

AP Calculus BC Summer Assignment

Cape Henlopen High School

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Dear BC Calculus Student,

Congratulations on your wisdom in taking the BC course! I know you will find it rewarding and a great way to spend next year. This course is primarily concerned with developing your understanding of the concepts of calculus and providing experience with its methods and applications. The course emphasizes a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. To be successful in this course you need the proper foundation (i.e. knowledge of algebra, geometry, trigonometry, analytic geometry, and elementary functions).

You **MUST** be very familiar with the basic families of functions, and all their representations, to be successful in your study of calculus. The concept of functions underlies everything that calculus considers. You will also need to be able to carry out certain computational tasks (i.e. algebra skills) with efficiency and accuracy. **Most problems are solved without the use of a calculator or Desmos. Do not use either on the summer packet.**

Please do not rely on AI when completing Practice Assignment (including this summer assignment).

Relying heavily on AI in the math classroom can hinder student learning and critical thinking. AI can solve problems quickly, but this can prevent students from developing their own understanding of the concepts and problem-solving processes, potentially leading to a lack of genuine understanding.

This packet is intended to prepare you for a successful year AP Calculus BC and should be taken seriously.

I am providing the graphs of the basic parent function. Make sure you know them and understand basic features of each. Domain, Range, Asymptotes and End Behaviors. **A short matching quiz will be given on the first day of class.**

As a calculus student you must know your unit circle, which I am including as well. In Calculus we do all trig work in radians. A blank copy of the Unit Circle is also included. Make sure you can fill it out in under 15 minutes. **You will be taking a quiz on the Unit Circle the second day of class. Your grade will be either 100 or zero.** You may take the quiz up to three times, but not on the same day.

You will have the opportunity to ask questions on the first day of school regarding any questions on the summer packet. A 45-minute quiz will be given on the third day of school with questions from the summer assignment. **Please let me stress not to use math AI tools, as it will not be of benefit when it is time to take the quiz. Phones will be put in a caddy and iPad put away. Quiz will be non-calculator.**

I look forward to working with each of you. BC Calculus will be challenging but I know we will have a great year of Math learning.

**SUMMER ASSIGNMENT WILL NOT BE COLLECTED OR GRADED.
IT SERVES AS A GUIDE FOR THE ASSESSMENTS GIVEN THE FIRST 3 DAYS OF SCHOOL.**

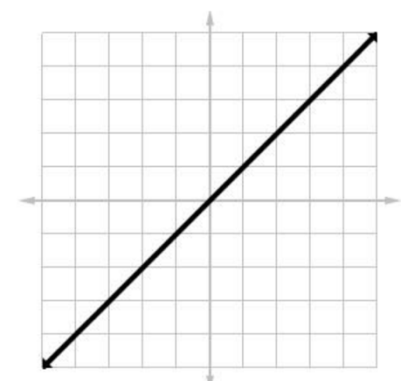
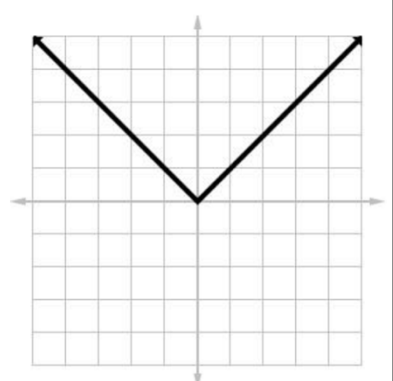
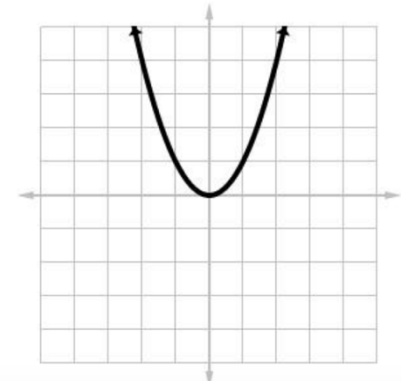
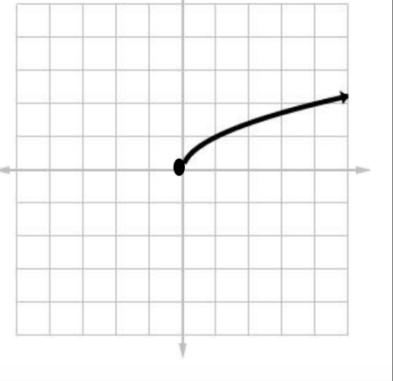
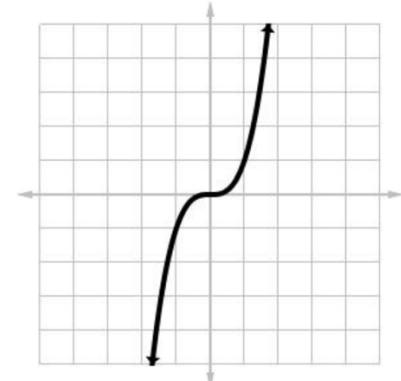
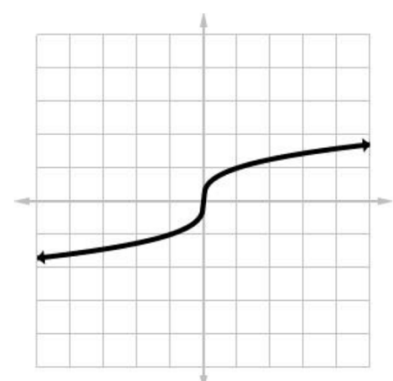
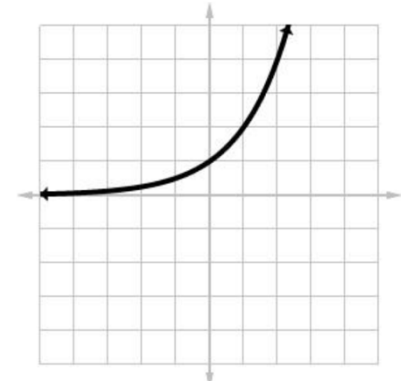
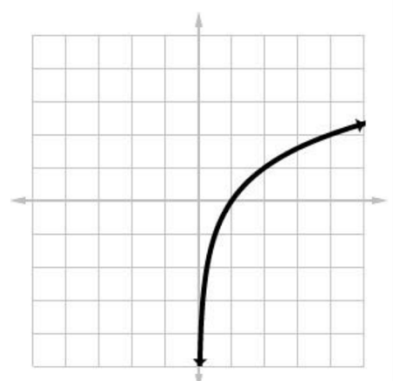
Have a restful summer and be ready to talk math again in August.

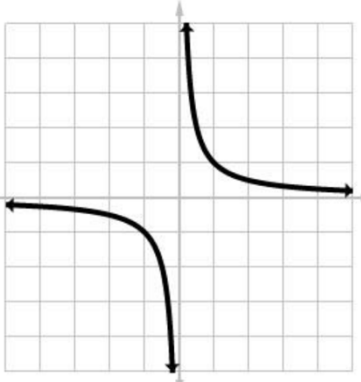
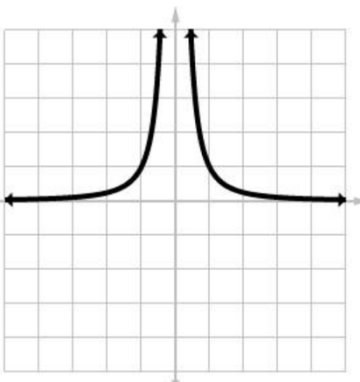
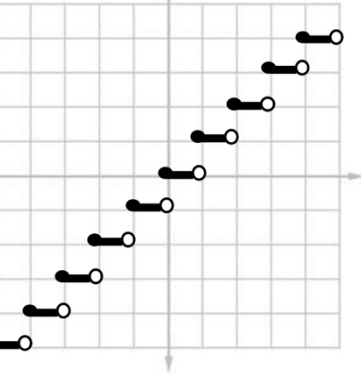
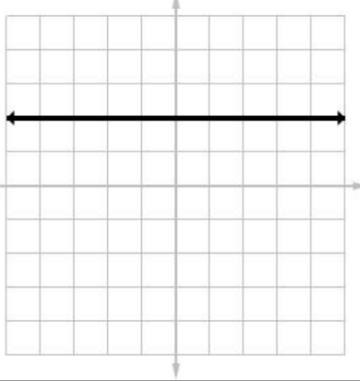
Sincerely,

Mrs. O'Brien

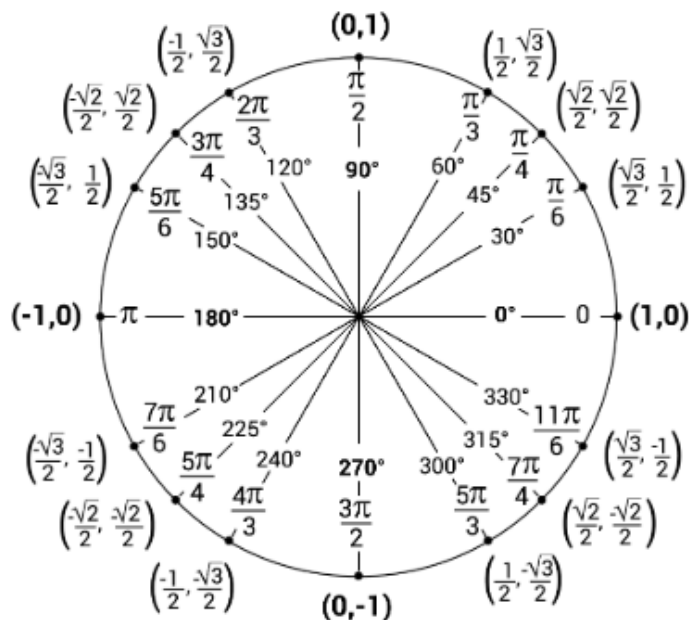
Jacquelyn.Obrien@cape.k12.de.us

10 Basic Parent Functions

Parent Function	Graph		Graph
$y=x$ Linear, Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ End Behavior: $x \rightarrow -\infty, y \rightarrow -\infty$ $x \rightarrow \infty, y \rightarrow \infty$ Critical points: $(-1, -1), (0, 0), (1, 1)$		$y = x $ Absolute Value, Even Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ End Behavior: $x \rightarrow -\infty, y \rightarrow \infty$ $x \rightarrow \infty, y \rightarrow \infty$ Critical points: $(-1, 1), (0, 0), (1, 1)$	
$y=x^2$ Quadratic, Even Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ End Behavior: $x \rightarrow -\infty, y \rightarrow \infty$ $x \rightarrow \infty, y \rightarrow \infty$ Critical points: $(-1, 1), (0, 0), (1, 1)$		$y = \sqrt{x}$ Radical, Neither (Square Root) Domain: $[0, \infty)$ Range: $[0, \infty)$ End Behavior: $x \rightarrow 0, y \rightarrow 0$ $x \rightarrow \infty, y \rightarrow \infty$ Critical points: $(0, 0), (1, 1), (4, 2)$	
$y=x^3$ Cubic, Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ End Behavior: $x \rightarrow -\infty, y \rightarrow -\infty$ $x \rightarrow \infty, y \rightarrow \infty$ Critical points: $(-1, -1), (0, 0), (1, 1)$		$y = \sqrt[3]{x}$ Cube Root, Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ End Behavior: $x \rightarrow -\infty, y \rightarrow -\infty$ $x \rightarrow \infty, y \rightarrow \infty$ Critical points: $(-1, -1), (0, 0), (1, 1)$	
$y = b^x, b > 1$ $(y = 2^x)$ Exponential, Neither Domain: $(-\infty, \infty)$ Range: $(0, \infty)$ End Behavior: $x \rightarrow -\infty, y \rightarrow 0$ $x \rightarrow \infty, y \rightarrow \infty$ Critical points: $(-1, 1/2), (0, 1), (1, b)$ Asymptote: $y = 0$		$y = \log_b x, b > 1$ $(y = \log_2 x)$ Log, Neither Domain: $(0, \infty)$ Range: $(-\infty, \infty)$ End Behavior: $x \rightarrow 0^+, y \rightarrow -\infty$ $x \rightarrow \infty, y \rightarrow \infty$ Critical points: $(1/b, -1), (1, 0), (b, 1)$ Asymptote: $x = 0$	

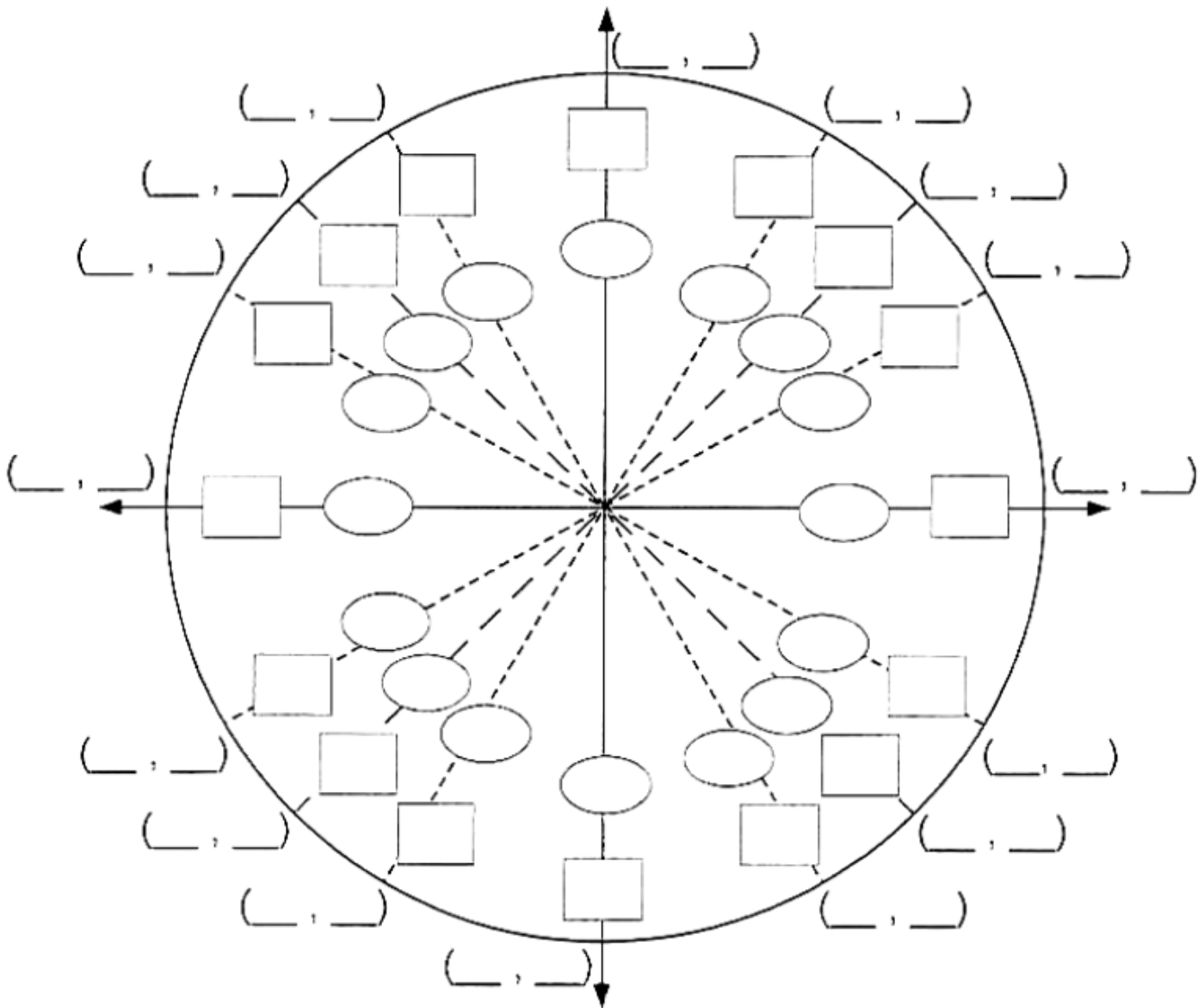
$y = -\frac{1}{x}$ Rational, Odd (Inverse) Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$ End Behavior: $x \rightarrow -\infty, y \rightarrow 0$ $x \rightarrow \infty, y \rightarrow 0$ Critical points: $(-1, -1), (1, 1)$ Asymptotes: $y = 0, x = 0$		$y = \frac{1}{x^2}$ Rational, Even (Inverse Squared) Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(0, \infty)$ End Behavior: $x \rightarrow -\infty, y \rightarrow 0$ $x \rightarrow \infty, y \rightarrow 0$ Critical points: $(-1, 1), (1, 1)$ Asymptotes: $x = 0, y = 0$	
$y = \text{int}(x) = \llbracket x \rrbracket$ Greatest Integer, Neither Domain: $(-\infty, \infty)$ Range: $\{y: y \in \mathbb{Z}\}$ ($\mathbb{Z}$ = integers) End Behavior: $x \rightarrow -\infty, y \rightarrow -\infty$ $x \rightarrow \infty, y \rightarrow \infty$ Critical points: $x: [-1, 0) \quad y: -1$ $x: [0, 1) \quad y: 0$ $x: [1, 2) \quad y: 1$		$y = C$ ($y = 2$) Constant, Even Domain: $(-\infty, \infty)$ Range: $\{y: y = C\}$ End Behavior: $x \rightarrow -\infty, y \rightarrow C$ $x \rightarrow \infty, y \rightarrow C$ Critical points: $(-1, C), (0, C), (1, C)$	

The Unit Circle



Video Link: An Easy Way to Memorize the Unit Circle: <https://www.youtube.com/watch?v=ao4EJzNWmK8>

THE UNIT CIRCLE



1. Fill out the chart.

	$0, 2\pi$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin x$							
$\cos x$							
$\tan x$							

2. Find the exact value.

a. $\sin \frac{11\pi}{2}$

b. $\cos(-3\pi)$

c. $\tan(-6\pi)$

d. $\cot \frac{7\pi}{2}$

e. $\csc \frac{11\pi}{3}$

f. $\sec\left(-\frac{3\pi}{4}\right)$

Reciprocal Identities:	$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$
Quotient Identities:	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$	
Pythagorean Identities:	$\sin^2 \theta + \cos^2 \theta = 1$	$\tan^2 \theta + 1 = \sec^2 \theta$	$\cot^2 \theta + 1 = \csc^2 \theta$
Even/Odd Identities:	$\sin(-\theta) = -\sin \theta$ $\csc(-\theta) = -\csc \theta$	$\cos(-\theta) = \cos \theta$ $\sec(-\theta) = \sec \theta$	$\tan(-\theta) = -\tan \theta$ $\cot(-\theta) = -\cot \theta$

3. Simplify.

a. $\frac{\tan \theta}{\sec \theta}$

b. $\frac{\cot \theta}{\csc \theta}$

c. $\frac{(\sin^2 \theta + \cos^2 \theta)}{\csc \theta}$

d. $\tan \theta \csc \theta$

e. $\frac{\sin(-\theta)}{\cos(-\theta)}$

4. Solve each equation on the interval $0 \leq \theta \leq 2\pi$.

a. $2 \sin \theta + 3 = 2$

b. $1 - \cos \theta = \frac{1}{2}$

c. $4 \cos^2 \theta = 1$

d. $2 \sin x \sec x = \sec x$

e. $3 \sec^2 x - 4 = 0$

f. $4 \sin \theta + 3\sqrt{3} = \sqrt{3}$

g. $\sqrt{3} \cot \theta + 1 = 0$

h. $4 \tan \theta + 4 = 0$

To solve a polynomial inequality:

- Rearrange the inequality so that it is of the form: $f(x) < 0$, $f(x) > 0$, $f(x) \leq 0$, or $f(x) \geq 0$
- Find the “critical numbers” by finding where $f(x) = 0$.
- Use the critical numbers to divide the real number line into regions.
- Test each region by evaluating the original inequality using any number within the region.
- Write the solution set using interval notation (be careful when choosing hard or soft brackets).

5. Solve the inequality and graph the solution set.

a. $x^3 - 4x \geq 0$

b. $x^2 + 7x < -12$

To solve a rational inequality:

- Find the “critical numbers” by finding where $f/g = 0$ and where f/g is undefined.
- Use the critical numbers to divide the real number line into regions.
- Test each region by evaluating the original inequality using any number within the region.
- Write the solution set using interval notation (be careful when choosing hard or soft brackets).

6. Solve the inequality. Graph the result and write the answer in interval notation.

a. $\frac{x-1}{2x+8} < 0$

b. $\frac{x^2 + 6x + 8}{x} < 0$

Operation	Laws of exponents	Laws of logs
Multiplication	$x^m \cdot x^n = x^{m+n}$	$\log(a \cdot b) = \log(a) + \log(b)$
Division	$\frac{x^m}{x^n} = x^{m-n}$	$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
Exponentiation	$(x^m)^n = x^{mn}$	$\log(a^n) = n \cdot \log(a)$ One of the most useful properties of logs
Zero property	$x^0 = 1$	$\log(1) = 0$
Inverse	$x^{-1} = \frac{1}{x}$	$\log(x^{-1}) = \log\left(\frac{1}{x}\right) = -\log(x)$

$$y = \log_a x \text{ is equivalent to } a^y = x$$

7. Convert each exponential equation into its logarithmic form.

a. $4^3 = a$

b. $b^3 = 27$

c. $e^x = 1$

8. Convert each log equation into its exponential form. Then solve for x.

a. $4 = \log_3 x$

b. $x = \log_5 125$

c. $-1 = \log_x 6$

9. Find the exact value of each logarithm.

a. $\log_2 32$

b. $\log_5 \frac{1}{125}$

c. $\log_{10} 0.01$

10. Solve each logarithmic equation.

a. $\log_4 (2x-1) = 3$

b. $\log_x 81 = 2$

11. Solve the exponential equation.

a. $e^{3x} = 6$

b. $e^{-2x} = \frac{1}{3}$

12. Writing a log expression as a sum, difference, or multiple of logs – AKA Expanding a log expression.

a. $\log_a (3xy^2)$

b. $\log_a (x\sqrt{x+3})$

c. $\log_a \frac{(x^2-4)}{(x+1)}$

Exponential Functions and Logarithmic Functions are inverses of each other.

13. Simplify.

a. $e^{\ln(x+3)}$

b. $\ln e^{2.4}$

c. $3 \ln e^2$

SEQUENCES: a function whose domain is the set of natural numbers expressed as a_n instead of $f(x)$, where n is the input and a_n is the output.

14. Write the first 3 terms of the sequence.

a. $a_n = \frac{2n+1}{3n}$

b. $a_n = \frac{(-1)^n}{2n-1}$

SUMMATION/SIGMA NOTATION: The sum of the first n terms of a sequence $\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \dots + a_n$

where i is called the index of summation, n is the upper limit of summation, and 1 is the lower limit of summation.

15. Find the sum of each sequence.

a. $\sum_{n=1}^5 (2n + 3)$

b. $\sum_{i=0}^4 3i^2$

FACTORIALS: If $n \geq 0$ is an integer, the factorial $n!$ is defined as $0! = 1$ $1! = 1$ $n! = n(n-1)(n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$

16. Evaluate the following factorials. **A calculator can be used on parts b and c due to large numbers.**

a. $5!$

b. $\frac{9!}{4!}$

c. $\frac{7!2!}{3!}$

Absolute Value Inequalities:

- Isolate the Absolute Value Expression
- Drop the absolute value and create a positive inequality and a negative inequality (flip the sign)
- Solve each inequality separately
- Graph.

16. Graph the solutions to each absolute value inequality.

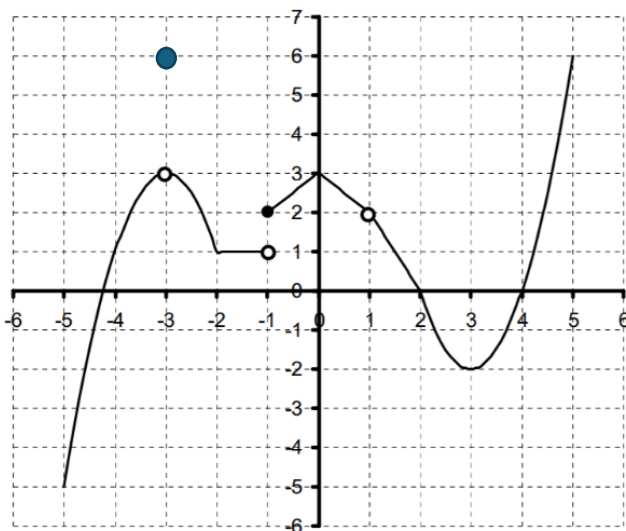
a. $|x| < 3$

b. $|x - 2| - 5 \leq 9$

c. $|3x - 6| \geq 9$

Limits

17. Use the graph of $f(x)$ to find the limits and $f(x)$.



- a. $\lim_{x \rightarrow -3^-} f(x)$ $\lim_{x \rightarrow -3^+} f(x)$ $\lim_{x \rightarrow -3} f(x)$ $f(-3)$
- b. $\lim_{x \rightarrow -1^-} f(x)$ $\lim_{x \rightarrow -1^+} f(x)$ $\lim_{x \rightarrow -1} f(x)$ $f(-1)$
- c. $\lim_{x \rightarrow 0^-} f(x)$ $\lim_{x \rightarrow 0^+} f(x)$ $\lim_{x \rightarrow 0} f(x)$ $f(0)$
- d. $\lim_{x \rightarrow 1^-} f(x)$ $\lim_{x \rightarrow 1^+} f(x)$ $\lim_{x \rightarrow 1} f(x)$ $f(1)$
- e. $\lim_{x \rightarrow 4^-} f(x)$ $\lim_{x \rightarrow 4^+} f(x)$ $\lim_{x \rightarrow 4} f(x)$ $f(4)$

18. Find each limit.

a. $\lim_{x \rightarrow -1} \frac{x^2 - 5x - 6}{x^2 - 1}$

b. $\lim_{x \rightarrow -1} \frac{x-1}{x^2-1}$

c. $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x^2-1}$

19. If $f(x) = \begin{cases} x+3, & x < 0 \\ x-3, & x \geq 0 \end{cases}$ evaluate,

a. $\lim_{x \rightarrow 0^-} f(x)$

b. $\lim_{x \rightarrow 0^+} f(x)$

c. $\lim_{x \rightarrow 0} f(x)$

20. Find each limit.

a. $\lim_{x \rightarrow 0^+} \left(\sin x + \frac{1}{x} \right)$

b. $\lim_{x \rightarrow 0^+} 3 \ln x$

c. $\lim_{x \rightarrow 1} \frac{x^2 - 3x}{x - 1}$

d. $\lim_{x \rightarrow 1^-} \frac{x^2 - 4}{x^2 - 3x + 2}$

21. Find each limit.

a. $f(x) = \frac{3x^2 + 4x + 3}{x^3 + x + 14}$

b. $g(x) = \frac{2x^2 + 6}{3x^2 - 4x + 1}$

7. $j(x) = \frac{2x^2 - 3x + 5}{x - 2}$

$\lim_{x \rightarrow \infty} f(x) =$

$\lim_{x \rightarrow \infty} g(x) =$

$\lim_{x \rightarrow \infty} j(x) =$

$\lim_{x \rightarrow -\infty} f(x) =$

$\lim_{x \rightarrow -\infty} g(x) =$

$\lim_{x \rightarrow -\infty} j(x) =$

DOD:

22. Use the Definition of the Derivative (DOD) to answer the following.

a. $\lim_{\Delta x \rightarrow 0} \frac{\cos\left(\frac{\pi}{3} + \Delta x\right) - \cos\left(\frac{\pi}{3}\right)}{\Delta x}$

b. $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

c. $\lim_{x \rightarrow -3} \frac{(4x^2 + 2x) - (30)}{x + 3}$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

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The Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$.

23. Find $f'(x)$.

a. $f(x) = x^3$

b. $g(x) = 5x^2$

c. $y = \frac{1}{x^2}$

d. $k(x) = \frac{7}{x}$

e. $y = \frac{2}{3}x^4$

f. $y = -3\sqrt{x}$

g. $y = \frac{4}{\sqrt{x}}$

h. $g(x) = 2\sqrt[5]{x}$

BASIC Derivatives

23. Find $f'(x)$.

a. $f(x) = x^2 + 3e^x + \ln x$

b. $y = \sin x - e^x$

c. $f(x) = 3^x - 4 \cos x$

d. $g(x) = -\sin x - \log_5 x$

e. $y = x - \tan x$

f. $y = 7x - \sec x$

g. $h(x) = \cot x + \csc x$

h. $f(x) = 5\sin^{-1}x$

g. $y = \tan^{-1}x$