Upper St. Clair High School Summer Assignment	
Course: Honors Precalculus	
Assignment Title:	Honors Precalculus Summer Assignment
Due Date:	See note
Purpose of the assignment:	☑ Review foundational material, concepts, or skills ☐ Build interest ☐ Provide a diagnostic measuring tool to help guide student differentiation & instruction ☐ Establish a supportive class culture & environment ☐ Expose students to required material, concepts, skills, and/or texts that will be referenced throughout the academic year
Resources needed to complete assignment:	☐ Textbook assigned by school ☐ Book(s) supplied by student ☑ Packet provided by teacher (please visit the Interactive Program of Studies) ☐ Other supplies: _____
Estimated time for Completion:	Students may average an estimated 4-6 hours hours to complete the assignment, but we encourage students to spend time on this assignment over several weeks.
Impact on Semester 1 Grade	Graded: ☐ Yes ☑ No Teacher Note: Although this assignment will not be collected, mastery of these Algebra 2 concepts is expected. Please be aware that this material will be included on your first unit exam.

Honors Precalculus

Summer Assignment

- Please use the following pages to review your Algebra 2 skills.
- Check your answers with the key included at the end of the packet.
- This assignment *will not be collected* in class but, you should be prepared for a quiz on these topics during the first few weeks of school.

ALGEBRA REVIEW

This appendix reviews the fundamental algebraic facts that are used frequently in this book. You must be able to handle these algebraic manipulations in order to succeed in this course and in calculus.

A.1 Integral Exponents

Exponents provide a convenient shorthand for certain products. If c is a real number, then c^2 denotes cc and c^3 denotes ccc. More generally, for any positive integer n

c^n denotes the product $ccc \cdots c$ (n factors).

In this notation c^1 is just c, so we usually omit the exponent 1.

Example 1

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$ and

$$(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32.$$

For every positive integer n, $0^n = 0 \cdots 0 = 0$.

Example 2

To find $(2.4)^9$ use the $\wedge$ (or a^b or x^y) key on your calculator:*

2.4 $\wedge$ 9 ENTER*

which produces the (approximate) answer 2641.80754.

Because exponents are just shorthand for multiplication, it is easy to determine the rules they obey. For instance,

$$c^3c^5 = (ccc)(ccccc) = c^8, \quad \text{that is,} \quad c^3c^5 = c^{3+5}.$$

$$\frac{c^7}{c^4} = \frac{ccccccc}{cccc} = \frac{\cancel{cccc}ccc}{\cancel{cccc}} = ccc = c^3, \quad \text{that is,} \quad \frac{c^7}{c^4} = c^{7-4}.$$

*The ENTER key is labeled EXE on Casio calculators.

Similar arguments work in the general case:

To multiply c^m by c^n, add the exponents: $c^m c^n = c^{m+n}$.

To divide c^m by c^n, subtract the exponents: $\frac{c^m}{c^n} = c^{m-n}$.

Example 3

$4^2 \cdot 4^7 = 4^{2+7} = 4^9$ and $\frac{2^8}{2^3} = 2^{8-3} = 2^5$.

The notation c^n can be extended to the cases when n is zero or negative as follows:

If $c \neq 0$, then c^0 is defined to be the number 1.

If $c \neq 0$ and n is a positive integer, then

$$c^{-n} \text{ is defined to be the number } \frac{1}{c^n}.$$

Note that 0^0 and negative powers of 0 are *not* defined (negative powers of 0 would involve division by 0). The reason for choosing these definitions of c^{-n} for nonzero c is that the multiplication and division rules for exponents remain valid. For instance,

$$c^5 \cdot c^0 = c^5 \cdot 1 = c^5, \quad \text{so that} \quad c^5 c^0 = c^{5+0}.$$

$$c^7 c^{-7} = c^7\left(\frac{1}{c^7}\right) = 1 = c^0, \quad \text{so that} \quad c^7 c^{-7} = c^{7-7}.$$

Example 4

$6^{-3} = \frac{1}{6^3} = \frac{1}{216}$ and $(-2)^{-5} = \frac{1}{(-2)^5} = -\frac{1}{32}$. A calculator shows that $(0.287)^{-12} \approx 3{,}201{,}969.857$.*

If c and d are nonzero real numbers and m and n are integers (positive, negative, or zero), then we have these

Exponent Laws

1. $c^m c^n = c^{m+n}$
2. $\frac{c^m}{c^n} = c^{m-n}$
3. $(c^m)^n = c^{mn}$
4. $(cd)^n = c^n d^n$
5. $\left(\frac{c}{d}\right)^n = \frac{c^n}{d^n}$
6. $c^{-n} = \frac{1}{c^n}$

* $\approx$ means "approximately equal to."

Example 5

Here are examples of each of the six exponent laws.

1. $\pi^{-5}\pi^2 = \pi^{-5+2} = \pi^{-3} = \dfrac{1}{\pi^3}$.
2. $\dfrac{x^9}{x^4} = x^{9-4} = x^5$.
3. $(5^{-3})^2 = 5^{(-3)2} = 5^{-6}$.
4. $(2x)^5 = 2^5x^5 = 32x^5$.
5. $\left(\dfrac{7}{3}\right)^{10} = \dfrac{7^{10}}{3^{10}}$.
6. $\dfrac{1}{x^{-5}} = \dfrac{1}{\left(\frac{1}{x^5}\right)} = x^5$.

CAUTION

$(2x)^5$ is not the same as $2x^5$. Part 4 of Example 5 shows that $(2x)^5 = 32x^5$ and not $2x^5$.

The exponent laws can often be used to simplify complicated expressions.

Example 6

a. $(2x^2y^3z)^4 = 2^4(x^2)^4(y^3)^4z^4 = 16x^8y^{12}z^4$.
Law (4) Law (3)

b. $(r^{-3}s^2)^{-2} = (r^{-3})^{-2}(s^2)^{-2} = r^6s^{-4} = \dfrac{r^6}{s^4}$.
Law (4) Law (3)

c. $\dfrac{x^5(y^2)^3}{(x^2y)^2} = \dfrac{x^5y^6}{(x^2y)^2} = \dfrac{x^5y^6}{(x^2)^2y^2} = \dfrac{x^5y^6}{x^4y^2} = x^{5-4}y^{6-2} = xy^4$. ■
Law (3) Law (4) Law (3) Law (2)

It is usually more efficient to use the exponent laws with the negative exponents rather than first converting to positive exponents. If positive exponents are required, the conversion can be made in the last step.

Example 7

Simplify and express without negative exponents

$$\frac{a^{-2}(b^2c^3)^{-2}}{(a^{-3}b^{-5})^2c}.$$

Solution

$$\frac{a^{-2}(b^2c^3)^{-2}}{(a^{-3}b^{-5})^2c} = \frac{a^{-2}(b^2)^{-2}(c^3)^{-2}}{(a^{-3})^2(b^{-5})^2c} = \frac{a^{-2}b^{-4}c^{-6}}{a^{-6}b^{-10}c}$$
Law (4) Law (3)

$$= a^{-2-(-6)}b^{-4-(-10)}c^{-6-1} = a^4b^6c^{-7} = \frac{a^4b^6}{c^7}.$$
Law (2)

Since $(-1)(-1) = +1$, any even power of -1, such as $(-1)^4$ or $(-1)^{12}$, will be equal to 1. Every odd power of -1 is equal to -1; for instance $(-1)^5 = (-1)^4(-1) = 1(-1) = -1$. Consequently, for every positive number c

$$(-c)^n = [(-1)c]^n = (-1)^n c^n = \begin{cases} c^n & \text{if } n \text{ is even} \\ -c^n & \text{if } n \text{ is odd} \end{cases}.$$

Example 8

$(-3)^4 = 3^4 = 81$ and $(-5)^3 = -5^3 = -125$.

CAUTION

Be careful with negative bases. For instance, if you want to compute $(-12)^4$, which is a positive number, but you key in (−) 12 ^ 4 ENTER the calculator will interpret this as $-(12^4)$ and produce a negative answer. To get the correct answer, you must key in the parentheses:

((−) 12) ^ 4 ENTER.

Exercises A.1

In Exercises 1–18, evaluate the expression.

1. $(-6)^2$
2. -6^2
3. $5 + 4(3^2 + 2^3)$
4. $(-3)2^2 + 4^2 - 1$
5. $\dfrac{(-3)^2 + (-2)^4}{-2^2 - 1}$
6. $\dfrac{(-4)^2 + 2}{(-4)^2 - 7} + 1$
7. $\left(\dfrac{-5}{4}\right)^3$
8. $-\left(\dfrac{7}{4} + \dfrac{3}{4}\right)^2$
9. $\left(\dfrac{1}{3}\right)^3 + \left(\dfrac{2}{3}\right)^3$
10. $\left(\dfrac{5}{7}\right)^2 + \left(\dfrac{2}{7}\right)^2$
11. $2^4 - 2^7$
12. $3^3 - 3^{-7}$
13. $(2^{-2} + 2)^2$
14. $(3^{-1} - 3^3)^2$
15. $2^2 \cdot 3^{-3} - 3^2 \cdot 2^{-3}$
16. $4^3 \cdot 5^{-2} + 4^2 \cdot 5^{-1}$
17. $\dfrac{1}{2^3} + \dfrac{1}{2^{-4}}$
18. $3^2\left(\dfrac{1}{3} + \dfrac{1}{3^{-2}}\right)$

In Exercises 19–38, simplify the expression. Each letter represents a nonzero real number and should appear at most once in your answer.

19. $x^2 \cdot x^3 \cdot x^5$
20. $y \cdot y^4 \cdot y^6$
21. $(0.03)y^2 \cdot y^7$
22. $(1.3)z^3 \cdot z^5$
23. $(2x^2)^3 3x$
24. $(3y^3)^4 5y^2$
25. $(3x^2y)^2$
26. $(2xy^3)^3$
27. $(a^2)(7a)(-3a^3)$
28. $(b^3)(-b^2)(3b)$
29. $(2w)^3(3w)(4w)^2$
30. $(3d)^4(2d)^2(5d)$
31. $a^{-2}b^3a^3$
32. $c^4d^5c^{-3}$
33. $(2x)^{-2}(2y)^3(4x)$
34. $(3x)^{-3}(2y)^{-2}(2x)$
35. $(-3a^4)^2(9x^3)^{-1}$
36. $(2y^3)^3(3y^2)^{-2}$
37. $(2x^2y)^0(3xy)$
38. $(3x^2y^4)^0$

In Exercises 39–42, express the given number as a power of 2.

39. $(64)^2$
40. $\left(\dfrac{1}{8}\right)^3$
41. $(2^4 \cdot 16^{-2})^3$
42. $\left(\dfrac{1}{2}\right)^{-8}\left(\dfrac{1}{4}\right)^4\left(\dfrac{1}{16}\right)^{-3}$

In Exercises 43–60, simplify and write the given expression without negative exponents. All letters represent nonzero real numbers.

43. $\dfrac{x^4(x^2)^3}{x^3}$

44. $\left(\dfrac{z^2}{t^3}\right)^4 \cdot \left(\dfrac{z^3}{t}\right)^5$

45. $\left(\dfrac{e^6}{c^4}\right)^2 \cdot \left(\dfrac{c^3}{e}\right)^3$

46. $\left(\dfrac{x^7}{y^6}\right)^2 \cdot \left(\dfrac{y^2}{x}\right)^4$

47. $\left(\dfrac{ab^2c^3d^4}{abc^2d}\right)^2$

48. $\dfrac{(3x)^2(y^2)^3x^2}{(2xy^2)^3}$

49. $\left(\dfrac{a^6}{b^{-4}}\right)^2$

50. $\left(\dfrac{x^{-2}}{y^{-2}}\right)^2$

51. $\left(\dfrac{c^5}{d^{-3}}\right)^{-2}$

52. $\left(\dfrac{x^{-1}}{2y^{-1}}\right)\left(\dfrac{2y}{x}\right)^{-2}$

53. $\left(\dfrac{3x}{y^2}\right)^{-3}\left(\dfrac{-x}{2y^3}\right)^2$

54. $\left(\dfrac{5u^2v}{2uv^2}\right)^2\left(\dfrac{-3uv}{2u^2v}\right)^{-3}$

55. $\dfrac{(a^{-3}b^2c)^{-2}}{(ab^{-2}c^3)^{-1}}$

56. $\dfrac{(-2cd^2e^{-1})^3}{(5c^{-3}de)^{-2}}$

57. $(c^{-1}d^{-2})^{-3}$

58. $[(x^2y^{-1})^2]^{-3}$

59. $a^2(a^{-1} + a^{-3})$

60. $\dfrac{a^{-2}}{b^{-2}} + \dfrac{b^2}{a^2}$

In Exercises 61–66, determine the sign of the given number without calculating the product.

61. $(-2.6)^3(-4.3)^{-2}$

62. $(4.1)^{-2}(2.5)^{-3}$

63. $(-1)^9(6.7)^5$

64. $(-4)^{12}6^9$

65. $(-3.1)^{-3}(4.6)^{-6}(7.2)^7$

66. $(45.8)^{-7}(-7.9)^{-9}(-8.5)^{-4}$

In Exercises 67–72, r, s, and t are positive integers and a, b, and c are nonzero real numbers. Simplify and write the given expression without negative exponents.

67. $\dfrac{3^{-r}}{3^{-s-r}}$

68. $\dfrac{4^{-(t+1)}}{4^{2-t}}$

69. $\left(\dfrac{a^6}{b^{-4}}\right)^t$

70. $\dfrac{c^{-t}}{(6b)^{-s}}$

71. $\dfrac{(c^{-r}b^s)^t}{(c^tb^{-s})^r}$

72. $\dfrac{(a^rb^{-s})^{-t}}{(b^tc^r)^{-s}}$

In Exercises 73–80, give an example to show that the statement may be false for some numbers.

73. $a^r + b^r = (a + b)^r$

74. $a^ra^s = a^{rs}$

75. $a^rb^s = (ab)^{r+s}$

76. $c^{-r} = -c^r$

77. $\dfrac{c^r}{c^s} = c^{\frac{r}{s}}$

78. $(a + 1)(b + 1) = ab + 1$

79. $(-a)^2 = -a^2$

80. $(-a)(-b) = -ab$

A.2 Arithmetic of Algebraic Expressions

Expressions such as

$$b + 3c^2, \qquad 3x^2 - 5x + 4, \qquad \sqrt{x^3 + z}, \qquad \frac{x^3 + 4xy - \pi}{x^2 + xy}$$

are called **algebraic expressions.** Each expression represents a number that is obtained by performing various algebraic operations (such as addition or taking roots) on one or more numbers, some of which may be denoted by letters.

A letter that denotes a particular real number is called a **constant;** its value remains unchanged throughout the discussion. For example, the Greek letter π has long been used to denote the number $3.14159\cdots$. Sometimes a constant is a fixed but unspecified real number, as in "an angle of k degrees" or "a triangle with base of length b."

A letter that can represent *any* real number is called a **variable.** In the expression $2x + 5$, for example, the variable x can be any real

number. If $x = 3$, then $2x + 5 = 2 \cdot 3 + 5 = 11$. If $x = \frac{1}{2}$, then $2x + 5 = 2 \cdot \frac{1}{2} + 5 = 6$, and so on.*

Constants are usually denoted by letters near the beginning of the alphabet and variables by letters near the end of the alphabet. Consequently, in expressions such as $cx + d$ and $cy^2 + dy$, it is understood that c and d are constants and x and y are variables.

The usual rules of arithmetic are valid for algebraic expressions:

Commutative Laws:

$$a + b = b + a \quad \text{and} \quad ab = ba$$

Associative Laws:

$$(a + b) + c = a + (b + c) \quad \text{and} \quad (ab)c = a(bc)$$

Distributive Laws:

$$a(b + c) = ab + ac \quad \text{and} \quad (b + c)a = ba + ca.$$

Example 1

Use the distributive law to *combine like terms;* for instance,

$$3x + 5x + 4x = (3 + 5 + 4)x = 12x.$$

In practice, you do the middle part in your head and simply write $3x + 5x + 4x = 12x$.

Example 2

In more complicated expressions, eliminate parentheses, use the commutative law to group like terms together, and then combine them.

$$(a^2b - 3\sqrt{c}) + (5ab + 7\sqrt{c}) + 7a^2b = a^2b - 3\sqrt{c} + 5ab + 7\sqrt{c} + 7a^2b$$

$$\textit{Regroup:} \qquad = \underbrace{a^2b + 7a^2b}\ \underbrace{- 3\sqrt{c} + 7\sqrt{c}} + 5ab$$

$$\textit{Combine like terms:} \qquad = \quad 8a^2b \quad + \quad 4\sqrt{c} \quad + 5ab.$$

CAUTION

Be careful when parentheses are preceded by a minus sign: $-(b + 3) = -b - 3$ and not $-b + 3$. Here's the reason: $-(b + 3)$ means $(-1)(b + 3)$, so that by the distributive law,

$$-(b + 3) = (-1)(b + 3) = (-1)b + (-1)3 = -b - 3.$$

Similarly, $-(7 - y) = -7 - (-y) = -7 + y$.

*We assume any conditions on the constants and variables necessary to guarantee that an algebraic expression does represent a real number. For instance, in $\sqrt{z}$ we assume $z \geq 0$ and in $\frac{1}{c}$ we assume $c \neq 0$.

The examples in the Caution Box illustrate the following.

Rules for Eliminating Parentheses

Parentheses preceded by a plus sign (or no sign) may be deleted.

Parentheses preceded by a minus sign may be deleted *if* the sign of every term within the parentheses is changed.

The usual method of multiplying algebraic expressions is to use the distributive laws repeatedly, as shown in the following examples. The net result is to *multiply every term in the first sum by every term in the second sum.*

Example 3

To compute $(y - 2)(3y^2 - 7y + 4)$, we first apply the distributive law, treating $(3y^2 - 7y + 4)$ as a single number:

$$(y - 2)(3y^2 - 7y + 4) = y(3y^2 - 7y + 4) - 2(3y^2 - 7y + 4)$$

Distributive law: $$= 3y^3 - 7y^2 + 4y - 6y^2 + 14y - 8$$

Regroup: $$= 3y^3 \underbrace{- 7y^2 - 6y^2} \underbrace{+ 4y + 14y} - 8$$

Combine like terms: $$= 3y^3 - 13y^2 + 18y - 8.$$

Example 4

We follow the same procedure with $(2x - 5y)(3x + 4y)$:

$$\begin{aligned}(2x - 5y)(3x + 4y) &= 2x(3x + 4y) - 5y(3x + 4y)\\ &= 2x \cdot 3x + 2x \cdot 4y + (-5y) \cdot 3x + (-5y) \cdot 4y\\ &= 6x^2 \underbrace{+ 8xy - 15xy} - 20y^2\\ &= 6x^2 - 7xy - 20y^2.\end{aligned}$$

Observe the pattern in the second line of Example 4 and its relationship to the terms being multiplied:

$$(2x - 5y)(3x + 4y) = 2x \cdot 3x + 2x \cdot 4y + (-5y) \cdot 3x + (-5y) \cdot 4y$$

First terms

$(2x - 5y)(3x + 4y)$

Outside terms

$(2x - 5y)(3x + 4y)$

Inside terms

$(2x - 5y)(3x + 4y)$

Last terms

CAUTION

The FOIL method can be used only when multiplying two expressions that each have two terms.

This pattern is easy to remember by using the acronym FOIL (First, Outside, Inside, Last). The FOIL method makes it easy to find products such as this one mentally, without the necessity of writing out the intermediate steps.

Example 5

$$(3x + 2)(x + 5) = 3x^2 + 15x + 2x + 10 = 3x^2 + 17x + 10.$$

First Outside Inside Last

Exercises A.2

In Exercises 1–54, perform the indicated operations and simplify your answer.

1. $x + 7x$
2. $5w + 7w - 3w$
3. $6a^2b + (-8b)a^2$
4. $-6x^3\sqrt{t} + 7x^3\sqrt{t} + 15x^3\sqrt{t}$
5. $(x^2 + 2x + 1) - (x^3 - 3x^2 + 4)$
6. $\left[u^4 - (-3)u^3 + \frac{u}{2} + 1\right] + \left(u^4 - 2u^3 + 5 - \frac{u}{2}\right)$
7. $\left[u^4 - (-3)u^3 + \frac{u}{2} + 1\right] - \left(u^4 - 2u^3 + 5 - \frac{u}{2}\right)$
8. $\left(6a^2b + 3a\sqrt{c} - 5ab\sqrt{c}\right) + \left(-6ab^2 - 3ab + 6ab\sqrt{c}\right)$
9. $[4z - 6z^2w - (-2)z^3w^2] + (8 - 6z^2w - zw^3 + 4z^3w^2)$
10. $(x^5y - 2x + 3xy^3) - (-2x - x^5y + 2xy^3)$
11. $(9x - x^3 + 1) - [2x^3 + (-6)x + (-7)]$
12. $\left(x - \sqrt{y} - z\right) - \left(x + \sqrt{y} - z\right) - \left(\sqrt{y} + z - x\right)$
13. $(x^2 - 3xy) - (x + xy) - (x^2 + xy)$
14. $2x(x^2 + 2)$
15. $(-5y)(-3y^2 + 1)$
16. $x^2y(xy - 6xy^2)$
17. $3ax(4ax - 2a^2y + 2ay)$
18. $2x(x^2 - 3xy + 2y^2)$
19. $6z^3(2z + 5)$
20. $-3x^2(12x^6 - 7x^5)$
21. $3ab(4a - 6b + 2a^2b)$
22. $(-3ay)(4ay - 5y)$
23. $(x + 1)(x - 2)$
24. $(x + 2)(2x - 5)$
25. $(-2x + 4)(-x - 3)$
26. $(y - 6)(2y + 2)$
27. $(y + 3)(y + 4)$
28. $(w - 2)(3w + 1)$
29. $(3x + 7)(-2x + 5)$
30. $(ab + 1)(a - 2)$
31. $(y - 3)(3y^2 + 4)$
32. $(y + 8)(y - 8)$
33. $(x + 4)(x - 4)$
34. $(3x - y)(3x + y)$
35. $(4a + 5b)(4a - 5b)$
36. $(x + 6)^2$
37. $(y - 11)^2$
38. $(2x + 3y)^2$
39. $(5x - b)^2$
40. $(2s^2 - 9y)(2s^2 + 9y)$
41. $(4x^3 - y^4)^2$
42. $(4x^3 - 5y^2)(4x^3 + 5y^2)$
43. $(-3x^2 + 2y^4)^2$
44. $(c - 2)(2c^2 - 3c + 1)$
45. $(2y + 3)(y^2 + 3y - 1)$
46. $(x + 2y)(2x^2 - xy + y^2)$
47. $(5w + 6)(-3w^2 + 4w - 3)$
48. $(5x - 2y)(x^2 - 2xy + 3y^2)$
49. $2x(3x + 1)(4x - 2)$
50. $3y(-y + 2)(3y + 1)$
51. $(x - 1)(x - 2)(x - 3)$
52. $(y - 2)(3y + 2)(y + 2)$
53. $(x + 4y)(2y - x)(3x - y)$
54. $(2x - y)(3x + 2y)(y - x)$

In Exercises 55–64, find the coefficient of x^2 in the given product. Avoid doing any more multiplying than necessary.

55. $(x^2 + 3x + 1)(2x - 3)$

56. $(x^2 - 1)(x + 1)$

57. $(x^3 + 2x - 6)(x^2 + 1)$

58. $(\sqrt{3} + x)(\sqrt{3} - x)$

59. $(x + 2)^3$

60. $(x^2 + x + 1)(x - 1)$

61. $(x^2 + x + 1)(x^2 - x + 1)$

62. $(2x^2 + 1)(2x^2 - 1)$

63. $(2x - 1)(x^2 + 3x + 2)$

64. $(1 - 2x)(4x^2 + x - 1)$

In Exercises 65–70, perform the indicated multiplication and simplify your answer if possible.

65. $(\sqrt{x} + 5)(\sqrt{x} - 5)$

66. $(2\sqrt{x} + \sqrt{2y})(2\sqrt{x} - \sqrt{2y})$

67. $(3 + \sqrt{y})^2$

68. $(7w - \sqrt{2x})^2$

69. $(1 + \sqrt{3}x)(x + \sqrt{3})$

70. $(2y + \sqrt{3})(\sqrt{5}y - 1)$

In Exercises 71–76, compute the product and arrange the terms of your answer according to decreasing powers of x, with each power of x appearing at most once.

Example: $(ax + b)(4x - c) = 4ax^2 + (4b - ac)x - bc$.

71. $(ax + b)(3x + 2)$

72. $(4x - c)(dx + c)$

73. $(ax + b)(bx + a)$

74. $rx(3rx + 1)(4x - r)$

75. $(x - a)(x - b)(x - c)$

76. $(2dx - c)(3cx + d)$

In Exercises 77–82, assume that all exponents are nonnegative integers and find the product.

Example: $2x^k(3x + x^{n+1}) = (2x^k)(3x) + (2x^k)(x^{n+1})$
$= 6x^{k+1} + 2x^{k+n+1}$.

77. $3^r3^43^t$

78. $(2x^n)(8x^k)$

79. $(x^m + 2)(x^n - 3)$

80. $(y^r + 1)(y^s - 4)$

81. $(2x^n - 5)(x^{3n} + 4x^n + 1)$

82. $(3y^{2k} + y^k + 1)(y^k - 3)$

In Exercises 83–92, find a numerical example to show that the given statement is false. Then find the mistake in the statement and correct it.

Example: *The statement* $-(b + 2) = -b + 2$ *is false when* $b = 5$, *since* $-(5 + 2) = -7$ *but* $-5 + 2 = -3$. *The mistake is the sign on the 2. The correct statement is* $-(b + 2) = -b - 2$.

83. $3(y + 2) = 3y + 2$

84. $x - (3y + 4) = x - 3y + 4$

85. $(x + y)^2 = x + y^2$

86. $(2x)^3 = 2x^3$

87. $(7x)(7y) = 7xy$

88. $(x + y)^2 = x^2 + y^2$

89. $y + y + y = y^3$

90. $(a - b)^2 = a^2 - b^2$

91. $(x - 3)(x - 2) = x^2 - 5x - 6$

92. $(a + b)(a^2 + b^2) = a^3 + b^3$

In Exercises 93 and 94, explain algebraically why each of these parlor tricks always works.

93. *Critical Thinking* Write down a nonzero number. Add 1 to it and square the result. Subtract 1 from the original number and square the result. Subtract this second square from the first one. Divide by the number with which you started. The answer is 4.

94. *Critical Thinking* Write down a positive number. Add 4 to it. Multiply the result by the original number. Add 4 to this result and then take the square root. Subtract the number with which you started. The answer is 2.

95. *Critical Thinking* Invent a similar parlor trick in which the answer is always the number with which you started.

A.3 Factoring

Factoring is the reverse of multiplication: We begin with a product and find the factors that multiply together to produce this product. Factoring skills are necessary to simplify expressions, to do arithmetic with fractional expressions, and to solve equations and inequalities.

The first general rule for factoring is

Common Factors

If there is a common factor in every term of the expression, factor out the common factor of highest degree.

Example 1

In $4x^6 - 8x$, for example, each term contains a factor of $4x$, so that $4x^6 - 8x = 4x(x^5 - 2)$. Similarly, the common factor of highest degree in $x^3y^2 + 2xy^3 - 3x^2y^4$ is xy^2 and

$$x^3y^2 + 2xy^3 - 3x^2y^4 = xy^2(x^2 + 2y - 3xy^2).$$

You can greatly increase your factoring proficiency by learning to recognize multiplication patterns that appear frequently. Here are the most common ones.

Quadratic Factoring Patterns

Difference of Squares	$u^2 - v^2 = (u + v)(u - v)$
Perfect Squares	$u^2 + 2uv + v^2 = (u + v)^2$
	$u^2 - 2uv + v^2 = (u - v)^2$

Example 2

a. $x^2 - 9y^2$ can be written $x^2 - (3y)^2$, a difference of squares. Therefore, $x^2 - 9y^2 = (x + 3y)(x - 3y)$.

b. $y^2 - 7 = y^2 - (\sqrt{7})^2 = (y + \sqrt{7})(y - \sqrt{7})$.*

c. $36r^2 - 64s^2 = (6r)^2 - (8s)^2 = (6r + 8s)(6r - 8s)$
$= 2(3r + 4s)2(3r - 4s) = 4(3r + 4s)(3r - 4s)$.

*When a polynomial has integer coefficients, we normally look only for factors with integer coefficients. But when it is easy to find other factors, as here, we shall do so.

Example 3

Since the first and last terms of $4x^2 - 36x + 81$ are perfect squares, we try to use the perfect square pattern with $u = 2x$ and $v = 9$:

$$\begin{aligned} 4x^2 - 36x + 81 &= (2x)^2 - 36x + 9^2 \\ &= (2x)^2 - 2 \cdot 2x \cdot 9 + 9^2 = (2x - 9)^2. \end{aligned}$$

Cubic Factoring Patterns

Difference of Cubes	$u^3 - v^3 = (u - v)(u^2 + uv + v^2)$
Sum of Cubes	$u^3 + v^3 = (u + v)(u^2 - uv + v^2)$
Perfect Cubes	$u^3 + 3u^2v + 3uv^2 + v^3 = (u + v)^3$
	$u^3 - 3u^2v + 3uv^2 - v^3 = (u - v)^3$

Example 4

a. $x^3 - 125 = x^3 - 5^3 = (x - 5)(x^2 + 5x + 5^2)$
$= (x - 5)(x^2 + 5x + 25).$

b. $x^3 + 8y^3 = x^3 + (2y)^3 = (x + 2y)[x^2 - x \cdot 2y + (2y)^2]$
$= (x + 2y)(x^2 - 2xy + 4y^2).$

c. $x^3 - 12x^2 + 48x - 64 = x^3 - 12x^2 + 48x - 4^3$
$= x^3 - 3x^2 \cdot 4 + 3x \cdot 4^2 - 4^3$
$= (x - 4)^3.$

When none of the multiplication patterns applies, use trial and error to factor quadratic polynomials. If a quadratic has two first-degree factors, then the factors must be of the form $ax + b$ and $cx + d$ for some constants a, b, c, d. The product of such factors is

$$\begin{aligned} (ax + b)(cx + d) &= acx^2 + adx + bcx + bd \\ &= acx^2 + (ad + bc)x + bd. \end{aligned}$$

Note that *ac is the coefficient of x^2* and *bd is the constant term* of the product polynomial. This pattern can be used to factor quadratics by reversing the FOIL process.

Example 5

If $x^2 + 9x + 18$ factors as $(ax + b)(cx + d)$, then we must have $ac = 1$ (coefficient of x^2) and $bd = 18$ (constant term). Thus, $a = \pm 1$ and $c = \pm 1$ (the only integer factors of 1). The only possibilities for b and d are

$$\pm 1, \pm 18 \quad \text{or} \quad \pm 2, \pm 9 \quad \text{or} \quad \pm 3, \pm 6.$$

We mentally try the various possibilities, using FOIL as our guide. For example, we try $b = 2$, $d = 9$ and check this factorization: $(x + 2)(x + 9)$. The sum of the outside and inside terms is $9x + 2x = 11x$, so this prod-

uct can't be $x^2 + 9x + 18$. By trying other possibilities we find that $b = 3$, $d = 6$ leads to the correct factorization: $x^2 + 9x + 18 = (x + 3)(x + 6)$.

Example 6

To factor $6x^2 + 11x + 4$ as $(ax + b)(cx + d)$, we must find numbers a and c whose product is 6, the coefficient of x^2, and numbers b and d whose product is the constant term 4. Some possibilities are

$ac = 6$

a	±1	±2	±3	±6
c	±6	±3	±2	±1

$bd = 4$

b	±1	±2	±4
d	±4	±2	±1

Trial and error shows that $(2x + 1)(3x + 4) = 6x^2 + 11x + 4$.

Occasionally the patterns above can be used to factor expressions involving larger exponents than 2.

Example 7

a. $x^6 - y^6 = (x^3)^2 - (y^3)^2 = (x^3 + y^3)(x^3 - y^3)$
$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$.

b. $x^8 - 1 = (x^4)^2 - 1 = (x^4 + 1)(x^4 - 1)$
$= (x^4 + 1)(x^2 + 1)(x^2 - 1)$
$= (x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$.

Example 8

To factor $x^4 - 2x^2 - 3$, let $u = x^2$. Then,

$$x^4 - 2x^2 - 3 = (x^2)^2 - 2x^2 - 3$$
$$= u^2 - 2u - 3 = (u + 1)(u - 3)$$
$$= (x^2 + 1)(x^2 - 3)$$
$$= (x^2 + 1)(x + \sqrt{3})(x - \sqrt{3}).$$

Example 9

$3x^3 + 3x^2 + 2x + 2$ can be factored by regrouping and using the distributive law to factor out a common factor:

$$(3x^3 + 3x^2) + (2x + 2) = 3x^2(x + 1) + 2(x + 1)$$
$$= (3x^2 + 2)(x + 1).$$

Exercises A.3

In Exercises 1–58, factor the expression.

1. $x^2 - 4$
2. $x^2 + 6x + 9$
3. $9y^2 - 25$
4. $y^2 - 4y + 4$
5. $81x^2 + 36x + 4$
6. $4x^2 - 12x + 9$
7. $5 - x^2$
8. $1 - 36u^2$
9. $49 + 28z + 4z^2$
10. $25u^2 - 20uv + 4v^2$
11. $x^4 - y^4$
12. $x^2 - \frac{1}{9}$
13. $x^2 + x - 6$
14. $y^2 + 11y + 30$
15. $z^2 + 4z + 3$
16. $x^2 - 8x + 15$
17. $y^2 + 5y - 36$
18. $z^2 - 9z + 14$
19. $x^2 - 6x + 9$
20. $4y^2 - 81$
21. $x^2 + 7x + 10$
22. $w^2 - 6w - 16$
23. $x^2 + 11x + 18$
24. $x^2 + 3xy - 28y^2$
25. $3x^2 + 4x + 1$
26. $4y^2 + 4y + 1$
27. $2z^2 + 11z + 12$
28. $10x^2 - 17x + 3$
29. $9x^2 - 72x$
30. $4x^2 - 4x - 3$
31. $10x^2 - 8x - 2$
32. $7z^2 + 23z + 6$
33. $8u^2 + 6u - 9$
34. $2y^2 - 4y + 2$
35. $4x^2 + 20xy + 25y^2$
36. $63u^2 - 46uv + 8v^2$
37. $x^3 - 125$
38. $y^3 + 64$
39. $x^3 + 6x^2 + 12x + 8$
40. $y^3 - 3y^2 + 3y - 1$
41. $8 + x^3$
42. $z^3 - 9z^2 + 27z - 27$
43. $-x^3 + 15x^2 - 75x + 125$
44. $27 - t^3$
45. $x^3 + 1$
46. $x^3 - 1$
47. $8x^3 - y^3$
48. $(x - 1)^3 + 1$
49. $x^6 - 64$
50. $x^5 - 8x^2$
51. $y^4 + 7y^2 + 10$
52. $z^4 - 5z^2 + 6$
53. $81 - y^4$
54. $x^6 + 16x^3 + 64$
55. $z^6 - 1$
56. $y^6 + 26y^3 - 27$
57. $x^4 + 2x^2y - 3y^2$
58. $x^8 - 17x^4 + 16$

In Exercises 59–64, factor by regrouping and using the distributive law (as in Example 9).

59. $x^2 - yz + xz - xy$
60. $x^6 - 2x^4 - 8x^2 + 16$
61. $a^3 - 2b^2 + 2a^2b - ab$
62. $u^2v - 2w^2 - 2uvw + uw$
63. $x^3 + 4x^2 - 8x - 32$
64. $z^8 - 5z^7 + 2z - 10$
65. *Critical Thinking* Show that there do *not* exist real numbers c and d such that $x^2 + 1 = (x + c)(x + d)$.

A.4 Fractional Expressions

Quotients of algebraic expressions are called **fractional expressions**. A quotient of two polynomials is sometimes called a **rational expression**. The basic rules for dealing with fractional expressions are essentially the same as those for ordinary numerical fractions. For instance, $\frac{2}{4} = \frac{3}{6}$ and the "cross products" are equal: $2 \cdot 6 = 4 \cdot 3$. In the general case we have

Properties of Fractions

1. *Equality rule:* $\frac{a}{b} = \frac{c}{d}$ exactly when $ad = bc$.*
2. *Cancellation property:* If $k \neq 0$, then $\frac{ka}{kb} = \frac{a}{b}$.

The cancellation property follows directly from the equality rule because $(ka)b = (kb)a$.

Example 1

Here are examples of the two properties:

1. $\frac{x^2 + 2x}{x^2 + x - 2} = \frac{x}{x - 1}$ because the cross products are equal:

$$(x^2 + 2x)(x - 1) = x^3 + x^2 - 2x = (x^2 + x - 2)x.$$

2. $$\frac{x^4 - 1}{x^2 + 1} = \frac{(x^2 + 1)(x^2 - 1)}{(x^2 + 1)} = \frac{x^2 - 1}{1} = x^2 - 1.$$

A fraction is in **lowest terms** if its **numerator** (top) and **denominator** (bottom) have no common factors except ± 1. To express a fraction in lowest terms, factor numerator and denominator and cancel common factors.

Example 2

$$\frac{x^2 + x - 6}{x^2 - 3x + 2} = \frac{(x - 2)(x + 3)}{(x - 2)(x - 1)} = \frac{x + 3}{x - 1}.$$

To add two fractions with the same denominator, simply add the numerators as in ordinary arithmetic: $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$. Subtraction is done similarly.

Example 3

$$\begin{aligned}\frac{7x^2 + 2}{x^2 + 3} - \frac{4x^2 + 2x - 5}{x^2 + 3} &= \frac{(7x^2 + 2) - (4x^2 + 2x - 5)}{x^2 + 3} \\ &= \frac{7x^2 + 2 - 4x^2 - 2x + 5}{x^2 + 3} \\ &= \frac{3x^2 - 2x + 7}{x^2 + 3}.\end{aligned}$$

*Throughout this section we assume that all denominators are nonzero.

To add or subtract fractions with different denominators, you must first find a common denominator. One common denominator for a/b and c/d is the product of the two denominators bd because both fractions can be expressed with this denominator:

$$\frac{a}{b} = \frac{ad}{bd} \quad \text{and} \quad \frac{c}{d} = \frac{bc}{bd}.$$

Consequently,

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd} \quad \text{and} \quad \frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad - bc}{bd}.$$

Example 4

$$\begin{aligned}\frac{2x+1}{3x} - \frac{x^2-2}{x-1} &= \frac{(2x+1)(x-1)}{3x(x-1)} - \frac{3x(x^2-2)}{3x(x-1)} \\ &= \frac{(2x+1)(x-1) - 3x(x^2-2)}{3x(x-1)} \\ &= \frac{2x^2 - x - 1 - 3x^3 + 6x}{3x^2 - 3x} \\ &= \frac{-3x^3 + 2x^2 + 5x - 1}{3x^2 - 3x}.\end{aligned}$$

Although the product of the denominators can always be used as a common denominator, it's often more efficient to use the *least common denominator*. The least common denominator can be found by factoring each denominator completely (with integer coefficients) and then taking the product of the highest power of each of the distinct factors.

Example 5

In the sum $\frac{1}{100} + \frac{1}{120}$, the denominators are $100 = 2^2 \cdot 5^2$ and $120 = 2^3 \cdot 3 \cdot 5$. The distinct factors are 2, 3, 5. The highest exponent of 2 in either denominator is 3, the highest of 3 is 1, and the highest of 5 is 2. So the least common denominator is $2^3 \cdot 3 \cdot 5^2 = 600$

$$\frac{1}{100} + \frac{1}{120} = \frac{6}{600} + \frac{5}{600} = \frac{11}{600}.$$

Example 6

To find the least common denominator of $\frac{1}{x^2 + 2x + 1}$, $\frac{5x}{x^2 - x}$, and $\frac{3x - 7}{x^4 + x^3}$, factor each of the denominators completely:

$$(x+1)^2, \qquad x(x-1), \qquad x^3(x+1),$$

The distinct factors are x, $x + 1$, and $x - 1$. The least common denominator is determined by the highest power of each factor:

$$x^3(x + 1)^2(x - 1).$$

To express one of several fractions in terms of the least common denominator, multiply its numerator and denominator by those factors in the common denominator that *don't* appear in the denominator of the fraction.

Example 7

The preceding example shows the least common denominator (LCD) of $\frac{1}{(x + 1)^2}$, $\frac{5x}{x(x - 1)}$, and $\frac{3x - 7}{x^3(x + 1)}$ to be $x^3(x + 1)^2(x - 1)$. Therefore,

$$\frac{1}{(x + 1)^2} = \frac{1}{(x + 1)^2} \cdot \frac{x^3(x - 1)}{x^3(x - 1)} = \frac{x^3(x - 1)}{x^3(x + 1)^2(x - 1)}$$

$$\frac{5x}{x(x - 1)} = \frac{5x}{x(x - 1)} \cdot \frac{x^2(x + 1)^2}{x^2(x + 1)^2} = \frac{5x^3(x + 1)^2}{x^3(x + 1)^2(x - 1)}$$

$$\frac{3x - 7}{x^3(x + 1)} = \frac{3x - 7}{x^3(x + 1)} \cdot \frac{(x + 1)(x - 1)}{(x + 1)(x - 1)} = \frac{(3x - 7)(x + 1)(x - 1)}{x^3(x + 1)^2(x - 1)}.$$

Example 8

To find $\frac{1}{z} + \frac{3z}{z + 1} - \frac{z^2}{(z + 1)^2}$ we use the LCD $z(z + 1)^2$:

$$\begin{aligned}\frac{1}{z} + \frac{3z}{z + 1} - \frac{z^2}{(z + 1)^2} &= \frac{(z + 1)^2}{z(z + 1)^2} + \frac{3z^2(z + 1)}{z(z + 1)^2} - \frac{z^3}{z(z + 1)^2} \\ &= \frac{(z + 1)^2 + 3z^2(z + 1) - z^3}{z(z + 1)^2} \\ &= \frac{z^2 + 2z + 1 + 3z^3 + 3z^2 - z^3}{z(z + 1)^2} \\ &= \frac{2z^3 + 4z^2 + 2z + 1}{z(z + 1)^2}.\end{aligned}$$

Multiplication of fractions is easy: Multiply corresponding numerators and denominators, then simplify your answer.

Example 9

$$\begin{aligned}\frac{x^2 - 1}{x^2 + 2} \cdot \frac{3x - 4}{x + 1} &= \frac{(x^2 - 1)(3x - 4)}{(x^2 + 2)(x + 1)} \\ &= \frac{(x - 1)(x + 1)(3x - 4)}{(x^2 + 2)(x + 1)} = \frac{(x - 1)(3x - 4)}{x^2 + 2}.\end{aligned}$$

Division of fractions is given by the rule:

Invert the divisor and multiply: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$.

Example 10

$$\frac{x^2 + x - 2}{x^2 - 6x + 9} \div \frac{x^2 - 1}{x - 3} = \frac{x^2 + x - 2}{x^2 - 6x + 9} \cdot \frac{x - 3}{x^2 - 1}$$
$$= \frac{(x + 2)(x - 1)}{(x - 3)^2} \cdot \frac{x - 3}{(x - 1)(x + 1)}$$
$$= \frac{x + 2}{(x - 3)(x + 1)}.$$

Division problems can also be written as fractions. For instance, $\frac{8}{2}$ means $8 \div 2 = 4$. Similarly, the compound fraction $\frac{\frac{a}{b}}{\frac{c}{d}}$ means $\frac{a}{b} \div \frac{c}{d}$. So, the basic rule for simplifying compound fractions is: *Invert the denominator and multiply it by the numerator.*

Example 11

a. $$\frac{\frac{16y^2z}{8yz^2}}{\frac{yz}{6y^3z^3}} = \frac{16y^2z}{8yz^2} \cdot \frac{6y^3z^3}{yz} = \frac{16 \cdot 6 \cdot y^5z^4}{8y^2z^3}$$
$$= 2 \cdot 6 \cdot y^{5-2}z^{4-3} = 12y^3z.$$

b. $$\frac{\frac{y^2}{y + 2}}{y^3 + y} = \frac{y^2}{y + 2} \cdot \frac{1}{y^3 + y} = \frac{y^2}{(y + 2)(y^3 + y)}$$
$$= \frac{y^2}{(y + 2)y(y^2 + 1)}$$
$$= \frac{y}{(y + 2)(y^2 + 1)}.$$

Exercises A.4

In Exercises 1–10, express the fraction in lowest terms.

1. $\frac{63}{49}$
2. $\frac{121}{33}$
3. $\frac{13 \cdot 27 \cdot 22 \cdot 10}{6 \cdot 4 \cdot 11 \cdot 12}$
4. $\frac{x^2 - 4}{x + 2}$
5. $\frac{x^2 - x - 2}{x^2 + 2x + 1}$
6. $\frac{z + 1}{z^3 + 1}$
7. $\frac{a^2 - b^2}{a^3 - b^3}$
8. $\frac{x^4 - 3x^2}{x^3}$
9. $\frac{(x + c)(x^2 - cx + c^2)}{x^4 + c^3x}$
10. $\frac{x^4 - y^4}{(x^2 + y^2)(x^2 - xy)}$

In Exercises 11–28, perform the indicated operations.

11. $\frac{3}{7}+\frac{2}{5}$

12. $\frac{7}{8}-\frac{5}{6}$

13. $\left(\frac{19}{7}+\frac{1}{2}\right)-\frac{1}{3}$

14. $\frac{1}{a}-\frac{2a}{b}$

15. $\frac{c}{d}+\frac{3c}{e}$

16. $\frac{r}{s}+\frac{s}{t}+\frac{t}{r}$

17. $\frac{b}{c}-\frac{c}{b}$

18. $\frac{a}{b}+\frac{2a}{b^2}+\frac{3a}{b^3}$

19. $\frac{1}{x+1}-\frac{1}{x}$

20. $\frac{1}{2x+1}+\frac{1}{2x-1}$

21. $\frac{1}{x+4}+\frac{2}{(x+4)^2}-\frac{3}{x^2+8x+16}$

22. $\frac{1}{x}+\frac{1}{xy}+\frac{1}{xy^2}$

23. $\frac{1}{x}-\frac{1}{3x-4}$

24. $\frac{3}{x-1}+\frac{4}{x+1}$

25. $\frac{1}{x+y}+\frac{x+y}{x^3+y^3}$

26. $\frac{6}{5(x-1)(x-2)^2}+\frac{x}{3(x-1)^2(x-2)}$

27. $\frac{1}{4x(x+1)(x+2)^3}-\frac{6x+2}{4(x+1)^3}$

28. $\frac{x+y}{(x^2-xy)(x-y)^2}-\frac{2}{(x^2-y^2)^2}$

In Exercises 29–42, express in lowest terms.

29. $\frac{3}{4}\cdot\frac{12}{5}\cdot\frac{10}{9}$

30. $\frac{10}{45}\cdot\frac{6}{14}\cdot\frac{1}{2}$

31. $\frac{3a^2c}{4ac}\cdot\frac{8ac^3}{9a^2c^4}$

32. $\frac{6x^2y}{2x}\cdot\frac{y}{21xy}$

33. $\frac{7x}{11y}\cdot\frac{66y^2}{14x^3}$

34. $\frac{ab}{c^2}\cdot\frac{cd}{a^2b}\cdot\frac{ad}{bc^2}$

35. $\frac{3x+9}{2x}\cdot\frac{8x^2}{x^2-9}$

36. $\frac{4x+16}{3x+15}\cdot\frac{2x+10}{x+4}$

37. $\frac{5y-25}{3}\cdot\frac{y^2}{y^2-25}$

38. $\frac{6x-12}{6x}\cdot\frac{8x^2}{x-2}$

39. $\frac{u}{u-1}\cdot\frac{u^2-1}{u^2}$

40. $\frac{t^2-t-6}{t^2-6t+9}\cdot\frac{t^2+4t-5}{t^2-25}$

41. $\frac{2u^2+uv-v^2}{4u^2-4uv+v^2}\cdot\frac{8u^2+6uv-9v^2}{4u^2-9v^2}$

42. $\frac{2x^2-3xy-2y^2}{6x^2-5xy-4y^2}\cdot\frac{6x^2+6xy}{x^2-xy-2y^2}$

In Exercises 43–60, compute the quotient and express in lowest terms.

43. $\frac{5}{12}\div\frac{4}{14}$

44. $\frac{\frac{100}{52}}{\frac{27}{26}}$

45. $\frac{uv}{v^2w}\div\frac{uv}{u^2v}$

46. $\frac{3x^2y}{(xy)^2}\div\frac{3xyz}{x^2y}$

47. $\frac{\frac{x+3}{x+4}}{\frac{2x}{x+4}}$

48. $\frac{\frac{(x+2)^2}{(x-2)^2}}{\frac{x^2+2x}{x^2-4}}$

49. $\frac{x+y}{x+2y}\div\left(\frac{x+y}{xy}\right)^2$

50. $\frac{\frac{u^3+v^3}{u^2-v^2}}{\frac{u^2-uv+v^2}{u+v}}$

51. $\frac{\frac{(c+d)^2}{c^2-d^2}}{cd}$

52. $\frac{\frac{1}{x}-\frac{3}{2}}{\frac{2}{x-2}+\frac{5}{x}}$

53. $\frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}+\frac{1}{y}}$

54. $\frac{\frac{x}{x+1}+\frac{1}{x}}{\frac{1}{x}+\frac{1}{x+1}}$

55. $\frac{\frac{6}{y}-3}{1-\frac{1}{y-1}}$

56. $\frac{\frac{1}{3x}-\frac{1}{4y}}{\frac{5}{6x^2}+\frac{1}{y}}$

57. $\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

58. $\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

59. $(x^{-1}+y^{-1})^{-1}$

60. $\frac{(x+y)^{-1}}{x^{-1}+y^{-1}}$

In Exercises 61–67, find a numerical example to show that the given statement is false. Then find the mistake in the statement and correct it.

61. $\frac{1}{a}+\frac{1}{b}=\frac{1}{a+b}$

62. $\frac{x^2}{x^2+x^6}=1+x^3$

63. $\left(\frac{1}{\sqrt{a}+\sqrt{b}}\right)^2=\frac{1}{a+b}$

64. $\frac{r+s}{r+t}=1+\frac{s}{t}$

65. $\frac{u}{v}+\frac{v}{u}=1$

66. $\frac{\frac{1}{x}}{\frac{1}{y}}=\frac{1}{xy}$

67. $\left(\sqrt{x}+\sqrt{y}\right)\frac{1}{\sqrt{x}+\sqrt{y}}=x+y$

Exercises A.1

In Exercises 1–18, evaluate the expression.

1. $(-6)^2$ **36**
2. -6^2 **−36**
3. $5 + 4(3^2 + 2^3)$ **73**
4. $(-3)2^2 + 4^2 - 1$ **3**
5. $\dfrac{(-3)^2 + (-2)^4}{-2^2 - 1}$ **−5**
6. $\dfrac{(-4)^2 + 2}{(-4)^2 - 7} + 1$ **3**
7. $\left(\dfrac{-5}{4}\right)^3$ $-\dfrac{125}{64}$
8. $-\left(\dfrac{7}{4} + \dfrac{3}{4}\right)^2$ $-\dfrac{25}{4}$
9. $\left(\dfrac{1}{3}\right)^3 + \left(\dfrac{2}{3}\right)^3$ $\dfrac{1}{3}$
10. $\left(\dfrac{5}{7}\right)^2 + \left(\dfrac{2}{7}\right)^2$ $\dfrac{29}{49}$
11. $2^4 - 2^7$ **−112**
12. $3^3 - 3^{-7}$ $26\dfrac{2186}{2187}$
13. $(2^{-2} + 2)^2$ $\dfrac{81}{16}$
14. $(3^{-1} - 3^3)^2$ $\dfrac{6400}{9}$
15. $2^2 \cdot 3^{-3} - 3^2 \cdot 2^{-3}$ $-\dfrac{211}{216}$
16. $4^3 \cdot 5^{-2} + 4^2 \cdot 5^{-1}$ $\dfrac{144}{25}$
17. $\dfrac{1}{2^3} + \dfrac{1}{2^{-4}}$ $\dfrac{129}{8}$
18. $3^2\left(\dfrac{1}{3} + \dfrac{1}{3^{-2}}\right)$ **84**

In Exercises 19–38, simplify the expression. Each letter represents a nonzero real number and should appear at most once in your answer.

19. $x^2 \cdot x^3 \cdot x^5$ x^{10}
20. $y \cdot y^4 \cdot y^6$ y^{11}
21. $(0.03)y^2 \cdot y^7$ $0.03y^9$
22. $(1.3)z^3 \cdot z^5$ $1.3z^8$
23. $(2x^2)^3 3x$ $24x^7$
24. $(3y^3)^4 5y^2$ $405y^{14}$
25. $(3x^2y)^2$ $9x^4y^2$
26. $(2xy^3)^3$ $8x^3y^9$
27. $(a^2)(7a)(-3a^3)$ $-21a^6$
28. $(b^3)(-b^2)(3b)$ $-3b^6$
29. $(2w)^3(3w)(4w)^2$ $384w^6$
30. $(3d)^4(2d)^2(5d)$ $1620d^7$
31. $a^{-2}b^3a^3$ ab^3
32. $c^4d^5c^{-3}$ cd^5
33. $(2x)^{-2}(2y)^3(4x)$ $8x^{-1}y^3$
34. $(3x)^{-3}(2y)^{-2}(2x)$ $\dfrac{1}{54x^2y^2}$
35. $(-3a^4)^2(9x^3)^{-1}$ a^8x^{-3}
36. $(2y^3)^3(3y^2)^{-2}$ $\dfrac{8y^5}{9}$
37. $(2x^2y)^0(3xy)$ $3xy$
38. $(3x^2y^4)^0$ **1**

In Exercises 39–42, express the given number as a power of 2.

39. $(64)^2$ 2^{12}
40. $\left(\dfrac{1}{8}\right)^3$ 2^{-9}
41. $(2^4 \cdot 16^{-2})^3$ 2^{-12}
42. $\left(\dfrac{1}{2}\right)^{-8}\left(\dfrac{1}{4}\right)^4\left(\dfrac{1}{16}\right)^{-3}$ 2^{12}

In Exercises 43–60, simplify and write the given expression without negative exponents. All letters represent nonzero real numbers.

43. $\dfrac{x^4(x^2)^3}{x^3}$ x^7
44. $\left(\dfrac{z^2}{t^3}\right)^4 \cdot \left(\dfrac{z^3}{t}\right)^5$ $\dfrac{z^{23}}{t^{17}}$
45. $\left(\dfrac{e^6}{c^4}\right)^2 \cdot \left(\dfrac{c^3}{e}\right)^3$ ce^9
46. $\left(\dfrac{x^7}{y^6}\right)^2 \cdot \left(\dfrac{y^2}{x}\right)^4$ $\dfrac{x^{10}}{y^4}$
47. $\left(\dfrac{ab^2c^3d^4}{abc^2d}\right)^2$ $b^2c^2d^6$
48. $\dfrac{(3x)^2(y^2)^3x^2}{(2xy^2)^3}$ $\dfrac{9x}{8}$
49. $\left(\dfrac{a^6}{b^{-4}}\right)^2$ $a^{12}b^8$
50. $\left(\dfrac{x^{-2}}{y^{-2}}\right)^2$ $\dfrac{y^4}{x^4}$
51. $\left(\dfrac{c^5}{d^{-3}}\right)^{-2}$ $\dfrac{1}{c^{10}d^6}$
52. $\left(\dfrac{x^{-1}}{2y^{-1}}\right)\left(\dfrac{2y}{x}\right)^{-2}$ $\dfrac{x}{8y}$
53. $\left(\dfrac{3x}{y^2}\right)^{-3}\left(\dfrac{-x}{2y^3}\right)^2$ $\dfrac{1}{108x}$
54. $\left(\dfrac{5u^2v}{2uv^2}\right)^2\left(\dfrac{-3uv}{2u^2v}\right)^{-3}$ $-\dfrac{50u^5}{27v^2}$
55. $\dfrac{(a^{-3}b^2c)^{-2}}{(ab^{-2}c^3)^{-1}}$ $\dfrac{a^7c}{b^6}$
56. $\dfrac{(-2cd^2e^{-1})^3}{(5c^{-3}de)^{-2}}$ $-\dfrac{200d^8}{c^3e}$
57. $(c^{-1}d^{-2})^{-3}$ c^3d^6
58. $[(x^2y^{-1})^2]^{-3}$ $\dfrac{y^6}{x^{12}}$
59. $a^2(a^{-1} + a^{-3})$ $a + \dfrac{1}{a}$
60. $\dfrac{a^{-2}}{b^{-2}} + \dfrac{b^2}{a^2}$ $\dfrac{2b^2}{a^2}$

In Exercises 61–66, determine the sign of the given number without calculating the product.

61. $(-2.6)^3(-4.3)^{-2}$ **negative**
62. $(4.1)^{-2}(2.5)^{-3}$ **positive**
63. $(-1)^9(6.7)^5$ **negative**
64. $(-4)^{12}6^9$ **positive**
65. $(-3.1)^{-3}(4.6)^{-6}(7.2)^7$ **negative**
66. $(45.8)^{-7}(-7.9)^{-9}(-8.5)^{-4}$ **negative**

In Exercises 67–72, r, s, and t are positive integers and a, b, and c are nonzero real numbers. Simplify and write the given expression without negative exponents.

67. $\dfrac{3^{-r}}{3^{-s-r}}$ 3^s
68. $\dfrac{4^{-(t+1)}}{4^{2-t}}$ $\dfrac{1}{64}$
69. $\left(\dfrac{a^6}{b^{-4}}\right)^t$ $a^{6t}b^{4t}$
70. $\dfrac{c^{-t}}{(6b)^{-s}}$ $\dfrac{6^sb^s}{c^t}$
71. $\dfrac{(c^{-r}b^s)^t}{(c^tb^{-s})^r}$ $\dfrac{b^{rs+st}}{c^{2rt}}$
72. $\dfrac{(a^rb^{-s})^{-t}}{(b^tc^r)^{-s}}$ $\dfrac{b^{2st}c^{rs}}{a^{rt}}$

In Exercises 73–80, give an example to show that the statement may be false for some numbers.

73. $a^r + b^r = (a + b)^r$
74. $a^ra^s = a^{rs}$
75. $a^rb^s = (ab)^{r+s}$
76. $c^{-r} = -c^r$
77. $\dfrac{c^r}{c^s} = c^{\frac{r}{s}}$
78. $(a + 1)(b + 1) = ab + 1$
79. $(-a)^2 = -a^2$
80. $(-a)(-b) = -ab$

73. possible answer: $3^2 + 4^2 = 9 + 16 = 25$, but $(3 + 4)^2 = 7^2 = 49$
74. possible answer: $2^3 \cdot 2^2 = 32$; but $2^{3 \cdot 2} = 64$
75. possible answer: $3^2 \cdot 2^3 = 9 \cdot 8 = 72$; but $(3 \cdot 2)^{2+3} = 6^5 = 7776$
76. possible answer: $1^{-1} = 1$; but $-1^1 = -1$
77. possible answer: $\dfrac{2^6}{2^3} = \dfrac{64}{8} = 8$, but $2^{\frac{6}{3}} = 2^2 = 4$
78. possible answer: $(1 + 1)(2 + 1) = 6$; but $1 \cdot 2 + 1 = 3$
79. possible answer: $(-3)^2 = (-3)(-3) = 9$, but $-3^2 = -9$
80. possible answer: $(-1)(-1) = 1$; but $-(1)(1) = -1$

Exercises A.2

In Exercises 1–54, perform the indicated operations and simplify your answer.

1. $x + 7x$ $8x$
2. $5w + 7w - 3w$ $9w$
3. $6a^2b + (-8b)a^2$ $-2a^2b$
4. $-6x^3\sqrt{t} + 7x^3\sqrt{t} + 15x^3\sqrt{t}$ $16x^3\sqrt{t}$
5. $(x^2 + 2x + 1) - (x^3 - 3x^2 + 4)$ $-x^3 + 4x^2 + 2x - 3$
6. $\left[u^4 - (-3)u^3 + \frac{u}{2} + 1\right] + \left(u^4 - 2u^3 + 5 - \frac{u}{2}\right)$ $2u^4 + u^3 + 6$
7. $\left[u^4 - (-3)u^3 + \frac{u}{2} + 1\right] - \left(u^4 - 2u^3 + 5 - \frac{u}{2}\right)$ $5u^3 + u - 4$
8. $\left(6a^2b + 3a\sqrt{c} - 5ab\sqrt{c}\right) + \left(-6ab^2 - 3ab + 6ab\sqrt{c}\right)$ $6a^2b - 6ab^2 + 3a\sqrt{c} + ab\sqrt{c} - 3ab$
9. $[4z - 6z^2w - (-2)z^3w^2] + (8 - 6z^2w - zw^3 + 4z^3w^2)$ $4z - 12z^2w + 6z^3w^2 - zw^3 + 8$
10. $(x^5y - 2x + 3xy^3) - (-2x - x^5y + 2xy^3)$ $2x^5y + xy^3$
11. $(9x - x^3 + 1) - [2x^3 + (-6)x + (-7)]$ $-3x^3 + 15x + 8$
12. $\left(x - \sqrt{y} - z\right) - \left(x + \sqrt{y} - z\right) - \left(\sqrt{y} + z - x\right)$ $x - 3\sqrt{y} - z$
13. $(x^2 - 3xy) - (x + xy) - (x^2 + xy)$ $-5xy - x$
14. $2x(x^2 + 2)$ $2x^3 + 4x$
15. $(-5y)(-3y^2 + 1)$ $15y^3 - 5y$
16. $x^2y(xy - 6xy^2)$ $x^3y^2 - 6x^3y^3$
17. $3ax(4ax - 2a^2y + 2ay)$ $12a^2x^2 - 6a^3xy + 6a^2xy$
18. $2x(x^2 - 3xy + 2y^2)$ $2x^3 - 6x^2y + 4xy^2$
19. $6z^3(2z + 5)$ $12z^4 + 30z^3$
20. $-3x^2(12x^6 - 7x^5)$ $-36x^8 + 21x^7$
21. $3ab(4a - 6b + 2a^2b)$ $12a^2b - 18ab^2 + 6a^3b^2$
22. $(-3ay)(4ay - 5y)$ $-12a^2y^2 + 15ay^2$
23. $(x + 1)(x - 2)$ $x^2 - x - 2$
24. $(x + 2)(2x - 5)$ $2x^2 - x - 10$
25. $(-2x + 4)(-x - 3)$ $2x^2 + 2x - 12$
26. $(y - 6)(2y + 2)$ $2y^2 - 10y - 12$
27. $(y + 3)(y + 4)$ $y^2 + 7y + 12$
28. $(w - 2)(3w + 1)$ $3w^2 - 5w - 2$
29. $(3x + 7)(-2x + 5)$ $-6x^2 + x + 35$
30. $(ab + 1)(a - 2)$ $a^2b - 2ab + a - 2$
31. $(y - 3)(3y^2 + 4)$ $3y^3 - 9y^2 + 4y - 12$
32. $(y + 8)(y - 8)$ $y^2 - 64$
33. $(x + 4)(x - 4)$ $x^2 - 16$
34. $(3x - y)(3x + y)$ $9x^2 - y^2$
35. $(4a + 5b)(4a - 5b)$ $16a^2 - 25b^2$
36. $(x + 6)^2$ $x^2 + 12x + 36$
37. $(y - 11)^2$ $y^2 - 22y + 121$
38. $(2x + 3y)^2$ $4x^2 + 12xy + 9y^2$
39. $(5x - b)^2$ $25x^2 - 10bx + b^2$
40. $(2s^2 - 9y)(2s^2 + 9y)$ $4s^4 - 81y^2$
41. $(4x^3 - y^4)^2$ $16x^6 - 8x^3y^4 + y^8$
42. $(4x^3 - 5y^2)(4x^3 + 5y^2)$ $16x^6 - 25y^4$
43. $(-3x^2 + 2y^4)^2$ $9x^4 - 12x^2y^4 + 4y^8$
44. $(c - 2)(2c^2 - 3c + 1)$ $2c^3 - 7c^2 + 7c - 2$
45. $(2y + 3)(y^2 + 3y - 1)$ $2y^3 + 9y^2 + 7y - 3$
46. $(x + 2y)(2x^2 - xy + y^2)$ $2x^3 + 3x^2y - xy^2 + 2y^3$
47. $(5w + 6)(-3w^2 + 4w - 3)$ $-15w^3 + 2w^2 + 9w - 18$
48. $(5x - 2y)(x^2 - 2xy + 3y^2)$ $5x^3 - 12x^2y + 19xy^2 - 6y^3$
49. $2x(3x + 1)(4x - 2)$ $24x^3 - 4x^2 - 4x$
50. $3y(-y + 2)(3y + 1)$ $-9y^3 + 15y^2 + 6y$
51. $(x - 1)(x - 2)(x - 3)$ $x^3 - 6x^2 + 11x - 6$
52. $(y - 2)(3y + 2)(y + 2)$ $3y^3 + 2y^2 - 12y - 8$
53. $(x + 4y)(2y - x)(3x - y)$ $-3x^3 - 5x^2y + 26xy^2 - 8y^3$
54. $(2x - y)(3x + 2y)(y - x)$ $-6x^3 + 5x^2y + 3xy^2 - 2y^3$

Exercises A.3

In Exercises 1–58, factor the expression.

1. $x^2 - 4$
   $(x + 2)(x - 2)$
2. $x^2 + 6x + 9$
   $(x + 3)^2$
3. $9y^2 - 25$
   $(3y + 5)(3y - 5)$
4. $y^2 - 4y + 4$
   $(y - 2)^2$
5. $81x^2 + 36x + 4$
   $(9x + 2)^2$
6. $4x^2 - 12x + 9$
   $(2x - 3)^2$
7. $5 - x^2$
   $(\sqrt{5} + x)(\sqrt{5} - x)$
8. $1 - 36u^2$
   $(1 - 6u)(1 + 6u)$
9. $49 + 28z + 4z^2$
   $(7 + 2z)^2$
10. $25u^2 - 20uv + 4v^2$
    $(5u - 2v)^2$
11. $x^4 - y^4$
    $(x^2 + y^2)(x + y)(x - y)$
12. $x^2 - \frac{1}{9}$
    $\left(x - \frac{1}{3}\right)\left(x + \frac{1}{3}\right)$
13. $x^2 + x - 6$
    $(x + 3)(x - 2)$
14. $y^2 + 11y + 30$
    $(y + 5)(y + 6)$
15. $z^2 + 4z + 3$
    $(z + 3)(z + 1)$
16. $x^2 - 8x + 15$
    $(x - 3)(x - 5)$
17. $y^2 + 5y - 36$
    $(y + 9)(y - 4)$
18. $z^2 - 9z + 14$
    $(z - 7)(z - 2)$
19. $x^2 - 6x + 9$
    $(x - 3)^2$
20. $4y^2 - 81$
    $(2y - 9)(2y + 9)$
21. $x^2 + 7x + 10$
    $(x + 5)(x + 2)$
22. $w^2 - 6w - 16$
    $(w - 8)(w + 2)$
23. $x^2 + 11x + 18$
    $(x + 9)(x + 2)$
24. $x^2 + 3xy - 28y^2$
    $(x + 7y)(x - 4y)$
25. $3x^2 + 4x + 1$
    $(3x + 1)(x + 1)$
26. $4y^2 + 4y + 1$
    $(2y + 1)^2$
27. $2z^2 + 11z + 12$
    $(2z + 3)(z + 4)$
28. $10x^2 - 17x + 3$
    $(5x - 1)(2x - 3)$
29. $9x^2 - 72x$
    $9x(x - 8)$
30. $4x^2 - 4x - 3$
    $(2x + 1)(2x - 3)$
31. $10x^2 - 8x - 2$
    $2(x - 1)(5x + 1)$
32. $7z^2 + 23z + 6$
    $(7z + 2)(z + 3)$
33. $8u^2 + 6u - 9$
    $(4u - 3)(2u + 3)$
34. $2y^2 - 4y + 2$
    $2(y - 1)^2$
35. $4x^2 + 20xy + 25y^2$
    $(2x + 5y)^2$
36. $63u^2 - 46uv + 8v^2$
    $(7u - 2v)(9u - 4v)$
37. $x^3 - 125$
38. $y^3 + 64$
39. $x^3 + 6x^2 + 12x + 8$
40. $y^3 - 3y^2 + 3y - 1$
41. $8 + x^3$
42. $z^3 - 9z^2 + 27z - 27$
43. $-x^3 + 15x^2 - 75x + 125$
44. $27 - t^3$
45. $x^3 + 1$
46. $x^3 - 1$
47. $8x^3 - y^3$
48. $(x - 1)^3 + 1$
49. $x^6 - 64$
50. $x^5 - 8x^2$
51. $y^4 + 7y^2 + 10$
52. $z^4 - 5z^2 + 6$
53. $81 - y^4$
54. $x^6 + 16x^3 + 64$
55. $z^6 - 1$
56. $y^6 + 26y^3 - 27$
57. $x^4 + 2x^2y - 3y^2$
58. $x^8 - 17x^4 + 16$

In Exercises 59–64, factor by regrouping and using the distributive law (as in Example 9).

59. $x^2 - yz + xz - xy$
60. $x^6 - 2x^4 - 8x^2 + 16$
61. $a^3 - 2b^2 + 2a^2b - ab$
62. $u^2v - 2w^2 - 2uvw + uw$
63. $x^3 + 4x^2 - 8x - 32$
64. $z^8 - 5z^7 + 2z - 10$
65. *Critical Thinking* Show that there do *not* exist real numbers c and d such that $x^2 + 1 = (x + c)(x + d)$.

Exercises A.3

ANSWERS

37. $(x - 5)(x^2 + 5x + 25)$
38. $(y + 4)(y^2 - 4y + 16)$
39. $(x + 2)^3$
40. $(y - 1)^3$
41. $(2 + x)(4 - 2x + x^2)$
42. $(z - 3)^3$
43. $(-x + 5)^3$
44. $(3 - t)(9 + 3t + t^2)$
45. $(x + 1)(x^2 - x + 1)$
46. $(x - 1)(x^2 + x + 1)$
47. $(2x - y)(4x^2 + 2xy + y^2)$
48. $x(x^2 - 3x + 3)$
49. $(x^3 + 2^3)(x^3 - 2^3)$
    $= (x+2)(x^2 - 2x + 4)(x - 2) \cdot (x^2 + 2x + 4)$
50. $x^2(x - 2)(x^2 + 2x + 4)$
51. $(y^2 + 5)(y^2 + 2)$
52. $(z - \sqrt{3})(z + \sqrt{3})(z - \sqrt{2})(z + \sqrt{2})$
53. $(9 + y^2)(3 + y)(3 - y)$
54. $(x + 2)^2(x^2 - 2x + 4)^2$
55. $(z+1)(z^2 - z + 1)(z - 1)(z^2 + z + 1)$
56. $(y+3)(y^2 - 3y + 9)(y - 1)(y^2 + y + 1)$
57. $(x^2 + 3y)(x^2 - y)$

In Exercises 55–64, find the coefficient of x^2 in the given product. Avoid doing any more multiplying than necessary.

55. $(x^2 + 3x + 1)(2x - 3)$ **3**

56. $(x^2 - 1)(x + 1)$ **1**

57. $(x^3 + 2x - 6)(x^2 + 1)$ **−6**

58. $(\sqrt{3} + x)(\sqrt{3} - x)$ **−1**

59. $(x + 2)^3$ **6**

60. $(x^2 + x + 1)(x - 1)$ **0**

61. $(x^2 + x + 1)(x^2 - x + 1)$ **1**

62. $(2x^2 + 1)(2x^2 - 1)$ **0**

63. $(2x - 1)(x^2 + 3x + 2)$ **5**

64. $(1 - 2x)(4x^2 + x - 1)$ **2**

In Exercises 65–70, perform the indicated multiplication and simplify your answer if possible.

65. $(\sqrt{x} + 5)(\sqrt{x} - 5)$
$x - 25$

66. $(2\sqrt{x} + \sqrt{2y})(2\sqrt{x} - \sqrt{2y})$
$4x - 2y$

67. $(3 + \sqrt{y})^2$
$9 + 6\sqrt{y} + y$

68. $(7w - \sqrt{2x})^2$
$49w^2 - 14w\sqrt{2x} + 2x$

69. $(1 + \sqrt{3x})(x + \sqrt{3})$
$\sqrt{3}x^2 + 4x + \sqrt{3}$

70. $(2y + \sqrt{3})(\sqrt{5y} - 1)$
$2\sqrt{5}y^2 + (\sqrt{15} - 2)y - \sqrt{3}$

In Exercises 71–76, compute the product and arrange the terms of your answer according to decreasing powers of x, with each power of x appearing at most once.

Example: $(ax + b)(4x - c) = 4ax^2 + (4b - ac)x - bc$.

71. $(ax + b)(3x + 2)$

72. $(4x - c)(dx + c)$

73. $(ax + b)(bx + a)$

74. $rx(3rx + 1)(4x - r)$

75. $(x - a)(x - b)(x - c)$

76. $(2dx - c)(3cx + d)$

In Exercises 77–82, assume that all exponents are nonnegative integers and find the product.

Example: $2x^k(3x + x^{n+1}) = (2x^k)(3x) + (2x^k)(x^{n+1})$
$= 6x^{k+1} + 2x^{k+n+1}$.

77. $3^r3^43^t$

78. $(2x^n)(8x^k)$

79. $(x^m + 2)(x^n - 3)$

80. $(y^r + 1)(y^s - 4)$

81. $(2x^n - 5)(x^{3n} + 4x^n + 1)$

82. $(3y^{2k} + y^k + 1)(y^k - 3)$

In Exercises 83–92, find a numerical example to show that the given statement is false. Then find the mistake in the statement and correct it.

Example: *The statement* $-(b + 2) = -b + 2$ *is false when* $b = 5$, *since* $-(5 + 2) = -7$ *but* $-5 + 2 = -3$. *The mistake is the sign on the 2. The correct statement is* $-(b + 2) = -b - 2$.

83. $3(y + 2) = 3y + 2$

84. $x - (3y + 4) = x - 3y + 4$

85. $(x + y)^2 = x + y^2$

86. $(2x)^3 = 2x^3$

87. $(7x)(7y) = 7xy$

88. $(x + y)^2 = x^2 + y^2$

89. $y + y + y = y^3$

90. $(a - b)^2 = a^2 - b^2$

91. $(x - 3)(x - 2) = x^2 - 5x - 6$

92. $(a + b)(a^2 + b^2) = a^3 + b^3$

In Exercises 93 and 94, explain algebraically why each of these parlor tricks always works.

93. *Critical Thinking* Write down a nonzero number. Add 1 to it and square the result. Subtract 1 from the original number and square the result. Subtract this second square from the first one. Divide by the number with which you started. The answer is 4.

94. *Critical Thinking* Write down a positive number. Add 4 to it. Multiply the result by the original number. Add 4 to this result and then take the square root. Subtract the number with which you started. The answer is 2.

95. *Critical Thinking* Invent a similar parlor trick in which the answer is always the number with which you started.

71. $3ax^2 + (3b + 2a)x + 2b$

72. $4dx^2 + (4c - dc)x - c^2$

73. $abx^2 + (a^2 + b^2)x + ab$

74. $12r^2x^3 + (4r - 3r^3)x^2 - r^2x$

75. $x^3 - (a + b + c)x^2 + (ab + ac + bc)x - abc$

76. $6cdx^2 + (2d^2 - 3c^2)x - cd$

77. 3^{4+r+t}

78. $16x^{n+k}$

79. $x^{m+n} + 2x^n - 3x^m - 6$

80. $y^{r+s} - 4y^r + y^s - 4$

81. $2x^{4n} - 5x^{3n} + 8x^{2n} - 18x^n - 5$

82. $3y^{3k} - 8y^{2k} - 2y^k - 3$

83. example: if $y = 4$, then $3(4 + 2) \neq (3 \cdot 4) + 2$; correct statement: $3(y + 2) = 3y + 6$

84. example: if $x = 0$, $y = 0$, then $0 - (3(0) + 4) \neq 0 - 3(0) + 4$; correct statement: $x - (3y + 4) = x - 3y - 4$

85. example: if $x = 2$, $y = 3$, then $(2 + 3)^2 \neq 2 + 3^2$; correct statement: $(x + y)^2 = x^2 + 2xy + y^2$

86. example: if $x = 5$, then $(2(5))^3 \neq 2(5)^3$; correct statement: $(2x)^3 = 8x^3$

87. example: if $x = 2$, $y = 3$, then $(7 \cdot 2)(7 \cdot 3) \neq 7 \cdot 2 \cdot 3$; correct statement: $(7x)(7y) = 49xy$

Exercises A.4

In Exercises 1–10, express the fraction in lowest terms.

1. $\frac{63}{49}$ **$\frac{9}{7}$**

2. $\frac{121}{33}$ **$\frac{11}{3}$**

3. $\frac{13 \cdot 27 \cdot 22 \cdot 10}{6 \cdot 4 \cdot 11 \cdot 12}$ **$\frac{195}{8}$**

4. $\frac{x^2 - 4}{x + 2}$ **$x - 2$**

5. $\frac{x^2 - x - 2}{x^2 + 2x + 1}$ **$\frac{x - 2}{x + 1}$**

6. $\frac{z + 1}{z^3 + 1}$ **$\frac{1}{z^2 - z + 1}$**

7. $\frac{a^2 - b^2}{a^3 - b^3}$ **$\frac{a + b}{a^2 + ab + b^2}$**

8. $\frac{x^4 - 3x^2}{x^3}$ **$\frac{x^2 - 3}{x}$**

9. $\frac{(x + c)(x^2 - cx + c^2)}{x^4 + c^3x}$ **$\frac{1}{x}$**

10. $\frac{x^4 - y^4}{(x^2 + y^2)(x^2 - xy)}$ **$\frac{x + y}{x}$**

14. $\frac{b-2a^2}{ab}$

15. $\frac{ce+3cd}{de}$

16. $\frac{r^2t+s^2r+st^2}{rst}$

17. $\frac{b^2-c^2}{bc}$

18. $\frac{ab^2+2ab+3a}{b^3}$

19. $\frac{-1}{x(x+1)}$

22. $\frac{y^2+y+1}{xy^2}$

23. $\frac{2x-4}{x(3x-4)}$

24. $\frac{7x-1}{(x-1)(x+1)}$

25. $\frac{x^2-xy+y^2+x+y}{x^3+y^3}$

27. $\frac{-6x^5-38x^4-84x^3-71x^2-14x+1}{4x(x+1)^3(x+2)^3}$

28. $\frac{(x+y)^3-2x(x-y)}{x(x-y)^3(x+y)^2}$

51. $\frac{cd(c+d)}{c-d}$

52. $\frac{-3x^2+8x-4}{14x-20}$

53. $\frac{y-x}{xy}$

61. example: if $a=1$, $b=2$, then $\frac{1}{1}+\frac{1}{2}\neq\frac{1}{1+2}$; correct statement: $\frac{1}{a}+\frac{1}{b}=\frac{b+a}{ab}$

62. example: if $x=1$, then $\frac{1^2}{1^2+1^6}\neq 1+1^3$; correct statement: $\frac{x^2}{x^2+x^6}=\frac{1}{1+x^4}$

63. example: if $a=4$, $b=9$, then $\left(\frac{1}{\sqrt{4}+\sqrt{9}}\right)^2\neq\frac{1}{4+9}$; correct statement: $\left(\frac{1}{\sqrt{a}+\sqrt{b}}\right)^2=\frac{1}{a+2\sqrt{ab}+b}$

In Exercises 11–28, perform the indicated operations.

11. $\frac{3}{7}+\frac{2}{5}$ **29/35** 12. $\frac{7}{8}-\frac{5}{6}$ **1/24** 13. $\left(\frac{19}{7}+\frac{1}{2}\right)-\frac{1}{3}$ **121/42**

14. $\frac{1}{a}-\frac{2a}{b}$ 15. $\frac{c}{d}+\frac{3c}{e}$ 16. $\frac{r}{s}+\frac{s}{t}+\frac{t}{r}$

17. $\frac{b}{c}-\frac{c}{b}$ 18. $\frac{a}{b}+\frac{2a}{b^2}+\frac{3a}{b^3}$ 19. $\frac{1}{x+1}-\frac{1}{x}$

20. $\frac{1}{2x+1}+\frac{1}{2x-1}$ **$\frac{4x}{(2x-1)(2x+1)}$**

21. $\frac{1}{x+4}+\frac{2}{(x+4)^2}-\frac{3}{x^2+8x+16}$ **$\frac{x+3}{(x+4)^2}$**

22. $\frac{1}{x}+\frac{1}{xy}+\frac{1}{xy^2}$ 23. $\frac{1}{x}-\frac{1}{3x-4}$

24. $\frac{3}{x-1}+\frac{4}{x+1}$ 25. $\frac{1}{x+y}+\frac{x+y}{x^3+y^3}$

26. $\frac{6}{5(x-1)(x-2)^2}+\frac{x}{3(x-1)^2(x-2)}$ **$\frac{5x^2+8x-18}{15(x-1)^2(x-2)^2}$**

27. $\frac{1}{4x(x+1)(x+2)^3}-\frac{6x+2}{4(x+1)^3}$

28. $\frac{x+y}{(x^2-xy)(x-y)^2}-\frac{2}{(x^2-y^2)^2}$

In Exercises 29–42, express in lowest terms.

29. $\frac{3}{4}\cdot\frac{12}{5}\cdot\frac{10}{9}$ **2** 30. $\frac{10}{45}\cdot\frac{6}{14}\cdot\frac{1}{2}$ **1/21**

31. $\frac{3a^2c}{4ac}\cdot\frac{8ac^3}{9a^2c^4}$ **2/3c** 32. $\frac{6x^2y}{2x}\cdot\frac{y}{21xy}$ **y/7**

33. $\frac{7x}{11y}\cdot\frac{66y^2}{14x^3}$ **3y/x²** 34. $\frac{ab}{c^2}\cdot\frac{cd}{a^2b}\cdot\frac{ad}{bc^2}$ **d²/bc³**

35. $\frac{3x+9}{2x}\cdot\frac{8x^2}{x^2-9}$ **12x/(x − 3)** 36. $\frac{4x+16}{3x+15}\cdot\frac{2x+10}{x+4}$ **8/3**

37. $\frac{5y-25}{3}\cdot\frac{y^2}{y^2-25}$ **5y²/3(y + 5)** 38. $\frac{6x-12}{6x}\cdot\frac{8x^2}{x-2}$ **8x**

39. $\frac{u}{u-1}\cdot\frac{u^2-1}{u^2}$ **(u + 1)/u**

40. $\frac{t^2-t-6}{t^2-6t+9}\cdot\frac{t^2+4t-5}{t^2-25}$ **(t + 2)(t − 1)/(t − 3)(t − 5)**

41. $\frac{2u^2+uv-v^2}{4u^2-4uv+v^2}\cdot\frac{8u^2+6uv-9v^2}{4u^2-9v^2}$ **(u + v)(4u − 3v)/(2u − v)(2u − 3v)**

42. $\frac{2x^2-3xy-2y^2}{6x^2-5xy-4y^2}\cdot\frac{6x^2+6xy}{x^2-xy-2y^2}$ **6x/(3x − 4y)**

In Exercises 43–60, compute the quotient and express in lowest terms.

43. $\frac{5}{12}\div\frac{4}{14}$ **35/24** 44. $\frac{\frac{100}{52}}{\frac{27}{26}}$ **50/27** 45. $\frac{uv}{v^2w}\div\frac{uv}{u^2v}$ **u²/vw**

46. $\frac{3x^2y}{(xy)^2}\div\frac{3xyz}{x^2y}$ **x/yz** 47. $\frac{\frac{x+3}{x+4}}{\frac{2x}{x+4}}$ **(x + 3)/2x** 48. $\frac{\frac{(x+2)^2}{(x-2)^2}}{\frac{x^2+2x}{x^2-4}}$ **(x + 2)²/x(x − 2)**

49. $\frac{x+y}{x+2y}\div\left(\frac{x+y}{xy}\right)^2$ **x²y²/(x + y)(x + 2y)** 50. $\frac{\frac{u^3+v^3}{u^2-v^2}}{\frac{u^2-uv+v^2}{u+v}}$ **(u + v)/(u − v)**

51. $\frac{(c+d)^2}{\frac{c^2-d^2}{cd}}$ 52. $\frac{\frac{1}{x}-\frac{3}{2}}{\frac{2}{x-2}+\frac{5}{x}}$ 53. $\frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}+\frac{1}{y}}$

54. $\frac{\frac{x}{x+1}+\frac{1}{x}}{\frac{1}{x}+\frac{1}{x+1}}$ **(x² + x + 1)/(2x + 1)** 55. $\frac{\frac{6}{y}-3}{1-\frac{1}{y-1}}$ **(−3y + 3)/y**

56. $\frac{\frac{1}{3x}-\frac{1}{4y}}{\frac{5}{6x^2}+\frac{1}{y}}$ **(4xy − 3x²)/(10y + 12x²)** 57. $\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$ **−1/x(x + h)**

58. $\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$ **(−h − 2x)/x²(x + h)²** 59. $(x^{-1}+y^{-1})^{-1}$ **xy/(x + y)**

60. $\frac{(x+y)^{-1}}{x^{-1}+y^{-1}}$ **xy/(x + y)²**

In Exercises 61–67, find a numerical example to show that the given statement is false. Then find the mistake in the statement and correct it.

61. $\frac{1}{a}+\frac{1}{b}=\frac{1}{a+b}$ 62. $\frac{x^2}{x^2+x^6}=1+x^3$

63. $\left(\frac{1}{\sqrt{a}+\sqrt{b}}\right)^2=\frac{1}{a+b}$ 64. $\frac{r+s}{r+t}=1+\frac{s}{t}$

65. $\frac{u}{v}+\frac{v}{u}=1$ 66. $\frac{\frac{1}{x}}{\frac{1}{y}}=\frac{1}{xy}$

67. $\left(\sqrt{x}+\sqrt{y}\right)\frac{1}{\sqrt{x}+\sqrt{y}}=x+y$