PreCalculus Summer Review Packet

The problems included in this packet are intended to be a review of some of the skills and concepts that you learned in Algebra I, Geometry, and Algebra II and will need to know to be successful in PreCalculus. **This assignment should not be a group effort, as students are expected to complete this assignment independently.**

Please do not wait until a day or two before school begins to begin working on this packet. If you plan ahead and work a few pages at a time throughout the summer, this packet should act as a gentle reminder of previously learned material.

There is not enough room to complete problems on the pages of the packet itself. **Please rewrite each problem and do your work on separate sheets of paper.** You can write on the backs of pages, but please write neatly and clearly. **You must show all of your work.**

This packet must be completed and ready to turn in on the FIRST day of your PreCalculus class. The work will be counted as three homework grades.

PreCalculus Summer Packet

1. What is the solution to the following system of equations?

$$2x - 3y = 4$$
$$4x + y = -6$$

2. Simplify.

$$(3x^2 + x - 4)(2x - 5)$$

3. Simplify.

$$(-2x^2 + 6x + 1) - 2\,(4x^2 - 3x + 1)$$

4. Simplify.

$$(6y^2 - 2)(6y + 2)$$

5. Simplify.

$$(5x^3 - 2x)(3x^2 + x - 8)$$

6. Factor completely.

$$8a^3 + c^3$$

7. Factor completely.

$$27x^3 - 125$$

8. Factor completely.

$$12x^2 - 147$$

9. Factor completely.

$$25x^2 - 40xy + 16y^2$$

10. Simplify.

$$\frac{x+3}{x+5} + \frac{6}{5x}$$

11. Simplify.

$$\frac{x}{x+1} - \frac{2}{x-1}$$

12. Simplify.

$$\frac{3x^2b^3c^{-2}}{(a^{-1}x^2c)^4}$$

13. Simplify.

$$\frac{20x^{-4}}{27y^2} \div \frac{8x^{-3}}{15y^{-5}}$$

14. Simplify.

$$\frac{4x^2 - 16}{x - 2}$$

15. Simplify.

$$\frac{5x^3y + 20x^2y^2 + 20xy^3}{5xy}$$

16. Simplify.

$$\frac{x^2 + 4x}{x + 3} \cdot \frac{x^2 - 9}{x^2 + x - 12}$$

17. Simplify.

$$\frac{2x^2 - 10x}{x^2 + 8x + 16} \cdot \frac{4x + 16}{x^2 - 25}$$

18. Simplify.

$$\frac{4\,(x + y)}{5x^2y^3} \div \frac{-2x - 2y}{10}$$

19. $i^4 = ?$

20. $(4i)\,(6i) = ?$

21. Simplify by rationalizing the denominator.

$$\frac{2}{3 + i}$$

22. Simplify.

$$(3 + i)(3 - i)$$

23. $(-3 - 2i) - (-5 + i) = ?$

24. What is the difference of $(12 - 5i)$ and $(-3 + 4i)$?

25. If $f(x) = 3x^2 - 4$ and $g(x) = 2x - 6$, what is $g(f(2))$?

26. Find $f(g(x))$ and $g(f(x))$.

$$f(x) = x^2 - 1 \qquad g(x) = x + 3$$

27. Convert $f(x)$ from the vertex form of a parabola to the standard form of a parabola.

$$f(x) = -3(x + 4)^2 - 2$$

28. Graph the parabola $f(x)$ on the attached graph paper. Find and include the vertex, axis of symmetry, the equation for the axis of symmetry and at least two symmetrical points on your graph.

$$f(x) = 2(x - 1)^2 + 5$$

29. What is a discriminant? How can the discriminant of a quadratic equation be used to tell us the number and types of solutions a quadratic function has? Use sketches to explain your answer.

30. Solve the system using substitution.

$$x^2 + y^2 = 10$$
$$y = -3x + 10$$

31. Simplify.

$$(x - 1)(3x - 1)(2x + 1)$$

32. Simplify.

$$(3x^2 + 6x - 1) + (5x^5 + 2x - 4)$$

33. Simplify.

$$(2x^3 + x - 5) - (3x^3 - 4x + 6)$$

34. Factor.

$$-2x^3 + x^2 + 6x$$

35. Find the degree of f(x).

$$f(x) = 4x^2 - 3x + x^3 + 5x^7$$

36. List all possible zeros of g(x).

$$g(x) = x^3 - 8x^2 + 11x + 20$$

37. Solve for x. Check for extraneous solutions.

$$1 - \frac{8}{x-5} = \frac{3}{x}$$

38. Solve for x. Check for extraneous solutions.

$$\frac{6}{x-3} = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$$

39. Solve the quadratic equation by completing the square.

$$x^2 - 2x - 5 = 0$$

40. Factor.

$$36 - x^2$$

41. Factor.

$$25y^2 - 100x^2$$

42. Factor.

$$16x^2 - 8x - 3$$

43. Factor.

$$22x^5y^8 - 99x^4$$

44. Simplify.

$$\frac{x+2}{x^2-9} - \frac{4}{x+3}$$

45. Simplify.

$$\frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}}$$

46. Solve for x. State any restrictions on x. Check for extraneous solutions.

$$\frac{3}{x+1} = \frac{x^2}{x+1}$$

47. List the possible zeros of the function *f(x)*. Using synthetic division, find all of the zeros of the function. Show your work.

$$f(x) = x^3 + 9x^2 + 23x + 15$$

48. Graph *f(x)* on the attached graph paper. Using open interval notation, describe how the function is increasing/decreasing.

$$f(x) = |x-2| + 4$$

49. If it takes 2 hours for hose A to fill a tank and 4 hours for hose B to fill the same tank, how many hours will it take for the two hoses working together to fill the tank?

50. Simplify and graph *f(x)* on the attached graph paper. What happens to your graph at x = -3? Explain.

$$f(x) = \frac{x^2 - 9}{x^2 + 6x + 9}$$

51. When solving the equation $x^2 - 8x = 9$ by completing the square, what number should be added to both sides of the equation to complete the square?

52. Two numbers have the following properties:

The second number is 3 more than the first number.

The product of the two numbers is 9 more than their sum.

Find the numbers.

53. What are the x-intercepts of $y = 12x^2 - 5x - 2$?

54. Simplify. Your final solution should have a common denominator.

$$(x^2)^3 - (x^5)^{-1}$$

55. Graph *f(x)* on the attached graph paper. Find the domain and range of *f(x).*

$$f(x) = x^2 + 4$$

56. Graph $g(x)$ on the attached graph paper. Find the domain and range of $g(x)$

$$g(x) = |x + 4|$$

57. Graph $h(x)$ on the attached graph paper. Find the domain and range of $h(x)$.

$$h(x) = \frac{1}{x}$$

58. Find the slope of the line through points (-1, 2) and (5, -6).

59. Find the equation of the line with a slope of -3 that passes through point (1, -1).

60. Find the equation of the line that is parallel to $y = -\frac{1}{2}x + 5$ and passes through the origin.

61. Find the equation of the line that is perpendicular to $-2x + 5y = 20$ and passes through the point (-3, -3).

62. Solve for x.

$$\frac{3x - 9}{3} = x - 3$$

63. Solve for x.

$$2x - 9 = 2x + 4$$

64. Graph $p(x)$ on the attached graph paper. What type of function is $p(x)$? Describe the asymptote associated with this function and include it on your graph.

$$p(x) = 2^x$$

65. When is $\sqrt{x} > 0$?

66. Find the distance between point (0, -3) and point (5, 5).

*Exact solution. No decimals please.

67. Find the midpoint between point (-4, 4) and point (2, 2).

68. Simplify.

$$(x^2 + 2y^3)^2$$

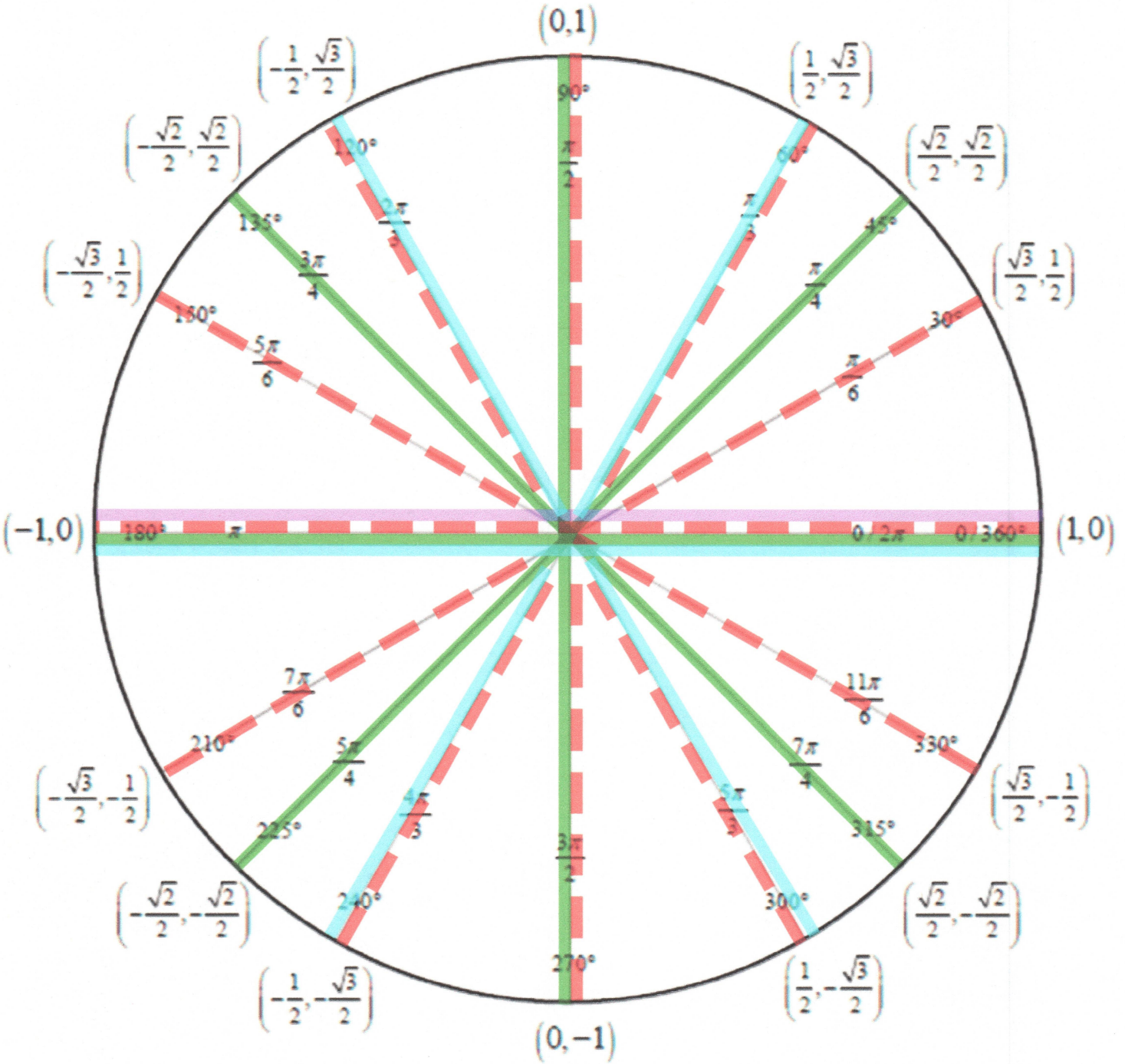
69. Simplify.

$$(3y - 1)^3$$

70. Find $f(x) - g(x)$. Find $g(x) - f(x)$.

$$f(x) = x^2 - 2x - 1$$

$$g(x) = 2\,(x + 1)^2$$


(0,1)
$\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
(−1,0)
(1,0)
$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
(0,−1)
90°
$\frac{\pi}{2}$
120°
$\frac{2\pi}{3}$
60°
$\frac{\pi}{3}$
135°
$\frac{3\pi}{4}$
45°
$\frac{\pi}{4}$
150°
$\frac{5\pi}{6}$
30°
$\frac{\pi}{6}$
180°
π
0 / 2π
0 / 360°
$\frac{7\pi}{6}$
210°
$\frac{11\pi}{6}$
330°
$\frac{5\pi}{4}$
$\frac{7\pi}{4}$
225°
315°
$\frac{4\pi}{3}$
$\frac{5\pi}{3}$
240°
300°
$\frac{3\pi}{2}$
270°

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1. $x = -1,\ y = -2$
2. $6x^3 - 13x^2 - 13x + 20$
3. $-10x^2 + 12x - 1$
4. $36y^3 + 12y^2 - 12y - 4$
5. $15x^5 + 5x^4 - 46x^3 - 2x^2 + 16x$
6. Remember:
   $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
7. Remember:
   $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
8, 9. Did you factor correctly? Expand your factorization and compare your solution with the original expression.
10. $\dfrac{5x^2 + 21x + 30}{5x(x + 5)}$
11. $\dfrac{x^2 - 3x - 2}{(x + 1)(x - 1)}$
12. $\dfrac{3b^3a^4}{x^6c^6}$
13. $\dfrac{25}{18xy^7}$

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14. $4(x+2)$

15. $(x+2y)^2$

16. x

17. $\dfrac{8x}{(x+4)(x+5)}$

18. $\dfrac{-4}{x^2y^3}$

19. 1

20. -24

21. $\dfrac{3-i}{5}$

22. 10

23. $2-3i$

24. $15-9i$

25. 10

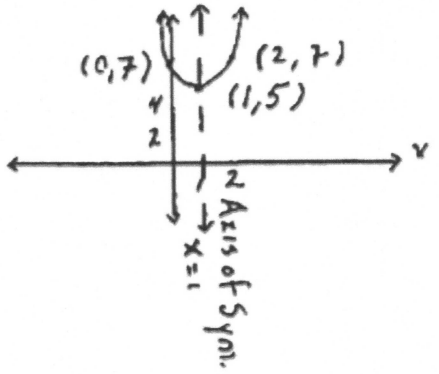
26. $f(g(x)) = x^2+6x+8$

$g(f(x)) = x^2+2$

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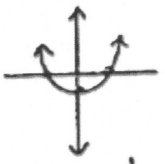
27. $f(x) = -3x^2 - 24x - 50$

28.



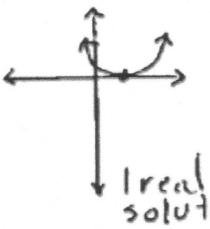
29. Discriminant $= b^2 - 4ac$

When $D > 0$



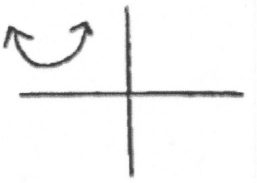
2 real solutions
2 x-intercepts

When $D = 0$



1 real solution
1 x-intercept

When $D < 0$



2 imag. solutions
No x-intercepts

30. $x = 3,\ y = 1$

31. $6x^3 - 5x^2 - 2x + 1$

32. $5x^5 + 3x^2 + 8x - 5$

33. $-1x^3 + 5x - 11$

34. $-x(2x + 3)(x - 2)$

35. Degree of $f(x)$: 7

36. Possible zeros:
$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

37. $x = 15$, $x = 1$

38. $x = \frac{3}{2}$ ~~$x = -3$~~

39. $x = 1 + \sqrt{6}$ $x = 1 - \sqrt{16}$

40-43. To check "factor" problems: simplify your factorization

44. $$\frac{-3x + 14}{(x-3)(x+3)}$$

45. $$\frac{5x}{3x + 8}$$

46. Restrictions on x: $x \neq -1$

Solve for x:

~~$x = -1$~~, $x = \pm\sqrt{3}$

47. Possible zeros: $\pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}$

Zeros: $x = -1, x = -3, x = -5$

48. Decreasing: $(-\infty, 2)$

Increasing: $(2, +\infty)$

49. $1\frac{1}{3}$ hrs.

50. $f(x) = \frac{(x-3)}{(x+3)}$ where $x \neq -3$

51. 16

52. $x = -4$, $y = -1$
or $x = 3$, $y = 6$

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53. x int: $\frac{2}{3}$ x int: $\frac{-1}{4}$

54. $\frac{x^{11} - 1}{x^5}$

55. D: $(-\infty, +\infty)$
R: $[4, +\infty)$

56. D: $(-\infty, +\infty)$
R: $[0, +\infty)$

57. D: $(-\infty, 0) \cup (0, +\infty)$
R: $(-\infty, 0) \cup (0, +\infty)$

58. $m = \frac{-4}{3}$

59. $y = -3x + 2$

60. $y = \frac{-1}{2}x$

61. $y = \frac{-5}{2}x - \frac{21}{2}$

62. $\mathbb{R}$ or All real numbers

63. No solution

64. $p(x)$ is an exponential function
Horizontal asymptote: $y = 0$

65. $\sqrt{x} > 0$ when x values are greater than 0. $x > 0$

66. distance = $\sqrt{89}$

67. midpoint : $(-1, 3)$

68. $x^4 + 9x^2y^3 + 4y^6$

69. $27y^3 - 27y^2 + 9y - 1$

70. $f - g = -x^2 - 6x - 3$

$g - f = x^2 + 6x + 3$