equation of the line that has a slope of 3 and passes through (1, -2). Graph the line on the attached graph paper.

29. Find the general form of the equation of a line that is parallel to $2x - 3y = 5$ and passes through the point (2, -1).

- General form: Ax + By + C = 0

30. Find the slope intercept form of the equation of a line that is perpendicular to $2x - 3y = 5$ and passes through the point (2, -1).

31. Find $f(b - 1)$.

$$f(x) = x^2 + 7$$

32. Graph the function $f(x)$ on the attached graph paper. Find the domain and range of $f(x)$.

$$f(x) = \sqrt{x - 1}$$

33. Graph $g(x)$ on the attached graph paper. Find the domain and range of $g(x)$. Describe how $g(x)$ is increasing / decreasing / constant.

$$g(x) = \begin{cases} 1 - x & x < 1 \\ (x-1)^{\frac{1}{2}} & x \geq 1 \end{cases}$$

34. Find $f(g(x))$ and $g(f(x))$.

$$f(x) = 2x - 3$$

$$g(x) = \cos x$$

35. Determine algebraically and graphically if $p(x)$ is even, odd, or neither.

$$p(x) = x^3 - x$$

36. Determine algebraically and graphically if $r(x)$ is even, odd, or neither.

$$r(x) = 1 + \cos x$$

37. Find the equation of the line that has an undefined slope and passes through the point (-5, -2).

38. Find the equation of the line that passes through the point (4, -4) and has a slope of 0.

39. Graph $h(x)$ on the attached graph paper. Find the domain and range of $h(x)$. Describe how $h(x)$ is increasing / decreasing / constant. Include asymptotes on your graph.

$$h(x) = \frac{3}{x}$$

40. Using your TI-84 calculator, find the intersection of y_1 and y_2.

$$y_1 = x^2 + 3$$

$$y_2 = 7$$

- 2nd trace ,5: intersect, enter, first curve, enter, second curve, enter, guess?, enter, intersection

41. Using your TI-84 calculator, find relative minimum and relative maximum values.

$$y = -x^4 + 4x^2 - 3x + 2$$

- 2nd trace min
- 2nd trace max

42. Solve the system of equations.

$$2x - y + 3z = 8$$

$$x - 6y - z = 0$$

$$-6x + 3y - 9z = 24$$

43. Using long division, divide $2x^4 + 21x^3 + 35x^2 - 37x + 46$ by $2x + 7$.

44. Simplify.

$$(3x^2 + x - 4)(2x - 5)$$

45. Simplify.

$$(-4x^2 + 5x + 1) - 3\,(4x^2 - 3x - 1)$$

46. Simplify.

$$(5y^2 - 2)(5y + 2)$$

47. The dimensions of a rectangular prism are given below. Find the volume of the prism.

Length x + 1

Width x + 4

Height x + 6

48. Simplify.

$$(5x^3 - 2x)(3x^2 + x - 8)$$

49. Factor.

$$8x^3 - y^3$$

- Remember: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

50. Factor.

$$27x^3 + 125y^3$$

- Remember: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

51. Simplify.

$$(x + 1)^2 - y^2$$

52. Factor.

$$12x^2 - 147$$

53. Factor.

$$25x^2 - 40xy + 16y^2$$

54. Simplify.

$$\frac{x + 3}{x + 5} + \frac{6}{x^2 + 3x - 10}$$

55. Simplify.

$$\frac{2x^2y^3z^{-2}}{(x^{-1}y^2c)^3}$$

56. Simplify.

$$\frac{20x^{-4}}{27y^2} \div \frac{8x^{-3}}{15y^{-5}}$$

57. Simplify.

$$\frac{4x^2 - 16}{2 - x}$$

58. Simplify.

$$\frac{x^2 + 4x}{x + 3} \cdot \frac{x^2 - 9}{x^2 + x - 12}$$

59. Simplify.

$$\frac{5x^3y + 20x^2y^2 + 20xy^3}{5xy}$$

60. Simplify.

$$\frac{2x^2 - 10x}{x^2 + 8x + 16} \cdot \frac{4x + 16}{x^2 - 25}$$

61. Simplify.

$$\frac{4\,(x + y)}{5x^2y^3} \div \frac{-2x - 2y}{10}$$

62. Find the value of i^5.

63. Simplify.

$$(4i)(6i)$$

64. Simplify. Rationalize the denominator.

$$\frac{2}{3+i}$$

65. Find the product of $(3 + i)$ and $(3 - i)$.

66. Simplify.

$$(-3 - 5i) - (-8 + i)$$

67. Solve for x.

$$x^2 + 2x + 2 = 0$$

68. Solve for x.

$$1 + \frac{1}{x^2} = \frac{3}{x}$$

69. Solve for x.

$$x^2 - 6x + 5 = 8$$

70. The point $(1, 0)$ is on a circle whose center is at $(3, 7)$. Write the standard form of the equation of the circle. Graph the circle on the attached graph paper.

71. Rewrite the equation of the circle in standard form. Graph the circle on the attached graph paper. Identify its center and radius.

$$x^2 + 4x + y^2 + 4y - 1 = 0$$

72. Find the standard form of the equation of the parabola with a vertex at the origin and a focus at (-2, 0). Graph the parabola on the attached graph paper. Include the vertex, focus, directrix, equation of the directrix and two symmetrical points of the parabola on your graph.

73. Find the standard form of the equation of the parabola with a vertex at (0, 4) and a directrix at y = 2. Graph the parabola on the attached graph paper. Include the vertex, focus, directrix, the equation of the directrix, and two symmetrical points of the parabola on your graph.

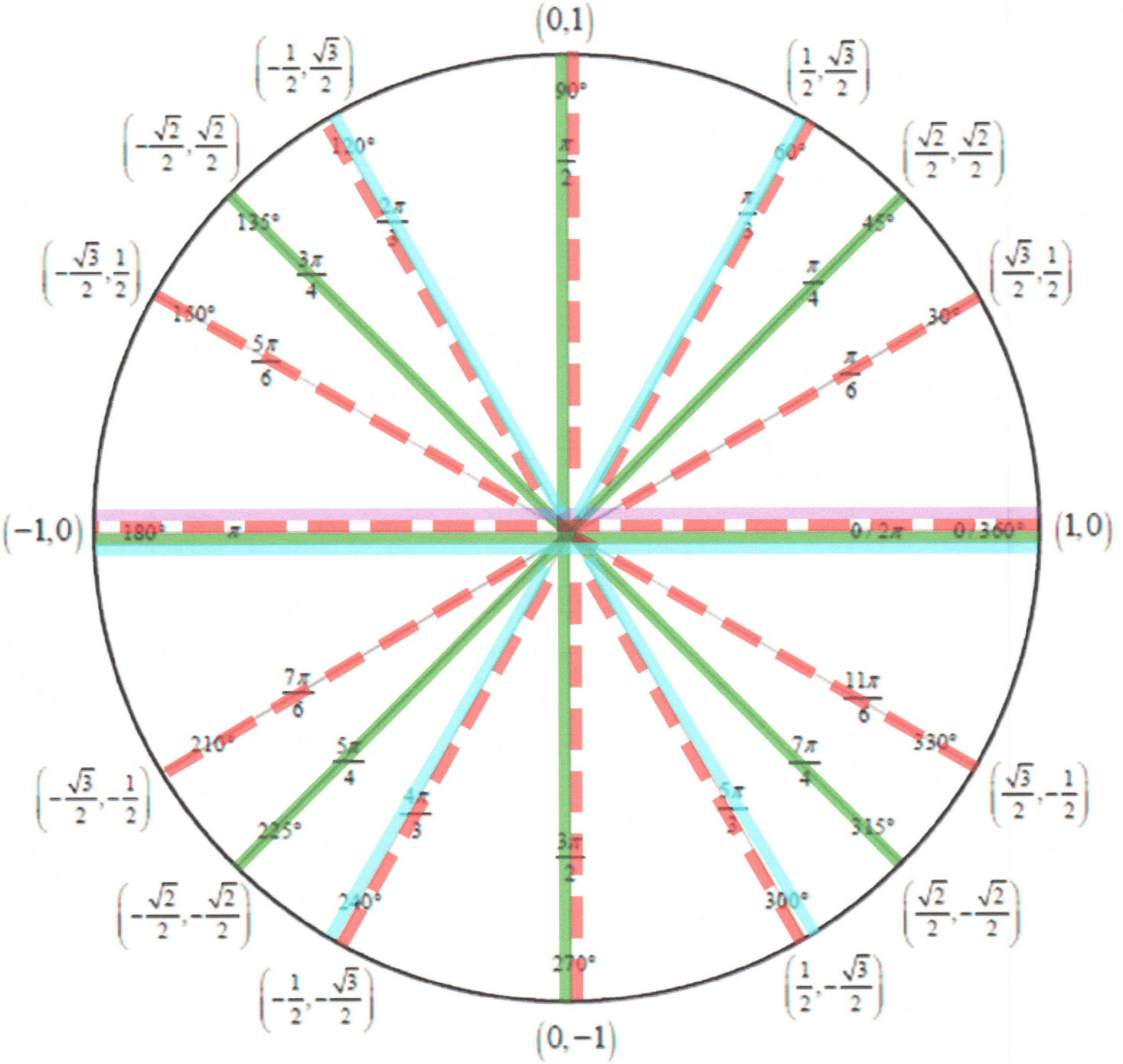
74. Find the standard form of the equation of the ellipse with a center at the origin, foci at (5, 0) and (-5, 0) and a major axis of length 14. Graph the ellipse on the attached graph paper. Include the center, the major and minor vertices and the foci on your graph.

75. Find the center, vertices, box points, and foci of the hyperbola. Graph the hyperbola on the attached graph paper. Include the center, vertices, box points, foci, and asymptotes on your graph.

$$\frac{y^2}{9} - \frac{x^2}{1} = 1$$

76. Find the standard form of the equation of the hyperbola. Graph the hyperbola on the attached graph paper. Include the center, vertices, box points, foci, and asymptotes on your graph.

$$9x^2 - 36x - y^2 - 6y + 18 = 0$$


(0,1)
$\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
(−1,0)
(1,0)
$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
(0,−1)
90°
$\frac{\pi}{2}$
120°
$\frac{2\pi}{3}$
60°
$\frac{\pi}{3}$
135°
$\frac{3\pi}{4}$
45°
$\frac{\pi}{4}$
150°
$\frac{5\pi}{6}$
30°
$\frac{\pi}{6}$
180°
π
0 / 2π
0 / 360°
$\frac{7\pi}{6}$
210°
$\frac{11\pi}{6}$
330°
$\frac{5\pi}{4}$
$\frac{7\pi}{4}$
225°
315°
$\frac{4\pi}{3}$
$\frac{5\pi}{3}$
240°
300°
$\frac{3\pi}{2}$
270°

AP Calculus Summer Review Packet Key

1. $\sin \frac{5\pi}{4} = \frac{-\sqrt{2}}{2}$

   $\cos \frac{5\pi}{4} = \frac{-\sqrt{2}}{2}$

   $\tan \frac{5\pi}{4} = 1$

   $\cot \frac{5\pi}{4} = 1$

   $\sec \frac{5\pi}{4} = -\sqrt{2}$

   $\csc \frac{5\pi}{4} = -\sqrt{2}$

2. $\sin \frac{-\pi}{3} = \frac{-\sqrt{3}}{2}$

   $\cos \frac{-\pi}{3} = \frac{1}{2}$

   $\tan \frac{-\pi}{3} = -\sqrt{3}$

   $\cot \frac{-\pi}{3} = \frac{-\sqrt{3}}{3}$

   $\sec \frac{-\pi}{3} = 2$

   $\csc \frac{-\pi}{3} = \frac{-2\sqrt{3}}{3}$

3. $\sin 5\pi = 0$

$\cos 5\pi = -1$

$\cos \frac{8\pi}{3} = \frac{-1}{2}$

$\sin \frac{9\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos \frac{-13\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin \frac{-7\pi}{4} = \frac{-1\sqrt{2}}{2}$

$\sin \frac{-19\pi}{6} = \frac{+1}{2}$

$\cos \frac{-8\pi}{3} = \frac{-1}{2}$

4. $\sin\theta = \frac{-15}{17}$

$\cos\theta = \frac{8}{17}$

$\csc\theta = \frac{-17}{15}$

$\sec\theta = \frac{17}{8}$

$\cot\theta = \frac{-8}{15}$

5. $\theta' = 60^\circ$

$\theta' = 60^\circ$

$\theta' = \frac{\pi}{5}$

6. Exponential form: $3^4 = 81$

7. Log form: $\log_5 125 = 3$

8. $x = 2$

9. $x = \frac{1}{10}$

10. $3x$

11. -4

12. $\frac{1}{2}$

13. $\log_2 3 + 2\log_2 x + 5\log_2 y$

14. $\log_8 x^{1/3}(x-1)^4$

15. $\frac{1}{2}\ln(2x-3) - \ln 6$

16. $\ln \frac{(x+8)^4}{x}$

17. $x = \ln 12$

18. $x = 5$

19. $x = 4$

20. $x = \frac{1}{e^2}$

21. $x = \frac{1}{10}$

22. $x = \frac{1}{2}(e^5 + 1)$

23. $x^2 + 6$

24. $x = 0$

25. $x = -\log_2 \frac{113}{5} + 3$

26. $x = 2 \pm \sqrt{2}$

27. Quadratic regression
$y = x^2 + 5$

28. $y = 3x - 5$
* line graph

29. $2x - 3x - 7 = 0$

30. $y = -\frac{3}{2}x + 2$

31. $f(b-1) = b^2 - 2b + 8$

32. D: $[1, +\infty)$ * Graph of $f(x)$
R: $[0, +\infty)$

33. D: $(-\infty, +\infty)$
R: $[0, +\infty)$
Decreasing $(-\infty, 1)$
Increasing $(1, +\infty)$
* Graph of $g(x)$

34. $f(g(x)) = 2\cos x - 3$
$g(f(x)) = \cos(2x - 3)$

35. $p(-x) = -x^3 + x$ odd, symmetric across origin.

36. $r(-x) = 1 + \cos(-x)$ Even, symmetric across y axis.
$= 1 + \cos x$
* cosx and secx are even functions.

37. $x = -5$

38. $y = -4$

39. Domain: $(-\infty, 0) \cup (0, +\infty)$
Range: $(-\infty, 0) \cup (0, +\infty)$
Decreasing from $(-\infty, 0), (0, +\infty)$

40. Points of intersection: $(2, 7)$
$(-2, 7)$

41. Relative max: $(-1.5737, 10.4940)$
Relative min: $(.4093, 1.4141)$
Relative max: $(1.1644, 2.0918)$

42. No solution

43. $x^3 + 7x^2 - 7x + 6 + \frac{4}{2x+7}$

44. $6x^3 - 13x^2 - 13x + 20$

45. $-16x^2 + 14x + 4$

46. $25y^3 + 10y^2 - 10y - 4$

47. $x^3 + 11x^2 + 34x + 24$

48. $15x^5 + 5x^4 - 46x^3 - 2x^2 + 16x$

49. $(2x - y)(4x^2 + 2xy + y^2)$

50. $(3x + 5y)(9x^2 - 15xy + 25y^2)$

51. $x^2 + 2x + 1 - y^2$

52. $3(2x - 7)(2x + 7)$

53. $(5x - 4y)^2$

54. $\dfrac{x(x+1)}{(x+5)(x-2)}$

55. $\dfrac{2x^5}{z^2y^3c^3}$

56. $\dfrac{25}{18xy^7}$

57. $-4(x+2)$

58. x

59. $(x+2y)^2$

60. $\dfrac{8x}{(x+4)(x+5)}$

61. $\dfrac{-4}{x^2y^3}$

62. $i^5 = i^2 \cdot i^2 \cdot i$
$= (-1)(-1)i$
$= i$

63. -24

64. $\frac{3-i}{5}$

65. 10

66. $5-6i$

67. $x = -1+i$
$x = -1-i$

68. $x = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$

69. $x = 3 \pm 2\sqrt{3}$

70. $(x-3)^2 + (y-7)^2 = 53$
* circle graph

71. $(x+2)^2 + (x+2)^2 = 9$
* circle graph

72. $y^2 = -8x$
* parabola graph

73. $x^2 = 8(y-4)$
* parabola graph

74. $\frac{x^2}{49} + \frac{y^2}{24} = 1$
* ellipse graph

75. Center: $(0,0)$
Vertices: $(0,3)$ $(0,-3)$
Box points: $(1,0)$ $(-1,0)$
Foci: $(0,\sqrt{10})$ $(0,-\sqrt{10})$